One-Step Approach for Two-Tiered Constrained Relay Node Placement in Wireless Sensor Networks

Ali Chelli, Miloud Bagaa, Djamel Djenouri, Ilangko Balasingham, and Tarik Taleb

***Abstract*— We consider in this letter the problem of constrained relay node (RN) placement where sensor nodes must be connected to base stations by using a minimum number of RNs. The latter can only be deployed at a set of predefined locations, and the two-tiered topology is considered where only RNs are responsible for traffic forwarding. We propose a One-Step constrained RN Placement (*OSRP*) algorithm which yields a network tree. The performance of *OSRP* in terms of the number of added RNs is investigated in a simulation study by varying the network density, the number of sensor nodes, and the number of candidate RN positions. The results show that *OSRP* outperforms the only algorithm in the literature for two-tiered constrained RNs placement.**

***Index Terms*—Constrained relay node placement, two-tiered topology, wireless sensor network.**

I. INTRODUCTION

Optimal relay node (RN) placement is a fundamental and challenging problem in wireless sensor network (WSN). Existing literature may be divided into two models [1]: $i)$ single-tiered model and $ii)$ two-tiered model. In the formal model, sensor nodes (SNs) participate in data forwarding towards the base station (BS), and the topology is enhanced with dedicated RNs to assure connectivity, or some other desired properties (fault tolerance, QoS, etc.). In the second model, only RNs forward data packets, while every SN is only used to acquire and transmit its own data. The two-tiered model ensures a longer network lifetime, as the SNs are generally power limited nodes, while dedicated RNs tend to have less limitation on energy (e.g. endowed with energy harvesting capabilities). Solutions belonging to the two-tiered model may also be split into two categories [1]: $i)$ constrained placement and $ii)$ unconstrained placement [2], [3]. The latter suppose that RNs may be deployed at any point of the deployment area, while the former considers constraints when selecting RNs positions. Although more challenging, constrained placement is more realistic and permits to capture possible physical constraints on the RNs positions. In real-world, the RNs potential positions may be restricted to certain regions or even to a limited set of particular locations.

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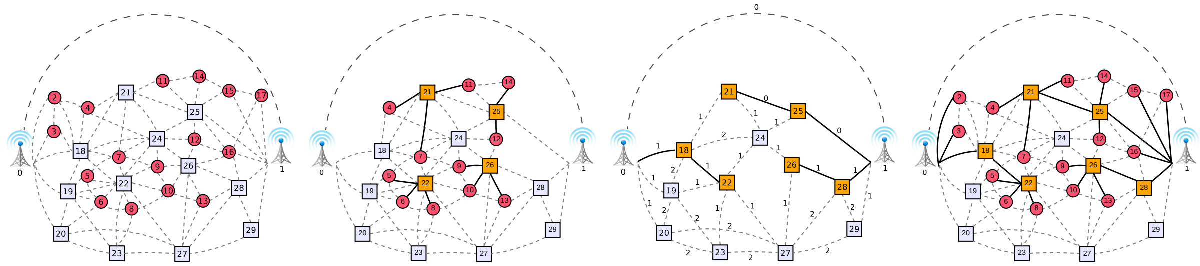
This work was carried out during the tenure of an ERCIM "Alain Bensoussan" Fellowship Programme.

This work was partly supported by the HOME Project, which is funded by the Research Council of Norway under the Idea Lab program (contract no. NFR 238848/O70), and by the CERIST's smart building project, which is funded by the Algerian Ministry of Higher Education and Scientific Research through the DGRSDT.

This problem has been recently treated in the literature. Zheng et al. [4] consider it jointly with sub-carrier allocation. They model it with mixed integer non-linear programming and solve it with heuristics. Zhang et al. [5] proposed a solution in a grid setting (discreetization of the space) and industrial applications, where the RN could be placed at any centre of grid cells. The proposed solution includes two phases, $i)$ calculation of the set of topologies that meet the requirements in terms of fault tolerance, energy consumption, and $ii)$ the selection of the best topology. This approach has a very high computation complexity. We are interested in particular in settings where the set of potential placements is finite, similarly to [1], [6], where Steiner trees have been used to determine RN positions. This problem has already been proved to be NP-hard [7]. Gao et al. [6] jointly consider minimum RNs addition with fault-tolerance and develop heuristics based on Steiner tree to connect SNs and BSs. Added RNs are supposed to be able to communicate with each other directly regardless of the distance and the physical environment conditions, which is generally unrealistic.

We are interested in the construction of a two-tiered topology for interconnecting SNs to BSs through RNs. This topology is more appropriate for limited resources WSN, which allows sensor nodes to preserve their energy for sensing and first mile transmissions. To the best of our knowledge, there is only one solution [1], dubbed *TTCR*, that addressed the two-tiered *constrained* relay node placement. Fig. 1 shows an example of *TTCR* execution. Fig. $1(a)$ depicts a network topology comprising two BSs, forming the set $\mathcal{B}$, twelve candidate relay positions, say the set $\mathcal{Y}$, and sixteen SNs, say the set $\mathcal{S}$. The circles and the squares represent the SNs $\mathcal{S}$ and candidate relay positions $\mathcal{Y}$, respectively, while a dashed edge between two nodes indicates that these two nodes can communicate together.

TTCR is executed on three steps to form two-tiered topology. In the first step, each SN that is neighbor to a BS selects it as its parent and then that SN is removed from the graph. Fig. $1(b)$ illustrates this step. In the second step, the rest of SNs are connected to a set of activated relay position $\mathcal{A} \in \mathcal{Y}$ that are selected using the 22-approximation algorithm for single-cover [8]. Fig. $1(b)$ illustrates this step. Before starting the last step, the rest of SNs are also removed from the graph. This is to construct two-tiered topology when connecting $\mathcal{A}$ to $\mathcal{B}$ by adding some extra relays in $\mathcal{Y}$. In the last step, a heuristic Algorithm for Steiner Tree Problem ($\mathcal{ASTP}$) is used to connect $\mathcal{A}$ to $\mathcal{B}$, where $\mathcal{A} \cup \mathcal{B}$ are sources, and the remaining candidate relay positions $(\mathcal{Y} \setminus \mathcal{A})$ are Steiner points, as depicted in Fig. $1(c)$. This figure shows the final tree constructed by *TTCR*, which adds six relays. This is far from the optimal solution as it will be demonstrated later. The



(a) (b) (c) (d)

Fig. 1. An example that shows the execution of *TTCR*.

execution of *TTCR* on three steps does not assure optimal solution. Although the selected RNs are locally optimal, the final set is not necessarily globally optimal (local optimum problem).

In this paper, we address the problem of two-tiered constrained RN placement and reduce the number of added ~~relays~~ compared with *TTCR*. This latter provides a suboptimal solution due to its three steps approach. The aim of this work is to deal with this problem and propose a one-step approach that construct a connected Steiner tree~~, which~~ intuitively reduce the number of RNs required to connect $\mathcal{S}$ to $\mathcal{B}$ (solution cost).

The rest of the letter is organized as follows. In Section II, we present the proposed solution. Section III presents the performance evaluation of the proposed solution *OSRP* compared to the sole base-line approach *TTCR*. Finally, the paper is concluded in Section IV.

II. Proposed Solution

A. Network Model

We consider a wireless sensor network comprising three types of nodes: the base stations, the sensor nodes, and the relay nodes. The latter can be placed only in certain predefined candidate positions. The objective is to connect the SNs to the BS using a minimum number of RNs. The SNs only generate the traffic while the RNs forward it (two-tiered topology). The communication range for the SNs and the RNs are referred to as r and R, respectively, such that $R \gg r$. The BSs are assumed to be connected directly with each other via the wired network.

Definition 1. *The communication between nodes is modeled as an undirected graph $G = (r, R, \mathcal{B}, \mathcal{Y}, \mathcal{S})$, with vertex set $V = \mathcal{B} \cup \mathcal{Y} \cup \mathcal{S}$, and edge set $E = E_{\mathcal{SS}} \cup E_{\mathcal{SB}} \cup E_{\mathcal{SY}} \cup E_{\mathcal{YY}} \cup E_{\mathcal{YB}} \cup E_{\mathcal{BB}}$. The notation $E_{\mathcal{UZ}}$ refers to the edges from the vertices of $\mathcal{U}$ to the vertices of $\mathcal{Z}$. An edge $(u, v) \in E$ iff the two nodes are within the transmission range of each other. E is formed as fellow: i) $\forall u \in \mathcal{B}, v \in \mathcal{B}$, $(u, v) \in E$. ii) $\forall u \in \mathcal{S}$, $v \in \mathcal{B} \cup \mathcal{Y} \cup \mathcal{S}$, $(u, v) \in E$ iff $d_{u,v} \leq r$. iii) $\forall u \in \mathcal{Y}, v \in \mathcal{Y} \cup \mathcal{B}$, $(u, v) \in E$ iff $d_{u,v} \leq R$. It is guaranteed that the formed graph G ensures bidirectional communications between the nodes in the network.*

Definition 2. *Let $\mathcal{G} = (r, R, \mathcal{B}, \mathcal{Y}, \mathcal{S})$ denotes the weighted communication graph to be used in the two-tiered topology, which is constructed from G by removing the edges between SNs. Moreover, for each edge $e = (u, v) \in \mathcal{E}$, where $\mathcal{E}$ denotes the set of edges in $\mathcal{G}$, the weight function is defined as:*

$$\mathcal{W}(e) = \begin{cases} |\mathcal{S} \cup \mathcal{Y} \cup \mathcal{B}| & if\ u \in \mathcal{S}\ and\ v \in \mathcal{Y} \\ |\{u, v\} \cap \mathcal{Y}| & Otherwise. \end{cases} \tag{1}$$

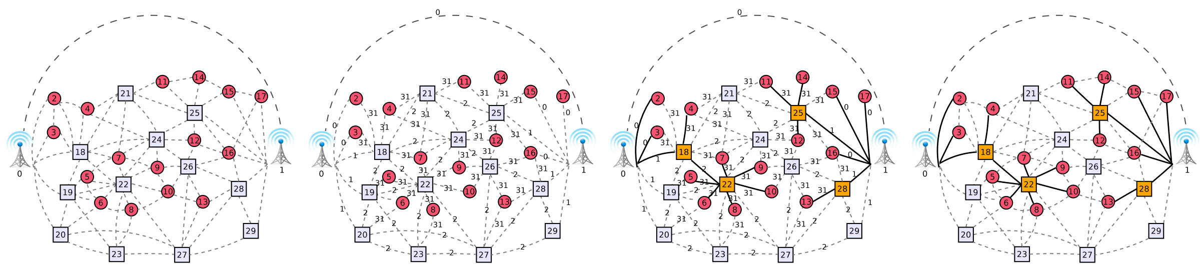
From (1), $\mathcal{W}(e)$ can take four values 0, 1, 2 and $|\mathcal{S} \cup \mathcal{Y} \cup \mathcal{B}|$. i) $\mathcal{W}(e) = 0$ iff $u, v \in \mathcal{B} \cup \mathcal{S}$; ii) $\mathcal{W}(e) = 1$ iff $u \in \mathcal{Y}$ and $v \in \mathcal{B}$; iii) $\mathcal{W}(e) = 2$ iff $u, v \in \mathcal{Y}$; iv) $\mathcal{W}(e) = |\mathcal{S} \cup \mathcal{Y} \cup \mathcal{B}|$ iff $u \in \mathcal{S}$ and $v \in \mathcal{Y}$.

B. Algorithm Description

In this section, the proposed solution *OSRP* is presented. The main idea in *OSRP* is to proceed in a single step and to calculate a Steiner tree that directly interconnects SNs to the BSs via RNs. This is as opposed to *TTCR* where first a set of activated RNs $\mathcal{A}$ is determined, then RNs in $\mathcal{A}$ are connected to the BSs via the RNs in $(\mathcal{Y} \setminus \mathcal{A})$. For *OSRP*, the union of SNs and BSs are the sources, while all possible positions for RNs ($\mathcal{Y}$) are utilised as Steiner points. The two tiered-topology is guaranteed by choosing appropriate values for the weights of the edges in the graph. A summary of *OSRP* is provided in Algorithm 1, while Fig. 2 serves as a detailed illustrative example. Given the positions of the BSs, the SNs, and the potential positions of the RNs, the communication graph $G = (r, R, \mathcal{B}, \mathcal{Y}, \mathcal{S})$ is generated using **Definition 1**. Fig. $2(a)$ depicts the communication graph $G = (r, R, \mathcal{B}, \mathcal{Y}, \mathcal{S})$, where the circles and the squares represent the SNs ($\mathcal{S}$) and candidate RN positions ($\mathcal{Y}$), respectively.

OSRP starts by forming the weighted communication graph $\mathcal{G} = (r, R, \mathcal{B}, \mathcal{Y}, \mathcal{S})$ using **Definition 2**. Firstly, the edges connecting SNs to each other are removed from $\mathcal{G}$ (Algorithm 1: lines $1-2$). This will ensure that a SN cannot use another SN to forward its data, which is a necessary condition to guarantee the two-tiered topology. Secondly, *OSRP* assigns the weights for the different edges in the graph using (1). For the edges connecting two BSs or a BS to a SN a weight of zero is assigned to this edge (Algorithm 1: lines $3-5$). This will allow connecting SNs that are in vicinity of a BS directly without going through a RN. If an edge is interconnecting a BS to a RN, the weight for this edge is set to one (Algorithm 1: lines $6-8$), whereas for an edge interconnecting two RNs, the edge weight is set to two (Algorithm 1: lines $9-11$). Even though the edges connecting the SNs together are removed, it might be possible that a RN uses a SN to forward its data, which violates the two-tiered topology condition. To avoid this, a weight equal to $|\mathcal{S} \cup \mathcal{Y} \cup \mathcal{B}|$ (the number of vertices in the graph) is assigned to the edges connecting the SNs to the RNs (Algorithm 1: lines $12-14$). Fig. $2(b)$ shows the graph $\mathcal{G}$ produced by *OSRP* algorithm.

An $\mathcal{ASTP}$ is now applied on the weighted graph where the set of source nodes is the union of the BSs and the SNs



(a) (b) (c) (d)

Fig. 2. Illustrative example that shows the execution of *OSRP*.

(Algorithm 1: lines 15−16). Any existing heuristic can be used in this step such as [9]. The output of the $\mathcal{ASTP}$ is a tree $(\mathcal{V}, \mathcal{E})$ which has a minimum total weight. The intersection between the vertices $\mathcal{V}$ and the set of candidate RNs $\mathcal{Y}$ represents the set of RN positions that should be activated $\mathcal{R}$ (Algorithm 1: lines $17-18$). Fig. $2(c)$ shows the tree $\mathcal{V}$ constructed by *OSRP* algorithm. Note that the initial network graph is the same in Fig. 1 and Fig. 2. By looking at the final tree produced by *TTCR* and *OSRP* in Fig. $1(d)$ and Fig. $2(d)$, respectively. It can be noticed that the number of added RNs is reduced from 6 to 4 by *OSRP*. This example illustrates that *OSRP* outperforms *TTCR* in terms of number of added RNs. The performance of *OSRP* will be investigated more extensively in the next section.

Algorithm 1 One Step constrained Relay node Placement (*OSRP*) Algorithm

Input:
$G = (V, E)$: Communication graph, where $E = E_{\mathcal{SS}} \cup E_{\mathcal{SY}} \cup E_{\mathcal{SB}} \cup E_{\mathcal{YY}} \cup E_{\mathcal{YB}} \cup E_{\mathcal{BB}}$.
$\mathcal{B}$: Set of base stations.
$\mathcal{S}$: Set of sensor nodes in the network.
$\mathcal{Y}$: Set of candidate relay nodes in the network.
$\mathcal{ASTP}(\mathcal{G}, \mathcal{W}, S)$: An approximation algorithm to obtain a Steiner tree from the weighted graph $\mathcal{G}$, where S are the source nodes and $\mathcal{W}$ represents the weight of the graph edges.
Output:
$\mathcal{R}$: The set of relay nodes positions that should be activated.

1: $\mathcal{G} = (V, \mathcal{E} = E)$; *Initially,* $\mathcal{E}$ *equals to* E.
2: $\mathcal{E} = \mathcal{E} \setminus \mathcal{E}_{\mathcal{SS}}$; *Remove the edges* $\mathcal{E}_{\mathcal{SS}}$ *from* $\mathcal{G}$.
3: **for all** $(u, v) \in \mathcal{E}_{\mathcal{BB}} \cup \mathcal{E}_{\mathcal{BS}}$ **do**
4: $\quad \mathcal{W}(u, v) = 0$;
5: **end for**
6: **for all** $(u, v) \in \mathcal{E}_{\mathcal{BY}}$ **do**
7: $\quad \mathcal{W}(u, v) = 1$;
8: **end for**
9: **for all** $(u, v) \in \mathcal{E}_{\mathcal{YY}}$ **do**
10: $\quad \mathcal{W}(u, v) = 2$;
11: **end for**
12: **for all** $(u, v) \in E_{\mathcal{SY}}$ **do**
13: $\quad \mathcal{W}(u, v) = |V|$;
14: **end for**
15: $Src = \mathcal{S} \cup \mathcal{B}$;
16: $(\mathcal{V}, \mathcal{E}) = \mathcal{ASTP}(\mathcal{G}, \mathcal{W}, Src)$;
17: $\mathcal{R} = \mathcal{V} \cap \mathcal{Y}$;
18: **Return** $\mathcal{R}$;

III. Simulation Evaluation

Given that the Steiner tree problem is NP-complete, existing heuristics are used as general solution for $\mathcal{ASTP}$ both in *TTCR* and *OSRP*. Most existing heuristics are based on the α-approximation concept, which ensures that the weight, $\mathcal{W}$, of the constructed tree is at most α times the optimal weight $\mathcal{W}_{OPT}$. Therefore, for any topology[1], it is guaranteed that the constructed tree has a weight $\mathcal{W} \in [\mathcal{W}_{OPT}, \alpha\, \mathcal{W}_{OPT}]$. However, the weight $\mathcal{W}$ randomly fluctuates in this interval with the topology change. Note that *OSRP* and *TTCR* use $\mathcal{ASTP}$ with different input topologies which will result in inaccurate comparison between *OSRP* and *TTCR*. To make a fair comparison, we eliminate the external effects due to $\mathcal{ASTP}$ by applying an exact solution for Steiner Tree Problem (STP). The exact solution is formulated through integer linear program (ILP), which is presented in the next subsection.

A. Exact Solution For Steiner Tree Problem (STP)

Let $\eta(u)$ denote the set of u's neighbor in $\mathcal{G}$, and χ, ζ the set of source nodes and the set of Steiner points, respectively. Three variables are also defined $i)$ $X_{u,v}$ is a boolean decision variable that is set to 1 if node u selects node v as a parent; (ii) $\mathcal{W}(u, v)$ is the weight of the edge $(u, v) \in \mathcal{E}$; (iii) F is a matrix of integers (with elements $F_{u,v}$) that introduces a flow to force the connectivity. The optimization problem is formulated with the following ILP.

$$\min \sum_{u \in V - \{\mathcal{B}\}} \sum_{v \in \eta(u)} \mathcal{W}(u, v) X_{u,v}. \tag{2}$$

Subject to,

$$\forall u \in \chi : \sum_{v \in \eta(u)} X_{u,v} = 1. \tag{3}$$

$$\forall u \in \zeta : \sum_{v \in \eta(u)} X_{u,v} \leq 1. \tag{4}$$

$$\forall u \in \chi : \sum_{v \in \eta(u)} F_{u,v} - \sum_{v \in \eta(u)} F_{v,u} = 1. \tag{5}$$

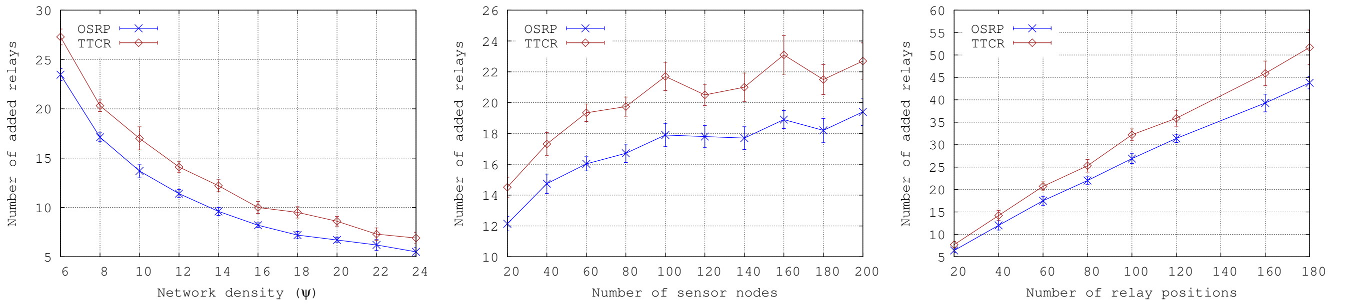
$$\forall u \in \zeta : \sum_{v \in \eta(u)} F_{u,v} = \sum_{v \in \eta(u)} F_{v,u}. \tag{6}$$

$$\sum_{u \in \mathcal{B}} \sum_{v \in \eta(u)} F_{v,u} = |\chi|. \tag{7}$$

$$\forall u, v \in V : 0 \leq F_{u,v} \leq X_{u,v} |\chi|. \tag{8}$$

The ILP in (2) applies both for *OSRP* and *TTCR* to find the optimal solution for to the given inputs (χ and ζ). Note that the inputs are not the same for both algorithms even though the initial network is the same. For $\mathcal{ASTP}$, $\chi = \mathcal{S} \cup \mathcal{B}$, $\zeta = \mathcal{Y}$, whereas for $\mathcal{TTCR}$, $\chi = \mathcal{A} \cup \mathcal{B}$ ($\mathcal{A}$ is the subset of $\mathcal{R}$ computed in the second step), $\zeta = (\mathcal{Y} \setminus \mathcal{A})$. The constraints are used to ensure the following conditions: constraint (3) ensures that each S has only one parent; constraint (4) ensures that a node (which does not belong to χ) can have at most 1 parent. The constraints (5), (6), (7), and (8) are used for modeling the network connectivity and to ensure that all SNs can transmit their data to the BSs. To guarantee that the formed topology connects all the χ to $\mathcal{B}$, a packet flow is mimicked, which is generated and routed from χ to $\mathcal{B}$. Each node in χ generates only one packet which is then forwarded through a set of ζ. The mimicked packet flow should be routed within the constructed tree. Constraint (5) ensures that each node in χ generates only one packet. Constraint (6) ensures that the nodes which belong to ζ should not generate any packet.

[1] The topology consists of the sources and the Steiner points.



(a) Network density (b) Number of sensor nodes (c) Number of relay nodes

Fig. 3. The solution cost measured as the number of relay nodes $\mathcal{R}$ which are added to ensure the connectivity between $\mathcal{S}$ and $\mathcal{B}$.

Constraint (7) captures the fact that the number of packets received at the BSs is equal to the number of sources ($|\chi|$). Constraint (8) forces the generated flow to be routed only within the constructed topology: There is a flow from each node u to its receiver v; otherwise if $X_{u,v} = 0 \Rightarrow F_{u,v} = 0$.

B. Numerical Analysis

In this section, we evaluate the proposed solution *OSRP* against *TTCR*. *OSRP* and *TTCR* are evaluated in terms of network cost which represents the number of RNs added to interconnect $\mathcal{S}$ to $\mathcal{B}$. The objective of any RN placement solution is to deploy the minimum number of RNs to interconnect $\mathcal{S}$ to $\mathcal{B}$. In the performance evaluation, we performed three set of experiments: i) We vary the number of SNs $|\mathcal{S}|$ and fix the network density ψ to 8 and the number of RN positions $|\mathcal{Y}|$ to 100; ii) We vary the number of RN positions $|\mathcal{Y}|$ while fixing $|\mathcal{S}|$ to 100 and ψ to 8; iii) We vary the network density ψ and fix $|\mathcal{S}|$ and $|\mathcal{Y}|$ to 100. Fig. 3 illustrates the cost as a function of the network density, the number of sensor nodes, and the number of RNs. The results clearly show that *OSRP* outperforms the *TTCR* for all considered investigations.

Fig. 3(a) shows the performance evaluation of *TTCR* and *OSRP* when varying $|\mathcal{S}|$. It can be seen from Fig. 3(a) that the number of required RNs increases with the number of SNs $|\mathcal{S}|$ both for *OSRP* and *TTCR*, but the number of deployed RNs by *OSRP* is clearly lower than that of *TTCR*. For example, when $|\mathcal{S}| = 160$ *OSRP* require 19 RNs while *TTCR* require only 23 to interconnect $\mathcal{S}$ to $\mathcal{B}$.

In Fig. 3(b), the number of added RNs is plotted vs. the number of candidate RN-positions. This figure shows that the number of required RN increases proportionally with the number candidate RN-positions $\mathcal{Y}$. As the number of candidate RN-positions increases while the network density is kept constant, the SNs become more spread apart, and consequently more RN nodes are needed to connect the SNs to the BSs. *OSRP* requires lower number of RNs than *TTCR* to interconnect $\mathcal{S}$ to $\mathcal{B}$. The gap between both algorithms increases proportionally with the number of candidate RN-positions $\mathcal{Y}$.

The network density ψ is varied in Fig. 3(c). It is clear from this figure that the increase in ψ has a positive impact on both algorithms. It is obvious that the increase in the network density gives nodes in S more chances to transmit their data without passing by $\mathcal{Y}$. It is also clear from the figure that the number of RNs required by *OSRP* is lower than the one required by *TTCR*. The gain achieved by *OSRP* increases as the network density decays. For instance, for a network density of 6, *OSRP* reduces the number of RNs deployed on average by 3.8 compared to *TTCR*. As the density of the network increases to 24, the gain achieved by using *OSRP* instead of *TTCR* decreases: *OSRP* deploys only 5.5 RNs on average while *TTCR* requires 6.9 RNs, i.e., a cost reduction of 1.4 RNs on average.

IV. Conclusion

In this letter, we consider the problem of constrained relay node placement. For this problem RNs can be placed only at a limited set of positions, and the sensor nodes should be connected to the base stations using a minimum number of RNs. In this work, we are interested in constructing two-tiered topology, where the sensor nodes do not participate in the data forwarding. In contrast to the sole solution in literature *TTCR*, we solved the problem using a single step to reduce the number of RNs that should be deployed. The performance evaluation confirms the targeted objective and shows that *OSRP* clearly outperforms *TTCR* in terms of the number of added relays.

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