

# Effect of Stiffeners on Nature of Post-critical Deformations of Thin-walled Composite Aircraft Structures

A combined numerical-experimental study

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**Abstract**—This paper presents results of numerical and experimental examination of thin-walled structures which model certain parts of aircraft structures subject to bending and torsion. The considered type of load and deformation corresponds to the state of such structures typical for in-flight conditions. A physical model of the analysed structure in several versions with and without stiffeners was made of composite materials. It was assumed that post-critical deformations of the structure are admissible within the limit loads. Results of non-linear numerical analyses carried out with the use of the finite element method applied to several variant structures with various types of stiffeners are compared with results of measurements taken on corresponding models.

**Keywords**- aircraft load-bearing structures, finite element method, loss of stability, non-linear numerical analysis, operating stability, thin-walled structures

## I. INTRODUCTION

Scientific studies devoted to the problem of stability loss occurring in component structures of load-bearing systems used in different branches of technology are usually focused on issues connected with determining critical load values. Post-critical states of structures are much more rarely selected as an object of analysis. This reflects the fact that in the majority of engineering disciplines, any structure losing its stability is considered ultimately destroyed.

In the aircraft design, in view of very specific nature of structures included within its scope of interest, equally specific standards have been adopted affecting the design development processes and assumptions concerning operating conditions. One of such rules, applicable to structures made of metallic materials still dominating in the aircraft industry, provides for admissibility of post-critical deformations in selected types of structures within the range of predetermined operating load limits (e.g. paragraph 23.305 in [1]).

It should be however underlined that, likewise in other disciplines of mechanical engineering, a commonly adopted rule provides that a bar structure subjected to buckling is considered destroyed. Stability and reliability of a structure depends therefore on accuracy to which geometrical

parameters for all structure components represented by bar frameworks are selected in the structure's model development process. Such components include e.g. stringers, spar flanges, members of lattices and frames etc. [2, 3].

A quite different rule applies to skin systems constituting elements of semi-monocoque structures, although also in this case there is a number of restrictions. In general, because of the overriding importance of minimising the mass of the structure, it is admissible to assume that a loss of stability of the skin may occur under in-flight conditions, provided that the phenomenon occurs within the elastic regime and only locally, i.e. within the area of a skin segment limited by skeleton components. Exceptions may include skins of such elements as wing torsion boxes and other substructures responsible for securing sufficient torsional rigidity of the structure as a whole. Another exceptions are those skin fragments in which deformations are not desirable in view of necessity to ensure that the macrostructure will maintain proper aerodynamic properties.

The problem of fuselage skins losing stability came up in the aircraft engineering as early as in the 1920s, when classical lattice constructions started to give way to metallic monocoque structures. Initially, design engineers strove to eliminate entirely the phenomenon of skin buckling by making skins thicker; this, however, resulted in excessive increase of aircraft weight and the related deterioration of service properties. The first fully reasonable solution of the problem should be attributed to Hugo Junkers who proposed to use corrugated sheet metal for skins on his newly designed aircraft models [4]. Junkers' solution was the first attempt to utilise the co-called integral stiffeners in skin constituting elements of airframe load-bearing structures.

As a result of following studies, the admissibility of certain forms of post-critical deformations in operating conditions corresponding to the admissible load regimes become a commonly accepted rule. It has been also found that the area of occurrence of post-critical deformation can be limited not only by increasing the number of skeleton components. In many cases, using different forms of integral stiffeners, as corrugated panels, turned out to be an equally effective way to ensure that post-critical

deformations will maintain a local nature. For high speed aeroplanes with thin wing sections integrally stiffened planks were developed [5].

Although lightweight metal constructions still prevail among components of currently operated aircraft structures, but almost every new design incorporates more and more various composite materials for main structure components. Bearing in mind insufficiency of currently available knowledge concerning evolution of mechanical properties of composite materials in the course of prolonged operation, load-bearing elements fabricated with the lamination technology used to be designed and manufactured as monocoque structures with intermediate layers preventing skins from losing their stability. Nevertheless, the research of postbuckling behaviour of composite thin-walled aeronautical structures attracts attention of the aerospace industry, as a vital issue for saving weight and fuel [6, 7, 8].

Today, structure design is developed almost exclusively with the use of numerical tools including computer aided design (CAD), computational fluid dynamics (CFD), flight loads and structures, available in the form of software packages supported mainly by algorithms based on the finite element method (FEM) [9]. If the assumption on admissibility of post-critical deformation is adopted, to determine displacement distribution patterns it is necessary carry out non-linear numerical analyses with geometrical non-linearities taken into account. The non-linear algorithms, for composites especially, are still being developed [10].

As accuracy of numerical tools is limited by their nature, one should expect that the obtained results might be still biased with serious errors [11, 12]. The only way to identify and eliminate such errors from models is to elaborate them on the grounds of results of appropriate experiments. In most of cases, such verification can be based on simplified tests carried out with the use of models where the behaviour of a structure is tracked by measuring displacement of specific reference points. If deformation patterns and magnitudes calculated numerically and those observed in the experiment are brought to conformity, one can assume that credible patterns of reduced stress distributions were obtained within the framework of a selected mechanical strength hypothesis. The base on which such credibility can be founded is the rule of uniqueness of solutions according to which there is a one-to-one correspondence between distribution of the reduced stress and the deformation state. This type of verification is not only important in the early stage of design, but also would be beneficial for structures which became airborne [13].

Presented results of research are consecutive follow-up of post-buckling analysis of composite thin-walled aeronautical structures [14, 15, 16].

## II. OBJECTIVE AND SCOPE OF RESEARCH

The objective of the study presented here consisted in performing a comparative analysis of several design solutions concerning a fragment of aeroplane wing with composite shell subject to post-critical deformations under a load acceptable in operating conditions. The subject of the

research were structures with the same dimensions (Fig. 1) but with different skeleton designs. In each variant, the front portion of the skin corresponding to the torsion box was made thicker to prevent it from the loss of stability. In the course of analyses, the area between the spar and the trailing edge was the subject of special attention.

In all variants, the same technology was used to produce models. To determine the effect of skin stiffness on the nature of post-critical deformations, some of the models were made in two variants differing with skin thickness.

In the experimental part of the study, measurements of skin displacement were taken to determine deformation distribution patterns and representative equilibrium paths. The results were used as a base for development and verification of numerical models subjected to non-linear analyses with the use of FEM-based software.

As a result of combined numerical and experimental studies, confirmation of appropriateness of the numerical models was assessed as well as usefulness of subsequent modifications of the analysed design solutions.

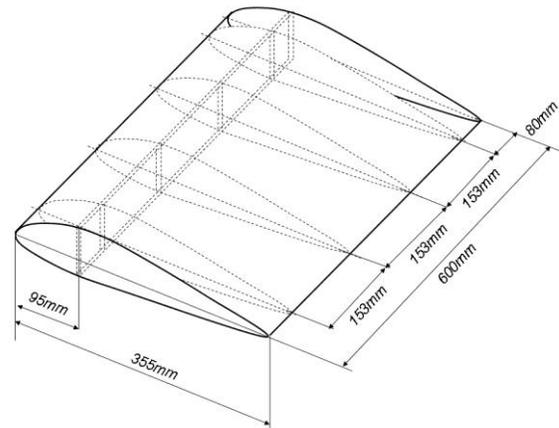


Figure 1. Schematic views and dimensions of the examined structures.

## III. EXPERIMENT

Skeleton structures for the models used in the experiment were made of plywood and wooden slats with known mechanical properties. The skin was made of an epoxy resin composite reinforced with glass fibre (GFRP).

The composite reinforcement was made of Interglass 02037 and 92110 glass fibre fabrics with weight ratios of 50 g/m<sup>2</sup> and 163 g/m<sup>2</sup>, respectively. The composite matrix was a saturating mix based on epoxy resin MGS L285/H286 with known mechanical properties. The resulting composite was characterised with the following mechanical parameters:  $E_{11} = 22,000$  MPa,  $E_{22} = 22,000$  MPa,  $\nu_{12} = 0.11$ ,  $G_{12} = 4600$ . The model skins in the torsion box area were made as a composite with layers of the symmetric fabric with the 50/50 reinforcement ratio. The composite's main directions of orthotropy were oriented at the angle of 45° with respect to spar flanges.

In the remaining portion of the structure, the shell was composed of three or two layers of fabric, depending on skin thickness variant (Fig. 2). Application of a larger number of layers in the torsion box zone was aimed at preventing the torsion box surface from the loss of stability and creating conditions for occurrence of post-critical deformations in the skin area between the spar and the trailing edge.

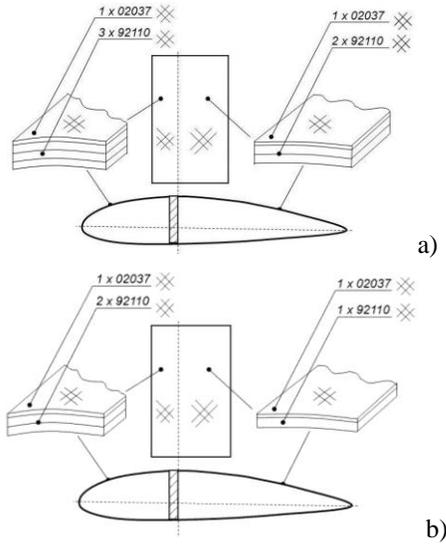


Figure 2. A schematic sketch of lamination structure: models with (a) thicker and (b) thinner shell.

The experiments were carried on a dedicated laboratory set-up shown in Fig. 3a. The tested models were subjected to torsion and bending corresponding to the actual characteristic of loads exerted on an aeroplane wing in the in-flight conditions. The structures were loaded gravitationally in way shown in Fig. 3b.

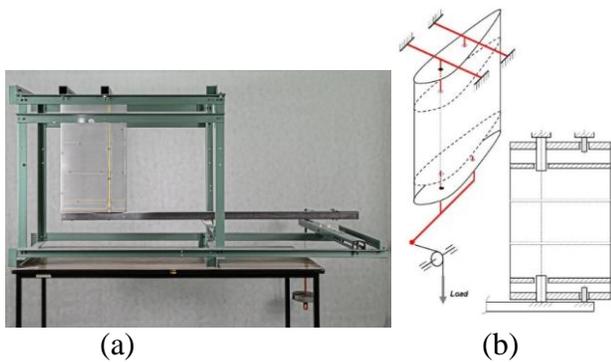


Figure 3. (a) The experimental set-up; (b) schematic diagram of structure fixing and load application.

In case of variants with thicker shell, two versions of the model were subject to examination (Fig. 4). In the first version, the tested shell fragment had no stiffening elements. The other model was provided with stiffeners

situated transversally with respect to the spar axis, made in the form of closed sections filled with polymer foam (Fig. 5).

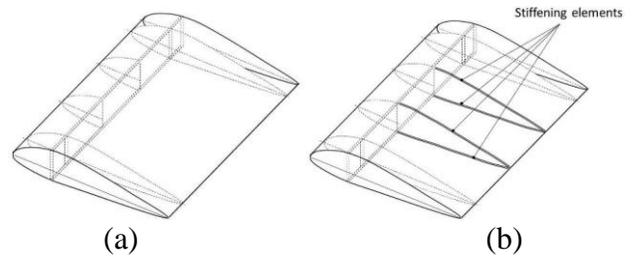


Figure 4. Schematic views of the examined model versions: (a) without stiffeners; (b) with integral stiffening elements.

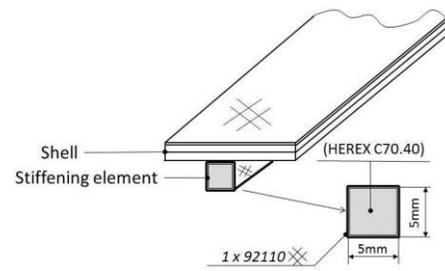


Figure 5. A schematic view of crosswise skin stiffeners.

The structure proposed in this variant does not fall directly under the category of integral stiffeners in the commonly adopted meaning of the term. However, it plays the same role as an element in the form of perimeter of figure created by proper shaping of outer layer of the shell. Moreover, such solution is much easier to build in practice. For that reason, such type of stiffening element will be referred to as the integral one for the purpose of this study.

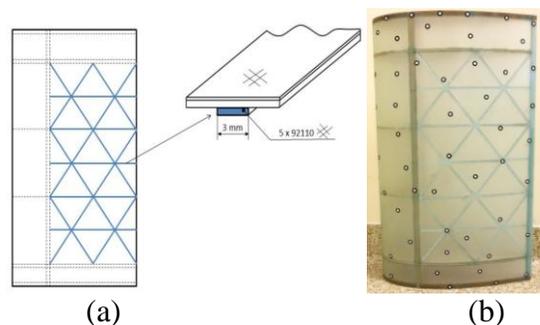


Figure 6. A model with stiffening in the form of geodetic structure: (a) schematic drawing; (b) actual model prepared for 3D scanning.

In case of variants with thinner shells, an additional third solution has been also considered which consisted in a composite skin joined with a geodetic structure formed out of glass fibre composite (Fig. 6). Also in this case, the skin

stiffeners can be numbered among the integral ones as they were made with the used of the same composite matrix material.

During the experiment, skin displacement measurements were taken in selected reference points in successive stationary deformation states of the structure. A system of micrometre gauges and PONTOS optical scanner (GOM Optical Measuring) was used for this purpose (Fig. 7). As a result, representative equilibrium paths were obtained representing the relationship between the structure's total angle of torsion and the load value (Figs. 8 and 9).

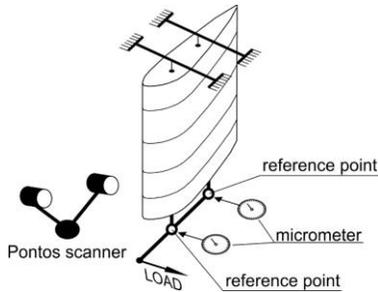


Figure 7. A schematic view showing the displacement measurement taking technique.

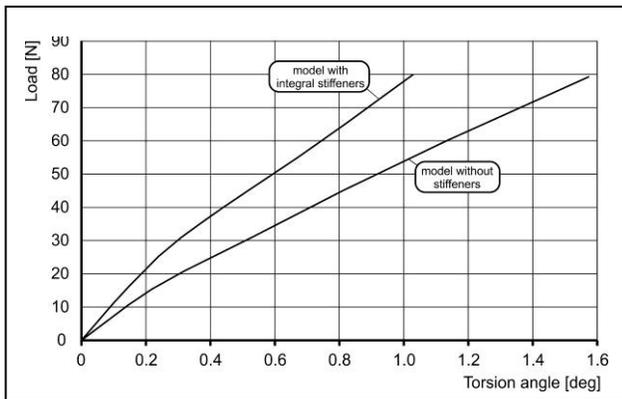


Figure 8. A comparison of representative equilibrium paths for variants with thicker shell.

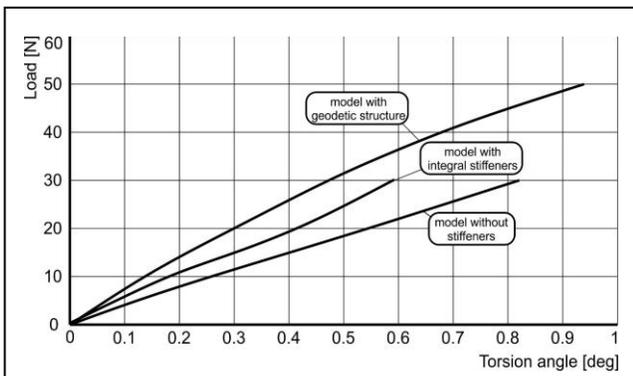


Figure 9. A comparison of representative equilibrium paths for variants with thinner shell.

The examined area of the deformed model was also scanned with the use of ATOS scanner for the target load value. Images of the deformation field shown in Fig. 10 were obtained this way and used further to verify results of numerical calculations.

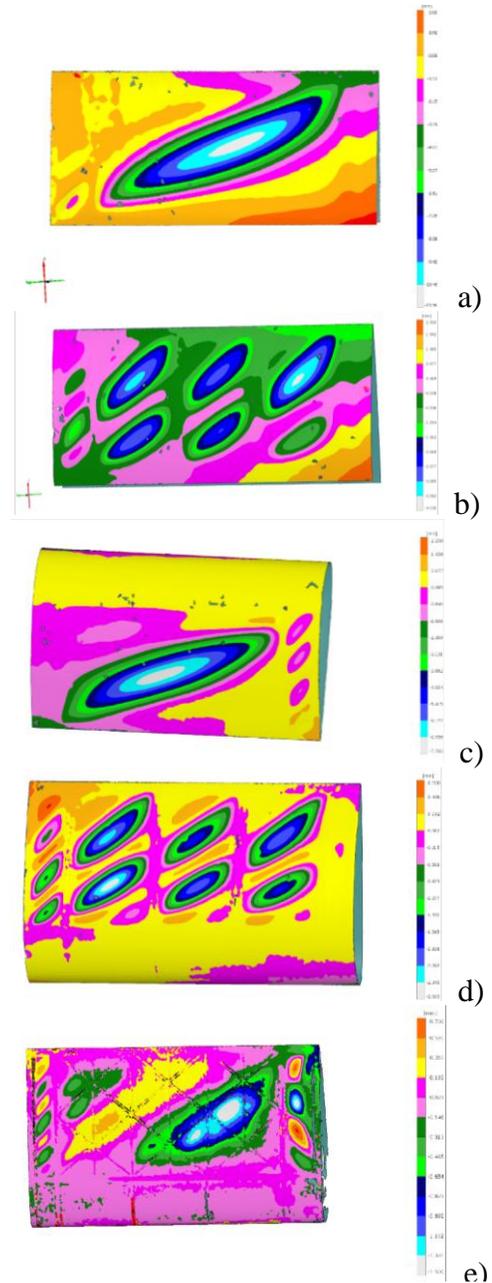


Figure 10. Resultant displacement patterns for a model:  
a) with thicker shell, w/o stiffeners,  
b) with thicker shell, with stiffeners,  
c) with thinner shell, w/o stiffeners,  
d) with thinner shell and crosswise stiffeners,  
e) with thinner shell and geodetic structure (upper skin, right rib fixed).

#### IV. NUMERICAL ANALYSES

The next stage of the study complementing the experimental phase consisted in developing appropriate and effective calculation models realised by means of the finite element method in order to obtain information about stress distribution patterns in the examined shells.

The structures were modelled numerically with the use of commercial MSC PATRAN/MARC software effectiveness of which was proved in case of analyses concerning post-critical deformations of skins made of isotropic materials [17]. In case of composite materials, the crucial model development phase consists in the use of an algorithm the purpose of which is to determine laminate properties based on sets of material constants characterising individual layers. The algorithm is an integral subroutine of the pre-processor and remains beyond the scope of any intervention of the user.

A specific feature of composite structures for which it is exceptionally difficult to reproduce them numerically is the non-homogeneity resulting not only from conditions in which individual layers are laminated but also from the fabrication technology itself, e.g. presence of local excesses of resin and/or diversified thickness of bonded joints. Such hardly controllable factors can result in local shell stiffness variations and have an effect on post-critical deformation patterns. Even when errors made in selection of geometrical parameters for the numerical model are small, such heterogeneities introduce definite deviations from actual boundary conditions characterising a skin segment and generate significant errors in the course of non-linear analysis.

In any non-linear problem, the fundamental feature determining quantitatively the relationship between condition of a structure and the load is the so-called equilibrium path of the system which represents a hypersurface in the hyperspace of states [18]. The relationship can be expressed in the form of the following matrix equation of residual forces:

$$\mathbf{r}(\mathbf{u}, \Lambda) = \mathbf{0}, \quad (1)$$

where  $\mathbf{u}$  is the vector of state containing displacement components of nodes of the structure corresponding to its current geometrical configuration,  $\Lambda$  is a matrix composed of control parameters corresponding to the actual load level, and  $\mathbf{r}$  is the residual vector of uncompensated force components related to the current system deformation state. Any set of control parameters can be assigned a single parameter  $\lambda$  being a function of the load. In such case, (1) takes the form

$$\mathbf{r}(\mathbf{u}, \lambda) = \mathbf{0}, \quad (2)$$

known as the single-parameter equation of residual forces.

The prediction-correction methods used in contemporary software routines to determine consecutive points of the equilibrium path include also a corrective phase. In this

stage it is checked whether the system satisfies an additional equation called the increment control equation or the equation of constraints:

$$c(\Delta\mathbf{u}_n, \Delta\lambda_n) = 0, \quad (3)$$

where

$$\Delta\mathbf{u}_n = \mathbf{u}_{n+1} - \mathbf{u}_n \quad \text{and} \quad \Delta\lambda_n = \lambda_{n+1} - \lambda_n \quad (4)$$

are the increments of  $\underline{\mathbf{u}}$  and  $\lambda$ , respectively, corresponding to transition from state  $n$  to state  $n + 1$ .

As it is rather difficult to represent equilibrium paths characterising systems with more than two degrees of freedom in a form of plots that could be readily interpreted, the so-called representative equilibrium paths are used in practice for the purpose of comparative analyses. An equilibrium path represents a functional relationship between a selected parameter characterising deformation of the system and a single control parameter related to the applied load. Results obtained from FEM-based non-linear numerical analyses are usually accepted and considered reliable when a satisfactory coherence is found between two representative equilibrium paths, of which one is determined in the course of an experiment and the other is calculated numerically. It is also necessary to achieve convergence between the deformation patterns following from the calculations [19] with results of corresponding experiment. If such convergence occurs, the obtained reduced stress distributions in the deformed skin can be considered reliable on the grounds of the above-mentioned rule of uniqueness of solutions [20].

Correctness of the non-linear numerical analysis, being an iterative process aimed at finding successive equilibrium states, is to a large degree determined by appropriate choice of the prognostic method, the correction strategy, and a set of control parameters. In case of the present study, the Newton-Raphson method was used in combination with the Crisfield hyperspherical correction strategy.

Otherwise than in the case of linear analyses where the objective is to have as large number of finite elements as possible, too dense mesh of elements in non-linear analyses leads frequently to faulty calculation results, with the calculation time significantly extended. After a number of numerical tests aimed at selection of a proper topology for the model it has been decided to define it with the use of 5000 four-node shell-type elements. The necessity to employ elements of that kind resulted from the fact that other types of elements available in the MSC MARC software library to which properties of laminated composites could be assigned do not offer the possibility to reproduce geometrically complex objects as far as the type and number of the degrees of freedom is concerned.

Numerical models of the materials were developed taking into account mechanical properties of composites used to make models used in the experimental phase and characterised with material constants quoted in the preceding section.

The key objective of non-linear numerical analyses consisted in selecting an optimally effective set of numerical methods which would allow to reproduce correctly post-critical deformations of the examined structures and confirm the possibility to obtain credible results with the use of general-purpose commercial software.

Two versions of the examined structures have been selected as subjects of preliminary analyses, i.e. models with thicker shell, one without stiffeners and the other with crosswise stiffening elements.

As a result, theoretical representative equilibrium paths have been determined from numerical models of both structures and compared with corresponding characteristics measured in the experiment with the result shown in Fig. 11.

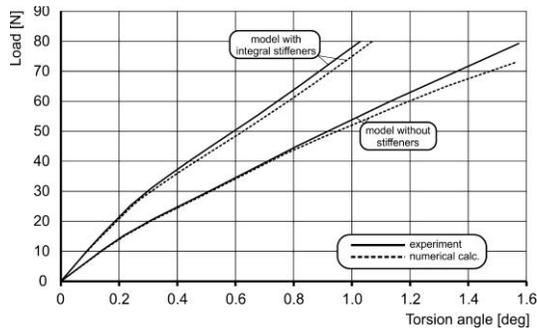


Figure 11. A comparison of representative equilibrium paths.

Figs. 12 and 13 show another output of the analyses which are distributions patterns of resultant displacements of the finite element mesh nodes constituting shell models.

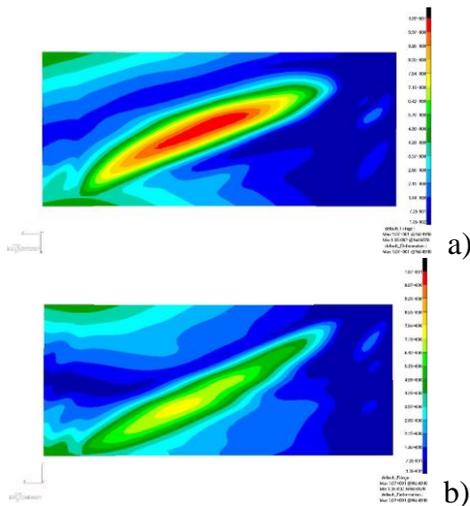


Figure 12. Resultant displacement patterns for the analysed skin fragments, model without stiffeners: (a) upper shell; (b) lower shell.

On the grounds of satisfactory convergence between results of numerical analyses on one hand and the experiment on the other observed in the scope of both representative equilibrium paths and resultant

displacements, it can be stated that properties of composites attributed to finite elements by PATRAN software, determined by the program based on data characterising individual layers of the composite, can be considered correct and reflecting their actual characteristics. It should be however emphasised that in case of occurrence of any defects in the real structure likely to occur in the process of lamination, it is may be necessary to introduce appropriate adjustments to the numerical model in order to take the effect of such flaws on local stiffness of the skin into account.

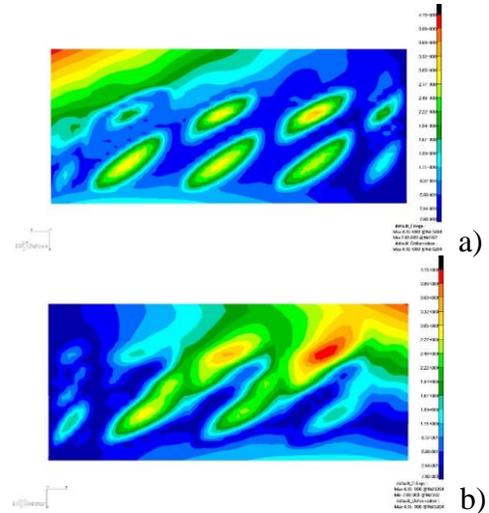


Figure 13. Resultant displacement patterns for the analysed skin fragments, model with ribs: (a) upper shell; (b) lower shell.

By comparing the representative equilibrium paths for models with and without stiffeners it can be claimed that the wing skin stiffening elements constitute a necessary component of the structure. In the model without stiffening elements, loss of stability occurred at a very low load value as a result of occurrence of a field of tensions within the area of the observed skin segment.

## V. SUMMARY AND CONCLUSIONS

Two fundamental objectives were set for the above-presented study devoted to experimental and numerical analysis of several design solutions of a typical fragment of thin-walled aircraft structure. The first goal consisted in determining and comparing the service properties of the examined structures to a maximum possible accuracy.

The study has demonstrated that integral crosswise skin stiffeners, employed in one version of the analysed models, although based on a very simple technological solution, proved to be very effective in comparison to reference structures lacking any stiffening elements. This allowed to chart the direction for further research efforts aimed at determining such types of integral stiffeners which would have properties most desirable in operating conditions specific for the structure in question. A criterion imposed on the target solution can be the maximum critical load value

or the minimum magnitude of a deformation considered representative one, with the overall structure mass increase as small as possible.

The idea of stiffening the skin by adding thereto an adjacent lightweight geodetic structure turned out to be a very effective solution. In this variant, division of the shell into a number of segments with relatively small surface areas results in significant reduction of the ratio of individual skin curvature radii to its overall dimensions. As a result, post-critical deformations appear and develop smoothly, without any noticeable bifurcation effects. At the same time, stiffness of the structure increases noticeably, and so its torsional strength, as can be seen from Fig. 9. This allows to regard the proposed direction adopted in search for most effective variants of providing composite skins with integral stiffening elements as a promising option. The following table summarises the weight-related data concerning the analysed models.

TABLE I.

Models with thicker shells		
Stiffeners type:	Mass:	Mass in reference to model without stiffeners
None	893 [g]	1
Integral, crosswise	933 [g]	1.044
Models with thinner shells		
None	748 [g]	1
Integral, crosswise	764 [g]	1.021
Geodetic structure	795 [g]	1.063

Although the figures presented in the table cannot be considered precisely comparable in view of the technology used to fabricate the experimental model and the related natural dispersion of results, it is still possible to use them to estimate effectiveness of the adopted solution. It can be stated in general that providing the shells with stiffening elements results in a very small increase of mass accompanied by significant increase of rigidity and major change in the nature of post-critical deformations. Definitely the most favourable balance was found in case of stiffening in the form of geodetic structure where the increase of stiffness by about seventy percent was demonstrated by the structure heavier by only six percent. In case of load corresponding to the maximum value for the model with crosswise stiffeners, the structure with geodetic lattice is subject to relatively small displacement, especially in the upper shell area.

The results of experiments and non-linear numerical analyses, as well as conclusions derived from them, should be assessed in the context of a larger-scale research project aimed at determining properties of a number of different stiffeners for aircraft composite skins subjected to post-critical deformations under operating load conditions.

A further objective of the study, representing an indispensable complement to the experimental phase and allowing to obtain knowledge of stress distribution patterns in the examined skins, consisted in development of appropriate and sufficiently effective calculation models with the use of finite element method. Properly verified

numerical models can be used as very effective tools in search for another design solutions representing various combinations of stiffening elements. However, it should be emphasised that the last step in the research procedure must consist in performing an experiment with the use of a model embodying the selected variant. This follows from the absolute necessity to verify numerical models by means of measurements taken on physical models embodying the final solution. Nevertheless, the adopted methodology allows to eliminate the experimental component from intermediate design development stages when a solution is identified as failing to meet the selected criteria.

The above conclusions allow to claim that the presented research methodology can be an effective tool allowing to design skin stiffening solutions most favourable from the point of view of mass vs. rigidity optimisation criteria.

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