Robust and stepwise optimization design for CO_2 pipeline transportation

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Abstract: Carbon capture, utilization, storage (CCUS) technology is an effective means to reduce the CO_2 emissions. It has been noted that engineering and economic design of the pipeline transportation are important components for CCUS. However, the uncertainties of the pipeline transportation model may make an infeasible design in practice and cause unnecessary cost. In this paper, a novel robust optimization method is proposed for CO_2 pipeline transportation design, which can deal with the multiple uncertainties. A stepwise method is presented to further improve the optimization performance. The proposed optimal algorithm is validated by using numerical studies, which show the proposed approach can deal with the multiple uncertainties and improve the design performance in comparison with the existing methods.

Keywords: CO_2 emissions; pipeline transportation; uncertainties; robust optimization; stepwise optimization

Nomenclature

C_{P_cap}	capital cost of pipelines
C_{C_cap}	capital cost of compressors
C_{B_cap}	capital cost of booster pumps
C_{T_OM}	annual operation and maintenance costs of pipelines, compressors and booster pumps
C_{T_energy}	annual energy costs of compressors and booster pumps
CRF_1	capital recovery factor of pipelines

CRF_2 capital recovery factor of compressors

CRF_3 capital recovery factor of booster pumps

C_{ps} price of steel pipeline

C_{PE} price of electricity

D_{inner} inner diameter of the pipeline

E longitudinal joint factor

F design factor.

$f_{\rho}(P_{ave}, T_{ave})$ function of density that depends on the P_{ave} and T_{ave}

$f_{\mu}(P_{ave}, T_{ave})$ function of viscosity that depends on the P_{ave} and T_{ave}

f_M material cost factor

$f_{BO\&M}$ percentage of the capital cost of booster pumps

$f_{PO\&M}$ percentage of pipelines capital cost

f Darcy-Weisbach friction

H_{ope} operation time of the transportation

I_0 base costs for calculating the compressor capital cost

ID_{id} optimal ideal inner diameter

ID_{NPS} inner diameter of the NPS

L length of the pipeline

L_{pum} maximum distance between the boosting pump stations

LC levelized cost of CO_2 pipeline transportation

M molar mass

N total number of compression stages

N_{pump} number of boosting pump stations

N_{Boost} largest number of boosting pump stations

n multiplication exponent

$O\&M$ operation and maintenance costs

P_{inlet} inlet pressure for each pipe segment

P_{outlet} outlet pressure for each pipe segment

P_{max} maximum pressure for CO_2 transportation

P_{inject} injection pressure

P_{ave} average pressure along the pipeline

P_{mop} maximum allowable operation pressure

P_{cap} suction pressure

P_{MOP} discharge pressure

Q_m CO_2 mass flow rate

R universal gas constant

r discount rate

S specified minimum yield stress for the pipe material

T_{ave} average temperature

T_{soil} soil temperature around the pipeline

T_{maxop} largest soil temperature around the pipeline

T_C operation time of compressor

T_B operation time of booster pump

T_1 suction temperature

t wall thickness

t_{id} optimal ideal wall thickness

t_{NPS} wall thickness of the NPS

V actual velocity

$V_{\min}$ minimum velocity

$V_{\max}$ maximum velocity

$W_{comp,0}$ base scale of the compressor

y scaling factor

z_1 lifetime of pipelines

z_2 lifetime of compressors

z_3 lifetime of booster pumps

ρ CO_2 density at average temperature along the pipeline

ρ_{ac} actual density along the pipeline

μ average CO_2 viscosity

γ specific heat ratio

η_{iso} isentropic efficiency of the compressor

η_{mech} mechanical efficiency of compressor

$\eta_{booster}$ booster pump efficiency

ΔP_{act} actual pressure drop

1 Introduction

The process of CO_2 capture, transportation, enhanced oil recovery (EOR) and storage is one of the best ways to reduce the CO_2 emissions, which not only can effectively prevent the increase of CO_2 concentration in the atmosphere, but also bring economic benefit with EOR (Marston 2013; Imtiaz et al. 2015). As a link between the capture source and the storage site, pipelines are attractive approach to transport large amount of CO_2 for long distance (Svensson et al. 2004; Luo et al. 2014; Martynov et al. 2015). It is obvious that engineering and economic pipeline design are important for the CCUS technology (Knoope et al. 2013). Most of the existing optimal approaches are model based, whose performance is affected by the uncertainties seriously (Zhang et al.

2012). Ignoring the uncertainties may lead to an infeasible design. Consequently, it is central to consider the uncertainties in optimization design for the pipeline transportation.

There are mainly two types of uncertainty of the CO_2 pipeline transportation: (1) Engineering model uncertainties. The CO_2 temperature along the pipeline changes with the seasons, even the pipeline has insulation and is buried in the soil (Zhang et al. 2012). Along with time, booster pumps, compressors and other equipment will be aging and their performance parameters will be varying. The impurities in CO_2 such as H_2S, SO_X and O_2 impact the density and viscosity. (2) Economic model uncertainties. There are many changes in labor, material, land prices, and regulations in pipeline lifetime (Middleton 2013). For easy calculation, the electricity cost is usually assumed to be constant over time. However, it is an significant uncertainty in the costs of CCUS power plants and in the electricity cost (Knoope et al. 2014). If these uncertainties are not considered in the pipeline transportation design, it may degrade the design performance.

To design the pipeline transportation easily, the existing researches always assume the temperature is constant along the pipeline (Chandel et al. 2010; Gao et al. 2011; Knoope et al. 2014), however, the temperature variation can significantly affect the transport cost (Chandel et al. 2010) and/or make the design not well (Zhang et al. 2012). To solve this problem, the highest temperature of the soil is used in the pipeline design. In order to simplify the calculation of the pipeline design, the linear optimization is introduced (Morbee et al. 2012; Middleton 2013), in which the modelling uncertainty is not considered. Effects of geologic reservoir uncertainties are analyzed on CO_2 transportation (Middleton et al. 2012), Monte Carlo trials are used to assess the sensitivity of transport cost to the uncertain model parameters (McCoy et al. 2008), but these approaches (McCoy et al. 2008; Middleton et al. 2012) have not discussed the design issues. Considering some uncertainties, an iteration method is proposed for the pipeline design (Knoope et al. 2015). However, the method is based on the designer's experience, which unavoidably exists one-sidedness in the design. In summary, the effects of uncertainties should not be ignored, the existing methods focus on the effects of single uncertainty (Chandel et al. 2010) or partial uncertainties (McCoy et al. 2008; Middleton et al. 2012; Knoope et al. 2015). But these approaches usually lack theoretical analysis. There have not been effective methods to deal with the multiple uncertainties. Therefore, it is necessary to present an approach to cope with multiple uncertainties and improve the design performance. The final selected inner diameter and wall thickness are the nominal pipe size (NPS) in the engineering practice which are larger than the ideal ones in general (McCoy et al. 2008; Zhang et al. 2012), therefore, in order to further improve the design performance, a new algorithm is desired to be explored.

In this paper, multiple uncertainties are transformed into bounded set, a new robust optimization model is initially developed to minimize the levelized cost of the CO_2 pipeline design, which is solved by using the linear matrix inequalities (LMI). The proposed robust optimization approach can deal with the effects of multiple uncertainties, which not only include the variable temperature, declined parameter performance, changeable density and viscosity, but also the change in labor, material, land prices etc. A stepwise optimization following the robust optimization is provided to further improve the optimization performance. The proposed approach is validated by using numerical studies. It should be mentioned that this paper further improves the results of (Zhao et al. 2016), which has not considered the effects of multiple uncertainties.

The rest of this paper is organized as follows. Uncertain optimization problem is formulated in Section 2. Solutions for robust optimization issue are given in Section 3. The stepwise optimization is presented in Section 4. The computation results and analysis are presented in Section 5. Finally, the conclusions are drawn in Section 6.

2 Uncertain optimization problem description

The optimal design for CO_2 pipeline transportation includes the inlet pressure, inner diameter, wall thickness and the number of boosting pump stations. Levelized cost and inlet pressure are selected as the objective function and design variable in the pipeline design, respectively.

The optimization model of CO_2 transportation is formulated as follows (Knoope et al. 2014):

$$\begin{aligned} \min \quad & LC \\ \text{s.t.} \quad & P_{outlet} < P_{inlet} < P_{max} \\ & V_{\min} < V < V_{\max} \\ & P_{out} = P_{inlet} - \Delta P_{act} L/(N_{pump}+1) \end{aligned} \tag{1}$$

$$LC = \frac{CRF_1 \times C_{P\_cap} + CRF_2 \times C_{C\_cap} + CRF_3 \times C_{B\_cap} + C_{T\_OM} + C_{T\_energy}}{Q_m \times 10^{-3} \times H_{ope} \times 3600} \tag{2}$$

$$CRF_x = \frac{r}{1-(1+r)^{-z_x}} \tag{3}$$

where LC is the levelized cost of CO_2 pipeline transportation ($€/t\ CO_2$); P_{inlet} is the inlet pressure for each pipe segment, which is selected as a decision variable (MPa); P_{outlet} is the outlet pressure for each pipe segment (MPa); P_{max} is the maximum pressure for CO_2 transportation (MPa); V, $V_{\min}$, $V_{\max}$ are the actual, minimum and maximum velocities, respectively (m/s); ΔP_{act} is the actual pressure drop (MPa/m); L is the length of the pipeline (m); N_{pump} is the number of boosting pump stations; C_{P_cap}, C_{C_cap}, C_{B_cap} are the capital costs of pipelines, compressors and booster pumps, respectively ($€$); C_{T_OM} are the annual operation and maintenance (O&M) costs of pipelines, compressors and booster pumps ($€$); C_{T_energy} are the annual energy costs of compressors and booster pumps ($€$). Q_m is the CO_2 mass flow rate (kg/s); H_{ope} is the operation time of the transportation ($hour/year$); CRF_1, CRF_2, CRF_3 are the capital recovery factors of pipelines, compressors and booster pumps, respectively ($x=1,2,3$); r is the discount rate (%); z_1, z_2, z_3 are the lifetime of pipelines, compressors and booster pumps, respectively (years). In order to make the paper clearly and easily to follow, Table 1 shows the detail models and the related literatures of C_{P_cap}, C_{C_cap}, C_{B_cap}, C_{T_OM}, C_{T_energy}.

Table 1 Detail models and the related literatures

Model	Literature
C_{P_cap}	(Gao et al. 2011; Knoope et al. 2013)
C_{C_cap}	(Knoope et al. 2014)

C_{B_cap}	(Chandel et al. 2010; Knoope et al. 2013)
C_{T_OM}	(Knoope et al. 2013)
C_{T_energy}	(Knoope et al. 2014)

Based on the first order Taylor series, the objective function can be linearized as:

$$LC = AP_{inlet} + b \tag{4}$$

where A is a coefficient, b is a constant.

Considering the uncertainties, (4) can be written as:

$$LC = (A_0 + \delta_{A_0})P_{inlet} + (b_0 + \delta_{b_0}) \tag{5}$$

where A_0 and b_0 are nominal, δ_{A0} and δ_{b0} are uncertainties, respectively.

Squaring equation (5), the optimization model can be re-written as:

$$\begin{aligned} \min \quad & P_{inlet} \; \tilde{\ } \ldots \tilde{\ } \ldots_{inlet} \ldots \tilde{\ } \ldots_{inlet} \ldots \\ \text{s.t.} \quad & P_{outlet} < P_{inlet} < P_{max} \\ & V_{\min} < V < V_{\max} \\ & P_{out} = P_{inlet} - \Delta P_{act} L/(N_{pump} + 1) \end{aligned} \tag{6}$$

where $\tilde{A} = A_0 + \sum_{i=1}^{L} u_i A_i$, $\tilde{B} = B_0 + \sum_{i=1}^{L} u_i B_i$, $\tilde{C} = C_0 + \sum_{i=1}^{L} u_i C_i$. A_0, B_0, C_0 are nominal parameters, $B_0 = A_0 b_0$, $C_0 = b_0^2$. $\sum_{i=1}^{L} u_i A_i = \delta_{A_0}$, $\sum_{i=1}^{L} u_i B_i = \delta_{A_0} b_0 + A_0 \delta_{b_0} + \delta_{A_0}\delta_{b_0} = \delta_{B_0}$, $\sum_{i=1}^{L} u_i C_i = 2b_0\delta_{b_0} + \delta_{b_0}^2 = \delta_{C_0}$. A_i, B_i, C_i are the uncertainty directions; $u = [u_1, u_2 \cdots u_L]^T$ are the uncertainties, $\|u\|_\infty \le \varepsilon$.

In order to realize the minimization of LC under the uncertainties, (6) can be rewritten as:

$$\begin{aligned} \min_{P_{inlet}} \; \max_{\|u\|_\infty \le \varepsilon} \quad & P_{inlet} \; \tilde{\ } \ldots \tilde{\ } \ldots_{inlet} \ldots \tilde{\ } \ldots_{inlet} \ldots \\ \text{s.t.} \quad & P_{outlet} < P_{inlet} < P_{max} \\ & V_{\min} < V < V_{\max} \\ & P_{out} = P_{inlet} - \Delta P_{act} L/(N_{pump} + 1) \end{aligned} \tag{7}$$

where $\|u\|_\infty = \max|u_i|$. (7) is named as robust optimal model of the original issue. The detailed formations of the robust models are given in Section 3 for the stepwise optimization.

Remark 1: $\|u\|_\infty \le \varepsilon$ denotes the uncertainties which is bounded.

Additional engineering models

The diameter is an intermediate variable, it can be calculated with the flow rate, variable pressure and temperature along the pipeline, it is implied in C_{P_cap} and C_{T_OM} of the objective function (Gao et al. 2011; Knoope et al. 2013; Knoope et al. 2014):

Pipeline inner diameter can be calculated as (Zhang et al. 2006):

$$D_{inner} = 0.363\left(\frac{Q_m}{\rho}\right)^{0.45}\rho^{0.13}\mu^{0.025} \tag{8}$$

where D_{inner} is the inner diameter of the pipeline (m); Q_m is the CO_2 mass flow rate in the pipeline (kg/s); ρ is the CO_2 density at average temperature along the pipeline (kg/m^3); μ is the average CO_2 viscosity ($Pa \cdot s$). For CO_2 pipeline, the average temperature, T_{ave}, is assumed to be the soil temperature, that is, $T_{ave}=T_{soil}$ (McCoy et al. 2008). Pressure and temperature affect the density and viscosity. The current research shows that the change of average temperature (soil temperature) is bounded along the buried pipeline (Zhang et al. 2012). This change is small, in this case, the density and viscosity are almost linear with pressure variation (NIST). The changes of density and viscosity caused by the variable temperature can be dealt with as the system uncertainties.

Based on the data from National Institute of Standards and Technology (NIST), (8) can be converted into:

$$D_{inner} = 0.363 Q_m^{0.45}[f_\rho(P_{ave}, T_{ave})]^{-0.32}[f_\mu(P_{ave}, T_{ave})]^{0.025} \tag{9}$$

where P_{ave} is the average pressure along the pipeline (MPa); T_{ave} is the soil temperature around the pipeline (°C). $f_\rho(P_{ave}, T_{ave})$ is the function of density that depends on P_{ave} and T_{ave} (kg/m^3); $f_\mu(P_{ave}, T_{ave})$ is the function of viscosity that depends on P_{ave} and T_{ave} ($Pa \cdot s$).

P_{ave} can be calculated as (Mohitpour et al. 2003):

$$P_{\text{ave}} = \frac{2}{3}\left(P_{inlet} + P_{outlet} - \frac{P_{inlet}P_{outlet}}{P_{inlet} + P_{outlet}}\right) \tag{10}$$

The density is given as a function of average pressure and temperature along the pipeline:

$$f_{\rho}(P_{ave}, T_{ave}) = (BT)^T P \tag{11}$$

The viscosity is given as a function of average pressure and temperature along the pipeline:

$$f_{\mu}(P_{ave}, T_{ave}) = (DT)^T P \tag{12}$$

where B and D are known constant matrixes; P is the matrix of P_{ave}; T is the matrix of T_{ave}.

$$B = \begin{bmatrix} b_{44} & b_{43} & b_{42} & b_{41} & b_{40} \\ b_{34} & b_{33} & b_{32} & b_{31} & b_{30} \\ b_{24} & b_{23} & b_{22} & b_{21} & b_{20} \\ b_{14} & b_{13} & b_{12} & b_{11} & b_{10} \\ b_{04} & b_{03} & b_{02} & b_{01} & b_{00} \end{bmatrix}, T = \begin{bmatrix} T_{ave}{}^4 \\ T_{ave}{}^3 \\ T_{ave}{}^2 \\ T_{ave} \\ 1 \end{bmatrix}, P = \begin{bmatrix} P_{ave}{}^4 \\ P_{ave}{}^3 \\ P_{ave}{}^2 \\ P_{ave} \\ 1 \end{bmatrix}, D = \begin{bmatrix} d_{44} & d_{43} & d_{42} & d_{41} & d_{40} \\ d_{34} & d_{33} & d_{32} & d_{31} & d_{30} \\ d_{24} & d_{23} & d_{22} & d_{21} & d_{20} \\ d_{14} & d_{13} & d_{12} & d_{11} & d_{10} \\ d_{04} & d_{03} & d_{02} & d_{01} & d_{00} \end{bmatrix}$$

By using (9-12), (8) can be re-written as:

$$D_{inner} = 0.363 Q_m^{0.45} \left((BT)^T P\right)^{-0.32} \left((DT)^T P\right)^{0.025} \tag{13}$$

The pipe wall thickness is given as (Chandel et al. 2010):

$$t = \frac{P_{mop} D_{inner}}{2(S \cdot F \cdot E - P_{mop})} \tag{14}$$

where t is the wall thickness (m); P_{mop} is the maximum allowable operation pressure (MP_a); S is the specified minimum yield stress for the pipe material (MP_a); E is the longitudinal joint factor; F is the design factor.

Liquid pipeline transportation is researched in this study. Compared with supercritical fluid transportation, liquid transportation is better energy efficiency and lower transportation cost over long distance (Zhang et al. 2006; Zhang et al. 2012; Knoope et al. 2014). The pressure drop is calculated for all liquid cases as follows (Knoope et al. 2014):

$$\Delta P_{act} = \frac{8 f Q_m{}^2}{\rho_{ac} \pi^2 D_{inner}^5} \tag{15}$$

where f is the Darcy-Weisbach friction; ρ_{ac} is the actual density along the pipeline (kg/m^3), it changes with the pressure drop and soil temperature, the change is dealt with as one of the multiple uncertainties.

In this paper, based on the levelized transport cost, the installation of boosting pump stations is an optimization design resulting from tradeoffs between increasing the inlet pressure, enlarging the pipeline diameter, or adding a boosting pump station. The number of boosting pump stations is calculated by (Knoope et al. 2014):

$$L_{pum} = \frac{P_{inlet} - P_{out}}{\Delta P_{act}} \tag{16}$$

$$N_{pump} = \left\lfloor \frac{L}{L_{pum}} \right\rfloor \tag{17}$$

where L_{pum} is the maximum distance between the boosting pump stations; N_{pump} is the number of boosting pump stations, $\lfloor \cdots \rfloor$ means the largest integer not greater than the enclosed ratio.

CO_2 velocity can be calculated as:

$$V = \frac{4Q_m}{\rho_{ac} \times \pi \times D_{inner}^2} \tag{18}$$

where V is the actual velocity (m/s), ρ_{ac} is the actual density along the pipeline.

3 Robust and stepwise optimization methods

In this Section, the robust optimization issue is solved by using LMI. Combined with robust and stepwise methods, the pipeline transportation optimization design algorithms are presented.

Theorem 1: Considering a robust optimization problem (7) with the uncertainties u, if there exist the decision variable P_{inlet}, auxiliary variables λ and τ_j such that

$$\begin{aligned}
&\begin{bmatrix} \lambda + F(x) - \sum_{j=1}^{J} \tau_j \varepsilon^2 & (N(x)+G)^T & E^T(x) \\ N(x)+G & \sum_{j=1}^{J} \tau_j Q_j & M^T(x) \\ E(x) & M(x) & I \end{bmatrix} \geq 0 \\
&P_{outlet} < P_{inlet} \\
&P_{inlet} < P_{max} \\
&V_{\min} < V \\
&V < V_{\max} \\
&P_{out} = P_{inlet} - \Delta P_{act} L/(N_{pump}+1) \\
&\tau_j \geq 0, j = 1,\ldots,J
\end{aligned} \tag{19}$$

(7) can be transformed into an optimization problem that will be an objective function λ with constraint (19). Some variables are defined as follows: $E(x)=A_0P_{inlet}$, $F(x)=-(2P_{inlet}^TB_0+C_0)$, $M(x)=\left[A_1P_{inlet}\quad A_2P_{inlet}\quad \cdots \quad \ldots_{et}\right]$, $N(x)=-\begin{bmatrix} P_{inlet}^TB_1 \\ \vdots \\ P_{inlet}^TB_1^L \end{bmatrix}$, $G=-\frac{1}{2}\begin{bmatrix} C_1 \\ \vdots \\ C_L \end{bmatrix}$. According to (19), the pipeline robust optimization problem can be solved by using LMI. The readers can find the proof for Theorem 1 in Appendix A.

Combined with the proposed robust optimization approach, a stepwise optimization method is presented for designing the pipeline transportation, which can be divided into two steps: (1) The robust optimization of inner diameter and wall thickness; (2) The re-robust optimization of inlet pressure and number of boosting bump stations.

Algorithm 1: The first step optimization

Step 1: Selecting the minimum operational temperature and inlet pressure as the initial values of T_{ave} and P_{inlet}, respectively.

Step 2: Substituting T_{ave} and P_{inlet} into (13) to compute D_{inner}; Substituting D_{inner} into (14) to compute t.

Step 3: Substituting D_{inner}, T_{ave} and P_{inlet} into (18) to compute V. If $V_{min}<V<V_{max}$, then go to next step, else if, letting $P_{inlet}=P_{inlet}+\Delta P_{inlet}$ and go to Step 2.

Step 4: Substituting D_{inner} into (17) to compute N_{pump}. If $N_{pump}\le N_{Boost}$, then go to next step, else if , letting $P_{inlet}=P_{inlet}+\Delta P_{inlet}$ and go to Step 2.

Step 5: Substituting all known parameters into (2) to get LC. If $P_{inlet}<P_{max}$, letting $P_{inlet}=P_{inlet}+\Delta P_{inlet}$, and go to Step 2, else if go to the next step.

Step 6: By using the enumeration method, comparing all the computed LC and selecting the minimum one as MLC. Then computing D_{inner}, t and N_{pump} according to MLC.

Step 7: If $T_{ave}<T_{maxop}$, then letting $T_{ave}=T_{ave}+\Delta T_{ave}$, $P_{inlet}=P_{min}$ and go to Step 2, else if go to next step.

Step 8: Dividing the optimal MLC function with different 'pieces' and representing each 'piece' with linear function; Obtaining A_0, δ_{A_0}, B_0, δ_{B_0}, C_0, δ_{C_0}, establishing the robust optimization model (7).

Step 9: Substituting the parameters of each 'piece' into (19), obtaining the robust optimization results by using LMI toolbox, selecting the smallest one.

Step 10: Calculating the wall thickness based on (14); Obtaining inner diameter of the NPS (ID_{NPS}) and wall thickness of the NPS (t_{NPS}) by selecting from the NPS.

End the program.

For the second step optimization, Algorithm 2 calculates the final P_{inlet} and N_{pump}.

Algorithm 2: The second step optimization

Step 1: Selecting the minimum operational temperature and inlet pressure as the initial values of T_{ave} and P_{inlet}, respectively.

Step 2: Substituting ID_{NPS}, t_{NPS}, T_{ave} and P_{inlet} into (18) to compute V. If $V_{min}<V<V_{max}$, then go to next step, else if, letting $P_{inlet}=P_{inlet}+\Delta P_{inlet}$ and go to Step 2.

Step 3: Substituting ID_{NPS}, t_{NPS} into (17) to calculate N_{pump}. If $N_{pump}\le N_{Boost}$, then go to next step, else if , letting $P_{inlet}=P_{inlet}+\Delta P_{inlet}$ and go to Step 2.

Step 4: Substituting all known parameters into (2) to get LC. If $P_{inlet}<P_{max}$, letting $P_{inlet}=P_{inlet}+\Delta P_{inlet}$, and go to Step 2, else if go to the next step.

Step 5: By using the enumeration method, comparing all the computed LC and selecting the minimum one as MLC.

Step 6: If $T_{ave}<T_{maxop}$, then letting $T_{ave}=T_{ave}+\Delta T_{ave}$, and go to Step 2, else if go to next step.

Step 7: Dividing the optimal MLC function with different 'pieces' and representing each 'piece' with linear function; Obtaining A_0, δ_{A_0}, B_0, δ_{B_0}, C_0, δ_{C_0}, establishing the robust optimization model (7).

Step 8: Substituting the parameters of each 'piece' into (19), obtaining the robust optimization results by using LMI toolbox, getting the optimal LC and related P_{inlet}, N_{pump}.

End the program.

After the executing the above algorithms, the optimal ID_{NPS} and t_{NPS} can be obtained from the Step 10 of Algorithm 1, the optimal P_{inlet}, N_{pump} can be obtained from the Step 8 of Algorithm 2.

Remark 2: Algorithm 1 and 2 are used to deal with the variable temperature, if considering the other multiple uncertainties, these uncertainties can be considered as perturbations of the nominal parameters.

4 Computation results and analysis

The basic parameters of the transportation are given in Table 2. The other detailed parameters are given in Table 3-5.

Table 2. Basic parameters of the transportation (Chandel et al. 2010; Gao et al. 2011; Zhang et al. 2012)

Parameter	Symbol	Value
Soil temperature (℃)	T_{soil}	2~17
CO_2 inlet pressure (MPa)	P_{inlet}	8.6~15.3
Pipeline length (km)	L	500
Injection pressure (MPa)	P_{inject}	10
Operation time ($hour$)	H_{ope}	8760

Table 3. Detailed parameter values of pipeline (McCoy et al. 2008; Vandeginste et al. 2008)

Parameter	Symbol	Value
Specified minimum yield stress for X70 steel (MPa)	S	483
Longitudinal joint factor	E	1.0

Design factor	F	0.72
Price of steel pipeline ($€/kg$)	C_{ps}	1.11
Material cost factor	f_M	22.4%
Percentage of pipelines capital cost	$f_{PO\&M}$	0.04

Table 4. Detailed parameter values of compressor and boosting pump stations (Zhang et al. 2006; Kuramochi et al. 2012; Knoope et al. 2014)

Parameter	Symbol	Value
Universal gas constant($J/mol\ K$)	R	8.3145
Suction temperature (K)	T_1	313.15
Specific heat ratio (c_p/c_v)	γ	1.294
Molar mass (g/mol)	M	44.01
Total number of compression stages	N	4
Isentropic efficiency	η_{iso}	80%
Mechanical efficiency	η_{mech}	99%
Suction pressure (MPa)	P_{cap}	0.101
Discharge pressure (MPa)	P_{MOP}	8.6
Base costs for calculating the compressor capital cost ($M€$)	I_0	21.9
Base scale of the compressor (MWe)	$W_{comp,0}$	13
Scaling factor	y	0.67
Multiplication exponent	n	0.9
Percentage of the capital cost of booster pumps	$f_{BO\&M}$	0.04
Booster pump efficiency	$\eta_{booster}$	0.5

Operation time of compressor (*hour*)	T_C	8760
Operation time of booster pump (*hour*)	T_B	8760
Price of electricity (€/ *per kilowatt hour*)	C_{PE}	0.0437
Number of boosting pump stations	N_{pump}	≤ 5
Actual velocity (m/s)	V	$0.5 < V < 6$

Table 5. Parameter values of the levelized cost model (Knoope et al. 2013; Knoope et al. 2014)

Parameter	Symbol	Value
Discount rate (%)	r	15
Design lifetime of the pipeline (*years*)	z_1	50
Design lifetime of compressors (*years*)	z_2	25
Design lifetime of the booster pumps (*years*)	z_3	25

Table 6 Robust optimization results for different design mass flow rates

Method \ Mass flow rate	Q_m (kg/s)	120	150	200	250
The first step	P_{inlet} (MPa)	13.7717	13.6839	13.2998	13.0346
	ID_{NPS} (m)	0.31115	0.33975	0.39055	0.44135
	t_{NPS} (m)	0.00635	0.007925	0.007925	0.007925
	Total cost (€)	1,290,803,170~ 1,293,016,997	1,659,755,448~ 1,662,475,428	2,024,327,376~ 2,027,654,424	2,383,628,850~ 2,387,551,140
The second step	P_{inlet} (MPa)	12.3079	12.2640	11.9307	11.5827
	Total cost	1,282,572,274~	1,649,762,478~	2,011,444,920~	2,366,520,570~

	(€)	1,284,114,384	1,651,666,464	2,013,715,512	2,369,004,030
------	Full Saving cost (€)	8,230,896~ 8,902,613	9,992,970~ 10,808,964	12,882,456~ 13,938,912	17,108,280~ 18,547,110

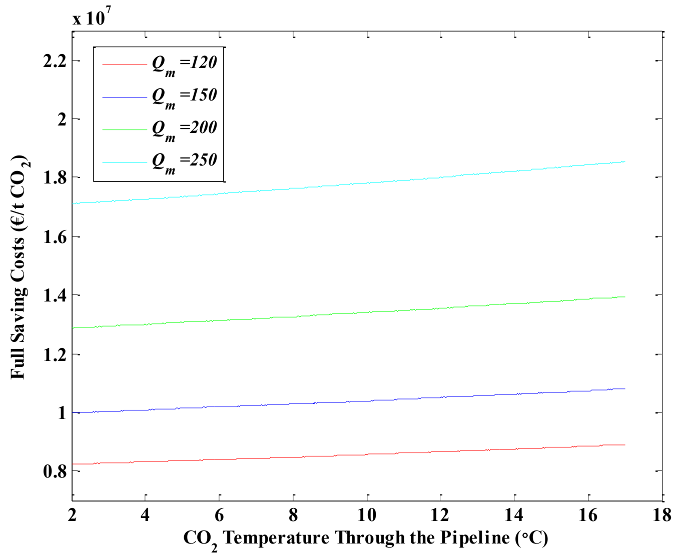

Figure 1. Full saving costs with different temperatures

Table 6 shows the robust optimization results for different design mass flow rates with variable temperature, compared with the first step optimization results, it can be seen that the second step saves the cost. To further illustrate this advantage, the full saving costs are given in Figure 1 for lifetime 25 years, note that full saving cost is the first step total cost minus the second step total cost. For instance, the design mass flow rate is 120 kg/s, the full saving costs are 8,230,896~8,902,613 € for 25 years. The reasons are given as: In the first step, the optimal ideal inner diameter (ID_{id}) and wall thickness (t_{id}) are computed by using the given design conditions. However, the final selected inner diameter and wall thickness are the NPS in the engineering practice which are larger than the ideal ones. Based on the first step optimization, the second stepwise can re-optimize the inlet pressure and the number of boosting pump stations, which can improve the optimal performance.

Table 7 shows the robust optimization results with multiple uncertainties. The flow rate is assumed to be 130

kg/s, the CO_2 temperature is variable with the seasons. The other multiple uncertainties are considered as perturbations, which are denoted as the random percentages (δ_P) of the nominal parameters. It can be seen that the levelized cost increases with the uncertainty increases.

Table 7 Robust optimization results with multiple uncertainties

$\delta_P \in$ / Parameter	$[-0.25\%, 0.25\%]$	$[-0.5\%, 0.5\%]$	$[-0.75\%, 0.75\%]$	$[-1\%, 1\%]$	$[-1.5\%, 1.5\%]$
ID_{NPS} (m)	0.31115	0.31115	0.31115	0.31115	0.31115
t_{NPS} (m)	0.00635	0.00635	0.00635	0.00635	0.00635
P_{inlet} (MPa)	12.6971	12.6969	12.6966	12.6951	12.6890
LC (€)	13.1123	13.1438	13.1764	13.2078	13.2688

To further illustrate the proposed approach, it will be compared with the existing methods. Two situations are presented as follows: (1) Considering the temperature uncertainty only (2) Not only considering the variable temperature but also the other multiple uncertainties.

Table 8 Comparison results of the existing and proposed methods with variable temperature

Method	Parameter	$Q_m = 123$ kg/s $L = 500$ km	$Q_m = 170$ kg/s $L = 380$ km	$Q_m = 200$ kg/s $L = 470$ km
The existing method (Chandel et al. 2010)	P_{inlet} (MPa)	13	13	13
	ID_{NPS} (m)	0.31115	0.33975	0.39055
	t_{NPS} (m)	0.00635	0.007925	0.007925
	P_{outlet} (MPa)	10.15900~9.90296	10.15905~9.90294	10.15901~9.90296
	Total cost (€)	1,301,231,319~	1,561,273,948~	1,964,157,458~

		1,303,784,612	1,563,843,186	1,968,193,020
The proposed method	P_{inlet} (MPa)	12.4857	12.1624	11.7914
	ID_{NPS} (m)	0.31115	0.3429	0.3937
	t_{NPS} (m)	0.00635	0.00635	0.00635
	P_{outlet} (MPa)	10.20830~10	10.18282~10	10.15295~10
	Total cost (€)	1,298,899,850~ 1,300,568,495	1,436,135,187~ 1,438,223,691	1785271765~ 1787428123
	Total saving cost (€)	2,331,469~ 3,216,117	125,138,761~ 125,619,495	178,885,693~ 180764897

Assuming the temperature is variable with seasons. The design should satisfy the following constraint: $P_{out} = P_{inlet} - \Delta P_{act} L / (N_{pump} + 1)$. It is important to note that $P_{out} = 10$ MPa is the minimum injection pressure (Zhang et al. 2012). The CO_2 temperature is assumed to be 12 $^\circ C$ by using the existing method (Chandel et al. 2010). Table 8 shows the comparison results of the existing and proposed methods with variable temperature. It can be seen that P_{out} may not satisfy the constraint based on the existing method, compared with the existing method, the proposed method saves the total cost. For example, assuming $Q_m = 123$ kg/s and $L = 500$ m, based on the existing method, the inlet pressure is 13 MPa, the optimized nominal inner diameter and wall thickness are 0.31115 m and 0.00635 m respectively, P_{out} decreases from 10.15900 to 9.90296 MPa as the temperature increases from 2~17 $^\circ C$. Therefore, if the optimization design is applied based on the existing method, P_{out} is smaller than 10 MPa at higher temperatures, this lead to an infeasible design. Based on the proposed approach, P_{out} decreases from 10.20830 to 10 MPa as the temperature increases. The proposed method satisfy the constraint. That's because the existing optimization design based on a constant temperature between the variable soil temperature, which ignore the effects of variable temperature. Over the life time of 25 years, the optimal total costs are 1,301,231,319~1,303,784,612 and 1,298,899,850~1,300,568,495 € by using the existing and proposed methods, respectively. The proposed method saves 2,331,469~3,216,117 €. The proposed method not only satisfies the constraint but also saves total cost. Therefore, the optimal results are more reasonable by using the proposed approach.

Table 9 (a) Comparison results of the existing and proposed methods with multiple uncertainties

Method	Parameter	Q_m (kg/s)			
		100	145	180	245
The existing research (Knoope et al. 2014)	ID_{NPS} (m)	0.26035	0.31115	0.33975	0.39055
	t_{NPS} (m)	0.00635	0.00635	0.007925	0.007925
	P_{inlet} (MPa)	14.1287	13.3920	13.2860	12.9213
The proposed method	ID_{NPS} (m)	0.31115	0.33975	0.39055	0.44135
	t_{NPS} (m)	0.00635	0.007925	0.007925	0.007925
	P_{inlet} (MPa)	11.6051	12.1170	11.56925	11.52536
Base optimal results with known uncertainties	ID_{id} (m)	0.26306	0.31547	0.351186	0.403641
	t_{id} (m)	0.00563	0.00647	0.007042	0.007857
	P_{id} (MPa)	14.2718	13.7047	13.4079	13.0316

Assuming the temperature is variable with seasons, other multiple uncertainties are bounded ($\delta_P \in [-3\%, 3\%]$). In order to further illustrate the effects for dealing with multiple uncertainties by using the proposed method, known perturbations are given as basic reference: the variable temperature is 17 ℃, other multiple uncertainties are 2% of nominal parameters, ID_{id} and t_{id} can be obtained. Table 9 (a) shows the optimization results of the existing and proposed methods with multiple uncertainties. Compared with the results from the basic reference, diameter and wall thickness not satisfy the design by using the existing method (Knoope et al. 2014). For example, assuming $Q_m = 100$ kg/s, $ID_{NPS} = 0.26035$ m is obtained by using the existing method, compared with $ID_{id} = 0.26306$ m, ID_{NPS} cannot satisfy the diameter design requirement. Assuming $Q_m = 145$ kg/s, $t_{NPS} = 0.00635$ m is obtained by using the existing method, compared with $t_{id} = 0.00647$ m, t_{NPS} cannot satisfy the wall thickness design requirement, there is no safety guarantee for the pipeline transportation.

The existing method may make an infeasible design, because it cannot deal with the effects caused by the variable temperature and other multiple uncertainties effectively. Note that the determination of diameter and wall thickness depends on temperature indeed (as shown in (13) and (14)). Compared with the results from the basic reference, the proposed method can deal with the multiple uncertainties well and get feasible results.

Table 9 (b) Comparison results of the existing and proposed methods with multiple uncertainties

Method	Parameter	Q_m (kg/s)			
		120	165	220	290
The existing research (Zhang et al. 2012)	ID_{NPS} (m)	0.31115	0.33975	0.39055	0.44135
	t_{NPS} (m)	0.00635	0.00635	0.007925	0.007925
	P_{inlet} (MPa)	13.9803	13.5245	13.1588	12.8461
	Total cost (€)	1,335,281,502	1,792,087,217	2,192,466,945	2,660,720,632
The proposed method	ID_{NPS} (. m .)	0.31115	0.33975	0.39055	0.44135
	t_{NPS} (m)	0.00635	0.007925	0.007925	0.007925
	P_{inlet} (MPa)	12.3098	12.6594	12.2705	12.0464
	Total cost (€)	1,325,140,505	1,784,887,923	2,183,147,847	2,648,879,152
	Total saving (€)	10,140,997	7,199,294	9,319,098	11,841,480

Table 9 (b) also shows the comparison results of the existing and proposed methods with multiple uncertainties. Compared with the existing method, the proposed method saves the total cost. For example, assuming $Q_m = 120$

kg/s, the optimal ID_{NPS}=0.31115 and t_{NPS}=0.00635 m are obtained by using the two methods. The optimal total costs are 1,335,281,502 and 1,325,140,505 € over the lifetime of 25 years for the existing and proposed methods, respectively. The proposed method saves 10,140,997 €, which improves the optimization performance.

5 Conclusion

In order to minimize LC for pipeline design, a novel robust optimization model is developed by considering multiple uncertainties. The solution for robust optimization problem is obtained by LMI. A stepwise optimization is given to improve the optimization performance. In the numerical studies, comparing with the existing optimization methods, it is verified that the proposed approach can improve the design performance and provides more securities for the pipeline transportation. In the future, the authors will focus on the applications of the proposed approach in the CO_2 pipeline design.

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Appendix A: Proof for Theorem 1

Lemma 1(El Ghaoui 1997): (*S*-procedure). Letting $F_0, \cdots$ be quadratic functions of the variable $\zeta \in R^m$:

$$F_i(\zeta) \square \quad \cdots$$

where $T_i = T_i^T$. If $F_0, \cdots$ satisfies the following condition:

$$F_0(\zeta) \geq 0 \quad \text{for all} \quad \zeta \quad \text{such that} \quad F_i(\zeta) \geq 0,\ i = 1, \cdots$$

there are $\tau_1 \geq 0, \cdots$ such that $\begin{bmatrix} T_0 & e_0 \\ e_0^T & v_0 \end{bmatrix} - \sum_{i=1}^{p} \tau_i \begin{bmatrix} T_i & e_i \\ e_i^T & v_i \end{bmatrix} \geq 0$.

Lemma 2 (Lin C 2007) (Schur complement) Letting S_1, S_2 and S_3 be appropriately dimensional matrices with S_1 and S_3 symmetric. Then,

$$\begin{bmatrix} S_1 & S_2 \\ S_2{}^T & S_3 \end{bmatrix} > 0$$

if and only if any of the following conditions holds:

(i) $S_1 > 0$ and $S_3 - S_2{}^T S_1{}^{-1} S_2 > 0$;

(ii) $S_3 > 0$ and $S_1 - S_2 S_3{}^{-1} S_2{}^T > 0$.

Proof: Introducing an auxiliary variable λ, (7) can be reformed as:

$$\begin{aligned} \min_{P_{inlet}} \ \max_{\|u\|_\infty \leq \varepsilon} \ & \lambda \\ \text{s.t.} \ & P_{inlet} \ldots \\ & P_{outlet} < P_{inlet} \\ & P_{inlet} < P_{max} \\ & V_{\min} < V \\ & V < V_{\max} \\ & P_{out} = P_{inlet} - \Delta P_{act} L / (N_{pump} + 1) \end{aligned} \tag{20}$$

Defining the following norm:

$$\|u\|_{Q_j} = \sqrt{u^T Q_j u} \qquad j = 1, 2 \cdots \tag{21}$$

where $Q_j \geq 0$, $\sum_{j=1}^{J} Q_j > 0$. Setting Q_j be an appropriately dimensional matrix, the element of the *j*th row and *j*th column of Q_j is 1, while the rest elements of Q_j are 0. Then $\|u\|_{Q_j} = |u_j|, j = 1, 2 \cdots$. $\|u\|_\infty \leq \varepsilon$ can be denoted as $|u_j| \leq \varepsilon, j = 1, 2 \cdots$. Therefore, $\|u\|_\infty \leq \varepsilon$ can be written as $\|u\|_{Q_j} \leq \varepsilon, j = 1, 2 \cdots$. (20) can be transformed into:

$$
\begin{aligned}
\min_{P_{inlet}} \quad \max_{u^T Q_j u \le \varepsilon^2} \quad & \lambda \\
\text{s.t.} \quad & P_{inlet} \text{[illegible]} \\
& P_{outlet} < P_{inlet} \\
& P_{inlet} < P_{max} \\
& V_{\min} < V \\
& V < V_{\max} \\
& P_{out} = P_{inlet} - \Delta P_{act} L / (N_{pump} + 1)
\end{aligned}
\tag{22}
$$

$u^T Q_j u \le \varepsilon^2$ is equivalent to:

$$
\begin{bmatrix} 1 \\ u \end{bmatrix}^T \begin{bmatrix} \varepsilon^2 & 0 \\ 0 & -Q_j \end{bmatrix} \begin{bmatrix} 1 \\ u \end{bmatrix} \ge 0 \tag{23}
$$

P_{inlet} [illegible] can be written as:

$$
P_{inlet}{}^T \left(A_0 + \sum_{i=1}^{L} u_i A_i \right)^T \left(A_0 + \sum_{i=1}^{L} u_i A_i \right) P_{inlet} + \left(B_0 + \sum_{i=1}^{L} u_i B_i \right)^T P_{inlet} + \left(C_0 + \sum_{i=1}^{L} u_i C_i \right) \le \lambda \tag{24}
$$

Defining variables transformation, $E(x) = A_0 P_{inlet}$, $F(x) = -\left(2P_{inlet}^T B_0 + C_0\right)$, $M(x) = \left[A_1 P_{inlet} \quad A_2 P_{inlet} \quad \cdots \quad \text{[illegible]} \right]$, $N(x) = -\begin{bmatrix} P_{inlet}^T B_1 \\ \vdots \\ P_{inlet}^T B_1^L \end{bmatrix}$, $G = -\frac{1}{2}\begin{bmatrix} C_1 \\ \vdots \\ C_L \end{bmatrix}$. (24) will be:

$$
\left[E(x) + M(x)u \right]^T \left[E(x) + M(x)u \right] \le 2\left[N(x) + G \right]^T u + F(x) + \lambda \tag{25}
$$

After some straight forward manipulations, (25) becomes:

$$
\begin{bmatrix} 1 \\ u \end{bmatrix}^T \begin{bmatrix} \lambda + F(x) - E(x)^T E(x) & \left(N(x) + G - M(x)^T E(x) \right)^T \\ N(x) + G - M(x)^T E(x) & -M(x)^T M(x) \end{bmatrix} \begin{bmatrix} 1 \\ u \end{bmatrix} \ge 0 \tag{26}
$$

Using the *S*-procedure (Lemma 1), for all u, (26) holds if there exist a scalar $\tau_j \ge 0$ such that

$$
\begin{bmatrix} \lambda + F(x) - E(x)^T E(x) & \left(N(x) + G - M(x)^T E(x) \right) \\ N(x) + G - M(x)^T E(x) & -M(x)^T M(x) \end{bmatrix} - \sum_{j=1}^{J} \tau_j \begin{bmatrix} \varepsilon^2 & 0 \\ 0 & -Q_j \end{bmatrix} \ge 0 \tag{27}
$$

After some straight forward manipulations, (27) is equivalent to:

$$
\begin{bmatrix} \lambda + F(x) - \sum_{j=1}^{J} \tau_j \varepsilon^2 & \left(N(x) + G \right)^T \\ N(x) + G & \sum_{j=1}^{J} \tau_j Q_j \end{bmatrix} - \left[E(x) \quad M(x) \right]^T \left[E(x) \quad M(x) \right] \ge 0 \tag{28}
$$

Using Schur complement (Lemma 2), (28) is transformed into:

$$\begin{bmatrix} \lambda + F(x) - \sum_{j=1}^{J} \tau_j \varepsilon^2 & \left(N(x)+G\right)^T & E^T(x) \\ N(x)+G_i & \sum_{j=1}^{J} \tau_j Q_j & M^T(x) \\ E(x) & M(x) & I \end{bmatrix} \geq 0 \tag{29}$$

According to (19), the pipeline robust optimization problem can be solved by using LMI.

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