

Evidence of negative Poisson's ratio in wood from finite element analysis and off-axis compression experiments

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Abstract

The microstructure of wood is highly anisotropic, which suggests that this material could exhibit unusual elastic properties. A few studies have suggested the possibility of auxeticity, or negative Poisson's ratio (NPR) in some wood samples. In order to conclusively confirm that wood does show NPR, we have used a combination of geometric modelling, finite element analysis (FEA) and off-axis compression testing. The geometric model indicates that idealised thin-walled wood cell structures should generate NPR of -1, FEA suggests that more realistic cell arrays can attain NPR, to -0.27, and compression tests show that some NPR –to -0.74– remains in real wood samples. These results could help design “engineered-wood” laminates products with tailored elastic properties.

Keywords: ~~Biomaterials~~, Structural, Negative Poisson's ratio, Simulation and modelling, Elastic properties

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1 Introduction

Auxetic materials, those materials with a negative Poisson's ratio (NPR), were long considered a rarity, but a recent study[1] has shown that around 37% of single crystals display NPR, sometimes lower than -1, often in off-axis directions, and that anisotropy correlates positively with NPR. Wood, or timber, are generic terms for a large number of natural materials with a range of mechanical properties, from balsa (density 0.16 g.cm^{-3}) to Black Ironwood (density 1.355 g.cm^{-3}). Always an important construction material, the emergence of engineered wood products[2] is leading to impressive uses in architecture such as wood skyscrapers[3, 4] and a renewed interest in optimising or fine-tuning the mechanical properties of wood structural elements (see for instance[5]). NPR has been shown to be beneficial for properties such as impact resistance[6], vibration absorption[7] reduced fibre pull-out in composites[8] and workability/synclasticity[9] for instance.

The evidence for NPR in wood is slim and scattered. Most experimental studies of elasticity in wood consider the three anatomical axes (longitudinal L, tangential T, radial R) only, and along these, no NPR has been found. A few studies have observed NPR in the LT plane. Using direct compression methods, Yamai[10] found NPR in 5 (out of 11) Japanese species, at around 30° off the L axis, for a minimum of -0.42 in *Cryptomeria Japonica* and Sliker[11] observed NPR in 3 hardwood, at 20° with values as low as -0.37 for Basswood. Using acoustic measurements, Bucur[12] identified NPR at -0.95 in Douglas Fir, but does not report any angle. Finally Kawahara[13] observed NPR at 30° off the L axis in Japanese cypress (a softwood) and Kalopanax (a hardwood), at -0.22 and -0.05 respectively.

2 Geometric modelling

Wood cells are generally slender hollow “tubes”, with cross sections of varying shape, mostly irregular hexagons or parallelograms. Elastically, wood is treated as an orthotropic medium. These two facts suggest that either rectangular or hexagonal tubes could approximate wood cells. In this study, we chose a hexagonal cross-section but a square cross-section leads to similar results. Consider the idealised wood cell (with thin walls) depicted in Fig. 1 (such a model can be created from a sheet of paper and scotch tape in a matter of minutes). Compressive loading at an angle, flattening the R and H (horizontal) directions, leads to an expansion in the T direction and to a positive Poisson’s ratio ν_{HT} . However, this response also generates a contraction on a given vertical line, corresponding to a negative Poisson’s ratio. It is possible to calculate ν_{HV} for any angle, as shown in Fig. 1, from the relation between the height Δh , the change in width Δw , the change in diagonal radius Δr , and the angle of tilt α

$$\Delta h \cdot \sin(\alpha) = \Delta r = \Delta w \cdot \cos(\alpha), \quad (1)$$

as

$$\nu_{HV} = -\frac{\varepsilon_V}{\varepsilon_H} = -\frac{\frac{\Delta w}{w}}{\frac{\Delta h}{h}} = -\frac{\frac{\Delta w}{\Delta h}}{\frac{w}{h}} = -\frac{\tan(\alpha)}{\tan(\alpha)} = -1. \quad (2)$$

In the limit cases of $\alpha = 0^\circ$ and $\alpha = 90^\circ$, either Δw or Δh is undefined, and the Poisson’s ratio of the idealised cell cannot be calculated. Otherwise, this simple model suggests that $\nu_{HV} = -1$, independently of the angle of tilt, and that off-axis NPR is *intrinsic* to anisotropic thin walled cellular structures.

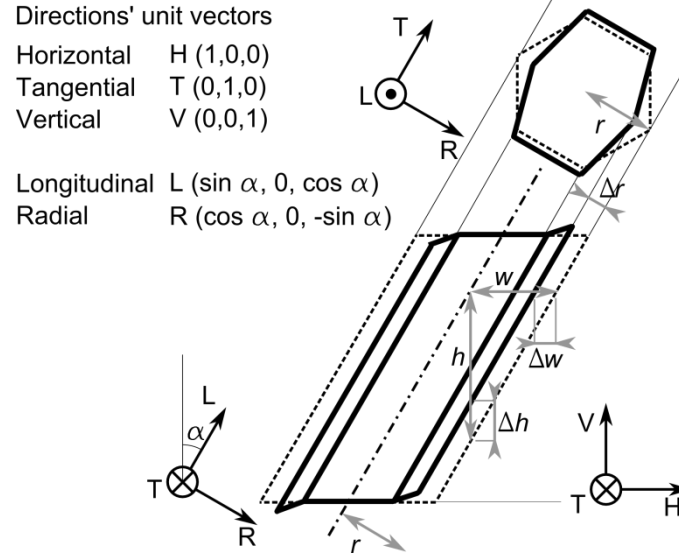


Fig. 1. Idealised wood cell model (hexagonal section): undeformed in dashed lines, compressed in full lines. The figure also displays the coordinate systems (LRT anatomical, VTH rotated).

3 Finite element analysis of wood cell arrays

Finite Element Analysis (FEA) of wood-like cell arrays increases model realism and complexity, and can determine whether NPR calculated from the simple geometric model is conserved with more realistic wall thicknesses, wall joints and periodicity.

3.1 Method

Blocks cut at varying angle from a honeycomb parent structures were subjected to simulated compression, using the FEA code Abaqus2016. The exact numbers of elements and nodes depends on the angle of tilt; the models were composed of between 113 thousand and 129 thousand elements, for between 206 thousand and 248 thousand nodes. The vast majority of the elements were of the quadratic tetrahedral type C3D10, with a few of the linear hexahedral elements type C3D8R.

The insert in Fig. 2 displays one such sample. The samples are 300mm long and wide and 150mm high. The diameter of the hexagons is 20mm, with a wall thickness of 0.5mm. The volume fraction is thus 10.6%, which according to Mitschke[14] is low enough for joint/hinge effects to dominate. In order to maintain elasticity and avoid

nonlinearities, the ratio of applied load to Young's modulus for the parent material was fixed at 0.1%. For simplicity and to emphasize the bending at the wall joints, the Poisson's ratio was set at 0 and the Shear modulus at 0.1% of the Young modulus.

The bottoms of the samples are constrained in the vertical direction. A uniform load is applied to the top of the sample by a rigid plate subjected to a body force. The interface between the sample and the plate allows for small displacements, without friction. The angle of tilt was increased from 0° to 90° in 9° increments.

The vertical strain is easily obtained from the displacement of the press plate, while the lateral strain is obtained by averaging the displacement of several points in the model.

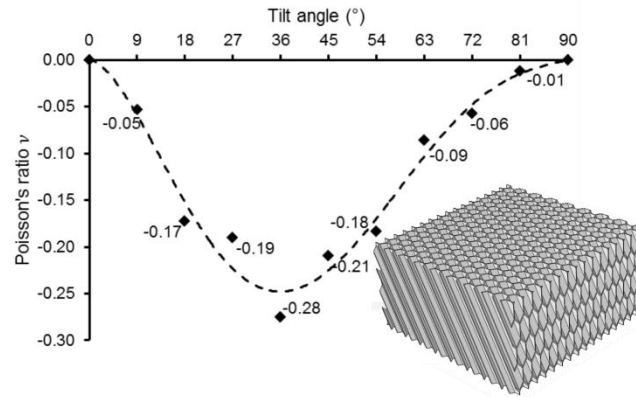


Fig. 2. Poisson's ratio from FEA simulations as a function of tilt angles. Dotted line: Polynomial fit (5th order). Insert: Wood cell structure model with 27° tilt angle

3.2 Results

The resulting Poisson's ratios are plotted in Fig. 2. With the exceptions of 0° and 90° , all samples show NPR, for a minimum of -0.28 around 36° . The results differ from the -1 predicted by the purely geometric model, but several assumptions are no longer valid: the tilting angle of cells changes with compression because the structure undergoes a sideward shear-like deformation and the rigidity of the side wall is no longer absolute. Still, some NPR is conserved.

4 Compression testing of off-axis wood

For experimental validation, wood samples were mechanically tested. As both the geometrical model and the FEA study suggest that NPR is present for thin walled structures, it is sensible to select a softwood of relatively low density. On the other hand, as our aim is to observe NPR in off-the-shelves, construction type wood, no effort is made to control for parameters such as moisture content.

4.1 Method

A beam of white pine (*pinus strobus*), of density 463 kg.m^{-3} was obtained from a reclamation yard and cut into $35\text{mm} \times 52\text{mm} \times 35\text{mm}$ clear wood blocks at 27° from the L direction, in the LR plane (rotation around the T axis). Four Micro-Measurements CEA-13-120CZ-120 strain gauges were bonded with a thin layer of M-Bond AE-10 epoxy to the VT side (“large”) and LR (or VH) side (“small”), as depicted in Fig. 3. The samples were subjected to compressive loading in the V direction by an Instron Universal Testing System, and the longitudinal and transverse strains were recorded as a function of time on Micro-Measurement P3 strain indicators.

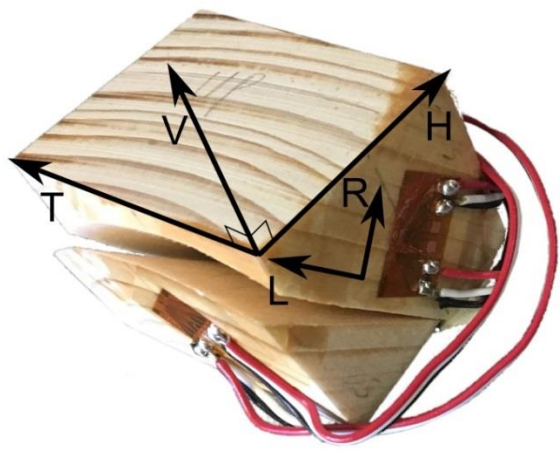


Fig. 3. Wood sample for compression test, with 4 strain gauges visible. Note that the directions of the strain gauges are oriented correctly along the V (twice), T and H axes (the orientation of the backing rectangles is misleading).

4.2 Results

The four strain rates for each sample were obtained by linear interpolation of the strain-time data[15] and are given in table 1. One would expect the two strain rates in the V direction to be comparable, but they vary in magnitude, if not in sign, for several samples. We have no real explanation for this discrepancy beyond the natural variability of wood and perhaps changes in tilt angle could be responsible, as for FEA.

The resulting Poisson's ratios are all negative in the H direction, between -0.02 and -0.74, and positive in the V direction, between 0.08 and 0.38. The sign of these results are consistent with those of the geometric and FEA models.

Table 1. Experimental results for small and large sample sides. Strain rates are in s^{-1} , Poisson's ratio have no units.

Sample	$\dot{\epsilon}_{V/VH}$	$\dot{\epsilon}_{H/VH}$	$\dot{\epsilon}_{V/VT}$	$\dot{\epsilon}_{T/VT}$	ν_{VH}	ν_{VT}
A	-33.55	-0.64	-37.42	13.94	-0.019	0.373
B	-6.23	-3.79	-32.34	2.45	-0.608	0.076
C	-18.60	-13.80	-35.04	10.04	-0.742	0.287

5 Conclusion

Using three complementary approaches, we show that NPR is present, off-axis, in standard construction pine timber. We also elucidate the simple geometric origin of this property and identify directions where it is most pronounced, essentially along the long diagonals of prismatic cell equivalents.

Due to the generality of the geometric and FEA models, we are confident that NPR is present in most, if not all, softwood species. We are currently undertaking a meta-analysis of published elastic properties for wood, but this is a non-trivial task due to the wildly varying formats in which they are reported.

Beside the structure of wood, our results also suggest that sandwich panel cores should exhibit off-axis NPR. However further work is needed to determine whether such auxeticity can be preserved if the cores are covered and restricted by outer skins.

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- [15] See Supplemental Material at [URL will be inserted by publisher] for strain-rates results and a discussion of the benefits of using them instead of strains to measure Poisson's ratio.