Agent-based Macroeconomics and Dynamic Stochastic General Equilibrium Models: Where do we go from here?*

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Abstract

Agent-based computational economics (ACE) has been used for tackling major research questions in macroeconomics for at least two decades. This growing field positions itself as an alternative to dynamic stochastic general equilibrium (DSGE) models. In this paper, we provide a much needed review and synthesis of this literature and recent attempts to incorporate insights from ACE into DSGE models. We first review the arguments raised against DSGE in the macroeconomic ACE (macro ACE) literature, and then review existing macro ACE models, their explanatory power and empirical performance. We then turn to the literature on behavioural New Keynesian models that attempts to synthesise these two approaches to macroeconomic modelling by incorporating insights of ACE into DSGE modelling. Finally, we provide a thorough description of the internally rational New Keynesian model, and discuss how this promising line of research can progress.

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1 Introduction

Agent based modelling is a computational research method that is frequently used in studies of complex social phenomena. As the name suggests, simple representations of decision-makers in social, economic, or political contexts are at the core of this method. By generating a high number of heterogeneous agents that can respond to internal and local as well as aggregate variables, researchers can simulate adaptive behaviour, interdependent decision making, spatial patterns and social networks in a broad range of contexts. In economics, such contexts include, but are not restricted to, asset price fluctuations (Horst (2005)), price bubbles (Duffy and Unver (2006)), bankruptcy cascades (Tedeschi et al. (2012)), and systemic risk related to the housing market (Geanakoplos et al. (2012)). The agent-based literature in economics has been given a number of names, but it is most commonly referred to as agent-based computational economics (ACE).

In macroeconomics, ACE models have been used for at least two decades, and they have continued to capture the attention of scholars, central banks, and policy makers since the Great Recession. An emerging literature attempts to incorporate the insights of macroeconomic ACE (macro ACE) into conventional dynamic stochastic general equilibrium (DSGE) models. The result of this literature is the 'behavioural New Keynesian model', which follows the work of Branch and McGough (2010), Branch et al. (2012), Branch and Evans (2011), De Grauwe and Katwasser (2012), De Grauwe (2011), and others. In this paper, we provide a much needed review and synthesis of this literature and its antecedents, linking it back to the macro ACE critiques of DSGE modelling. This synthetic perspective allows us to clarify the arguments and contributions of recent research, and discuss how the program as a whole can develop to advance macroeconomic modelling. In particular, we propose that a version of the behavioural New Keynesian model that incorporates the internal rationality approach of Adam and Marcet (2011) is a fruitful way forward, and we provide a thorough description of the resulting New Keynesian model.

The paper is organized as follows. In Section 2, we provide an overview of the relevant criticisms of DSGE modelling from the macro ACE literature. We then critically review key models in the macro ACE literature in section 3, and examine their empirical performance and explanatory power in Section 4. This part of the paper clarifies the main arguments and theoretical background of macro ACE studies for readers who are not yet familiar with this literature, and sheds light on the avenues by which conventional DSGE models can be improved. Then, in Section 5, we discuss the extent to which the limitations highlighted in macro ACE studies have been addressed in the behavioural New Keynesian model. Finally, we present very recent work following Deak et al. (2016) on the internally rational New Keynesian model, which provides the behavioural New

Keynesian model with more robust microfoundations.

We hope that this overview and synthesis of the literature on macro ACE and behavioural New Keynesian economics will be useful for macroeconomists who are not yet familiar with the two literatures, or the manner in which they are related. We also hope that it will provide signposts for extending both the macro ACE and DSGE literatures, and provide a possible framework for a more realistic macroeconomics.

2 ACE Critiques of DSGE

In this section, we review some of the criticisms of DSGE modelling from an ACE perspective. We do not present a comprehensive review of the criticisms directed at DSGE from macro ACE scholars. Rather, we present those criticisms that seem most important in achieving a constructive re-evaluation of the DSGE and macro ACE approaches. An alternative, complementary critique of DSGE from the ACE perspective can be found in Fagiolo and Roventini (2016). Specifically, we consider four arguments: that DSGE models ignore heterogeneity, disequilibrium, complex dynamics, and bounded rationality.

2.1 Representative Agent versus Heterogeneous Interacting Agents

The representative agent (RA) assumption is, arguably, at the core of both DSGE modelling and criticism towards it. Essentially, it is the idea that a single agent can stand for an entire sector of the economy, and that any aggregation bias that results from this is negligible. Alongside a number of epistemological concerns (see Delli Gatti et al. (2005), Delli Gatti et al. (2010)), macro ACE critics argue that the RA assumption ignores significant heterogeneities in preferences and endowments, including non-normal distributions and interactions between agents (Delli Gatti et al. (2005)) and, for this reason, DSGE models do not allow any room for emergent macroscopic patterns (Delli Gatti et al. (2005), Gaffeo et al. (2007), see also Gabaix (2011) in this respect).

These ACE criticisms of the RA assumption are rather different from the contemporary critiques of aggregation within the mainstream macroeconomic community, which tend to focus on approximate aggregation results (see Heathcote et al. (2009) for a survey). Notwithstanding this, contemporary dynamic general equilibrium modelling has gone a long way in answering critiques of representative agent assumptions, particularly with the work on incomplete market models that started in the real business cycle literature (e.g. Aiyagari (1994), Krusell and Smith (1998), Heathcote et al. (2009)). Overlapping generations models also remain important - although more in growth theory than short run macroeconomics - and there also exist DSGE models that incorporate asset market segmentation and multiple countries (see e.g. Alvarez et al. (2002) for a model which incorporates both of these elements). Finally, one might also mention DSGE models with simpler forms of heterogeneity, including the rule-of-thumb consumers introduced in Gali et al. (2007). This approach retains the benefits of heterogeneous agents whilst significantly reducing the computational burden of, for example, incomplete market models.

In section 5 of this paper we outline a different approach to heterogeneity which we believe is closer to the ACE critiques of DSGE than those mentioned above. Finally, however, it is worth pointing out that macro ACE studies also criticise the RA assumption in terms of the purpose it serves. Traditionally, following the Lucas critique, one of the major reasons for the popularity of DSGE models is their ability to deliver robust predictions of the effects of policy shifts. However, in cases where preferences are influenced by policy regimes (Bowles (1998), Delli Gatti et al. (2010)), the RA approach may fail to deliver robust predictions given unpredictable changes in behaviour.

2.2 General Equilibrium versus Disequilibrium

As their name suggests, DSGE models rely heavily on equilibrium methods. Whilst the definition of equilibrium is contested in economic theory (Milgate 1987), at the most basic level it implies a collapse in the time ordering of events. Thus, for example, the predicted values of endogenous variables in a DSGE model, within any given time period, are the solution to a system of simultaneous equations. Usually the system does not have a unidirectional causal structure within the period, and no time ordering of events can be unambiguously assigned[1]. As a result, the stability of the intra-period equilibrium is usually not an object of study in contemporary macroeconomics, and instead the stability properties of sequences of temporary equilibria are studied.

The macro ACE literature is critical of the equilibrium assumption in DSGE modelling. These criticisms, however, tend to focus on the Walrasian equilibria used in the real business cycle literature and older DSGE models. It is, for example, pointed out that the set of competitive prices that clear Walrasian market systems is not necessarily unique, with obvious implications for stability (Gaffeo et al. (2007), Lengnick (2013)). More generally, it is argued that the reliance on systems of simultaneous equations within time periods leads to a focus, not on 'real', but on 'hypothetical' time in reference to the *tâtonnement* process, and the Walrasian auctioneer. In this

[1] Consider the system, $A(x_t) = B(x_{t-1}, x_{t-2}, ...)$. If A represents a directed acyclic graph (i.e. a graph in which one cannot loop back to any vertex from itself), then there is a simple unidirectional contemporaneous causal structure between the variables in x. We could also interpret this as a unidirectional time ordering of events within the period, and use it as a formal definition of disequilibrium. As far as the authors are aware, all ACE models are of this form; DSGE models are not often of this form.

respect, the macro ACE literature argues that by focusing on equilibria that an economy reaches in hypothetical time, DSGE models fail to tackle mechanisms of coordination and learning that corresponded to a failure to understand how exactly the economy works (Howitt (2012); also see Mosini (2007)).

2.3 Exogenous Shocks versus Complex Dynamics

In light of the arguments that DSGE modelling ignores heterogeneity, local interaction, and disequilibrium, macro ACE scholars criticise mainstream macroeconomics for ignoring the complexity of economic dynamics. Whilst complexity is often only approximately defined, the basic determinants are a high dimensional state space and a degree of non-linearity such that superposition is not present. Superposition is a property of linear systems, whereby the net response of a system to two or more simultaneous impulses is given by the sum of the responses of the system to the same impulses separately. In particular, two identical impulses, differing only in sign, will cancel each other out. With non-linear models this property fails to hold, and as such the response of a non-linear system to an impulse is not necessarily proportional to the size of the shock, and the state of the system will matter in determining the response to any given shock (see also Chen and Wang (2011); Al-Suwailem (2011)). In addition, some scholars argue that complexity requires neutral equilibria, such that complex dynamics are a mid-point between chaos and asymptotic stability (Foley (2005)).

The macro ACE community argues that this type of complexity is pervasive in macroeconomics, with particular importance being given to endogenous business cycles, or business cycles that are not driven by aggregate shocks. Dosi et al. (2008), for example, argue that real business cycle models and New Keynesian DSGE models are both inadequate because a large part of the dynamics are driven by aggregate technology shocks, and "both streams of literature dramatically underestimate the role of endogenous technological shocks occurring at the microeconomic level" (*ibid.*). Gaffeo et al. (2007), similarly, argue that "complexity arises because of the dispersed, localized, non-linear interactions of a large number of heterogeneous components", and that the economy should be modelled as such.[2] This criticism is related to a more basic criticism that DSGE papers tend to focus on saddle-path stable parameterisations of linearised models.

As with the ACE critique of representative agents outlined in section 2.1, the critique of complexity and non-linearity has been addressed to a certain extent in recent DSGE models. First,

[2] It should be noted that endogenous business cycles have a history that long pre-dates ACE - Hicks, Kalecki, and Kaldor were early advocates, and Goodwin (1967) is a classic example. This literature continues in contemporary Post Keynesian and Marxian economics.

non-linearity and chaotic dynamics had been incorporated into optimising models in the 1980s (see Benhabib (1992) for a collection of the early papers), but this approach faded from view during the 'Great Moderation'. Second, standard DSGE models are highly non-linear in their structural form, and it is possible to solve and estimate higher order approximations given modern computing power. Examples of where this is necessary include any situation in which risk affects decision making - e.g. asset pricing or optimal policy problems. A useful discussion in the context of simulated method of moments is presented in Ruge-Murcia (2012), and work on estimating the non-linear model presented in section 5 of the present paper (using Bayesian techniques) is ongoing work in progress.

2.4 Rational Expectations versus Bounded Rationality

The final ACE criticism of the DSGE paradigm that we wish to highlight is concerned with rational expectations (RE). This criticism is similar to existing critiques of the rational expectations hypothesis, which extend back to the early days of New Classical economic theory, but arises as an almost necessary conclusion from the focus on heterogeneity, disequilibrium, and complexity. An interesting extension of the rational expectations critique from the ACE community is linked to the concept of "emergence" - since real-life versions of economic agents are clearly not equipped with perfect information and foresight, rational expectations are not a property of individual agents but of the system as a whole. In other words, rational expectations supposes that expectations are correct *on average*, and economic theory refers to an instance of "emergence" in the RE hypothesis. This, of course, is contested. For example, Howitt argues that, "even blind faith in internal rationality does not guarantee that the system as a whole will find this fixed point [of rational expectations]" (Howitt (2012)).

As before, modern DSGE models often go beyond the rational expectations benchmark. Various approaches have been proposed in the literature, including least squares learning, rational inattention, restricted perceptions, and near-rational expectations (see Woodford (2013) for a very useful survey). As a move away from rational expectations is at the core of the model presented in section 5, we postpone further discussion of this issue until then - although it is important to note that alternative approaches exist which dull the impact of the ACE critique.

2.5 Summary

Four important critiques of DSGE from an ACE perspective revolve around heterogeneity, disequilibrium, complexity, and rationality. Modern DSGE models often answer one or more of these critiques, and we have mentioned some important examples in the foregoing. Notwithstanding this, ACE models attempt to move away from DSGE models along all of these lines simultaneously. Before the behavioural New Keynesian models that aim to incorporate the insights of ACE are considered in section 5, the key macroeconomic ACE models are examined in section 3, and their empirical performance is reviewed in section 4.

3 Key Macro ACE Models

Although it is difficult to strictly classify and compare ACE models, it is possible to identify four major families of models within the macro ACE literature. These are the Keynes meets Schumpeter (K&S) model (Dosi et al. (2006), Dosi et al. (2008), Dosi et al. (2010), Dosi et al. (2013)), the CATS model[3] (Delli Gatti et al. (2005), Delli Gatti et al. (2007), Delli Gatti et al. (2010), Gaffeo et al. (2007), Russo et al. (2007), Ricetti et al. (2013)), the Eurace model (Deissenberg et al. (2008), Dawid et al. (2009), Cincotti et al. (2010), Dawid and Neugart (2011), Raberto et al. (2012)) and the Strategy Switching (SS) models following Brock and Hommes (1997). The following subsections aim to introduce these families of models, explain how they differ from each other in terms of addressing the abovementioned criticisms of the DSGE approach and give examples of decision making heuristics that they use to give a general idea about their modelling approach.

3.1 The K&S Model

The K&S family of models has been developed by Dosi, Fagiolo and Rovetini for over a decade. The first paper that we could identify was published in 2006 (Dosi et al, 2006), although the first model that the authors called a K&S model is dated 2010 (Dosi et al. (2010)). Across various versions, two properties of K&S models are useful when comparing them with other macroeconomic models. Firstly, from a theoretical standpoint, the authors position their models in evolutionary perspectives (Nelson and Winter (1982)) and as the name K&S implies, they refer to influential paradigms in the history of economic thought aiming to develop a synthetic perspective of issues that are usually studied separately. The model is used to study Keynesian approaches to demand dynamics together with Schumpeterian approaches to innovation (Dosi et al. (2010)), and in later versions (Dosi et al. (2013)), Minskyan credit dynamics are also added to the analysis. Secondly, in

[3] Also referred to as the MBU model in Delli Gatti et al. (2011). MBU stands for “Macroeconomics from the Bottom Up”, whereas CATS stands for “Complex AdapTive System” - and is a play on the names of two of the model’s chief authors, apparently suggested by Hyman Minsky.

terms of the model conceptualisation, K&S models are critical of exogenous technological shocks in mainstream DSGE models and represent technological progress as endogenous to macroeconomic dynamics. Endogeneity of technological progress is implemented in K&S models with a detailed capital goods industry that produces machinery and equipment for the consumer goods industry, invests in research and innovation and thus populates the economy with different vintages of capital goods. In this respect, K&S models also differ from most other macro ACE models that do not represent innovation explicitly.

As is common in agent-based models, agents are heterogenous in K&S models and their behaviour is boundedly rational. Whilst households consume all of their income (an extreme form of the Keynesian consumption heuristic), firms price and produce in an approximately Post Keynesian manner. For example, production levels are determined by naive or adaptive expectations over demand levels, and pricing is given by a mark-up over unit costs, where the mark-up evolves according to the following heuristic:

$$\mu_{jt} = \mu_{jt-1}\left(1 + \frac{f_{jt-1} - f_{jt-2}}{f_{jt-2}}\right). \tag{1}$$

In (1), μ_j denotes firm j's mark-up, whilst f_j denotes firm j's market share. Whilst this heuristic could be interpreted as pursuing profit maximisation, there is no attempt in the Dosi et al papers to formally justify this. Given the above, the K&S model produces macroeconomic complexity in the sense of endogenous business cycles, which appear to be driven by pervasive non-linearity and idiosyncratic shocks.

3.2 The CATS Model

The CATS family of macro ACE models has also been developed over a long period of time, with publications dating back to early 2000s. Although these publications jointly have a long list of authors, two names in Domenico Delli Gatti and Mauro Gallegati appear to be at the core of the development of CATS models. Like the K&S models, CATS models incorporate heterogeneity and boundedly rational agents. Unlike K&S models that are positioned in evolutionary economics, however, CATS models make frequent references to complexity theory and methodological developments in statistical physics (see for example, Bianchi et al. (2008), Delli Gatti et al. (2010)). While K&S models emphasise endogeneity of technological progress, CATS models focus on emergent macroeconomic outcomes, in particular, the role of firm heterogeneity in the transmission and amplification of shocks, and position the concept of emergence as an alternative to equilibrium theorising.

In terms of the model structures, CATS models are simpler than K&S models in terms of agent types, and more complex in terms of direct interaction. Many CATS models (for example Delli Gatti et al. (2003), Delli Gatti et al. (2005), Russo et al. (2007)) have one production sector that interacts with a banking sector, although some (for example Delli Gatti et al. (2009), Delli Gatti et al. (2010)) also incorporate an intermediate goods sector.

In Russo et al. (2007) the economy consists of one sector with idiosyncratic R&D shocks at the firm level. Again, firms' pricing strategies are boundedly rational, and evolve according to the following heuristic:

$$P_{jt}^{s} = \begin{cases} P_{jt-1}(1+\eta_{jt}) & \text{if } S_{jt-1} = 0 \\ P_{jt-1}(1-\eta_{jt}) & \text{if } S_{jt-1} > 0 \end{cases} \tag{2}$$

In (2), S_j denotes the firm's stock of unsold goods, and η_j is a firm specific idiosyncratic shock. Hence the firm raises its price if it sells all its produced output in the previous period, and lowers it otherwise. This is not associated with profit maximisation, but is associated with Simon's "satisficing" approach to firm behaviour (P_j^s denotes the firm's "satisfying" price; this is equal to the selling price if it covers unit cost). As with the K&S models, the CATS models produce macroeconomic complexity in the sense of endogenous business cycles driven by idiosyncratic shocks and pervasive non-linearity.

3.3 The Eurace Model

The Eurace model was produced by a project that attempted to construct an agent-based model of the European economy, by representing four markets (consumer goods, investment goods, labour, and financial markets), incorporating spatial elements (the NUTS-2 regions of the EU-27 countries) and generating an unusually large number of agents (Deissenberg et al. (2008)). In this larger model, the heuristic behaviour of at least some of the agents is brought closer to internal rationality - that is, explicit utility and/or profit maximisation in the context of bounded rationality. In particular, the pricing decision of consumption goods firms is predicated on the belief in a CES demand function. Denoting the expected price elasticity as ε^e, firms set prices in the Eurace model as follows:

$$p_{jt} = \frac{\bar{c}_{jt-1}}{1 + 1/\varepsilon^e_{jt}} \tag{3}$$

In (3), $\bar{c}_{jt-1}$ is a measure of unit costs that takes into account past costs and inventory levels. Household behaviour is also based on internal rationality, where the decision rule is justified by appeal to prospect theory, and in particular the theory of loss aversion.

3.4 Strategy Switching Models

The SS models follow the general approach of Brock and Hommes (1997) in representing different strategies agents use to form their expectations. These models usually distinguish between agents who use sophisticated but costly strategies and those who use simple, less costly heuristics. The former are usually thought of as agents who think in line with and are capable of applying mainstream economic theory. Fundamentalists, for example, expect asset prices to reflect an unbiased reflection of news about future income streams related to the asset (see, for example, Lux and Marchesi (1999)). The latter are often thought of as noise traders, agents with naive expectations, or chartists. Yet, they have the capability of switching to the more sophisticated strategy. In effect, the core dynamics of the model are shaped around agents switching between the two (or more) strategies based on fitness measures and costs of strategies. In addition to showing the emergent properties resulting from interdependencies between agents' strategies, or the "rational route to randomness", strategy switching is used in macro ACE models for incorporating learning, usually in the context of learning based on fitness measures or by observing performances of other agents.

The macro ACE models that use strategy switching usually focus on macroeconomic dynamics related to financial markets. While the other modelling families discussed in this section mainly developed around the work of a core group of authors, the progress of SS models has been distributed with authors independent from each other developing their versions of strategy switching and apply it to a broad range of contexts.

3.5 Summary

The overview of some of the key elements of the K&S, CATS, Eurace, and SS models provided above can only scratch the surface of what are vibrant and continuing research programs. Later versions of the K&S and CATS models, for example, incorporate credit and banking networks, and examine the role of government policy in controlling fluctuations and growth. Nevertheless, we hope to have given an indication of the manner in which existing macro ACE models incorporate heterogeneity, disequilibrium, complexity, and bounded rationality.

4 Explanatory Power and Empirical Performance of Macro ACE Models

This section reviews the explanatory power and empirical performance of the models examined in section 3. Macro ACE models investigate various dynamics and contexts and it is not possible to fully describe the contribution of the literature here. Our aim instead is to provide some insights about how these models can be used to explain macroeconomic phenomena, and to provide examples of how the main modelling families we identified in the previous section have been empirically validated. As above, we consider the four most prominent families of macro ACE models sequentially: the Dosi et al K&S model, the Delli Gatti et al CATS model, the Eurace model, and the SS model.

In addition, we examine two more recent models: the Lengnick (2013) model, and the Assenza et al. (2015) model. Each of these models, to varying degrees, is subject to a calibration and moment comparison exercise in at least one published paper.

4.1 The K&S Model

The early K&S models (see, for example, Dosi et al. (2006)) focused on studying business cycles together with several stylised facts concerning the distributions of macroeconomic variables, and showed that interactions between boundedly rational firms in an environment characterised by endogenous technological shocks and demand waves can create cyclical patterns. Later versions of the model (Dosi et al. (2010)) demonstrated the complemantarity between Schumpeterian innovation and Keynesian demand-management policies for achieving sustained growth. In this respect, the models suggest that even though Schumpeterian policies create beneficial structural change, they alone are not adequate to foster long term growth. In a more recent (Dosi et al. (2013)) version of the model with Minskyan credit dynamics, the authors investigate the interactions between distribution of income (between wages and profits) and monetary and fiscal policies in terms of their effects on output and employment. The simulation results indicate that the effects of monetary policy depend on the functional income distribution, as the higher the distribution is skewed towards profits, the less the need of firms for external finance and so the smaller the effect of monetary policy. Regarding fiscal policies, like in the 2010 paper, authors conclude fiscal policy is necessary to control business cycles.

Out of the papers studying the K&S model, Dosi et al. (2008) provides the most in depth empirical validation. Dosi et al. (2008) initially identifies a number of stylised facts to facilitate basic qualitative validation of the model. These include the standard US business cycle facts, the lumpiness and finance dependent nature of individual firm investment expenditure, pronounced and persistent productivity dispersion across firms, and the distinctive distributions of firm size and firm growth rates. The basic calibration follows the antecedent models in Dosi et al. (2005) and Dosi et al. (2006), but is otherwise unexplained. Nevertheless, the model reproduces the basic stylised facts that the authors target, and this result appears to be robust to the exact parameterisation used (see Dosi et al. (2006)).

Of greater interest are the cross-correlograms presented. These compare the correlations at plus and minus four lags of band pass filtered consumption and GDP, investment and GDP, stock accumulation and GDP, employment and GDP, and unemployment and GDP. Interestingly, given the detailed modelling of firm level investment in the model, and the ability of the model to match cross-sectional stylised facts, aggregate investment still performs poorly in comparison to the other time series. This is in line with the failure of standard New Keynesian DSGE models to match aggregate investment data satisfactorily.

4.2 The CATS Model

Early CATS models focus on the emergent macroeconomic outcomes of heterogeneity of agents. In this respect, while the K&S models focus on heterogeneity in terms of subclasses of agents (consumer and capital good) and accumulated capital stocks (levels and mix of different technologies), CATS models tend to focus on behavioural heterogeneity, such as the distribution of variables related to the financial structure of firms. In a 2003 version of the model (Delli Gatti et al. (2003)), the focus is on entry and exit dynamics and how they affect the financial fragility of the system as a whole by changing the distribution of firms with respect to their equity ratio, which affects their ability to use credit. By not assuming exiting firms will be replaced by new firms with similar properties, the model allows the generation of procyclical behaviour of entries and exits, which then contributes to the formation of business cycles in a Minskyan manner. The 2005 paper builds on this approach of allowing for endogenous exit dynamics, this time focusing on firm size which tends to have a power law distribution, and presents a possibility proof for the existence of a Laplace distribution of growth rates of firm-level and aggregate output. In a 2010 (Delli Gatti et al. (2010))

paper, however, the authors employ a different approach. Unlike the previous papers mentioned above, they focus on the endogenous evolution of credit networks between banks, upstream, and downstream firms. The network evolves through preferential attachment while agents search for better prices or interest rates in limited random subsets of potential counterparts and the paper shows that these interactions can create bankruptcy avalanches.

Out of the papers studying the CATS model family, Gaffeo et al. (2008) provides the most in depth empirical validation. This paper is particularly interesting, from our perspective, in that it explicitly attempts to "rival the explanatory power of DSGE models" (*ibid.*: 443). As with Dosi et al. (2008), the calibration is unexplained. However, instead of a list of stylised facts, the authors regard endogenous business cycles as a basic explandum, and compare the model's time series co-movements with US data. Unfortunately the correlograms presented are mostly not comparable with those of Dosi et al. (2008), describing the correlations at plus and minus four lags of Hodrick-Prescott filtered employment and GDP, productivity and GDP, price index and GDP, interest rate and GDP, and the real wage and GDP. Neither is there an attempt to compare these correlations with a standard New Keynesian DSGE model - although on balance, it seems fair to say that the model performs relatively poorly compared to Dosi et al. (2008).

4.3 The Eurace Model

Due to its large scale and detailed nature, published studies using the Eurace model tend to focus on part of the model with only some of the markets being activated and the foci of these studies have been on specific policy instruments. There appears to be two main streams of studies in this regard; one (led by Dawid) studies labour market dynamics and skills within the main structure of the Eurace model and the other (by Cincotti, Raberto and Teglio) studies dynamics in the financial and credit markets. Regarding the first stream, a 2008 paper (Dawid et al. (2008)) puts together a stochastic process for technological progress of capital goods, spatial elements in terms of two regions with different average skill levels and commuting costs, policy options of investing heavily in one region versus more equally in both regions and the dynamics related to consumer goods and labour markets with respect to diverging costs and prices between regions due to differences in productivity. The study found that although investing heavily in improving skills in one region yields higher output in the short run, this policy performs worse in the long run due to labour shortage in the well performing region. In their 2009 paper, the authors showed that the effect of commuting costs on the above mentioned cost, price and output dynamics are not monotonic. Although in both no and high commuting cost scenarios investing heavily in one region yields worse results in terms of output growth, in the case of low commuting costs it performs better than investing equally in both regions. The 2014 study used a social network context instead of a spatial distribution and studied the impact of the density of workers' networks on wage inequality in the context of referral hiring. The model in this study allows for a gap between productivity of firms to emerge endogenously as firms decide on which capital goods to invest in based on the average skill level of their workers. The wage firms offer reflect this productivity gap, and firms with higher productivity offer higher wages. In this context, although firms do not discriminate between referral and non-referral hirings, wage inequality emerges between workers who obtain their jobs through referrals and those who obtain their jobs via other avenues.

Like CATS and K&S models, the second stream of Eurace papers focused on understanding business cycles, in particular through the interplay between real economic variables such as output and employment and the structure and dynamics of the credit market. A 2010 paper (Cincotti et al. (2010) see also Raberto et al. (2012) and Teglio et al. (2012)), for example, represents the interdependencies between financial and credit markets through firms' dividend payment decisions. These affect the demand for credit, and the quantitative easing policy of government affects the supply of money, and the authors show how these dynamics can produce endogenous business cycles.

Unsurprisingly, the Eurace model, given its size, is also not subject to formal estimation. Given this, the Eurace model builders approach the calibration problem in the same manner as early versions of the K&S and CATS models - a set of stylised facts is identified, and the region of the parameter space that can reproduce those facts is identified. In general, as before, this seems to be a relatively informal method, but Dawid et al. (2009) cite a number of varied empirical studies to justify the choice of calibration. It is difficult, from the available literature, to judge the empirical performance of the calibrated Eurace model.

4.4 The Strategy Switching Models

SS models are used mostly for exploring the results of interdependencies between agents' strategies. Lux and Marchesi's (1999) study, for example, indicates a critical value for the number of *noise traders* a market can accommodate before starting to show high volatility. This volatile phase of the market, however, is temporary as it creates perceived opportunities for fundamentalists whose actions re-stabilise markets. Similarly, in Landini et al. (2015) the SS approach is placed in a simple macro ACE model with households, consumer good firms and a representative bank. Firms in the model need to choose a value for a financial coefficient which affects their output level. They can learn which strategy performs better with or without interaction. Like Lux and Marchesi (1999), the paper shows that phase transitions in the economy depend on the number of agents using each strategy, and indicates the critical rate of strategy switching that generates further phase transitions. The authors call these recursive dynamics re-configurative learning. In Salle et al (2013) there is a similar interaction, but this time it is households who learn from each other while they are trying to adjust their reservation wage and consumption patterns based on expectations of inflation. The model also has a central bank that applies inflation targeting and the study focuses on the role of precision and credibility of the central bank's inflation target in a learning economy. The results of the study indicate that perceived noise surrounding the central bank's choice of inflation target may lead to co-ordination failures between households and the central bank, resulting in a trade-off between unemployment and the inflation target.

Although most SS models are theoretical, simplified versions of the Brock and Hommes 1998 model have been used as examples in papers on validation of ACE models (Recchioni et al. (2015); Kukacka and Barunik (2016)). Notably, the type of learning interaction modelled in SS models has been extensively studied in laboratory experiments, where the explanatory power of these models can be rigorously tested. This is not unique in agent based modelling, but it is relatively unusual in models that pertain to macroeconomic dynamics. See, for example, Hommes (2011). This type of empirical work is of obvious importance to agent based modelling as a research programme, given the critique of rational expectations discussed above.

4.5 Recent Models

There exist two prominent macro ACE models that attempt to combine the insights of the K&S and CATS frameworks into simplified, more manageable models. The aim of Lengnick (2013), the first of these, is to "take the most prominent ACE macro models and reduce them in complexity" (pp.104). Again, the calibration is unexplained, but the model succeeds in generating artificial Phillips curves, Beveridge curves, and the long run neutrality and short run non-neutrality of money. The only cross-correlogram presented is between the price level and GDP, and the model appears to perform as least as well as Gaffeo et al. (2008) along this dimension.

The second attempt to combine the K&S and CATS frameworks is presented in Assenza et al. (2015) - in this case, by including capital goods in a zero growth CATS framework. Again, the calibration is relatively arbitrary, although a small number of the parameter choices are explained (e.g. the desired level of capacity utilisation is chosen to match average capacity utilisation in the USA). However, this paper presents by far the most in depth moment comparison exercise, comparing autocorrelation functions and cross-correlograms of HP filtered GDP, consumption, investment, and unemployment, against equivalent US time series, as well as absolute standard deviations. The model performs strikingly well along these dimensions. Again, however, it is interesting to note that investment still performs poorly, in a similar manner to DSGE models - it is considerably more volatile than in the data. Interestingly, the model correlogram between unemployment and total debt is qualitatively similar to that in US data, although there appears to be a small phase shift, and the model correlations are lower at each lag than in the data.

4.6 Summary

Two major conclusions stand out from sections 3 and 4: first, that macro ACE models are very rich, if rather varied, and second, that their complexity and analytical intractability make them very difficult to validate empirically. This goes some way in explaining the absence of formal estimation procedures in the vast majority of macroeconomics ACE papers.[4] However, the available evidence - particularly that presented in Assenza et al. (2015), suggests that ACE is a promising modelling approach in macroeconomics. Given the relative ease of estimating and validating DSGE models, and given the significant amount of expertise in DSGE modelling and estimation in the profession as a whole, this suggests that a fruitful way forward might be the incorporation of ACE insights into DSGE modelling. This has been attempted in the behavioural New Keynesian model, which we turn to next.

5 Bridging the Gap

Section 2 of this paper isolated what we believe to be the four most important criticisms of DSGE modelling from the ACE community. To recap, these are:

- The representative agent assumption, as opposed to heterogeneous interacting agents.
- The general equilibrium assumption, as opposed to disequilibrium and temporal structure.
- Dynamics driven by exogenous shocks, as opposed to complexity and endogenous cycles.
- Rational expectations, as opposed to bounded rationality.

[4] See Grazzini and Richiardi (2015) for a discussion of the prospects for formal estimation of agent based models.

This section reviews studies within the DSGE framework that aim to incorporate the insights of ACE models discussed in section 2. In particular, we review those studies that incorporate the Brock-Hommes complexity framework into DSGE models, and in so doing respond to three of the four critiques of DSGE identified here. They do not, however, properly address the issue of disequilibrium, which is an issue we return to in section 5.5.

In the sense that the models reviewed in this section have features that characterize ACE, they bridge the gap between the two modelling approaches. In section 5.1 we review the Brock and Hommes complexity approach, and in section 5.2 the behavioural New Keynesian model. In section 5.3 we review Adam and Marcet's approach to internal rationality, and explain how this can be incorporated into the behavioural New Keynesian model to improve the microfoundations of the latter. Section 5.4 discusses the theoretical and numerical properties of the internally rational New Keynesian model, following Deak et al. (2016).

5.1 Brock-Hommes Complexity

The complexity framework comprehensively described in Hommes (2013), and first introduced in Brock and Hommes (1997), provides a minimal way of generating complex dynamics via heterogeneous agents with varying degrees of rationality. As such it provides a simple method of answering the major critiques of DSGE outlined above, but until recently was only explored in the context of partial equilibrium models and the agent based models considered in sections 3.4 and 4.4. A simple cob-web model demonstrates the main features. The model is of a partial equilibrium with two types of producers. A proportion $n_{1,t}$ form rational expectations of the price level p_t at time t, denoted by $\mathbb{E}_t(p_t)$. This amounts to perfect foresight so $\mathbb{E}_t(p_t) = p_t$. The remaining proportion of producers, $1 - n_{1,t}$, are boundedly rational in a manner to be defined. Their expectations, formed at time $t-1$, are denoted by $\mathbb{E}^*_{t-1}(p_t)$. We assume linear demand and supply curves subject to random shocks $\epsilon_{d,t}$ and $\epsilon_{s,t}$ respectively. Given $n_{1,t}$ and our definition of $\mathbb{E}^*_{t-1}(\cdot)$, the market-clearing price is given by:

$$D(p_t) = a - dp_t + \epsilon_{d,t} \tag{4}$$

$$S(p_t, \mathbb{E}^*_{t-1}(p_t), n_{1,t}) = s(n_{1,t}p_t + (1 - n_{1,t})\mathbb{E}^*_{t-1}(p_t)) + \epsilon_{s,t} \tag{5}$$

$$D(p_t) = S(p_t, \mathbb{E}^*_{t-1}(p_t), n_{1,t}) \tag{6}$$

where a, d and s are fixed parameters. These pin down the deterministic steady state of the price level denoted by p. We assume that boundedly rational agents eventually forecast correctly, so that in the steady state $p_t = \mathbb{E}^*_{t-1}(p_t) = p$ which is given by $p = \frac{a}{d+s}$.

The learning literature adopts two basic approaches to modelling boundedly rational expectations. The first is usually referred to as statistical learning, where agents are competent econometricians who make observations of the price (in this example), have some idea of the data generating process and estimate it using standard techniques. We leave a discussion of this approach to our later application to a macroeconomic model. Here we adopt the second approach, which assumes that agents use simple heuristic forecasting rules.[5] A general formulation that nests particular examples found in the literature is an adaptive expectations rule of the form

$$\mathbb{E}^*_{t-1}(p_t) \quad = \quad \mathbb{E}^*_{t-2}(p_{t-1}) + \lambda(p_{t-1} - \mathbb{E}^*_{t-2}(p_{t-1})\,;\;\; \lambda \in [0,1] \tag{7}$$

The key component of the Brock-Hommes framework giving rise to complex dynamics is the method by which the proportions of rational and non-rational producers are updated over time. Here the literature adopts a basic general framework set out in Young (2004). To limit the departure from rationality, the approach of reinforcement learning proposes that, although adaptation can be slow and there can be a random component of choice, the higher the 'payoff' (defined appropriately) from taking an action in the past, the more likely it will be taken in the future. Here the payoff is defined as minus the last period's squared forecasting error plus the cost of obtaining that forecasting rule. Then the updated fraction of rational producers is given by a discrete logit model:

$$\begin{aligned} n_{1,t} &= \frac{\exp(-\gamma[(p_t - \mathbb{E}_t(p_t))^2 + C])}{\exp(-\gamma([p_t - \mathbb{E}_t(p_t))^2 + C]) + \exp(-\gamma([p_t - \mathbb{E}^*_{t-1}(p_t)]^2)} \\ &= \frac{\exp(-\gamma C)}{\exp(-\gamma C) + \exp(-\gamma([p_t - \mathbb{E}^*_{t-1}(p_t)]^2)} \end{aligned} \tag{8}$$

The key features of (8) is that the best-performing rule will attract the most followers, and that there is a fixed per period cost, C, of making rational predictions. The parameter γ is referred to in the literature as the intensity of choice and dictates how quickly agents will switch to the best-performing rule. The steady state proportion of rational producers is given by $n_1 = \frac{\exp(-\gamma C)}{\exp(-\gamma C)+1}$.

The stability properties of this model depend on the parameter values s, d, a, C that determine the steady state and λ, γ that determine the speed of learning. For a high proportion of rational producers the model exhibits local stability: in response to an exogenous shock price and output return to their steady state values. As C increases above zero the proportion of rational producers falls and we enter regions of local instability. However, depending on λ and γ, the trajectories are locally unstable, but do not explode. Rather they show chaotic patterns: random-like complex be-

[5]Such rules are *misspecified* and do not converge to a RE equilibrium. Hommes and Zhu (2014) study more general parsimonious rules of this type.

haviour. As forecast errors under adaptive expectations become large, non-rational but intelligent producers switch behaviour by investing the amount C needed to make rational forecasts. Then forecast errors fall and they switch back to non-rational forecasting.

5.2 The Behavioural New Keynesian Model

The Brock-Hommes framework has been used by a number of authors to propose a behavioural version of the standard New Keynesian (NK) model with RE (see e.g. Woodford (2003), Gali (2008)). These include Branch and McGough (2010), Branch et al. (2012), Branch and Evans (2011), De Grauwe and Katwasser (2012), De Grauwe (2011), De Grauwe (2012a), De Grauwe (2012b), Jang and Sacht (2012), Massaro (2013) and Jang and Sacht (2014). Branch and McGough (2016) provides a recent survey. The basic three-equation linearized work-horse NK model used in this literature in its rational expectations form is as follows:

$$y_t = \mathbb{E}_t y_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}) + u_{1,t} \tag{9}$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \lambda(y_t + u_{2,t}) \tag{10}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\theta_\pi \pi_t + \theta_y y_t) + u_{3,t} \tag{11}$$

where y_t, π_t and r_t are the output gap, the inflation rate and the nominal interest rate respectively. The shock processes $u_{i,t}$, $i = 1, 3$ should be interpreted as shocks to preferences, marginal costs and monetary policy, respectively, and are usually AR(1) processes. (9) is the linearized Euler equation for consumption which is equated with output in equilibrium (there is no government expenditure). (10) is the NK Phillips curve and (11) is the nominal interest rate rule with persistence responding to current inflation and the output gap. Expectations up to now are formed assuming rational expectations and perfect information of the state vector (which includes the shock processes).[6]

As in the cob-web model example, the model becomes behavioural by a departure from RE and the introduction of two groups of agents forming expectations through different learning rules. In De Grauwe (2012b) there are two groups using fundamentalist (f) and extrapolative (e) rules with (possibly) non-RE market expectations denoted by $\mathbb{E}^*$. The market forecasts are assumed to be simple weighted averages:

$$\mathbb{E}^*_t y_{t+1} = \alpha_{f,t} \mathbb{E}^*_t y^f_{t+1} + (1-\alpha_{f,t}) \mathbb{E}^*_t y^e_{t+1} \tag{12}$$

$$\mathbb{E}^*_t \pi_{t+1} = \beta_{f,t} \mathbb{E}^*_t \pi^f_{t+1} + (1-\beta_{f,t}) \mathbb{E}^*_t \pi^e_{t+1} \tag{13}$$

[6] Habit in consumption and price indexing result in additional lags in y_t in (9) and in π_t in (10) providing additional persistence mechanisms that help to fit the model to data.

We refer to this approach to non-rational expectations as the Euler Learning (EL) approach. The model is completed by the expressions for the weights $\alpha_{f,t}, \beta_{f,t}$, and the learning rules for the output gap and inflation. The former follows the Brock-Hommes framework as follows:

$$\alpha_{f,t} = \frac{\exp(\gamma U_{f,t}(\{y_t\})}{\exp(\gamma U_{f,t}(\{y_t\})) + \exp(\gamma U_{e,t}(\{y_t\}))} \tag{14}$$

$$\beta_{f,t} = \frac{\exp(\gamma U_{f,t}(\{\pi_t\}))}{\exp(\gamma U_{f,t}(\{\pi\}_t)) + \exp(\gamma U_{e,t}(\{\pi\}_t))} \tag{15}$$

where $U_{f,t}(\{x_t)\})$ is the payoff measure of the fundamentalist rule for outcome $\{x_t\} = \{y_t\}, \{\pi_t\}$, given by a MSE predictor:

$$U_{f,t}(\{x_t\})) = \rho U_{f,t-1}(\{x_t\})) - (1-\rho)[x_{t-1} - \mathbb{E}_{f,t-2}\, x_{t-1}]^2 \tag{16}$$

Equations (9)-(11), with $\mathbb{E}$ replaced with $\mathbb{E}^*$, and equations (12)-(16) complete the behavioural New Keynesian model without specifying the forecasting rules. Whilst De Grauwe, for example, uses a selection of boundedly rational predictors, rules in the spirit of Brock and Hommes (1997), Hommes (2013), and Branch and McGough (2010) are

$$\mathbb{E}^*_t y^f_{t+1} = \mathbb{E}_t y_{t+1} \tag{17}$$

$$\mathbb{E}^*_t y^e_{t+1} = \mathbb{E}^*_{t-1} y^e_t + \lambda_y (y_t - \mathbb{E}^*_{t-1} y^e_t)\,; \quad \lambda_y \in [0,1] \tag{18}$$

$$\mathbb{E}^*_t \pi^f_{t+1} = \mathbb{E}_t \pi_{t+1} \tag{19}$$

$$\mathbb{E}^*_t \pi^e_{t+1} = \mathbb{E}^*_{t-1} \pi^e_t + \lambda_\pi (\pi_t - \mathbb{E}^*_{t-1} \pi^e_t)\,; \quad \lambda_\pi \in [0,1] \tag{20}$$

This assumes fundamentalists are rational and the extrapolative learners use a general adaptive expectations rule. As before, we have:

$$\alpha_{f,t} = \frac{\exp(\gamma(U_{f,t}(\{y_t\}) - C))}{\exp(\gamma(U_{f,t}(\{y_t\}) - C)) + \exp(\gamma U_{e,t}(\{y_t\}))} \tag{21}$$

$$\beta_{f,t} = \frac{\exp((\gamma U_{f,t}(\{\pi_t\}) - C))}{\exp((\gamma U_{f,t}(\{\pi\}_t) - C)) + \exp(\gamma U_{e,t}(\{\pi\}_t))} \tag{22}$$

where C represents the relative costs of being rational. Thus, by incorporating the Brock-Hommes complexity framework into the workhorse New Keynesian DSGE model, the behavioural New Keynesian model incorporates heterogeneity and bounded rationality into a DSGE framework. In addition, and as with the simple Cobweb model presented in section 5.1, the behavioural NK model can generate persistent and asymmetric fluctuations in response to small shocks, and generate endogenous business cycles characterised by bounded instability and chaos (see e.g. Branch et al.

(2012)). Hence the behavioural NK model answers - at least to some extent - three out of the four critiques of DSGE modelling outlined above.

5.3 Internal Rationality versus Euler Learning

As Graham (2011) has pointed out, the form of learning implied by the NK behavioural model above follows the 'Euler equation learning' approach (henceforth EL) and in effect assumes that agents forecast their own decisions - for the household their consumption decision, and for the firm their price decision. These beliefs only become heuristic forecasts $\mathbb{E}_t^* y_{t+1}$ and $\mathbb{E}_t^* \pi_{t+1}$ of *aggregate* output and inflation respectively, as implied by the behavioural model of the preceding sub-section, if we limit the bounded nature of rationality by assuming that, in the symmetric equilibrium, agents *know they are identical.*[7] This is also true when forecasting rules are well-specified and can potentially converge to the RE equilibrium in the statistical learning approach pioneered in Evans and Honkapohja (2001a). Agents then know the minimum state variable (MSV) form of the equilibrium (equivalent to the saddle-path under rational expectations) and use direct observations of these states to update their parameter estimates each period using a discounted least-squares estimator. Then a statistical learning equilibrium is one where this perceived law of motion and the actual law of motion coincide.

An alternative approach, following Adam and Marcet (2011), was first introduced by Eusepi and Preston (2011) into an RBC model.[8] This assumes that agents are internally rational (IR) given their beliefs over aggregate states and prices. As with the Euler equation learning approach, agents cannot form model-consistent expectations and instead learn about these variables using their knowledge of the MSV form of the equilibrium forming well-specified rules that lead to a statistical learning equilibrium.

The internal rationality-anticipated utility (henceforth IR) approach in NK behavioural models with heuristic forecasting rules has been used by Massaro (2013) and Deak et al. (2016), and here we follow the latter where full details can be found.[9] In the rest of this subsection we first describe the construction of a pure IR equilibrium. Then we demonstrate that the modelling decision of EL

[7] See Deak et al. (2016) for further discussion of this point.

[8] A recent survey is provided in Eusepi and Preston (2016). Note they use the term 'anticipated utility' instead if internal rationality and provide a useful discussion of how this modelling assumption differs from Euler equation learning. We adopt the general definition of internal rationality used by Adam and Marcet (2011): namely that "agents maximize utility under uncertainty, given their constraints and given a consistent set of probability beliefs about payoff-relevant variables that are *beyond their control or external*". Then beliefs can take the form of a well-defined probability measure over a stochastic process (the 'fully Bayesian' plan), or they can adopt an 'anticipated utility framework of Kreps (1998). Adam and Marcet (2011) adopt the former approach whereas this paper and the other applications mentioned adopt the latter. Cogley and Sargent (2008) compares the two and encouragingly find that anticipated utility can be seen as a good approximation to fully Bayesian optimization.

[9] See also the appendix in the working paper version of this paper, Dilaver et al. (2016).

versus IR matters for monetary policy by examining the policy space for Taylor-rule parameters in (11) in the two cases. Finally we study the dynamic properties of the heterogeneous RE-IR model through simulations.

The construction of an IR equilibrium for an NK model goes through the following steps:

- Solve the household budget constraint forward in time and impose the transversality condition.
- Use the first-order conditions and either linearize or assume point expectations to obtain consumption as a function of expected nominal interest rates, inflation, wages, tax rates and profits.
- For monopolistically competitive retail firms express the Calvo contract for the price optimizing firm as a function of expected aggregate demand, aggregate inflation, real marginal cost and mark-up shocks (again either linearizing or assuming point expectations).
- To close the IR model, we need to specify the manner in which internally rational households and firms form their expectations.

In modelling the beliefs of IR agents, we assume that variables which are local to the agents, in a geographical sense, are observable within the period, whereas variables that are strictly macroeconomic are only observable with a lag. This categorization regarding information about the current state of the economy follows Nimark (2014). He distinguishes between the local information that agents acquire directly through their interactions in markets and statistics that are collected and summarised, usually by governments, and made available to the wider public.[10] The only exception to this is the nominal interest rate, which we assume is observable within the period given the timing structure of New Keynesian models. There are three exogenous autoregressive processes which drive the uncertainty in the model as a whole. One of these is government spending which is assumed, via a balanced government budget constraint, to be equal to taxes. Households observe this per head as a local variable without a lag, but do not know the stochastic process. There are also technology and mark-up shocks which are unobserved. Given these information assumptions, we assume a general adaptive expectations rule with one-step ahead forecasts given by:

$$\mathbb{E}_t^* x_{t+1} = \mathbb{E}_{t-1}^* x_t + \lambda_x (x_{t-j} - \mathbb{E}_{t-1}^* x_t)\,; \quad j = 0, 1 \tag{23}$$

where the lag $j = 0$ applies to local variables and $j = 1$ applies to macroeconomic variables. Internally rational households then make intertemporal decisions for their consumption demand and

[10]His paper actually focuses on a third category, information provided by the news media, and allows for imperfect information in the form of noisy signals, issues which go beyond the scope of our paper.

hours supply given adaptive expectations of the wage rate, taxation rate, the nominal interest rate and profits observed with no lag ($j = 0$), as these are all local variables, and of aggregate inflation observed with a lag ($j = 1$). Similarly, IR price-setting retail firms form adaptive expectations of the local real marginal costs observed with no lag, and of aggregate demand and aggregate inflation observed with a lag. For example, for households the forecast of the future nominal interest is given by (23) with $x = r$ so that

$$\mathbb{E}^*_t r_{t+1} = \mathbb{E}^*_{t-1} r_t + \lambda_r (r_{t-j} - \mathbb{E}^*_{t-1} r_t) = \sum_{i=1}^{\infty} \lambda^i_r r_{t-j-i}\,; \quad j = 0, 1 \tag{24}$$

Finally, the model is closed in the same way as the behavioural NK model considered above, with two groups of households and firms, one adopting rational expectations and one internally rational. The proportions of rational households and firms are given by

$$\begin{aligned} n_{h,t} &= \frac{\exp(\gamma \Phi^{RE}_{h,t})}{\exp(\gamma \Phi_{h,t})^{RE} + \exp(\gamma \Phi^{IR}_{h,t})} \\ n_{f,t} &= \frac{\exp(\gamma \Phi^{RE}_{f,t})}{\exp(\gamma \Phi^{RE}_{f,t}) + \exp(\gamma \Phi^{IR}_{f,t})} \end{aligned}$$

where fitness for households is given by

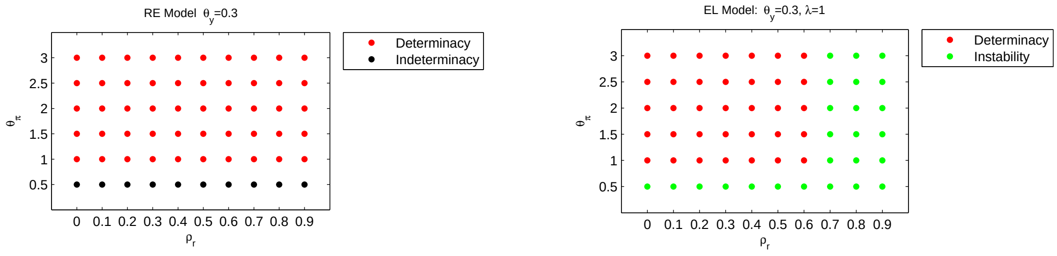
$$\begin{aligned} \Phi^{RE}_{h,t} &= \mu^{RE}_h \Phi^{RE}_{h,t-1} - \Big(\text{weighted sum of forecast errors} + C_h\Big) \\ \Phi^{IR}_{h,t} &= \mu^{IR}_h \Phi^{IR}_{h,t-1} - \Big(\text{weighted sum of forecast errors}\Big) \end{aligned}$$

As before, C_h is a fixed cost of the rational expectations operator for households and firms.

We now have three possible models of expectations, rational (i.e. model consistent), boundedly rational with Euler learning and boundedly but internally rational. We denote these three cases by RE, EL and IR respectively. In the next subsection we first consider homogeneous expectations for which all agents (households and firms) form either RE or IR or EL expectations. We then allow for the possibility that households and firms are heterogenous across these groups (but retain intra-group homogeneity). Then we allow for intra-group heterogeneity in a full Brock-Hommes NK model with IR and RE agents.

5.3.1 Stability Analysis

In the numerical results below we fix parameters at their priors used in the Bayesian estimation reported in Deak et al. (2016) apart from the adaptive learning parameter λ_x which we set at unity. As stated above we make the following information assumptions: for observations of aggregate



(a) RE

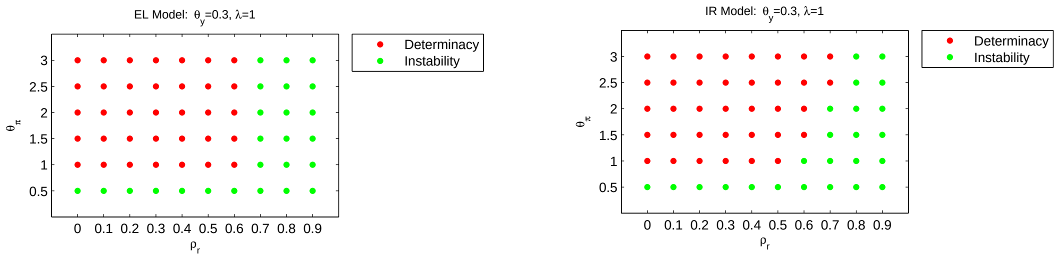
(b) EL

Figure 1: **Comparison of Stability Properties of RE and EL Models.** $\rho_r > 0$, $\lambda_x = 1$.

output and inflation $j = 1$, which is assumed in the EL approach. Later in the IR approach we need to model observations of local variables consisting of factor prices, profits and marginal costs. These we assume to be observed without a lag and therefore $j = 0$. Note that this only applies to the EL and IR agents; the RE equilibrium assumes perfect information where agents observe all current values of the state variables.

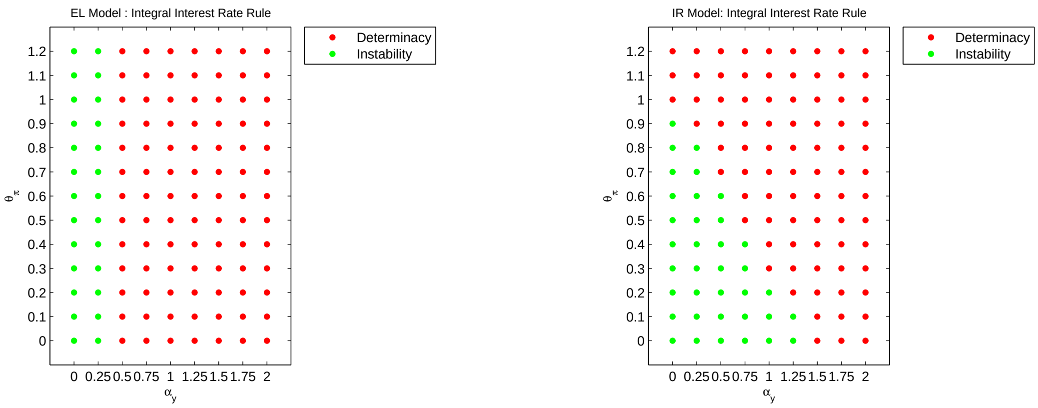
Figures 1 and 2 compare the models in (ρ_r, θ_π) space with $\theta_y = 0.3$. Figure 3 sets $\rho_r = 1$ and compares EL and IR models in (α_y, α_π) space having re-parameterized the rule as $r_{n,t} = \rho_r r_{n,t-1} + \alpha_\pi \pi_t + \alpha_y y_t$. Note that this rule reduces to a price-level rule when $\alpha_y = 0$. The differences in the sizes of the policy spaces that result in a saddle-path stable equilibrium are significant. Furthermore a clear ranking of the sizes of these spaces emerges with $RE \supset EL \supset IR$. This means that unless the policy rule is designed for the IR model, uncertainty as to which model of expectations is correct can lead to a rule that is unstable or has infinitely many equilibria (i.e. is indeterminate).

Note here that determinacy refers to a locally stable model in which a unique rational expectations consumption function exists, and a unique rational expectations pricing function exists, following Blanchard and Kahn (1980). These functions will include the proportions of non-rational households and firms - and their bounded rational expectations - as arguments in the general case. Thus we refer to models as determinate even though the expectations of a subset of the agents are state variables. This terminology, of course, would cease to make sense if there were no agents with rational expectations in the model.



(a) EL (b) IR

Figure 2: **Comparison of Stability Properties of EL and IR Models.** $\rho_r > 0$**,** $\lambda_x = 1$**.**



(a) EL (b) IR

Figure 3: **Comparison of Stability Properties of EL and IR Models.** $\rho_r = 1$**,** $\lambda_x = 1$**.**

5.3.2 Internal Rationality in a Heterogeneous New Keynesian Model

We now turn to the heterogeneous agent NK model with proportions $n_{h,t}$ and $n_{f,t}$ of RE households and firms respectively, with the remaining agents internally rational.[11] In our set-up (and that of Massaro (2013)) RE agents know the full RE-IR composite model and are therefore aware of the existence of IR agents and of the reinforcement learning process that determines the evolution of $n_{h,t}$ and $n_{f,t}$.

Figure 4 provides insights into the model's dynamic behaviour by plotting the impulse response functions for the monetary policy shock with $n_h = n_f = 0.5$, $\lambda_x = 0.04$. With the fundamental parametrization exactly the same for the RE model and the IR model, the latter produces high frequency cycles superimposed on the business cycle frequency generated by the shock. Thus, the internally rational NK model exhibits larger responses and greater volatility following small shocks.

[11] See Branch and McGough (2016) for a recent survey.

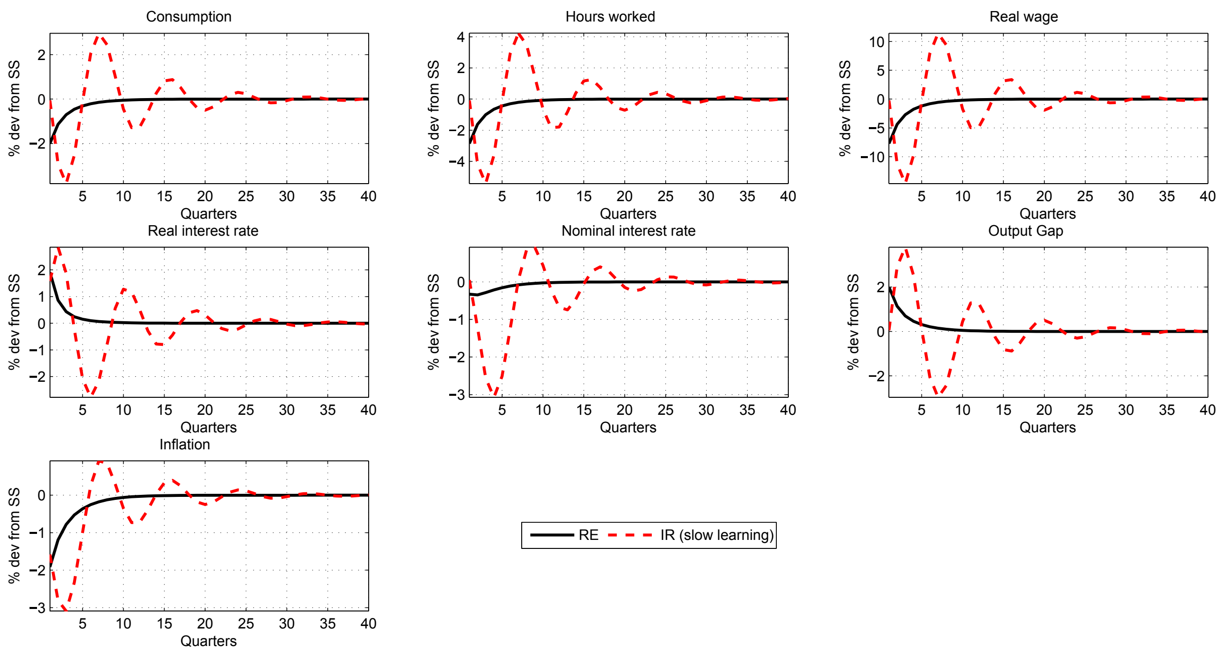


Figure 4: RE versus IR-RE Composite Expectations with $n_h = n_f = 0.5$, $\lambda_x = 0.04$; lagged observations: Monetary Policy Shock

In this sense, the stable parametrization here produces business cycles that are *more* endogenous than those generated by rational expectations NK models. These features of the model are well-supported by the data: in a Bayesian estimation of the heterogeneous agent model, Deak et al. (2016) show in a likelihood race that the IR component with heuristic rules provides a far better fit than the pure RE case.

The highly nonlinear nature of the model results in a highly asymmetric and non-normal joint distribution in the stochastic steady state. Table 1 describes the steady state distribution for a third order solution of the internally rational NK model described in section 5.3 with the following parameterisation: $\mu_h^{RE} = \mu_h^{IR} = \mu_f^{RE} = \mu_f^{IR} = 0.7$, $C_h = C_f = 0$, $\sigma_A = \sigma_{MS} = 0.01$; $\sigma_{MPS} = 0.001$, where the last three parameters are the standard deviations of the technology, mark-up, and monetary policy shocks, respectively. Aggregate consumption, hours, inflation, and interest rates exhibit extremely high kurtosis, or fat tails, and high skewness. Although the stochastic means of most endogenous variables are close to their deterministic steady state values, the stochastic means of the proportions of rational and boundedly rational agents are quite different from their deterministic steady state values. If volatility increases as a result of more volatile exogenous shocks then the stochastic mean will shift further in favour of rationality. Counter-intuitively, this raises the possibility that injecting more uncertainty into the economy by raising the standard deviation of the monetary policy shock can actually be welfare improving - although this would depend on the size of the welfare loss stemming from the cost of rationality.

	Deterministic Mean	Stochastic Mean	SD	Skewness	Kurtosis
$\frac{C_t}{C}$	1.0000	0.9912	0.0260	-2.037	15.56
$\frac{H_t}{H}$	1.0000	1.0036	0.0166	2.595	19.09
$\frac{W_t}{W}$	1.0000	0.9981	0.0204	0.560	1.951
$\frac{\Pi_t}{\Pi}$	1.0000	1.0054	0.0169	1.644	9.904
$\frac{R_{n,t}}{R_n}$	1.0000	1.0054	0.0145	1.708	10.25
$n_{h,t}$	0.5000	0.5795	0.1872	1.964	12.48
$n_{f,t}$	0.5000	0.5195	0.0290	2.169	12.09

Table 1: Third Order Solution of NK IR-RE Model

This subsection has examined the case where monetary policy induces saddle-path stability thus ruling out indeterminacy and local instability. But if rules are badly chosen the model will exhibit globally bounded dynamics with chaotic dynamics as in Brock and Hommes (1997) and Branch and McGough (2010). Whether such patterns of output and inflation are compatible with the data is an open question.

5.4 Incorporating Disequilibrium in DSGE Models

We now turn to the remaining area highlighted earlier, concerning the criticism that DSGE models fail to deal with disequilibrium. As Howitt (2012) puts it,

> "...the macroeconomic learning literature of Sargent (1999), Evans and Honkapohja (2001b) and others goes a long way towards understanding disequilibrium dynamics. But understanding how the system works goes well beyond this. For in order to achieve the kind of coordinated state that general equilibrium analysis presumes, someone has to find the right prices for the myriad of goods and services in the economy, and somehow buyers and sellers have to be matched in all these markets. More generally someone has to create, maintain and operate markets, holding buffer stocks of goods and money to accommodate other transactors' wishes when supply and demand are not in balance, providing credit to deficit units with good investment prospects, especially those who are maintaining the markets that others depend on for their daily existence, and performing all the other tasks that are needed in order for the machinery of a modern economic system to function."

In the most general sense, ACE scholars are concerned that DSGE models, and indeed most temporary equilibrium models, rely too heavily on simultaneous determination of prices and quantities within any given period. Instead, ACE models tend to have explicit time orderings of events and unidirectional causal structure within any given period, allowing them to be interpreted as disequilibrium models (see section 2.2 and footnote 1 above).

Given the above, the behavioural NK models surveyed here can incorporate this type of disequilibrium, but do not usually do so. In addition, this basic critique of temporary equilibrium is often conflated with criticisms of the more restrictive Walrasian equilibria used in early DSGE models and real business cycle models, which tends to be replaced with search and matching, buffer stocks, and imperfect competition in contemporary models. The incorporation of search and matching mechanisms started with the contributions of Mortensen and Pissarides (1994) and Pissarides (2000), and there has been significant progress in embedding Mortensen-Pissarides search-matching frictions into otherwise standard NK models. Examples include Campolmi et al. (2010), Faia et al. (2010) and Monacelli et al. (2010). Many models featuring MPSM frictions focus only on the extensive margin, but there also are examples of models (for instance Thomas (2008), Krause et al. (2008) and Cantore et al. (2014)) that model the intensive margin in addition to the unemployment rate.

Regarding output market frictions and buffer stocks of goods and services, only recently have these been incorporated into RBC or DSGE models. Examples are Khan and Thomas (2007), which provides a micro-founded theory of inventories that succeeds in reproducing stylized facts regarding inventory investment in the USA, and Den Haan (2014), which combines an inventory model with a MPSM model of labour markets. These are equilibrium models in both the Nash sense and the sense described above, but disequilibrium models in the Walrasian sense. They also assume rational expectations; combining the goods and labour market frictions in these models with bounded rationality as above is a possible route for behavioural NK models to take.

5.5 Summary

This section reviewed the behavioural New Keynesian model that incorporates Brock-Hommes complexity into the conventional New Keynesian DSGE model. Existing behavioural NK models use a form of Euler equation learning, which essentially means that households and firms forecast their own future decisions. Thus an internally rational NK model was discussed, following recent work by Deak et al. (2016), which can be considered an advance on this literature. Both the behavioural and internally rational NK models incorporate heterogeneity and bounded rationality into the standard New Keynesian framework, and in so doing generate complex dynamics.

As discussed in section 2, the DSGE literature has gone a long way in answering the criticisms of ACE and other scholars. However, we believe that the New Keynesian model discussed in this survey incorporates the insights of ACE in a fruitful manner. Unfortunately, it does not incorporate

AB	Standard DSGE	Internally Rational NK Model
Computational Solution	Computational Solution	Computational Solution
Non-optimising Agents	Dynamic Optimizing Agents	Dynamic Optimizing Agents
Heterogeneous Agents	Representative Agent	Heterogeneous Agents
Boundedly Rational Agents	Rational Expectations	Combination of RE and non-RE
Direct Interaction	Interaction through Market Prices	Interaction through Market Prices
Networks	Implied Fully Connected Networks	Implied Fully Connected Networks
Non-Market Clearing	Market Clearing	Non-Market Clearing
Disequilibrium	Equilibrium	Equilibrium
Distribution Calibration	Steady State Calibration	Systems Estimation

Table 2: Summary of Differences Between ACE, standard DSGE, and IR NK Models

local interaction and disequilibrium - the remaining criticisms of DSGE that we have reviewed in this paper. Table 2 summarises the progress that has been made in bridging the gap between ACE and DSGE with the internally rational NK model.

6 Concluding Remarks

This paper has considered the major points of contention between macro ACE and DSGE modelling, and has reviewed a literature which attempts to synthesise, and thus bridge the gap between, these two approaches. First, we considered the major criticisms directed at DSGE modelling by the ACE community, and then reviewed the main families of mature macro ACE models. We then reviewed the empirical performance of these models. We then reviewed the literature surrounding the behavioural New Keynesian model, and provided a detailed description of the internally rational New Keynesian model. This model improves the microfoundations of the behavioural New Keynesian model, whilst answering some of the criticisms directed at DSGE modelling in the ACE literature.

We hope that the synthesis and review presented in this paper will be useful for macroeconomists who are not yet familiar with macro ACE or behavioural New Keynesian models. We also hope that the research program outlined here will constitute a profitable direction for the economics profession following the perceived failure of DSGE modelling in the light of the 2008 financial crisis. Two avenues immediately suggest themselves. First, local interaction, in the shape of matching functions or replicator dynamics, could be incorporated into the internally rational NK model. Second, a similar model could be constructed using the heterogeneous agent methodology of Krusell and Smith (1998), which would bring the methodology closer to the microsimulation structure of a lot of agent-based models.

The DSGE literature reviewed in the present paper is not, of course, the only way in which ACE and DSGE models can be brought closer together. Sinitskaya and Tesfatsion (2015) work from the opposite direction by introducing forward-looking optimising agents into an ACE framework. They use an equivalent to internal rationality which they refer to as “constructive rational decision making”. This is a novel macro ACE model in having internally rational optimisers: households maximise expected intertemporal utility over an infinite time-horizon and firms do the same with their utility being taken as profit. Notably, the model is closer to a pure disequilibrium model, with the time interval divided into 6 sub-intervals, and agents adopt optimised parameterised decision rules proportional to expected market-clearing prices and then update these parameters through reinforcement learning. A third line of research, therefore, would be to explore the similarities between the internally rational NK model presented here and the Sinitskaya and Tesfatsion (2015) model.

Finally, a limitation that affects any macroeconomics that seeks to incorporate bounded rationality is the gap in our empirical knowledge with respect to the microfoundations of economic behaviour. For example, Lengnick (2013) argues that identity should be considered as part of individual decisions, and concepts such as reciprocity, fairness, and loss aversion should be incorporated into macroeconomic models. Yet, he notes that simple behavioural rules in ACE models are usually either derived from survey studies or “common-sensical reasoning”. A profitable way forward here may be a sustained effort to incorporate the results of experimental economics into macroeconomic analysis - extending work already done with robust maxmin decision rules (Hansen and Sargent (2008)), smooth ambiguity utility (Ilut and Schneider (2014), Ju and Miao (2012)), prospect theory (Kahneman and Tversky (1979), Kahneman et al. (1990), Barberis (2013) Dhami and Al-Nowaihi (2010)), and hyperbolic discounting (Harris and Laibson (2001), Krusell and Smith (2002)). This ambitious project is more in keeping with the inter-disciplinary nature of agent-based modelling and awaits future research.

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A Response to Referees

Referee 1

1. The only remaining comment I have is with regard to the explanation of the analysis of the new section 5.3.1. It is unclear what the exercise intends to show. What are the criteria for determinacy and stability that are being used here? For determinacy, are the figures showing when the RE beliefs are saddle-path stable in all cases? Or are you expanding the definition of determinacy to refer to something about a multiplicity of steady-states in the case of different types of beliefs? Because with the inclusion of any kind of adaptively formed beliefs, the notion of unique paths to steady that determinacy usually implies under rational expectations doesn't really make sense. It always depends on initial beliefs or initial conditions in the case where $\lambda_x = 1$ *as done* *here. With learning, beliefs are state variables. I think you likely mean that the RE beliefs still satisfy the standard determinacy conditions for a solution surrounding a unique steady state that takes into account the adaptive learning beliefs when* $\lambda_x = 1$. *Whether or not that outcome is stable under learning is not being addressed. But it isn't clear exactly what you want readers to take away from this section. Clarification here would be helpful to readers.*

We have now added an extra paragraph to the end of the section you refer to, which we agree was somewhat brief. As it explains, we use determinacy in the standard way described in Blanchard and Kahn (1980) - i.e. in the state space form there should be the same number of eigenvalues greater than one in modulus as forward looking variables and the same number of eigenvalues smaller than one in modulus as predetermined variables. This ensures that, in our model, the expectations terms in the state space RE consumption and pricing functions will converge and ensure unique reduced form RE consumption and pricing functions. The state space form will then be saddle path stable, and the reduced form will be locally stable. Indeed, as you point out, the RE consumption and pricing functions will have to take into account both the proportions and beliefs of non-RE households and firms, inasmuch as these (pre-determined) variables help predict aggregate output, wages, prices, etc.

Note, however, that this is the case even when $\lambda_x \neq 1$ - all λ_x governs is the speed at which non-RE households and firms update their expectations. Finally, we have noted in the added paragraph that all of the above is completely irrelevant when there are no RE households or firms, because in this case there are no forward looking variables in the state space form so determinacy (or the lack thereof) is not an issue.