

1 NETWORK COARSENING DYNAMICS IN
2 A PLASMODIAL SLIME MOULD: MODELLING
3 AND EXPERIMENTS

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11 The giant unicellular slime mould *Physarum polycephalum* forms an
12 extended network of stands (veins) that provide for an effective intracellular
13 transportation system, which coarsens in time. The network coarsening was
14 investigated numerically using an agent-based model and the results were
15 compared to experimental observations. The coarsening process of both
16 numerical and experimental networks was characterised by analyses of the
17 kinetics of coarsening, of distributions of geometric network parameters
18 (as, for instance, the lengths and widths of vein segments) and of network
19 topologies.

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21 **1. Introduction**

22 *Physarum polycephalum* is a well-studied, giant, multinucleated, single
23 amoeboid cell, which has developed into a prototypical system for inves-
24 tigating two-dimensional transportation networks. The morphology of the
25 plasmodium of *P. polycephalum* consists of an apical zone and an adaptive
26 vein network [1], through which protoplasm and nutrients are continuously
27 pumped back and forth. This peristalsis-driven phenomenon is known as
28 shuttle streaming. The adaptive vein network of *P. polycephalum* forms a
29 regular graph (in the mathematical sense) [2, 3], which is known to solve sev-
30 eral graph theoretical problems, like finding the shortest path in a maze [4],
31 constructing Steiner minimum trees [5], or even mimicking the topology of
32 road and railway networks [6–10]. The biological functionality of the vein
33 network is to provide for an effective transport of protoplasm. Recently, it

34 has been shown that the self-similar vein network is hierarchically structured
35 with respect to its transport efficiency [11]. Such a functionality demands a
36 continuous and well-organised optimisation of the vein network. To under-
37 stand the criteria according to which these networks are optimised, numerical
38 simulations have been performed using a variety of models [6–8, 12–22].

39 The proposed models of the network topology of *P. polycephalum* were
40 developed in order to address different questions. Hence, the nature of these
41 models varies. A first group of models has been developed to study the op-
42 timisation of the transportation capabilities (*i.e.*, the protoplasmic flow) of
43 the plasmodial vein networks. These models consider the networks as follow-
44 ing Kirchhoff’s rules and supporting Poiseuille flow of protoplasm. Usually,
45 a network of a preselected topology is given and the change in the weights
46 of the connectivities between nodes (mimicking the intensity of the flux be-
47 tween two nodes) is studied as some conditions are varied [6, 8, 12–15]. These
48 simulations provide networks with altered vein strengths, however, they do
49 neither consider the annihilation of veins nor any topology changes during
50 the development of the network.

51 A second group of models has been developed to investigate the synchro-
52 nisation of peristaltic pumping in a network [13, 16]. These models treat
53 the nodes (branching points of veins) as coupled oscillators and focus on
54 the nature of the synchronisation patterns obtained in the network. Again,
55 modifications of the network topology are generally not addressed by such
56 networks.

57 A third group of models has been proposed to study changes in the
58 topology of the vein networks of *P. polycephalum* [17–22]. These models are
59 either cellular automata [17], agent-based models [18, 19], or even hybrid
60 agent–cellular automaton schemes [20–22]. Numerical studies using these
61 models focused on the morphologies of developing networks [22], mimicking
62 the growth and morphology of the plasmodium either under different envi-
63 ronmental conditions [17], or in presence of multiple food sources [20–22].
64 These models have been used to simulate *P. polycephalum*’s ability to solve
65 mazes and to approximate Steiner minimum trees [20].

66 The multi-agent model introduced in Ref. [18] uses a mobile particle
67 approach to approximate the self-assembly, formation and subsequent adap-
68 tation of *P. polycephalum* transport networks. The model was introduced
69 to explore the potential role of spatially implemented material-based uncon-
70 ventional computing substrates [23–25]. The motivation for this approach
71 was inspired by the *P. polycephalum* plasmodium itself, which exhibits com-
72 plex behaviour emerging from only simple component parts and interactions
73 (and, importantly, has no special or critical components). It may thus be
74 described as a ‘bottom-up’ modelling approach. Although other modelling
75 approaches, notably cellular automata, also share these motivations and

76 properties, the direct mobile behaviour of the agent particles renders it more
77 suitable to reproduce the flux within the plasmodium. The model is notable
78 for the self-assembly of transport networks which emerge from an initially
79 random distribution of particles. These networks were found to exploit Local
80 Activation and Lateral Inhibition (LALI) dynamics, (where Lateral Inhibi-
81 tion was indirectly implemented by substrate depletion) and, subsequently,
82 reproduced a wide range of Turing-type reaction-diffusion patterns [26]. Fur-
83 thermore, these networks also exhibit physical properties such as network
84 minimisation, the formation of Plateau angles and the observation of von
85 Neumann's law [27].

86 In contrast to the first two groups of models, the cellular automata and
87 agent-based models not only consider the formation of novel veins but also
88 consider their subsequent morphological adaptation and annihilation. An-
89 nihilation of veins, in fact, is a hallmark of the coarsening of networks. In
90 contrast to flow optimisation, the coarsening of the vein networks of *P. poly-*
91 *cephalum* has, so far, received much less attention in simulation studies.
92 When monitoring a fixed area of the network, coarsening is observed when
93 the plasmodium propagates. It coarsens continuously until, eventually, the
94 slime mould leaves the monitored domain altogether. During coarsening
95 several morphological parameters of the vein network change, for instance,
96 the density of veins, the number of nodes (*i.e.*, branching points of veins)
97 and the mean length of the vein segments. Therefore, one should require
98 that any model for the coarsening dynamics reproduces three main features,
99 namely (*i*) the changes in the geometry of veins, (*ii*) the annihilation and
100 disappearance of nodes, and (*iii*), in the long run, the network should dis-
101 appear completely or collapse to a single point.

102 In the present paper, we investigate the evolution of *P. polycephalum* vein
103 networks and focus on the coarsening dynamics of originally dense networks.
104 We consider the morphology of the network, the distributions of typical net-
105 work parameters, and how these properties change during the coarsening of
106 the network. We perform numerical simulations using the multi-agent based
107 model, recently proposed by Jones [18], and compare the numerical results
108 to findings obtained from experiments. To this purpose, we first briefly in-
109 troduce the multi-agent based model [18]. Next, we present the materials
110 and methods used in the experiments and to perform the network graph
111 analysis. The subsequent section reports on the results on the coarsening
112 processes in both the simulated and experimental networks, respectively.
113 Finally, we discuss the obtained results.

114 2. Multi-agent model of the *P. polycephalum* plasmodium

115 The multi-agent model of *P. polycephalum* uses a population of coupled
 116 mobile particles with very simple behaviours within a diffusive lattice [18].
 117 The lattice stores particle positions and the concentration of a local diffusive
 118 factor referred to generically as chemoattractant.

119 The function of this chemoattractant is to reproduce the sol flux within
 120 the plasmodium. Particles deposit the chemoattractant factor when they
 121 move and also sense the local concentration of the factor during the sensory
 122 stage of the particle algorithm. The particles are thus indirectly coupled by
 123 the diffusive factor. This is a simple approximation of the changing composi-
 124 tion of the *P. polycephalum* plasmodium whereby collective particle positions
 125 represent the global *structure* of the material (gel phase), and collective par-
 126 ticle movement represents the flux within the plasmodium (sol phase).

127 In this article, the particles reside within a circular virtual ‘Petri dish’
 128 inside a lattice 400×400 pixels in size. The initial population size was com-
 129 posed of 25 000 particles, initialised at random positions and with random
 130 orientations.

131 2.1. Generation of model plasmodium cohesion 132 and morphological adaptation

133 The behaviour of each particle occurs in two distinct stages, the sensory
 134 stage and the motor stage. In the sensory stage, the particles sample their
 135 local environment using three forward biased sensors whose angle from the
 136 forward position (the sensor angle parameter, SA), and distance (sensor
 137 offset, SO) may be parametrically adjusted (Fig. 1 (a)). The sampling area
 138 A is thus given as

$$A = \frac{SA}{360^\circ} (SO)^2 \pi. \quad (1)$$

139 The offset sensors generate local indirect coupling of sensory inputs and
 140 movement to generate the cohesion of the material. The SO parameter acts
 141 as a scaling parameter and distance is measured in pixels. A minimum dis-
 142 tance of 3 pixels is required for coupling to occur and coupling strength
 143 increases with SO. For the experiments in this article, we fixed the values of
 144 SA and RA to 67.5° and varied the values of SO. During the sensory stage,
 145 each particle changes its orientation to rotate (via the parameter rotation
 146 angle, RA) towards the strongest local source of chemoattractant (for ex-
 147 ample, rotating to the right in Fig. 1 (b)). Variations in both SA and RA
 148 parameters have been shown to generate a wide range of reaction-diffusion
 149 patterns [26] and for these experiments, we concentrate on a particular range
 150 of SA and RA parameters which have been shown to generate network as-
 151 sembly and adaptation [27].

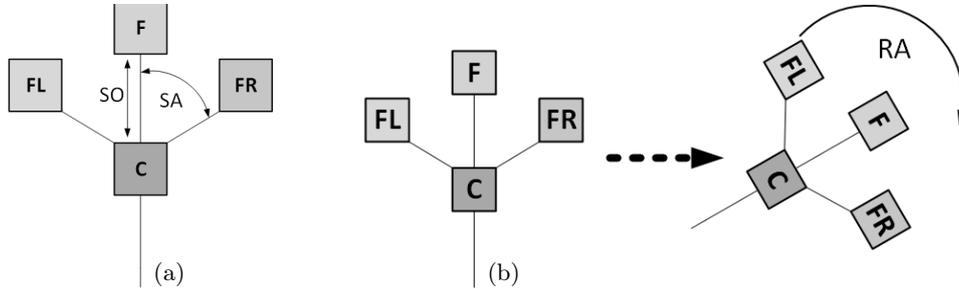


Fig. 1. Architecture of a single particle detailing the three sensory parameters. (a) Morphology showing agent position ‘C’ and offset sensor positions (FL, F, FR, that stand for forward left, forward, and forward right, respectively) and the SO and SA parameters, (b) Effect of the RA parameter of agent orientation.

152 After the sensory stage, each particle executes the motor stage and at-
 153 tempts to move forwards in its current orientation (an angle from 0–360°)
 154 by a single pixel. Each lattice site may only store a single particle and par-
 155 ticles deposit chemoattractant into the lattice (5 arbitrary units per step)
 156 only in the event of a successful forwards movement. If the next chosen site
 157 is already occupied by another particle, move is abandoned and the parti-
 158 cle selects a new randomly chosen direction. Selection of a new direction
 159 in response to obstruction prevents the build-up of momentum within the
 160 particle population. This ensures fluid network adaptation but prevents the
 161 accumulation of different regions of flux within the population, and so the
 162 emergence of oscillatory movement is not generated. This can be achieved
 163 by removing the condition of changing direction, causing oscillatory domains
 164 to emerge and grow [28], however this is outside the scope of this article.

165 Diffusion of the attractant left by particle movement in the lattice was
 166 implemented at each scheduler step and at every site in the lattice in parallel
 167 via a simple mean filter of kernel size 3×3 . Damping of the diffusion distance,
 168 which limits the distance of chemoattractant gradient diffusion, was achieved
 169 by multiplying the mean kernel value by 0.9 per scheduler step.

170 2.2. Adaptation of model plasmodium population size

171 Adaptation of the population size was implemented via tests at regular
 172 intervals. The frequency at which the growth and shrinkage of the population
 173 was executed determines the turnover rate for the population. The frequency
 174 of testing for growth was given by the G_f parameter and the frequency for
 175 testing for shrinkage is given by the S_f parameter. Both G_f and S_f were
 176 set to 5. Growth of the population was implemented as follows: If there
 177 were between G_{\min} (0) and G_{\max} (10) particles in a local neighbourhood

178 (window size given by G_w , in this case 9×9) of a particle, and the particle
179 had moved forward successfully, a new particle was created if there was a
180 space available at a randomly selected empty location in the immediate 3×3
181 neighbourhood surrounding the particle.

182 Shrinkage of the population was implemented as follows: If there were
183 between S_{\min} (0) and S_{\max} (22) particles in a local neighbourhood (window
184 size given by S_w , in this case 5×5) of a particle the particle survived, other-
185 wise it was deleted. Deletion of a particle left a vacant space at this location
186 which was filled by nearby particles (due to the emergent cohesion effects),
187 thus causing the population to shrink slightly. As the process continues, the
188 model plasmodium continues to adapt its shape and shrink further.

189 The model runs within a multi-agent framework running on a Windows 7
190 PC system. The particles act independently and iteration of the particle
191 population is performed randomly to avoid any artifacts from sequential
192 ordering.

193 3. Material and methods

194 The dehydrated form of *P. polycephalum* strain HU195×HU200, the sclero-
195 rotia, was stored up to 24 months. Sclerotia were placed on a 1.0% w/v
196 (weight per volume) plain, non-nutritive agar gel (Difco BactoAgar) in a
197 polystyrene box (size: $18 \times 25 \times 35$ cm³) at a constant temperature of 21°C
198 in the dark. The sclerotia germinated and transformed into plasmodia which
199 expanded over the agar matrix. During growth, oat flakes (Kölln Flocken)
200 were used to feed the plasmodium, in order to increase the plasmodial mass.

201 An area of about 1 cm × 4 cm of the frontal zone of the expanding
202 plasmodium was carefully cut off, and transferred into the centre of a square
203 polystyrene Petri dish of 12 cm diameter, which contained 1.0% w/v plain,
204 non-nutritive agar gel (Difco BactoAgar). After several hours, a network
205 of tubular strands (veins) developed, that coarsened as the plasmodium
206 propagated forwards. From this network, the evolution of a region of interest
207 was observed over time. The coarsening process was monitored with a CCD
208 Camera (Hamamatsu C3077) at a resolution of 768×576 px, where 1 px =
209 0.0456 mm (*i.e.* an area of 3.5 cm × 2.6 cm). Images were acquired at a
210 frame rate of 1/6 Hz and stored in a computer for later analysis.

211 The experimental and simulated networks were extracted from the stored
212 or calculated images, respectively, and subsequently analysed according to
213 the methods described in references [29, 30].

214 4. Results

215 A typical network coarsening, as produced by the model, is depicted
216 in Fig. 2. After the initialisation of the model, the network is formed
217 (Fig. 2 (a)), and subsequently it begins to coarsen (Fig. 2 (b)–(d)). During

218 coarsening the number of veins is reduced, and the area of the network
 219 decreases, as indicated by the red circle which encloses the vein network.
 220 During the entire coarsening process, none of the nodes or edges remain in
 221 their position.

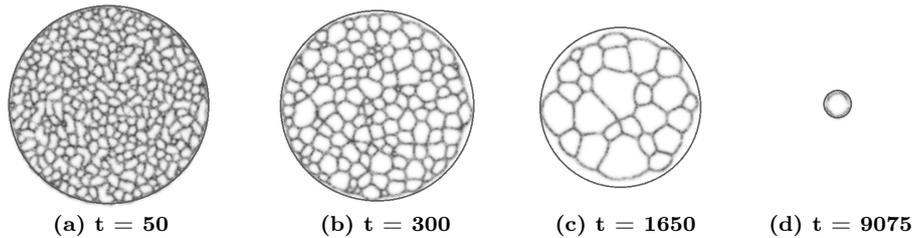


Fig. 2. Coarsening in the model. $SO = 5$, $RA = SA = 67.5^\circ$. The red circle enclosing the network indicates the circular shape of the network. This allows the measurement of the network diameter and thus the area, where the vein network is embedded. (a) At 50 time steps (time units, t.u.), a dense network has formed. (b) At 300 t.u., the number of veins has decreased. (c) At 1650 t.u., the vein network has almost lost its circularity. (d) At 9075 t.u., the vein network has vanished, due to the coarsening.

222 The coarsening of an experimentally observed vein network is shown in
 223 Fig. 3. At the beginning of the experiment, the plasmodium propagates over
 224 the agar. When it completely covers the observed area (Fig. 3 (a)), the vein
 225 network is very dense. As the plasmodium propagates, it keeps its mass
 226 (as the plasmodium migrates over a non-nutrient gel). This leads to the
 227 coarsening of the vein network (Fig. 3 (b), (c)). The coarsening is monitored
 228 until the plasmodium has moved out of the observation area.

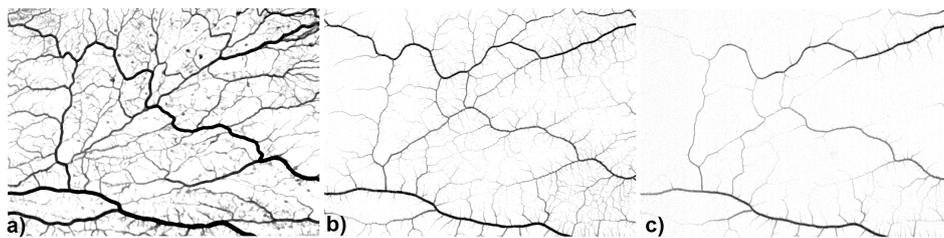


Fig. 3. Coarsening of a *P. polycephalum* vein network. The plasmodium propagates from left to right. (a) 16 h after beginning the experiment, the dense vein network is found in the observation area. (b) 17.5 h. The vein network begins to coarsen. (c) 19.5 h. Further coarsening of the vein network. After 20.0 h (not shown), the vein network has almost vanished.

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4.1. Morphology of coarsening networks

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At the beginning of the simulation, the model network covers the maximum area as the particles are distributed over the entire area of interest A_{net} . The densely and randomly distributed particles form a dense and extended network of veins. With time, this network coarsens and the area A_{net} is covered by the network shrinks. These phenomena are associated with a continuous decrease in the number of veins, a situation that is also observed in the experiments. A notable difference between the morphologies of the experimentally observed networks and the model networks is found for the widths of the strands: whether the widths of the veins are log-normally distributed in the real *P. polycephalum* networks [2], the width of the stands is uniform and invariant in the model networks.

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The log-normal distribution of vein widths observed in the experiments contributes to the generation of *P. polycephalum* networks that are hierarchically and self-similarly organised with respect to their transport efficiency [11]. Hence, these networks are optimised to provide for an efficient transport of protoplasm. Similar network structures are not found in the model.

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Another morphological aspect studied is the type of graph that is realised by the model and experiment. It was recently reported that the plasmodial vein networks of *P. polycephalum* form regular graphs with the unique node degree $k = 3$ [2, 3]. In the model, by contrast, nodes of degree $k = 3$ predominate, however, during the contraction of the lacunar areas delimited by the veins, nodes of higher degree (up to $k = 5$) can also be found. Hence, model networks possess node degree distributions, and therefore they do not form regular graphs as the real networks do.

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4.2. Network area coverage

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We define the area of the smallest circle that covers the entire network as the network area A_{net} , and the number of all pixels belonging to veins of the network as vein area A_{v} . The network coverage ρ

$$\rho = \frac{A_{\text{v}}}{A_{\text{net}}} \quad (2)$$

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describes the density of veins in the network area, and it is given by the ratio of the vein area to the network area. At the beginning of the simulation, the network coverage $A_{\text{v}}/A_{\text{net}}$ is large, as almost 70% of the space is covered by veins or cell mass, thus yielding $\rho \approx 0.7$. With time the network coverage decreases until any branches of veins have disappeared and the shape of the last remaining vein has become circular. During this process, the area coverage converges to $\rho \approx 0.20$. Once the network consists only of a single

266 circular vein, coarsening continue further and the network density ρ increases
 267 again, as the circle shrinks to a point, such that $\rho \rightarrow 1$ in the long term
 268 (Fig. 4).

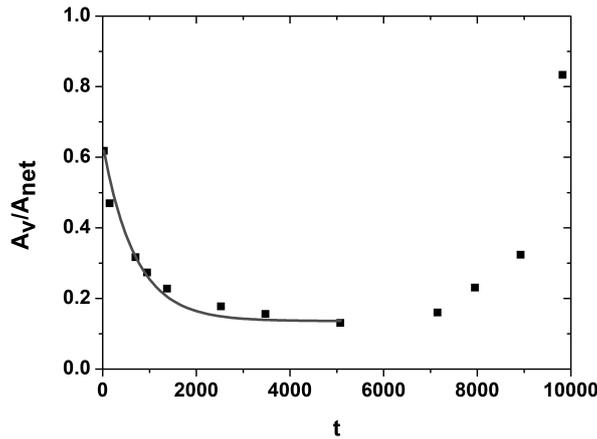


Fig. 4. Temporal evolution of the network coverage $\rho = A_v/A_{\text{net}}$ for a model network with $\text{SO} = 5$ and $\text{RA} = \text{SA} = 67.5^\circ$. The evolution is determined by two processes: initially, the coarsening process, where ρ decays exponentially until it reaches a minimum at $t_c = 6850$ t.u. (time units). Thereafter, the evolution is given by the shrinkage of the remaining circular vein to a single spot. This leads to an increase of ρ .

269 The evolution of the network coverage ρ is governed by two processes,
 270 namely the coarsening of the network and, in the last stage of the coarsen-
 271 ing, the subsequent collapse of a circular vein (Fig. 2 (d)) to a single point.
 272 These processes are reflected in Fig. 4, where the network coverage ρ at first
 273 decreases exponentially with time

$$\rho = \rho_0 e^{-\kappa t}, \tag{3}$$

274 until it reaches a minimum. In Eq. (3), κ is the decay constant. The time
 275 required to reach this minimum is the coarsening time t_c that is defined as
 276 the instant where all branching points of the network have been annihilated.
 277 In Fig. 4, which was obtained using a sensor offset $\text{SO} = 5$, the coarsening
 278 time was $t_c = 6850$ t.u., where t.u. stands for time units (or time steps).
 279 The second process is the collapse of a circular vein to a single spot, and it
 280 occurs at $t > t_c$. This process is associated with an increase in ρ .

281 In experiments, a similar shrinkage of the network coverage was also
 282 reported [2]. Initially, the network coverage was high, however, its value
 283 decreased as the network coarsened and finally settled to an asymptotic
 284 value of $\rho \approx 0.20$.

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4.3. Coarsening time

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The dependence of the coarsening time t_c on the range of the sensor offset was determined from networks generated with various values of SO. Figure 5 shows that t_c shortens with increasing SO. An analysis reveals that the coarsening time t_c is proportional to $1/\text{SO}^2$, according to

$$t_c = \gamma \frac{1}{\text{SO}^2} + t_c(0), \quad (4)$$

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as demonstrated in the inset of Fig. 5. γ is the coarsening constant, which is determined as $\gamma = 140352 \text{ t.u.} \times \text{px}^{-2}$, and the offset $t_c(0) = 1422 \text{ t.u.}$ It is worth noticing that the dimension of SO is that of a reciprocal diffusion constant D .

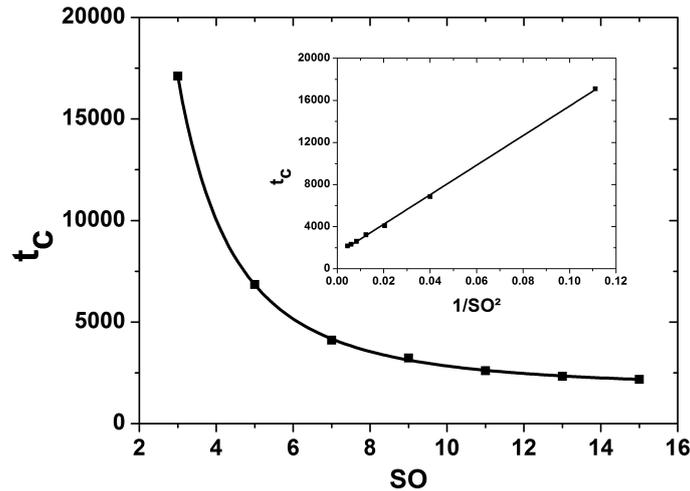


Fig. 5. Dependence of the coarsening time t_c on the sensor offset SO. A plot of t_c as a function of γ/SO^2 is shown in the inset. This correlation is linear (see Eq. (4)) with the slope γ as the coarsening constant obtained as $\gamma = 140352 \text{ t.u.} \times \text{px}^{-2}$ and the offset $t_c(0) = 1422 \text{ t.u.}$

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The coarsening time t_c depends on the sampling area A which is probed by each particle during the sensory stage. This can be shown by substituting Eq. (1) into Eq. (4), such that

$$t_c = \gamma \left(\pi \frac{\text{SA}}{360^\circ} \right) \frac{1}{A} \quad (5)$$

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which states that the coarsening time decreases (increases) as the sampling area A is increased (decreased).

299 Hence, the area of the sensing domain may be interpreted as a measure
 300 for the “range” of influence of nearby flux on an individual particle (relating
 301 to the position and distance of nearby veins with greater flux). The more
 302 extended the sampling area A is, the faster any merging of veins may occur,
 303 thus leading to a faster coarsening process.

304 4.4. Number of veins

305 As the network coarsens, the number of veins in the network diminishes.
 306 However, the coarsening takes place at different time scales in experiment
 307 and simulations. To enable the comparison of kinetic data obtained from
 308 both experiments and simulations, we introduce the normalised time t/t_{\max} .
 309 Here t_{\max} is the time at which the network had either collapsed to a single
 310 point or completely disappeared from the region of observation.

311 In model networks, the number N of veins decreases following the bi-
 312 exponential function (Fig. 6)

$$N = N_1 e^{-\alpha_1 t} + N_2 e^{-\alpha_2 t}, \quad (6)$$

313 due to coarsening. $N_1 + N_2 = N_0$ is the number of veins at the beginning
 314 of the simulation (*i.e.*, at time $t = 0$). In other words, in the model, the
 315 coarsening takes place at two time scales that are characterised by the decay
 316 rate constants α_1 and α_2 . The fast decay rate constant α_1 is associated
 317 with the rearrangement of the densely distributed particles to form veins.
 318 This process is fast and leads to a drop in the network density ρ . Once
 319 the first veins are formed, the network coarsens at a slower rate, which is
 320 dependent on the rate constant α_2 . This means that in Fig. 6, the fast
 321 process (associated to α_1) lasts until $t/t_{\max} \approx 0.06$, and the coarsening
 322 process of veins which is associated with the decay rate constant α_2 becomes
 323 dominant at $t/t_{\max} > 0.06$.

324 The coarsening of real vein networks of *P. polycephalum* follows different
 325 kinetics than that of the model networks. The annihilation of veins was
 326 found to decrease mono-exponentially in time, as described by

$$N = N_0 e^{-\alpha t}, \quad (7)$$

327 suggesting that reduction in the number of veins follows a single process.
 328 The kinetics of this process is characterised by the decay rate constant α .
 329 The physical process accounted by the (mono-exponential) decay constant α
 330 resembles that described by the (bi-exponential) decay constant α_2 in coars-
 331 ening model networks.

332 The decreases in the number of veins during coarsening in both model
 333 and real vein networks are plotted in Fig. 6. Here, a normalised number of
 334 veins N/N_0 and a normalised time t/t_{\max} were used to allow for a convenient
 335 comparison of the behaviours of the model and experimental networks.

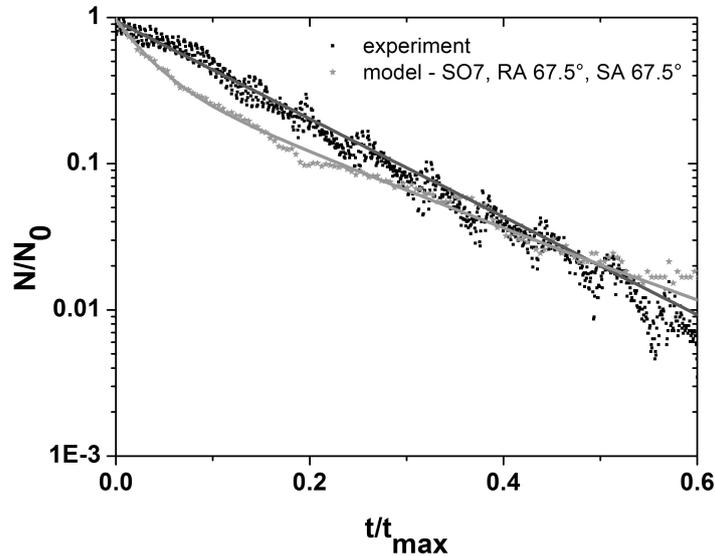


Fig. 6. Evolution of the number of veins in a coarsening network. The decay of the normalised number of veins N/N_0 in dependence of the normalised time t/t_{\max} . The coarsening of the model network (red stars) with $SO = 7$ is best fitted by a bi-exponential function (light grey/red line, with the decay constants $\alpha_1 = -31.25 \text{ t.u.}^{-1}$ and $\alpha_2 = -6.25 \text{ t.u.}^{-1}$). By contrast, the experimental network (black squares) presents a mono-exponential coarsening dynamics (grey/blue/line: mono-exponential fit, with $\alpha = -7.69 \text{ s}^{-1}$).

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4.5. Evolution of the mean length of veins

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The lengths of the veins in the network are distributed log-normally, in both, the model networks (Fig. 7) and in the real networks [2]. This functional form remains constant during the entire coarsening process, only the parameter values change in time. In numerically generated networks, the log-normal function fits to the length distribution of veins to a good agreement as long as SO is kept small (*i.e.* $SO \leq 7$). With increasing SO values the peak of the function becomes sharper, increasingly deviating from the typical log-normal distribution.

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With time, the mean length $\langle l \rangle$ of the veins increases almost linearly in both model and real networks (Fig. 8). This can be explained by the removal of nodes from the network, which leads to both a reduction in the number of veins and an increase in their lengths. *P. polycephalum* continuously optimises its plasmodial vein network, resulting in the annihilation of several nodes of the vein network, such that mean vein length $\langle l \rangle$ increases. In the model, lacunar areas between the veins shrink and nodes are continuously

352 removed, causing an increase in the lengths of the adjacent veins. As the
 353 network coarsens, these processes lead to a decrease of the number N of
 354 veins and to an increase of the mean vein length $\langle l \rangle$.

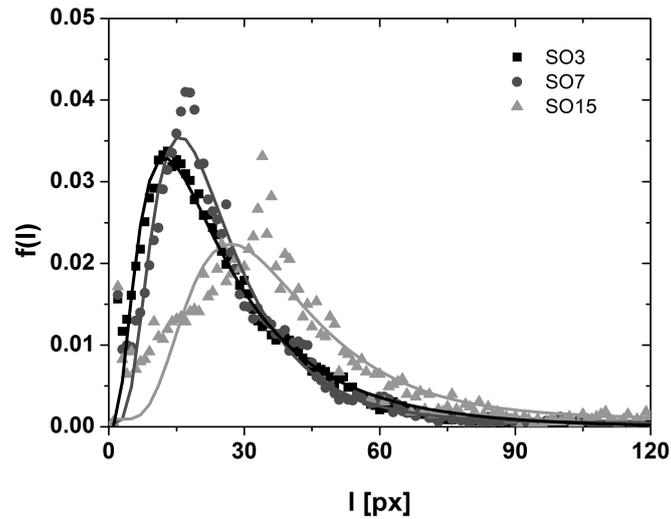


Fig. 7. Distribution of lengths of veins in simulations for $SO = 3$ (squares), $SO = 7$ (circles), and $SO = 15$ (triangles). The values of $RA = 67.5^\circ$ and $SA = 67.5^\circ$ were held constant. Log-normal distributions were fitted to the data (lines).

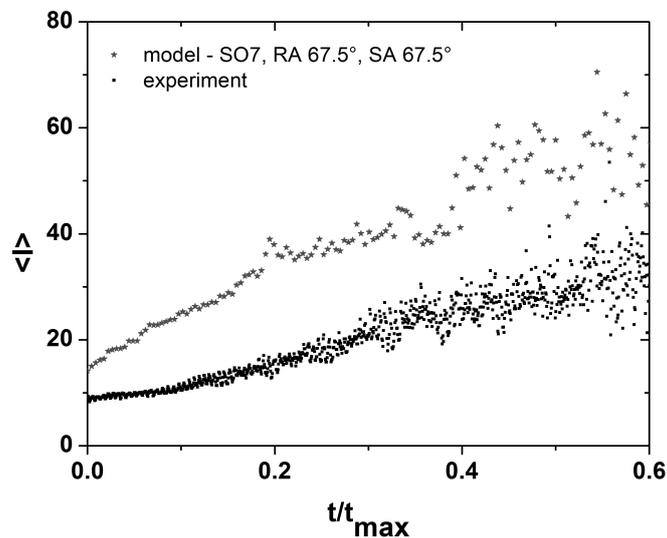


Fig. 8. Evolution of the mean vein length $\langle l \rangle$ as a function of the normalised time t/t_{\max} . Red stars show the data obtained from model networks, whereas black squares represent data obtained from experiments.

355 The mean length $\langle l \rangle$ and the number of veins N were found to be corre-
 356 lated through the power law

$$\langle l \rangle = \eta N^\beta, \quad (8)$$

357 as revealed by Fig. 9. The exponents β obtained from the coarsening model
 358 and experimental networks were $\beta = -0.41$ and $\beta = -0.35$, respectively,
 359 suggesting a similar, but not identical coarsening dynamics.

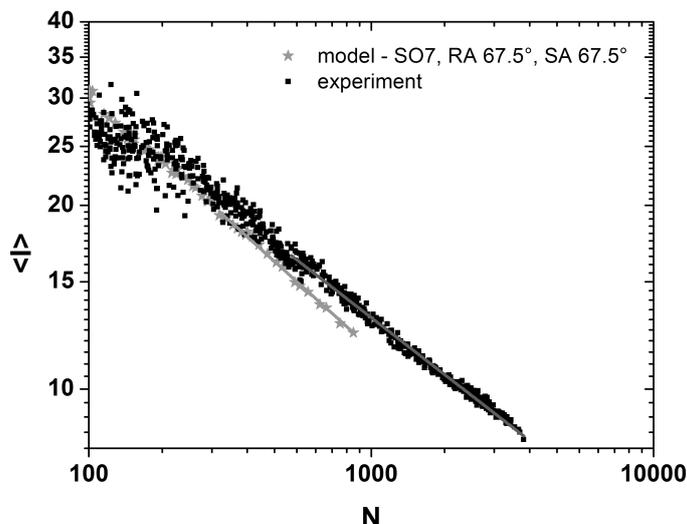


Fig. 9. Correlation between the number of veins N and the mean vein length $\langle l \rangle$. Red stars and lines represent the model data and the corresponding fit of Eq. (8) to the data, respectively, whereas black squares and the light grey/blue line represent the experimental data and the corresponding fit of Eq. (8) to the data, respectively.

360 4.6. Mean width of the veins

361 The mean width $\langle w \rangle$ of veins remains constant in time in both the model
 362 and the experiment (Fig. 10). However, the mechanisms leading to a con-
 363 stant mean vein width $\langle w \rangle$ are different in the model and experimen-
 364 tal networks. In the model, the width of veins is determined by the values of SA,
 365 RA and SO. Once these values are set, they remain fixed during the entire
 366 simulation, and so does the mean width $\langle w \rangle$ of the veins. This contrasts with
 367 the situation encountered in the experiments where the widths of the veins
 368 are distributed log-normally [2, 3] at all stages of the coarsening process.
 369 Interestingly, however, the mean width $\langle w \rangle$ of veins in the experiment also
 370 remains nearly constant during the coarsening process, since the log-normal
 371 distributions of the vein widths are narrow and the mean of the distribution
 372 always settles at a small value of w .

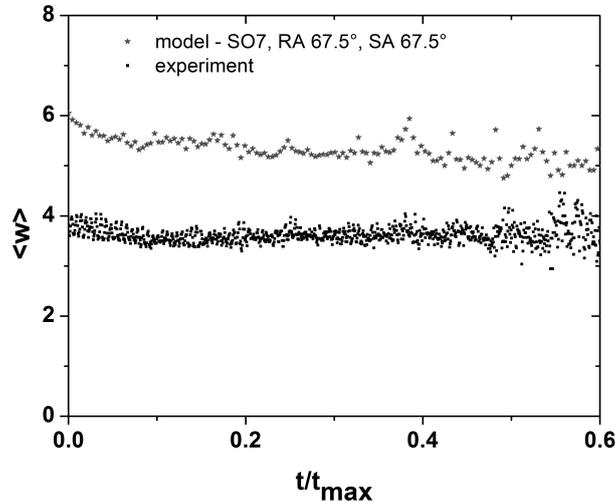


Fig. 10. Temporal evolution of the mean vein width $\langle w \rangle$ in model (red stars) and experimental (black squares) networks. Interestingly, $\langle w \rangle$ is constant in both cases.

373

5. Discussion

374 Coarsening is a process that was observed in studies of *P. polycephalum*
 375 vein networks, which are optimised with respect to the transport efficiency
 376 of the protoplasm and nutrients transporting veins. Whereas the coarsen-
 377 ing process and its dynamics have been characterised in a series of stud-
 378 ies [2, 22, 29, 30], simulation studies of this process are scant. In fact, this
 379 has been attempted by Gunji and coworkers, who have presented a model
 380 (the vacant particle-shrinkage model) that accounts for the coarsening of an
 381 initially very dense to a sparse network that connects nutrient sources de-
 382 posited on the arena (Petri dish) [22]. This setting reproduces experiments
 383 as reported in Ref. [31]. Gunji *et al.* have also compared the coarsening dy-
 384 namics obtained in numerical simulations to that of laboratory experiments
 385 by studying the temporal evolution of some network parameters, for instance
 386 the network area, the number of loops, and area closure of the experimental
 387 and simulated networks [22].

388 In the present article, we have performed a detailed examination of the
 389 coarsening dynamics as presented by a frequently used agent-based model
 390 for *P. polycephalum* network [18, 19]. Results of the numerical studies were
 391 compared to those obtained from experiments. We found that the model
 392 reproduces a series of features seen in the coarsening of *P. polycephalum*
 393 to good agreement, while some discrepancies remain. Good agreement was
 394 achieved for the distribution of the lengths of the veins in the network,
 395 that were found to obey log-normal distributions in both, experiments and

396 numerical simulations. Furthermore, a good agreement was also observed
397 in the evolution of the mean vein length $\langle l \rangle$, which was found to correlate
398 to the number of veins N in the network by a power-law function in both,
399 experiments and simulations. The values of the exponents β were quite
400 similar as well.

401 Another point where numerical and real networks behave similarly is the
402 development of the network coverage ρ . Both experiments and simulations
403 reveal that the network coverage decreases in time until it approaches a
404 value of ≈ 0.2 in both experimental and numerical networks. However, in
405 experiments this value of $\rho \approx 0.2$ is asymptotic, whereas in the numeri-
406 cal simulations the area coverage ρ increases again as soon as all branching
407 points have been removed from the network, and only a single shrinking
408 circular vein remains. This difference can be explained by different problem
409 settings studied in the experiments at the one hand, and in the numerical
410 studies at the other. The experiments were designed to elucidate the dynam-
411 ics of a freely migrating giant plasmodium on a nutrient-free gel substrate.
412 Ultimately, the scarce, propagating network leaves the region of observation.
413 On the other hand, the agent-based model was originally designed to repro-
414 duce a scenario where a dense matrix of protoplasm is spread on a substrate
415 that contains a few nutrient sources. In such a situation, the plasmodium
416 does not migrate. In the long term, a plasmodium located in a nutrient- and
417 stimulus-free setting (as studied in this paper) rather contracts to a single
418 spot.

419 These different settings lead to some disparities in the coarsening of ex-
420 perimental and numerical networks. The most pronounced difference lies in
421 the kinetics of the number of veins N in the network: whereas N decays
422 mono-exponentially in the experiments, the decay of N in numerical net-
423 works is bi-exponential. In the experiments, one considers the evolution of
424 the number of veins in the network area. That is, the formation of veins in
425 the transition zone between the apical and the network zones of the plas-
426 modium [1] are not taken into account. By contrast, the initial condition
427 used in numerical simulations corresponds a plasmodium that is entirely
428 and densely covered by tiny veins, as it is the case of the transition zone.
429 Therefore, the simulated networks account for two processes, namely the
430 formation of the veins and their fate in a coarsening network. Following this
431 reasoning, the kinetics observed in the experimental networks corresponds
432 to the network decay described by α_2 (Eq. (6)) in the simulated networks.

433 One of the factors determining the kinetics of coarsening in the simulated
434 networks is the area of the domain A that is sensed by any agent. In fact,
435 the coarsening occurs faster as the size of the sensing domain A (and hence
436 the value of the sensor offset parameter SO) increases. This suggests that
437 the rate of network coarsening augments with the area from which any agent

438 (*i.e.*, any position in the network) draws information about its environment.
 439 This further suggests, that an agent approaches a more efficient vein in a
 440 more directed way as the sampling range A increases.

441 In conclusion, the present study has provided insights in the coarsen-
 442 ing dynamics of both the plasmodial vein network of *P. polycephalum*, and
 443 networks produced by the multi-agent model proposed in Ref. [18]. Even
 444 though the modelling approach was developed for other purposes than the
 445 study of the contemplative migration of a plasmodial vein network, the net-
 446 work coarsening in experiments and numerical simulations show remarkable
 447 similarities. Nevertheless, the mechanistic origins leading to the remaining
 448 differences between experiments and numerical simulations constitute an in-
 449 teresting challenge for further studies.

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