Pseudo-Goodwin Cycles in a Minsky Model

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Abstract: Goodwin cycles result from the dynamic interaction between a profit-led demand regime and a reserve army effect in income distribution. The paper proposes the concept of a pseudo-Goodwin cycle. We define this as a counter-clockwise movement in output and wage share space which is not generated by the usual Goodwin mechanism. In particular, it does not depend on a profit-led demand regime. To illustrate this, a simple Minsky-type model is extended by a reserve army distribution adjustment. In this model the cycle is generated by the interaction of financial fragility and demand. The wage share rises at higher levels of output but this generates no feedback so that, by design, demand does not react to changes in income distribution. But the model does exhibit a pseudo-Goodwin cycle in the output-wage share space. This holds true even if we introduce a wage-led demand regime. This demonstrates that the existence of a counter-clockwise movement of output and the wage share cannot be regarded as proof of the existence of a Goodwin cycle and a profit-led demand regime.

Keywords: business cycles, Goodwin cycle, Minsky cycle, financial fragility, distribution cycles, Post Keynesian economics

JEL classifications: E11, E12, E32

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Introduction

The question of how aggregate demand and income distribution interact to generate business cycles continues to generate considerable debate in the heterodox economics literature. One influential view is derived from Goodwin's (1967) model, in which the cycle is generated by the interaction between a profit-led demand function and a reserve army distribution function. The Goodwin cycle posits that an increase in the wage share will have a negative effect on demand because investment is driven by the profit share. At the same time, higher levels of output will result in a rising wage share due to the increased bargaining power of labour as unemployment falls. The interaction of these two elements generates a counter-clockwise cycle in output-wage share space.

There is a related, ongoing, debate about the nature of the demand regime in advanced economies. On one side are those who regard the demand regime as profit-led, in line with the assumptions of the Goodwin model (e.g. Taylor, 2012). On the other are those who argue that aggregate demand is wage-led (e.g. Stockhammer et al., 2009). These two strands of the literature intersect in studies which look for Goodwin-like patterns in empirical data and argue that such patterns provide support for the profit-led demand hypothesis: "A general finding is that 'profit squeeze' cycles exist for the US economy." (Barbosa-Filho & Taylor, 2006, p. 392)

We take issue with this last claim, that the existence of a counter-clockwise movement in output-wage share space is proof of the existence of a Goodwin cycle and a profit-led demand regime. Instead, we demonstrate the existence of a pseudo-Goodwin cycle which we define as a counter-clockwise movement in output and wage share space that is not generated by the standard Goodwin mechanism. Specifically, it does not rely on a profit-led demand regime - or indeed on any feedback from distribution to demand. Simply put, a pseudo-Goodwin cycle is something that looks like a Goodwin cycle but isn't.

To illustrate this we take a simple two equation predator-prey version of the Minsky model and add a distribution adjustment a la Marx and Goodwin. In Minsky's model, the cycle results from an interaction between financial and real variables. In its simplest version, the theory states that growing demand will give rise to higher debt ratios and that higher fragility will have a negative effect on demand. This will generate clockwise movements in output-fragility space.

In the Minsky model with a reserve army effect, the wage share rises with higher levels of output. This model exhibits a pseudo-Goodwin cycle in output-wage share space. However this cycle is not a Goodwin cycle because, by design, there is no feedback from distribution to demand. Instead, the cycle is generated entirely by the interaction between financial fragility and demand. Further, we demonstrate that such pseudo-Goodwin cycles can still arise if we additionally assume a wage-led demand regime.

There have been some attempts at a synthesis of Goodwin and Minsky models (Keen 1995). These models have a feedback from distribution, as well as from finance, on aggregate demand. Our paper, however, is asking a different question. We are interested in a model that has a Minsky cycle and a reserve army effect, but *no feedback from distribution to demand*. We know, by design, that the business cycle in this model will be driven by the finance-demand interaction. What the literature has not so far analysed is the question of what cyclical properties such a system will exhibit in output-distribution space. In other words, we are asking what a researcher looking for a Goodwin cycle would see, if he encountered a Minsky world with a reserve army distribution function.

The purpose of the paper is one of theoretical clarification, not of realism. The models presented are thus of the reduced-form variety. We are also not interested in determining the conditions under which stable cycles (closed orbits) – as opposed to damped or explosive oscillations – are generated. The paper therefore uses simple predator-prey models which generate stable cycles and, for the sake of clarity, we keep the number of parameters used to a minimum.

The paper is structured as follows. Section 2 presents a benchmark Goodwin model and discusses the related literature. Section 3 presents a simple Minsky model. Section 4 introduces a distribution function with a reserve army effect into the Minsky model and demonstrates the existence of pseudo-Goodwin cycles. Section 5 introduces a weak wage-led effect in the demand function. Section 6 concludes.

The Goodwin cycle

Goodwin (1967, 1972) presents a simple dynamic model of the cyclical interaction of the growth rates of employment and income distribution. Two key Marxian relationships generate the results of the model. The first is the relationship between the growth of the wage rate and level of unemployment. This relies on the reserve army assumption that as unemployment increases the bargaining power of labour is diminished, leading to a fall in the wage rate. This implies a relationship between output and the wage share such that higher levels of output will be associated with higher employment and a rising wage share. The second key relationship is the profit squeeze theory of accumulation, in which output growth is constrained by higher real wage rates because investment is assumed to be driven by profits so that as wages rise and profits fall, investment is curtailed and the rate of growth falls.[1]

These two relationships can be combined to give the following system of differential equations, in which *y* represents output and *w* the wage share and $\dot{y}$ and $\dot{w}$ signify the derivatives of each variable with respect to time:

$$\dot{y} = y(1 - w) \tag{1}$$

$$\dot{w} = w(-c + ry) \tag{2}$$

The first equation captures the profit squeeze relationship and the second the reserve army effect. In the formalisation presented here, some of parameters have been normalised to one for the sake of simplicity. A number of further simplifying modifications are made to the original Goodwin model.

[1] The result relies on the following assumptions: workers consume all wages and capitalists invest all profits, the marginal productivity of labour is constant and the capital-output ratio is fixed. Given these assumptions, a rise in the real wage will lead to a reduction in the growth of profits and - since investment is equal to profit - a reduction in the rate of output growth

In particular, in the original model, it is assumed that labour productivity and the labour force grow at a steady exogenous rate while in our formulation we assume instead that labour productivity is constant. Our model thus generates cycles in output around the steady state, while the original Goodwin model is a growth model.

This pair of equations is analogous to the Lotka-Volterra biological model of the dynamic interaction of the populations of two species: a predator and a prey (Lotka 1925, Volterra 1926). In the model, higher populations of the prey species lead to growth predator population. However, the rise in the predator population eventually causes the prey population to fall. The interaction of these two elements generates the cyclical dynamics of the Lotka-Volterra system. In the Goodwin system, the prey is represented by employment levels which increase with output. The predator is represented by the wage share since, according to the profit squeeze theory, a higher wage rate reduces profits and thus causes a fall in investment and growth.

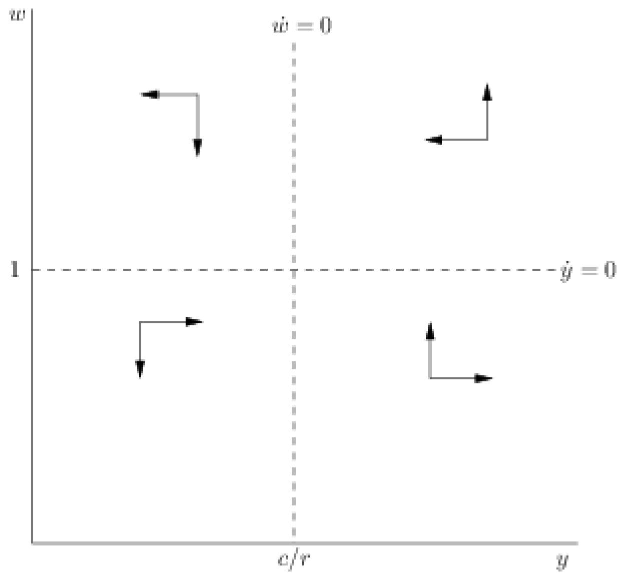

Figure 1. Phase diagram for the Goodwin Model

The Goodwin system has two stationary points, the first where both variables are equal to zero, and the second at:

$$y = \frac{c}{r}, w = 1$$

The first stationary point can be shown to be an unstable saddle-point, while the second is the centre of a family of concentric closed orbits.[2] The shapes of the orbits are defined by the parameters of the model, while the specific orbit generated by the system is determined by the starting values of the variables. The phase diagram for the model is shown in Figure 1 and the Jacobian matrix of the system evaluated at the non-zero stationary point can be calculated as the following,

[2]Both roots of the characteristic equation of this system are imaginary when evaluated at the non-zero fixed point. The system is a simple conservative oscillator which generates a family of closed orbits given by the following equation: $\ln w - w + c \ln y - ry = C$, where C is some constant.

$$J = \begin{bmatrix} 0 & -1 \\ +r & 0 \end{bmatrix} = \begin{bmatrix} 0 & - \\ + & 0 \end{bmatrix}$$

The non-zero stationary point occurs at the intersection of the two nullclines of the system: $y = \frac{c}{r}$ and $w = 1$. The direction of rotation around this point is determined by the signs of the two coefficients. In the current system, the direction of rotation is counter-clockwise in (y,w) space, so that output peaks before the wage share in the cycle.[3]

Figure 2 shows the result of simulating this pair of equations with parameter values of $c = 1.2$ and $r = 0.9$ and starting values $y = 0.5$ and $w = 1.25$. The system generates an anticlockwise cycle in (w,y) space as shown in the bottom pane. The top pane plots each variable against time, illustrating that output peaks before the wage share in the cycle.

The Goodwin model has given rise to a rich literature. On one hand, the model has been used as the starting point for a range of more complex models (e.g. Skott 1989, Shah and Desai 1981). On the other, an empirical literature has developed which either looks for the existence of Goodwin-like patterns in historical data or uses econometric techniques in order to estimate the main behavioural relationships of the model and other related models. Most recently, the empirical literature, inspired by Kaleckian models, has focused on the issue of whether demand is wage-led or profit-led, and the resulting implications for the generation of business cycles.

[3]Given that the system generates closed orbits around the non-trivial stationary point, in order to determine the direction of rotation of the system, we can examine the signs of $\dot{w}$ and $\dot{y}$ at a point close to this point in one of the quadrants defined by the intersection of the nullclines. If we examine a point in the upper-right quadrant of the system: $y = \left(\frac{c}{r}\right) + \delta$ and $w = 1 + \delta$, where δ is a small positive integer. $\dot{w}$ can be calculated as follows:

$$\dot{w} = w(-c + r\left(\frac{c}{r} + \delta\right))$$

$$= \delta w$$

Similarly, $\dot{y}=-\delta y<0$, (and thus $dw/dy<0$). The direction of rotation of the system can also be inferred from the signs of the Jacobian of the system evaluated at the non-trivial steady-state

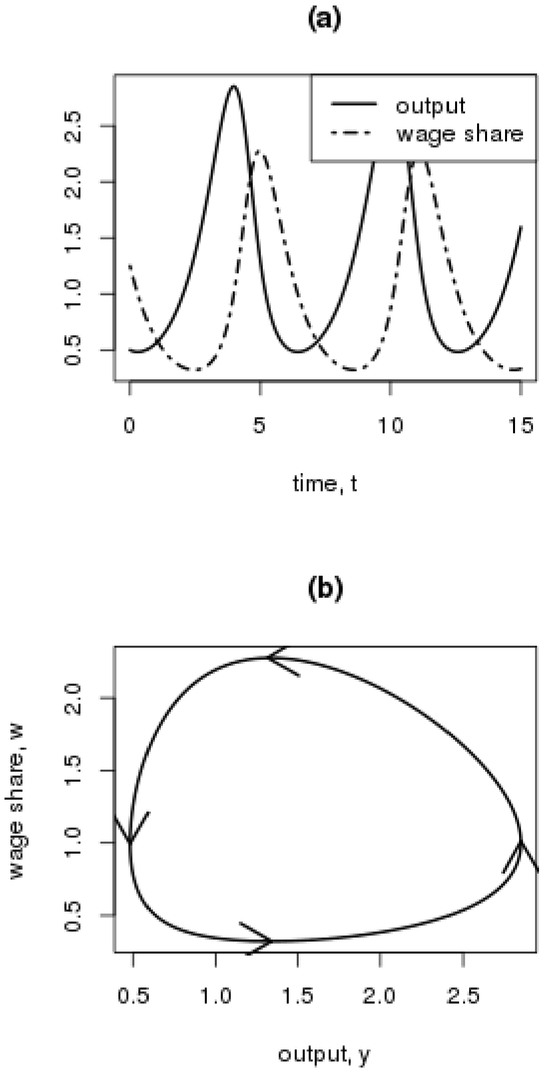

Figure 2. A simulation of the Goodwin model

This literature has some parallels with an earlier debate in the Marxian literature which focused on the mechanism generating the upper turning point of business cycles in the post-war US economy. This debate took place between, on the one hand, those who follow Goodwin in viewing the upper turning point as resulting from a profit squeeze due to rising wages (e.g. Goldstein, 1999a, 1999b) and, on the other, those who argue that insufficient aggregate demand is the primary mechanism (e.g. Sherman, 1997;)[4] This debate is focused on the correct way to interpret the historical data on business cycles and, in particular, on the timing of changes in profits and wages in relation to output. Van Lear (1999) regards the data as lending support to the underconsumptionist view, while Goldstein (1999b; 2002) argues that the data supports the profit squeeze theory.

[4] Sherman (1979) presents an early overview of the competing theoretical positions, from a Marxian viewpoint.

More recently, there has been debate among authors in the Kaleckian tradition between those inspired by the Goodwin literature and those who emphasise the role of aggregate demand. One part of this debate connects with the earlier Marxian debate in that it focuses on the relative timing of peaks in the wage share and output over the business cycle. It is argued by a number of authors that graphical plots of two key Goodwin variables – the wage share and unemployment – show counter-clockwise patterns that are consistent with the dynamics of the Goodwin model. Within this literature, authors have looked for evidence of both long-run trends, and shorter-run cyclical behaviour. Solow (1990) examines the data and concludes that it provides, at best, very weak evidence for long-run interaction of the Goodwin type. Flaschel (2010), updating previous research by Flaschel and Groh (1995) which examined data for eight OECD countries, concludes that strong evidence does exist for a long-run Goodwin mechanism. Mohun and Veneziani (2006) argue that the assumption made by these authors – of long-phase secular waves generated by a Goodwin mechanism – is theoretically problematic, and these longer run trends are more likely to be the result of structural changes. The evidence for shorter-run Goodwin-shaped cycles of 10-15 years in length appears to be stronger. Desai (1984) presents evidence of such cycles for the UK post-war period. Harvie (2000) examines data from 10 OECD countries, finding evidence of incomplete (approximately three-quarter-length) shorter Goodwin-type cycles of around 10—15 years, in most of the ten countries examined. Mohun and Veneziani (2006) concur, arguing that evidence for short-run Goodwin cycles is strong.

The other stream of empirical literature consists of contributions which use econometric testing to try and pin down macroeconomic behavioural relationships such as investment and consumption functions. Depending on the relative strength of the reactions of consumption and investment to other macroeconomic variables, in particular income distribution and profit rates, regimes can be classified as either wage-led or profit-led.[5] The relevance of this is that the Goodwin model relies on a profit-led system to generate business cycles. Within this stream, Stockhammer et al (2009), Stockhammer and Stehrer (2011), Hein and Vogel (2008), Naastepad and Storm (2006) argue that aggregate demand (or at least its domestic components) are wage-led. On the other side of the debate, Barbosa-Filho and Taylor (2006) and Flaschel et al (2007) present evidence that aggregate demand in the USA is profit led. Barbosa-Filho and Taylor (2006) estimate a two equation VAR with a demand equation and a distribution equation (without contemporaneous interaction) for the US economy using quarterly data and the cyclical component of the HP filter. The effects for individual components of demand are then decomposed from the aggregate results (rather than estimated as behavioural equations). This gives a strongly profit-driven consumption function. The (negative) effect of an increase in the wage share on consumption is larger than those on investment and net exports combined. They conclude that the US economy is in a profit-led demand regime.[6]

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[5] There is a close, but not an exact, correspondence between such comparative-static exercises and the dynamic relationships of the original Goodwin model. Taylor (2012) notes that "[t]he Kaleckian literature often draws attention to wage-led and profit-led macro responses to changes in the income distribution. ... The labels have been applied to comparative static responses of output, growth and other variables; slopes of nullclines in a phase diagram; and other aspects of the macro system. The distributive shifts may involve movements in the real wage or profit rate, wage and profit shares, etc."(p. 43)

[6] Stockhammer and Stehrer (2011) highlight econometric problems and argue that these findings are very sensitive to lag lengths.

A recent contribution by Taylor (2012) connects the theoretical and empirical strands of the literature. Taylor contrasts two models of distribution and demand. The first combines a locally stable "Domar-style" investment function (that leads to a profit-led regime) with a Goodwin-type reserve-army distribution function. This is contrasted with a model which combines an unstable "Harrod-style" investment function (which produces a wage-led demand regime) with a distribution function based on a pro-cyclical profit-share. The first model generates Goodwin-type counter-clockwise cycles in output-distribution space, while the second generates cycles which turn in the opposite direction. On the basis of the empirical evidence, Taylor argues that the anticlockwise Goodwin-type movement is what is being observed. He thus concludes, "Overall, Domar-style investment and a profit squeeze appear to fit the data better than Harrod and short-run wage-led aggregate demand" (p. 50). Taylor thus cites the apparent existence of counter-clockwise cycles in the empirical data as evidence for a Goodwin mechanism and a profit-led demand regime.[7] He then extends the analysis by discussing models where cyclical behaviour is generated by the interaction of financial variables and demand in a Minskian framework in which asset prices and debt interact. However, these financial models are left only partially specified. In particular, the role of income distribution is not investigated, thus the question what type of cycles the financial group of models gives rise to in the wage share-income space is not investigated.

It is this question that we consider in the remainder of the paper. In particular, we take issue with the claim that the observation of what look like Goodwin cycles in the data imply that the mechanism generating those cycles is a profit-led demand regime. We demonstrate that such cycles can instead be generated by the interaction of financial fragility, demand and distribution. In particular, models can be constructed which generate Goodwin-type cycles but do not rely on a profit-led demand function.

A Minsky cycle

Unlike the Goodwin model, there is no canonical version of the Minsky cycle. Minsky's own work (Minsky 1975, 1986) presents a verbal description of the cycle mechanism, but no formal model. Consequently, several different models purporting to summarise his argument have been proposed (e.g. Skott 1994, Asada 2001, Fazzari et al 2008, Charles 2008). However, the basic structure of the argument for our purposes is clear enough. The cycle results from the interaction of goods market demand and financial fragility of non-financial businesses. This section will present a minimalistic version of the Minsky model because the aim of the paper is to investigate the properties of the model when it is extended to include a reserve army function. To ensure comparability with the structure of the Goodwin we use a predator-prey model.

The model consists of two differential equations.

$$\dot{f} = f(-1 + py) \tag{3}$$

[7] A similar approach is taken by Diallo et al (2011) who construct a model with four possible regimes: clockwise and counter-clockwise cyclical regimes, and two saddle-point cases which do not generate cyclical behaviour. Diallo et al (2011) also conclude that the empirical evidence supports the profit-led version of the model.

$$\dot{y} = y(1 - f) \tag{4}$$

Equation (3) depicts the rate of increase of financial fragility, f, as a positive function of the level of demand. The assertion is that in the course of the boom firms adopt a more optimistic outlook and therefore take on higher levels of debt (relative to cash flows). Financial fragility is usually thought of as the inverse of the debt-to-income ratio of firms. Banks share the optimistic outlook and are thus willing to lend. The balance sheets of economic units in the Minsky model endogenously become more fragile. Equation (4) specifies an inverse relationship between aggregate demand growth and the degree of fragility of the system. This is because the degree of indebtedness of firms makes them vulnerable to bankruptcy once interest rates rise and/or a larger share of cash flows is absorbed by debt services, which negatively impacts on investment. Skott (1994), Asada (2001), Fazzari et al (2008), and Charles (2008) derive such models from behavioural functions and discuss them further.

Our formulation captures Minsky's basic argument while sidestepping a number of controversies surrounding his model. The source of disagreement in the literature is twofold. Firstly, there is no consensus on the mechanism which generates the negative feedback from financial fragility to demand. In Fazzari et al (2008) this is due to the effect of the central bank increasing interest rates in response to rising inflation at the peak of the cycle. Much of the literature, however, argues that it is the increasing debt ratio of firms that eventually leads banks to increase interest rates, thereby causing the downturn (e.g. Charles 2008). Second, while Minsky regarded the upswing as being accompanied by an increasing leverage ratio of firms, Lavoie and Seccareccia (2001) have demonstrated that Minsky's argument is based microeconomic principles that need not hold in the Kaleckian macroeconomic framework that Minsky is assuming (see also Michell, 2014). In such a framework, it is not clear why the debt ratio should rise during the boom. This is because, under the Kaleckian assumption that workers do not save out of wage income, aggregate profits at the macroeconomic level are determined by the investment of firms. As such, the high rate of investment growth during the boom should be expected to lead to decreasing leverage ratios.

Our model avoids both issues by design. First, Equation 3 states that financial fragility increases with output. We thus follow Minsky's argument of a pro-cyclical leverage ratio. The reason for this is conceptual rather than empirical accuracy: we wish to model a Minsky cycle, and pro-cyclical leverage ratio is necessary for this. Second, Equation 4 does not specify what brings about the rise in interest rates. Rather it simply posits that higher fragility leads to lower growth of demand. Again, for the purpose of this paper, we are interested in the simplest representation of the Minsky cycle because we will investigate in the next section whether such a model can give rise to pseudo-Goodwin cycles.

The non-trivial steady state solution of the Minsky model is

$$f^* = 1, y^* = 1/p$$

and the Jacobian at the steady state is

$$J = \begin{bmatrix} 0 & p \\ -1/p & 0 \end{bmatrix} = \begin{bmatrix} 0 & + \\ - & 0 \end{bmatrix}.$$

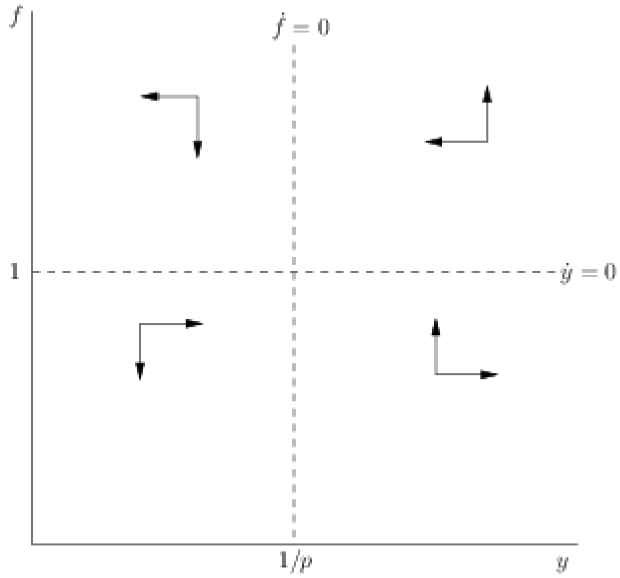

Figure 3 Phase diagram for Minsky model

Figure 4 simulates the dynamics of the system with $p = 0.75$ and initial values $y = 0.5, f = 1$. The system generates cycles such that peaks in output precede peaks in financial fragility (Figure 2a). Alternatively, this could be expressed by saying that the system produces anti-clockwise motion in output-fragility space (clockwise motion in fragility-output space) (Figure 4b).

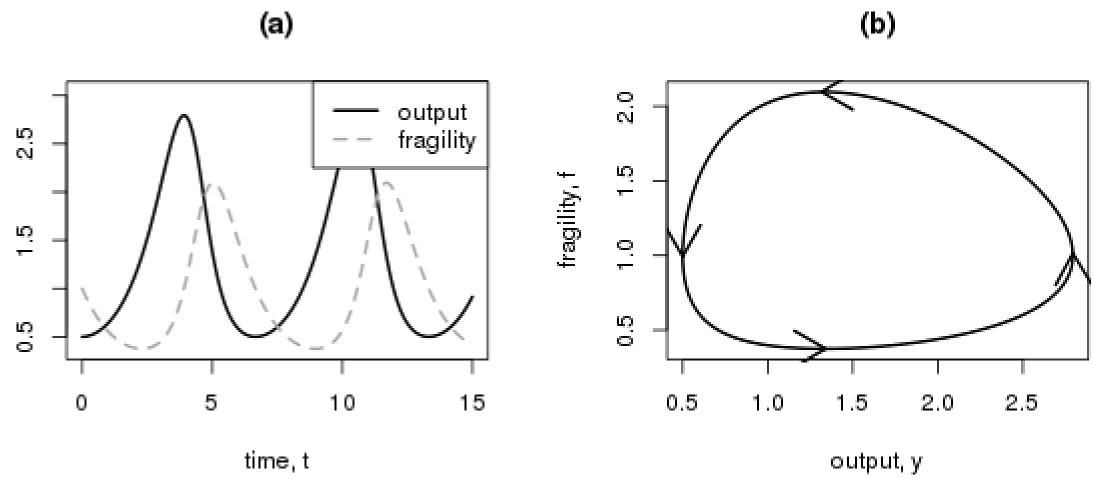

Figure 4 Simulation of Minsky model

A Minsky cycle with a reserve army effect

In this section we discuss a Minsky model that includes a reserve army effect. The dynamics of this system will be driven entirely by the interaction between financial fragility and demand. The distribution function has, by design, no feedback on demand. We retain equations 3 and 4. To this we add a distribution equation with a reserve army effect:

$$\dot{w} = w(-c + ry - w) \quad (5)$$

This equation is similar to eq. (2), but additionally includes a negative feedback effect upon itself: this serves to contain the rate of real wage increases and counteract the wage demands of workers at higher output.

By design, the distribution equation has no feedback onto f and y: it is auxiliary to the cyclical dynamics of the system which are fully determined by the interaction between f and y. In the language of biological models distribution is a scavenger: its population growth depends on the population of another species, but the causality is unidirectional (Chauvet et al 2002, Nolting et al 2008).[8] In this model the cycle is, by design, generated from the interaction of financial fragility and demand.

This model may appear similar to models that try to integrate Goodwin and Minsky mechanisms, such as Keen (1995). However, there is an important difference in terms of the synthesis as well as in terms of the questions these models are supposed to answer. Keen (1995) constructs a model with the intention of examining the properties of models which have a feedback from distribution as well as from finance to aggregate demand. By design the feedback goes from demand to finance and vice versa as well as from distribution to demand and vice versa. In this section, however, we are asking a different question. We are interested in a model that has a Minsky cycle and a Marxian distribution function *but has no feedback from distribution to demand*.

It should be noted that, as in Goodwin's original, our model is specified such that the wage share is not bounded within a specific range of values and, in particular, may exceed unity. We have chosen the current specification for the sake of simplicity. As shown by Desai et. al (2006), by replacing the linear distribution function of the Goodwin model with a non-linear form in which the rate of growth of the wage share becomes infinite as the economy approaches full employment, the model may be transformed so that wage share values remain within plausible bounds. We could apply such a transformation to our model but the qualitative behaviour of the system will not be altered and the mathematics becomes more complex. For this reason we choose to retain the linear distribution function.

The interior steady state of the model is the following:

$$y^* = \frac{1}{p}, f_* = 1 + s, w^* = -c + \frac{r}{p}.$$

[8] Strictly speaking a scavenger population depends on deaths of prey, i.e. the kill rate. Our wage share depends on the population of output rather than its death rate. Biologically speaking distribution thus plays the role of a harmless parasite rather than a scavenger.

The dynamics of the model are simulated with parameters $c = 1.5, p = 2, r = 5$ and starting values of $f = 0.75, y = 0.5, w = 1$. The model generates oscillations in all three variables (Figure 5a). The peak in output precedes those in fragility and in the wage share. The cycle that drives the system is, identically to the Minsky model, the interaction between fragility and demand (Figure 5b). The interesting property of this system is that the system produces limit cycles in y and w that look like a Goodwin cycle (Figure 5c). However this cycle is simply a side effect of the cycle between financial fragility and output. This is what we call a pseudo-Goodwin cycle: a cycle that rotates counter-clockwise in y, w space but is not due to the mechanisms of the Goodwin model. The model also generates cycles in distribution and fragility. Like the pseudo-Goodwin cycles these do not arise out of direct causal links between these variables but as side effects of other causal relations.

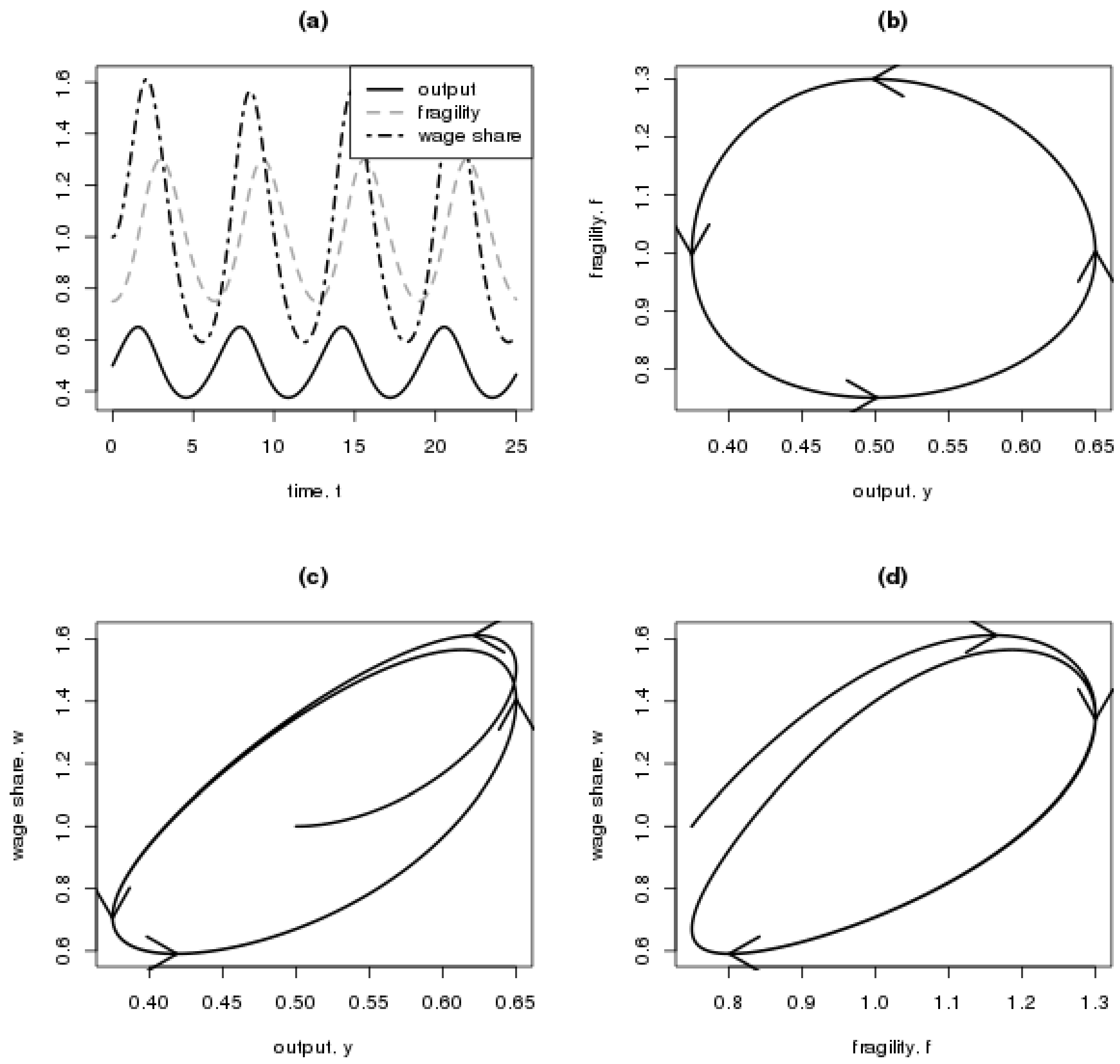

Figure 5 Simulation of the augmented Minsky model

To illustrate that there is no true Goodwin cycle in the model, assume that f is fixed. If $f > 1$ then both y and w will increase exponentially without fluctuations. If $f < 1$ then both y and w will collapse to zero, again without fluctuations. In isolation from f, the equations for y and w cannot generate a cycle. Thus, when the full three-variable system exhibits cycles in the wage share-output space, it is not a Goodwin cycle. However, in the absence of evidence of the role played by f, the cycle in output and distribution could be interpreted as arising out of direct Goodwin-type interaction between the two variables.

The simulations of our augmented Minsky model (Figure 5) exhibit limit cycles in both output-distribution space and fragility-distribution space. Since there is no feedback from w to either y or f, these two variables continue to interact in closed orbits as in the original Lotka-Volterra formulation. This periodic orbit for demand and fragility is by design.

If Equations (3) and (4) are augmented with Equation (2) instead of Equation (5) - i.e. if the distribution function does not have an own effect - the system will still exhibit pseudo-Goodwin cycles, i.e. counterclockwise fluctuations in y-w space. However, limit cycles in (y, w) space will no longer be generated. Instead a closed orbit in distribution and output is possible under certain parameter restrictions, while in the more general case we either obtain either dampened or explosive oscillations in distribution.[9]

A Minsky model with distribution function and a wage-led effect in the demand function

The previous section has demonstrated that pseudo-Goodwin cycles can arise in a system where there is no effect from income distribution on demand. In this section we introduce one more ingredient into our model: a positive feedback from the wage share to output. In other words, we investigate whether a pseudo-Goodwin cycle can arise in wage-led economy.

To do so, we retain equations (3) and (5) but replace Equation (4) with Equation (4'), which includes a positive feedback from the wage share to demand.

$$\dot{y} = y(1 - f + sw) \qquad (4')$$

The interior steady state solution of this system is:

$$y^* = \frac{1}{p}, f^* = 1 + s\left(-c + \frac{r}{p}\right), w^* = -c + \frac{r}{p}.$$

The dynamics of this system are depicted in Figure 6, assuming parameter values of $c = 1.5, p = 2, r = 5, s = 0.02$ and starting values $f = 0.75, y = 0.5, w = 1$. The cyclical behaviour of the model is still driven by the interaction between y and f (Figure 6b). However, this no longer produces a closed orbit because of the instability introduced by the wage-led demand term. As before, if f were fixed, the system in (4') and (5) would exhibit explosive non-oscillatory growth: the feedback from

[9] This is not surprising. Distribution has no feedback in this system and it is carried along with output, but has no equilibrium value rooted in the system. It will exhibit cycles that reflect the cycles in y, but its value over the cycle may exhibit a drift (see Appendix A.1). The evolutionary biology literature thus often introduces self-stabilising mechanisms (Nolting et al 2008) as we have done.

distribution to output as well as from output to distribution are positive. Figure 6c shows that our model still exhibits a pseudo-Goodwin cycle – counter-clockwise motion in output-wage share space. However this cyclical motion is now unstable, so that outward spirals are generated in the phase space of all three pairs of variables. The introduction of a wage-led demand function into the system thus does not prevent the generation of pseudo-Goodwin cycles. Instead it simply switches the system from generating limit cycles to unstable oscillations.[10] Evidence of counter-clockwise motion in output-wage-share space thus does *not* provide sufficient evidence to rule out a wage-led demand regime.

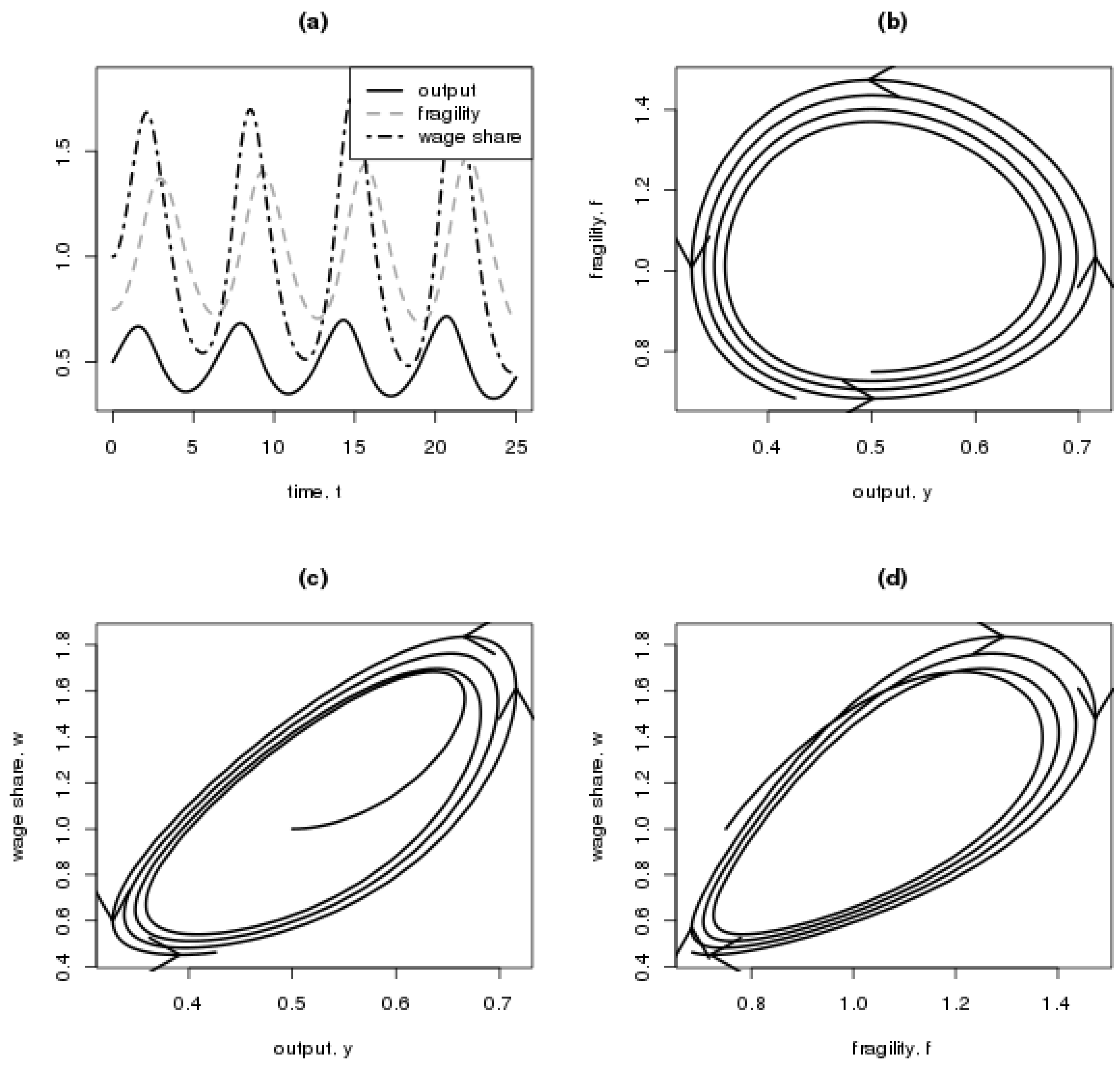

Figure 6 Simulation of Pseudo-Goodwin model with wage-led demand.

[10] The simulations do not impose bounds on the wage share. As the wage share is increasing in this model it would eventually hit its upper bound at which point the dynamics of the system would change. Our finding thus is that this model will exhibit pseudo-Goodwin cycles as long the wage share remains below its upper bound.

Conclusion

Goodwin's (1967) model generates counter-clockwise cycles in output-wage-share space due to the interaction of a profit-led demand regime and a reserve army distribution function. The paper has demonstrated that counter-clockwise cycles in output-wage-share space can arise for reasons that are unrelated to the Goodwin mechanisms. We call such counter-clockwise cycles that are not based on a profit-led demand regime pseudo-Goodwin cycles. As an illustration, a Minsky model in financial fragility and output was extended to include a reserve army function. In a second step we introduced a wage-led effect in the demand equation. In both cases we show that pseudo-Goodwin cycles can arise.

This lends itself to the following conclusions. First, any cycle that is not based on feedbacks from income distribution can produce pseudo-Goodwin cycles if the wage share adjustment is a la Marx and Goodwin. Second, the existence of a counter-clockwise movement of in output wage share space is not a sufficient condition for the existence of actual Goodwin cycles and can therefore not be interpreted as evidence in supporting the existence of a profit-led demand regime. Indeed a wage-led demand regime can be perfectly consistent with pseudo-Goodwin cycles.

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Appendix

A.1 Minsky with reserve army eq (2)

This appendix presents the simulations of a Minsky model with reserve army effect. The system consists of equations (2), (3), and (4). Compared to the model discussed in section 4 the wage equation does not contain an own effect.