Three dimensional solid structures simulation on Isogeometric B-rep analysis

Daniel Herrero-Adán*, Riccardo Rossi[1], Tobias Teschemacher[2], Rui Pedro Cardoso[3]

*Department of Engineering Design and Mathematics, University of the West of England, BS16 1QY Bristol, UK
[1] International Centre for Numerical Methods in Engineering (CIMNE), Technical University of Catalonia, Campus Norte UPC, 08034, Spain
[2] Lehrstuhl für Statik, Technische Universität München, Arcisstr. 21, 80333 München, Germany.
[3]Department of Mechanical, Aerospace and Civil Engineering, Brunel University London, UB8 3PH Uxbridge, London, UK

UWE Bristol | University of the West of England

daniel.herreroadan@uwe.ac.uk

00.- INTRODUCTION

01.- IMPORTING CAD SOLIDS TO IGA
02.- PATCHES COUPLING AND BOUNDARY CONDITIONS
03.- INTEGRATION OF TRIMMED SOLIDS

04.- EXAMPLES

05.- CONCLUSIONS

00.- INTRODUCTION

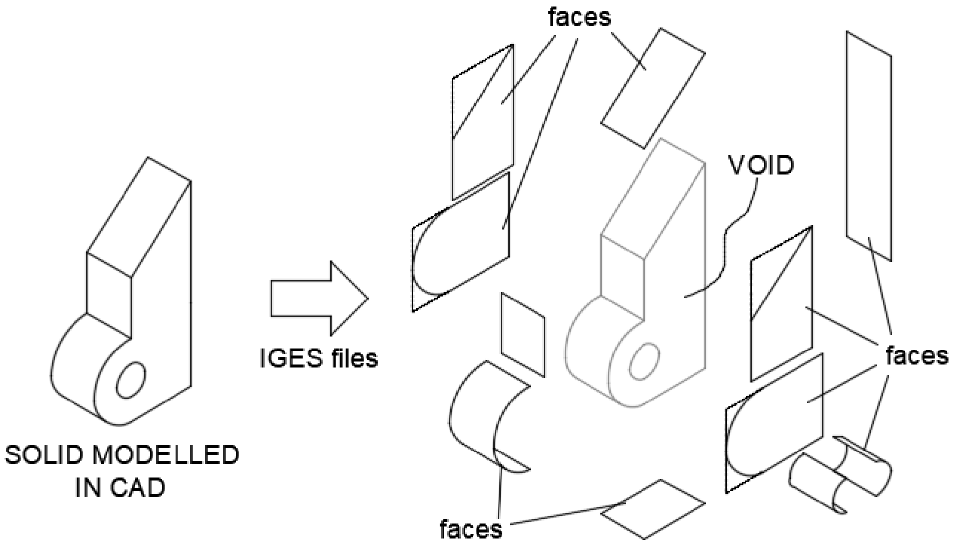
importation of solid from CAD

CAD => IGES FILES: contain NURBS information

One of the main purposes of IGA is transfer directly those IGES to analysis

For solids, IGES provides faces =>
=> No parametrization of the solid

section 1



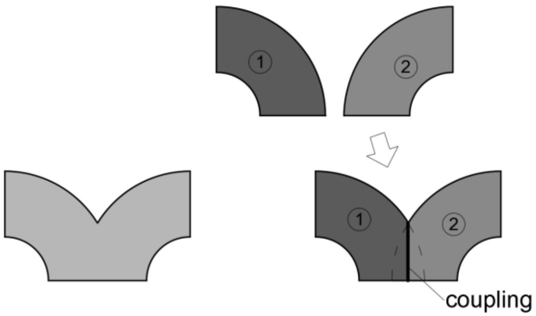
00.- INTRODUCTION

sections 2 and 3

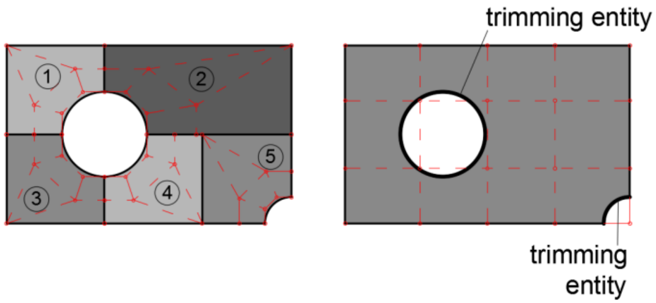
domain definition with NURBS

To facilitate domain definition we can use:

Multi-patching: coupling (on trimmed or un-trimmed boundaries)



Trimming for shape or boundary conditions



Using both together provides flexibility

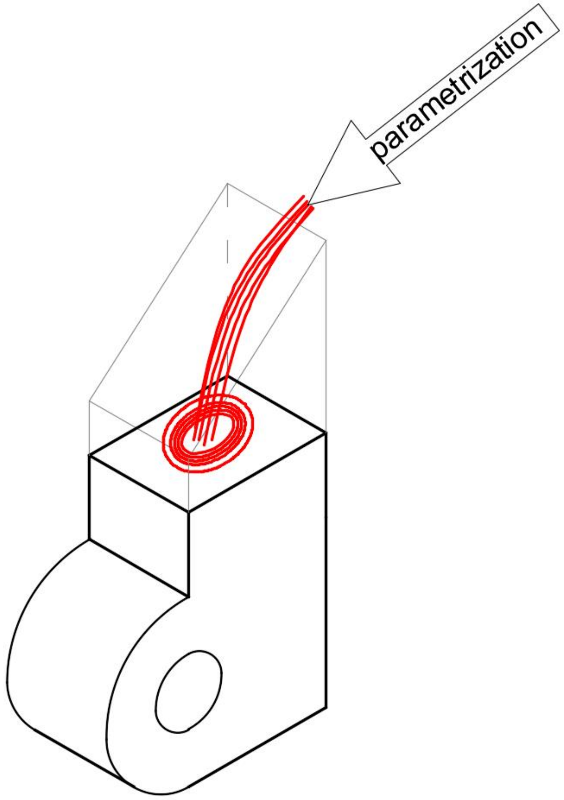
01.- IMPORTING CAD SOLIDS TO IGA

PURPOSE:
extract IGES files and use them to define domain for IGA

Problem:

from IGES files we get faces, but no the solid itself

We need to 'parametrize' or 'fill' the solid with parameter space



01.- IMPORTING CAD SOLIDS TO IGA

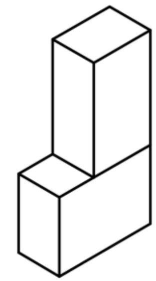
Solution strategy:

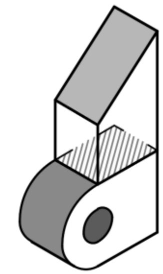
User CAD operations

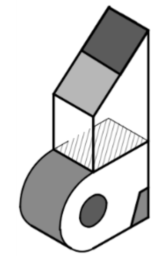
1.- define 'gross patches' with initial shape under a set of constraints. → Export IGES files

2.- trim and couple to obtain the final domain shape

3.- define any other boundary surface to apply BC's. These are the 'bounded patches'. → Export IGES files


1


2


3

Algorithm operations

4.- use 'gross patches' to generate solid parametrization (´filling')

5.- use 'bounded patches' to extract NURBS of domain limits and:
- integrate only the domain within the limits
- apply BC's

01.- IMPORTING CAD SOLIDS TO IGA

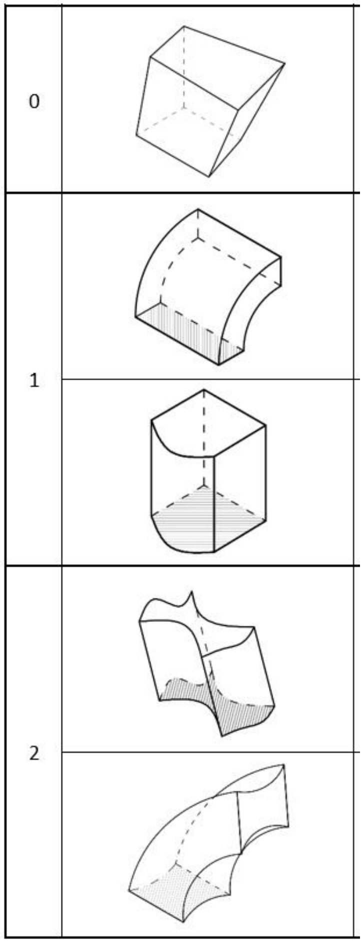
GROSS DOMAIN DEFINITION IN CAD:

The idea is to get as close as possible to the final shape

↓

that will save time during integration of the domain

However constrains are imposed to enable the algorithm to understand the faces. Any gross patch must belong to one of the shown three cases



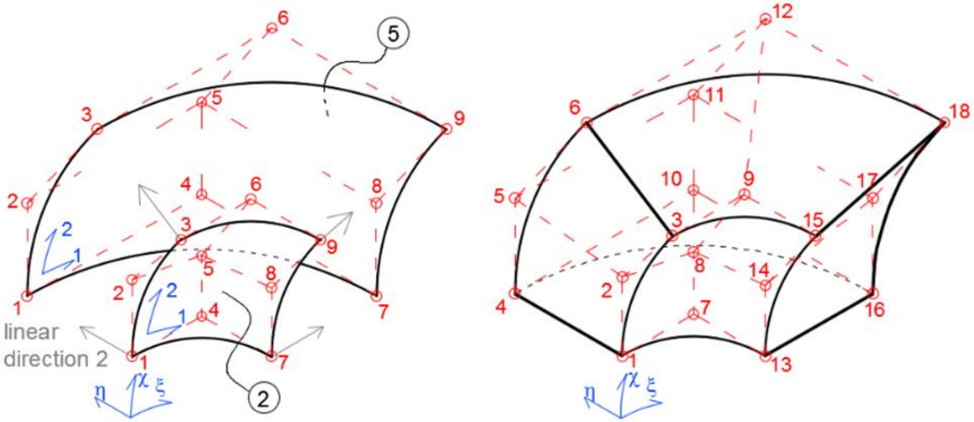
Constrains:

-Six faces

Allows to identify faces and their relative position

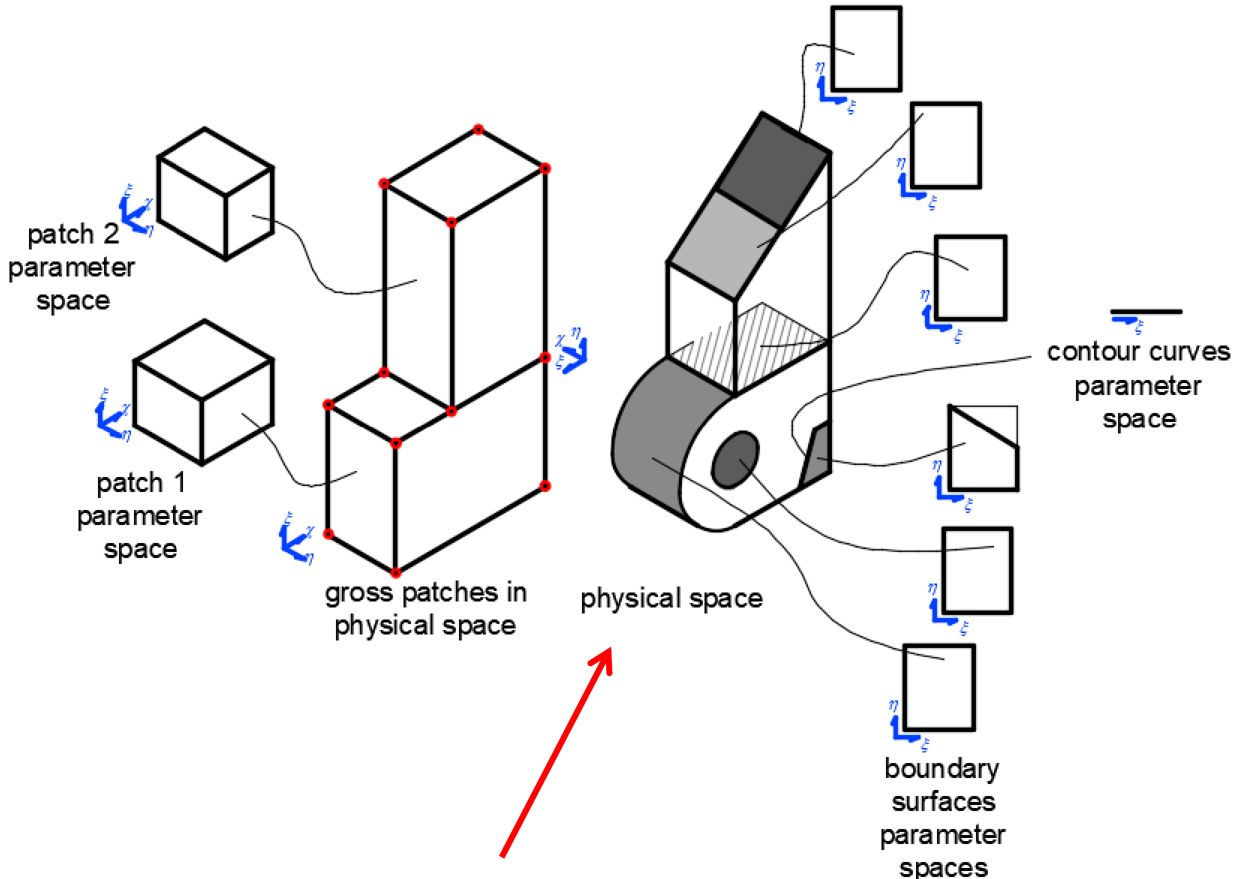
-At least one linear parameter direction

Use the two faces perpendicular to linear direction for defining the solid ('sandwich')



01.- IMPORTING CAD SOLIDS TO IGA

SPACES GENERATED:



All entities meet in physical space, but each has its own parameter space

02.- PATCHES COUPLING AND BC's

COUPLING CONSTRAINT

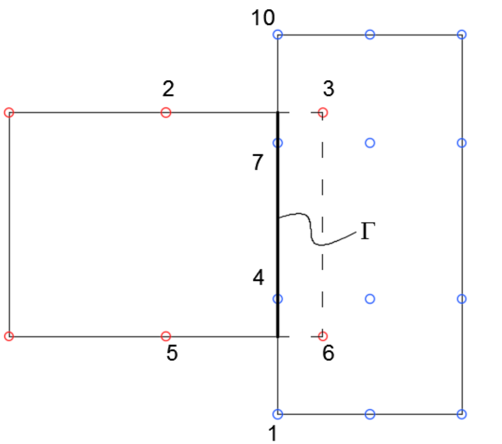
(Same principles as [1])

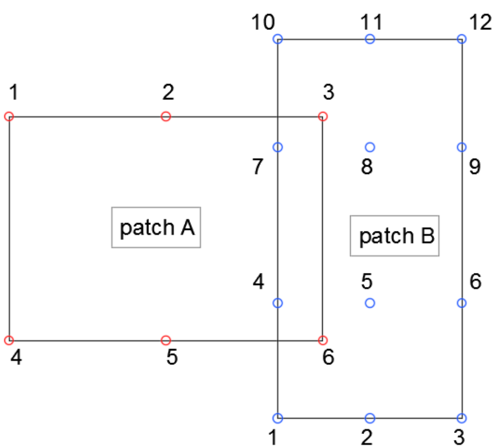
$$\int_{\Gamma} (\mathbf{u}^{A} - \mathbf{u}^{B})\, d\Gamma = 0$$

$$\mathbf{R}^{A}{}_{i} = \begin{bmatrix} R^{A}{}_{i} & 0 \\ 0 & R^{A}{}_{i} \end{bmatrix} \qquad \mathbf{u}_{i}^{A} = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix}$$

$$\int_{\Gamma} \mathbf{R}^{A}{}_{i} \left(\mathbf{R}^{A}{}_{i}^{T}\, \mathbf{u}_{i}^{A} - \mathbf{R}^{B}{}_{j}^{T}\, \mathbf{u}_{j}^{B} \right) d\Gamma = 0$$

$$i = 2,3,5,6 \quad j = 1,4,7,10$$





$$\int_{\Gamma} \left[\begin{bmatrix} R_2^A R_2^A & & \cdots & R_2^A R_6^A & \\ & R_2^A R_2^A & & & R_2^A R_6^A \\ \vdots & & \ddots & & \vdots \\ R_6^A R_2^A & & \cdots & R_6^A R_6^A & \\ & R_6^A R_2^A & & & R_6^A R_6^A \end{bmatrix} - \begin{bmatrix} R_2^A R_1^B & & \cdots & R_2^A R_{10}^B & \\ & R_2^A R_1^B & & & R_2^A R_{10}^B \\ \vdots & & \ddots & & \vdots \\ R_6^A R_{10}^B & & \cdots & R_{n6}^A R_{10}^B & \\ & R_6^A R_{10}^B & & & R_6^A R_{10}^B \end{bmatrix} \right] d\Gamma \begin{Bmatrix} u_{2x}^A \\ u_{2y}^A \\ \vdots \\ u_{6x}^A \\ u_{6y}^A \\ u_{1x}^B \\ u_{1y}^B \\ \vdots \\ u_{10x}^B \\ u_{10y}^B \end{Bmatrix} = \mathbf{0}$$

$$\boxed{\mathbf{H}_{C}\, \mathbf{u} = \mathbf{0}}$$

[1] T. Teschemacher, A. M. Bauer, T. Oberbichler, M. Breitenberger, R. Rossi, R. Wüchner and K.-U. Bletzinger, Realization of CAD-integrated Shell Simulation based on Isogeometric B-Rep Analysis, Submitted and accepted in *Advanced Modeling and Simulation in Engineering Sciences*.

02.- PATCHES COUPLING AND BC's

DISPLACEMENT CONSTRAINT

(Same principles as [1])

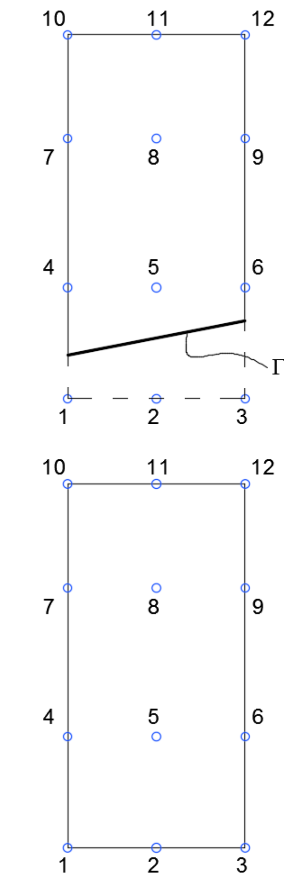
$$\int_\Gamma \mathbf{u}\, d\Gamma = 0$$

$$\mathbf{R}_i = \begin{bmatrix} R_i & 0 \\ 0 & R_i \end{bmatrix}$$

$$\mathbf{u}_i = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix}$$

$$\int_\Gamma \mathbf{R}_i \left(\mathbf{R}_i{}^T \; \mathbf{u}_i \right) d\Gamma = 0$$

$$i = 1, 2, 3, 4, 5, 6$$



$$\int_\Gamma \begin{bmatrix} R_1\, R_1 & & \cdots & R_1\, R_6 & \\ & R_1\, R_1 & & & R_1^A R_6 \\ \vdots & & \ddots & & \vdots \\ R_6\, R_1 & & \cdots & R_6\, R_6 & \\ & R_6\, R_1 & & & R_6\, R_6 \end{bmatrix} d\Gamma \begin{Bmatrix} u^A_{1x} \\ u^A_{1y} \\ \vdots \\ u^A_{6X} \\ u^A_{6y} \end{Bmatrix} = \mathbf{0}$$

$$\boxed{\mathbf{H_D}\, \mathbf{u} = \mathbf{0}}$$

02.- PATCHES COUPLING AND BC's

constraints into the structure system – Lagrange multipliers

$$\begin{bmatrix} \mathbf{K}^A & \mathbf{0} \\ \mathbf{0} & \mathbf{K}^B \end{bmatrix} \begin{Bmatrix} \mathbf{u}^A \\ \mathbf{u}^B \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^A \\ \mathbf{f}^B \end{Bmatrix} = \mathbf{K}\mathbf{u} = \mathbf{f}$$

$$g(u) = \begin{cases} \mathbf{H}_C\,\mathbf{u} = \mathbf{0} \\ \mathbf{H}_D\,\mathbf{u} = \mathbf{0} \end{cases} = \mathbf{H}\,\mathbf{u} = \mathbf{0}$$

(Same principles as [1])

$$\Psi(\mathbf{u}, \boldsymbol{\lambda}) = \mathbf{f}(\mathbf{u}) + \boldsymbol{\lambda}\, g(u)$$

Find the minimum by Lagrange multipliers

Minimize the functional Ψ , that implies : $\Pi = \mathrm{d}\Psi = 0$

Use Newton-Raphson to find the root of $\mathrm{d}\Psi$

Assume $\mathbf{f}(\mathbf{u})$ such that $\frac{\partial \mathbf{f}(\mathbf{u})}{\partial \mathbf{u}} = \mathbf{K}\mathbf{u} - \mathbf{f}$

$$\begin{Bmatrix} \mathrm{d}\mathbf{u} \\ \mathrm{d}\boldsymbol{\lambda} \end{Bmatrix} = \frac{-\Pi(\mathbf{u}, \boldsymbol{\lambda})}{\mathrm{d}\Pi(\mathbf{u}, \boldsymbol{\lambda})}$$

$$\Pi = \begin{Bmatrix} \frac{\partial \Psi}{\partial \mathbf{u}} \\ \frac{\partial \Psi}{\partial \boldsymbol{\lambda}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{K}\mathbf{u} - \mathbf{f} + \mathbf{H}^T\boldsymbol{\lambda} \\ \mathbf{H}\mathbf{u} \end{Bmatrix}$$

$$\mathrm{d}\Pi = \begin{bmatrix} \frac{\partial^2 \Psi}{\partial \mathbf{u}\partial \mathbf{u}} & \frac{\partial^2 \Psi}{\partial \mathbf{u}\partial \boldsymbol{\lambda}} \\ \frac{\partial^2 \Psi}{\partial \boldsymbol{\lambda}\partial \mathbf{u}} & \frac{\partial^2 \Psi}{\partial \boldsymbol{\lambda}\partial \boldsymbol{\lambda}} \end{bmatrix} = \begin{Bmatrix} \mathbf{K} & \mathbf{H}^T \\ \mathbf{H} & \mathbf{0} \end{Bmatrix}$$

Linear systems & initial displacements 0

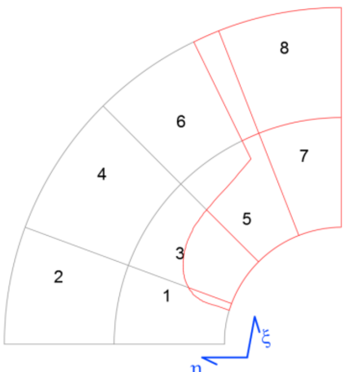
$$\begin{bmatrix} \mathbf{K} & \mathbf{H}^T \\ \mathbf{H} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}$$

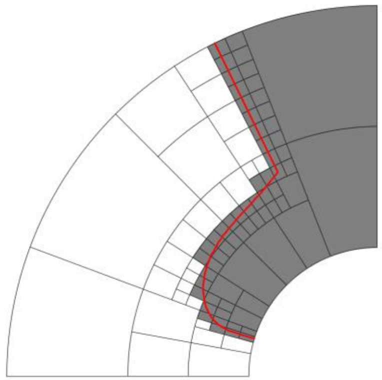
03.- INTEGRATION OF TRIMMED SOLIDS

$$\begin{bmatrix} \mathbf{K} & \mathbf{H}^T \\ \mathbf{H} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}$$

K requires integration of solid domain

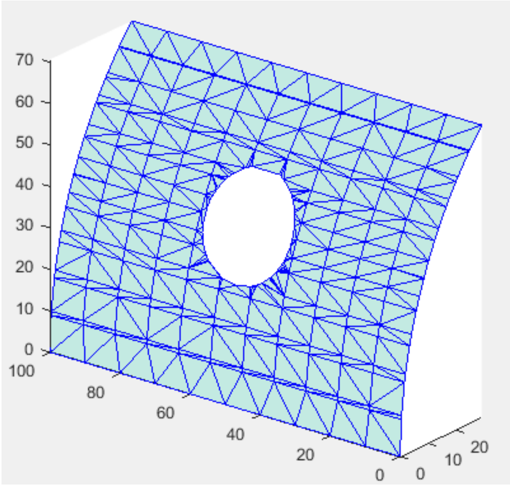
If trimmed => sequential partition of knot spans until desired level of partition (based on [2]).





H requires integration on boundary surfaces

Use triangular elements scheme, as it is used also for representation

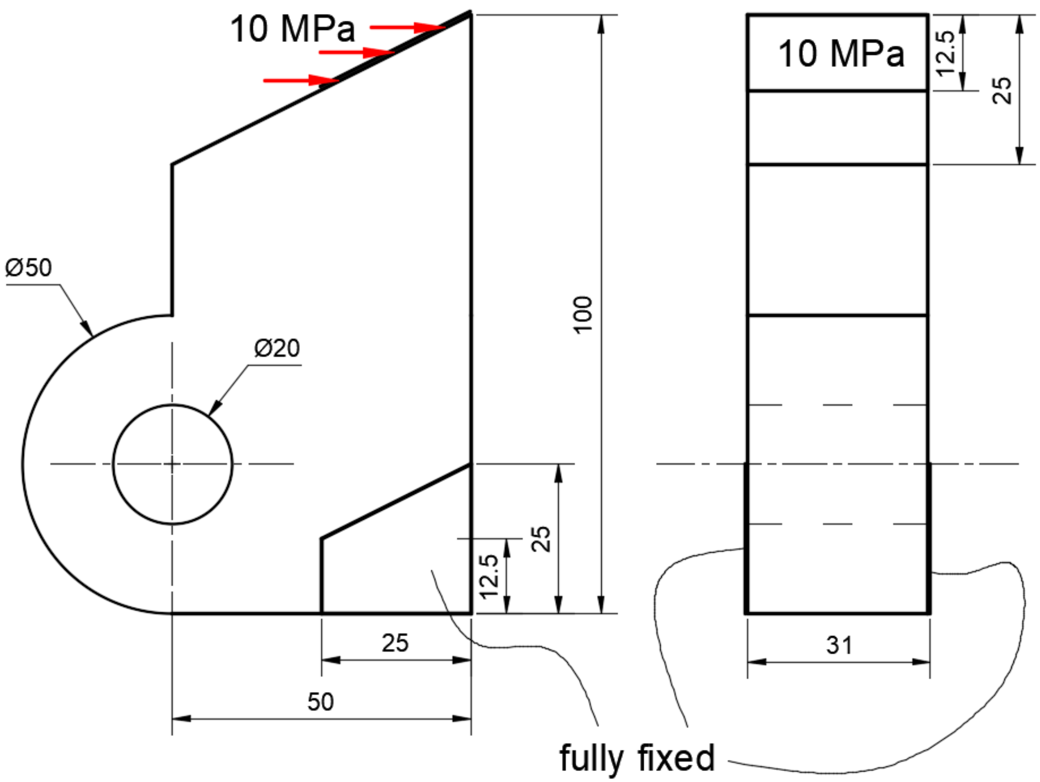


To know if one knot span is trimmed: find if any trimming surface point is inside the knot span =>
=> involves point inversion => **expensive**

[2] J. Parvizian, A. Düster and E. Rank, Finite cell method. *h*- and *p*-extension for embedded domain problems in solid mechanics, *Comput. Mech.*, 41:121–133, 2007.

04.- EXAMPLES

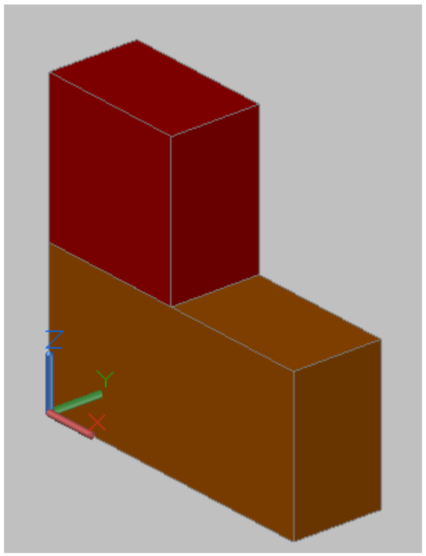
Example 1



Isotropic material
E = 210000 MPa
ν = 0.30

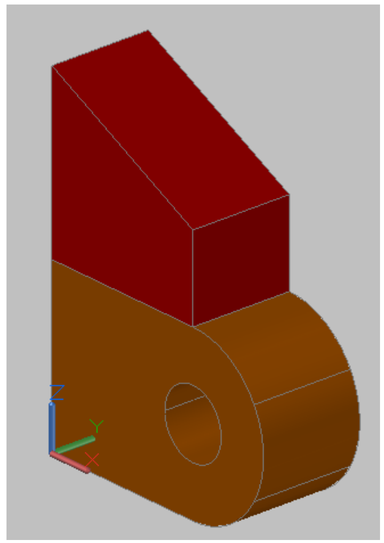
04.- EXAMPLES

Example 1: CAD modelling



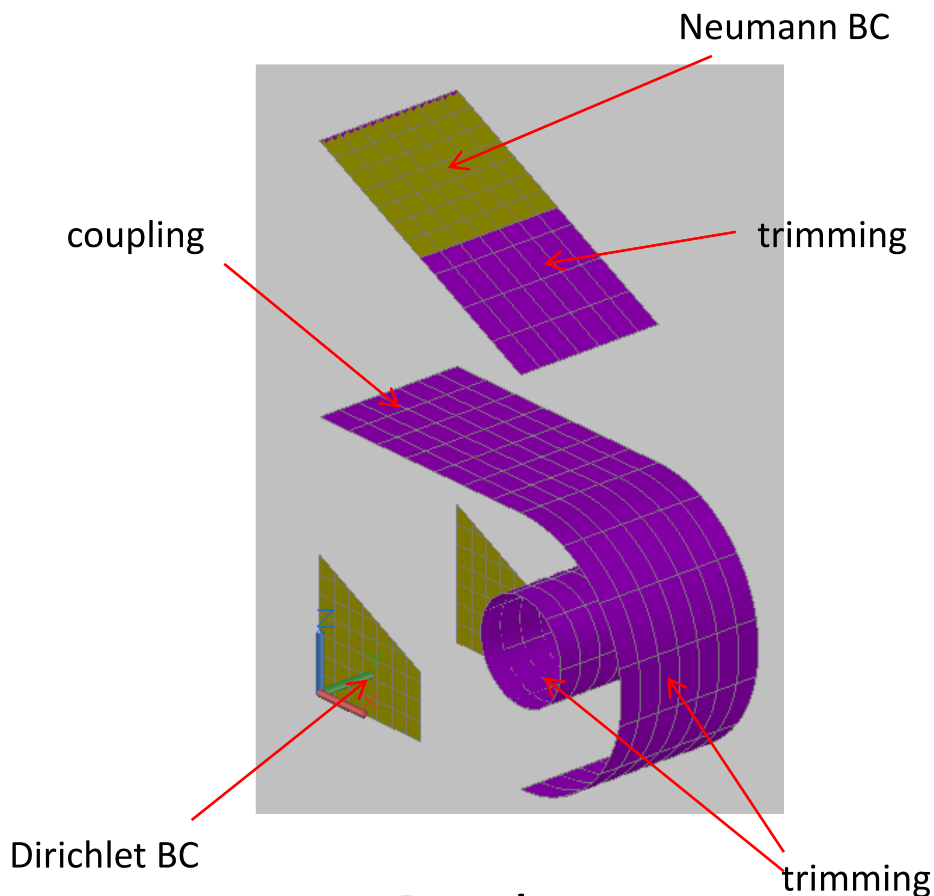
Gross patches

Export IGES to algorithm



Bounded patches

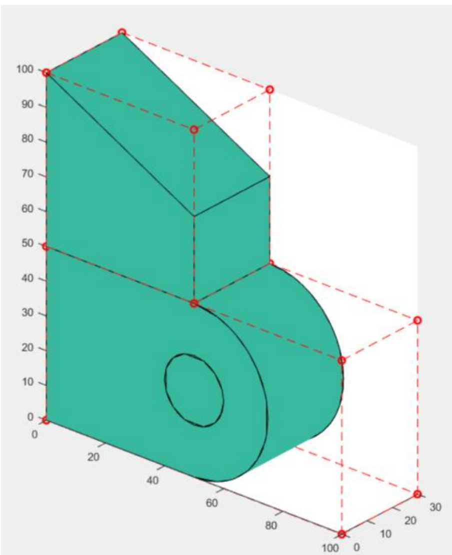
Export IGES to algorithm



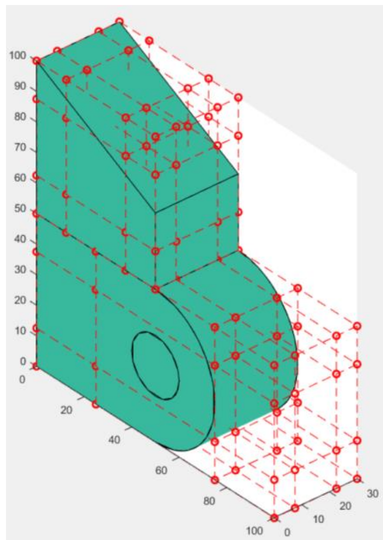
Boundary entities

04.- EXAMPLES

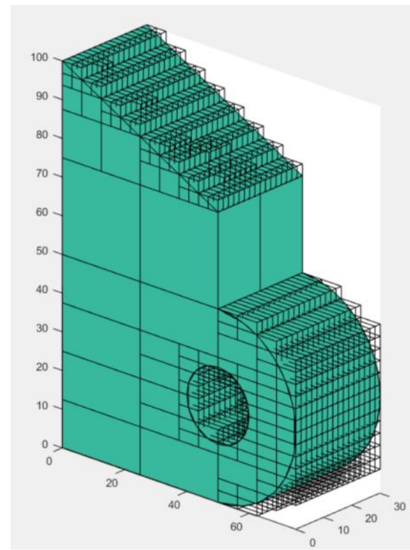
Example 1: IGA algorithm



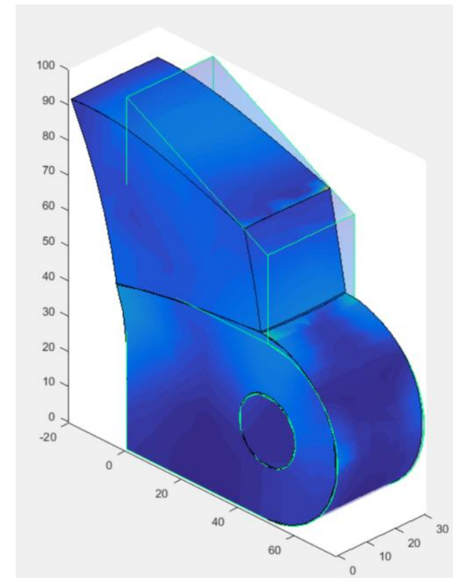
Solid parametrization



k-refinement



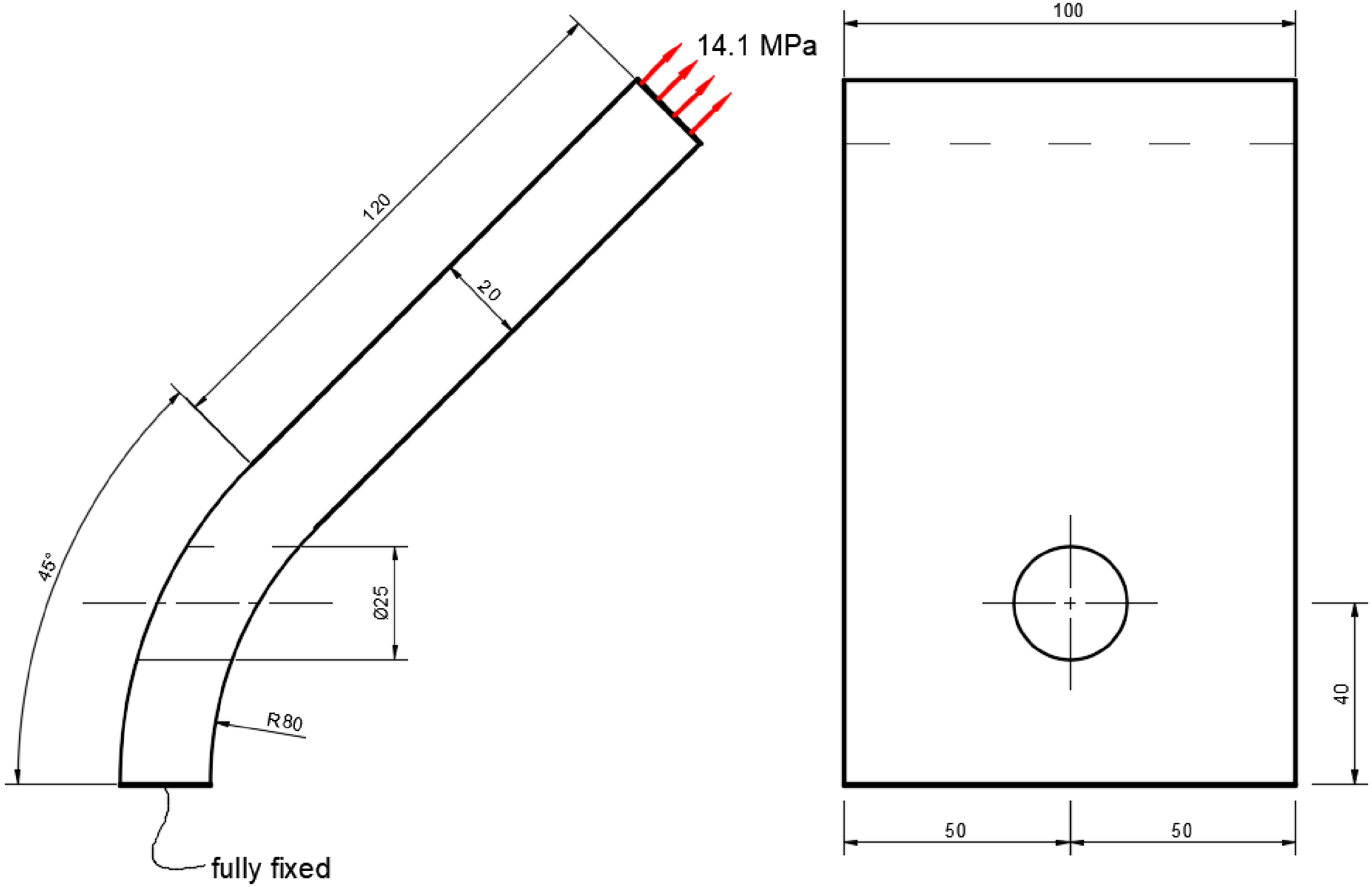
Integration mesh



Analysis result

04.- EXAMPLES

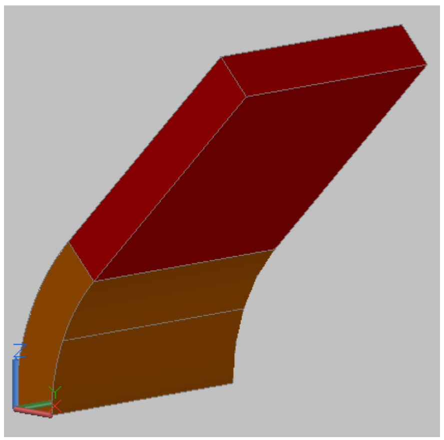
Example 2

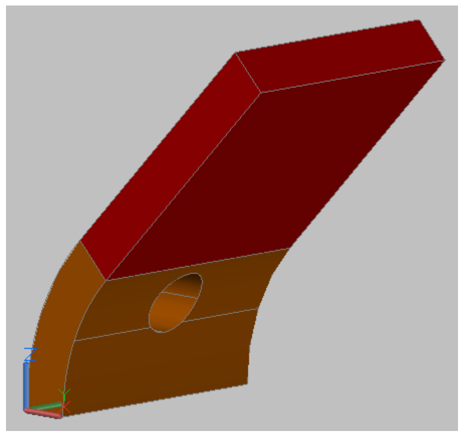


Isotropic material
E = 210000 MPa
ν = 0.30

04.- EXAMPLES

Example 2: CAD modelling

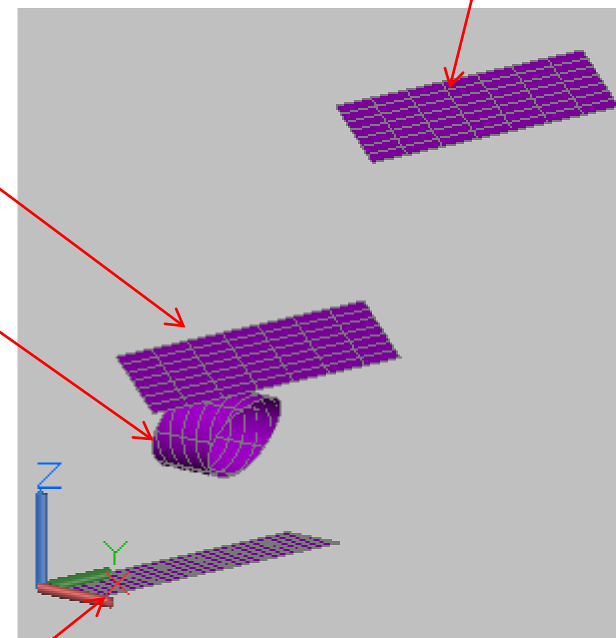




Neumann BC

coupling

trimming



Gross patches

Bounded patches

Export IGES to algorithm

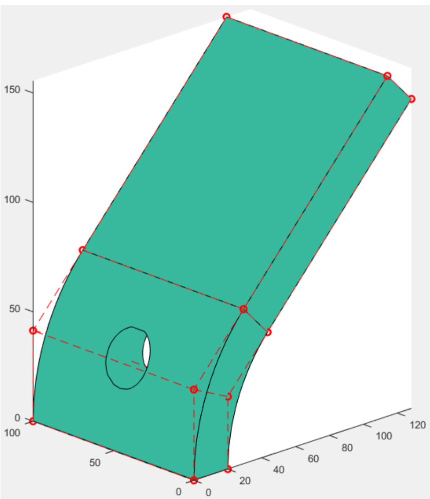
Export IGES to algorithm

Dirichlet BC

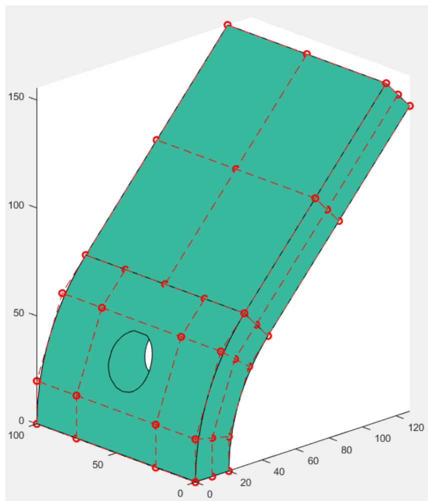
Boundary entities

04.- EXAMPLES

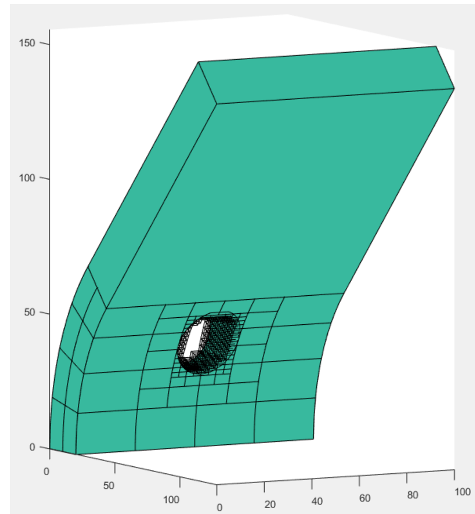
Example 2: IGA algorithm



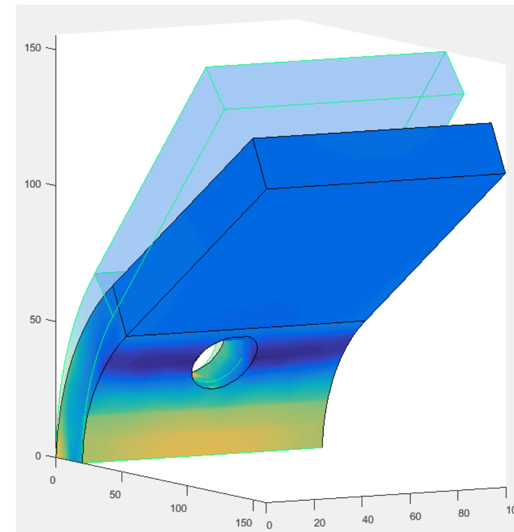
Solid parametrization



k-refinement



Integration mesh



Analysis result

05.- CONCLUSIONS

- Procedures to:

  1 - transform 3D solid domains from CAD into analysis suitable domains
  2 - construct any domain by applying coupling and BC's to trimmed surfaces

- Results seem in the right direction, BUT still testing and need validation

- Gross patches No. cases should be increased

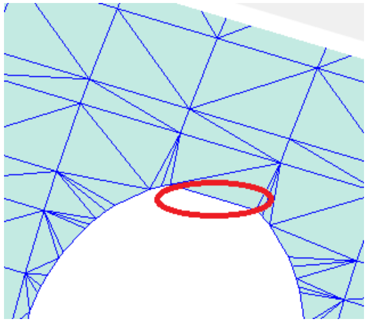
- Integration of trimmed domains is expensive

- Accuracy of integration of boundary entities can be improved

e.g.:
-gross patch with no linear direction
-No. faces different than 6

Alternatives:
-Improve elem's division techniques
-Tetrahedral

Use curved edge in the triangle trimmed side



% THANK YOU FOR

YOUR ATTENTION_