A Composite feedback approach to stabilize nonholonomic systems with time varying time delays and nonlinear disturbances

Somayieh Rasoolinasab a, Saleh Mobayen a, b, [[1]](#footnote-1), Afef Fekih c, Pritesh Narayan d, Yufeng Yao d

a Advanced Control Systems Laboratory, Department of Electrical Engineering, Faculty of Engineering, University of Zanjan, Zanjan 3879145371, Iran

b Future Technology Research Center, National Yunlin University of Science and Technology, 123 University Road, Section 3, Douliou, Yunlin 64002, Taiwan, R.O.C.

c Electrical and Computer Engineering Department, University of Louisiana at Lafayette, P.O. Box 43890, Lafayette, LA 70504, USA

d Department of Engineering Design and Mathematics, University of the West of England, Bristol, BS16 1QY, UK

**Abstract**: In this work, we propose a robust stabilizer for nonholonomic systems with time varying time delays and nonlinear disturbances. The proposed approach implements a composite nonlinear feedback structure in which a linear controller is designed to yield a fast response and a nonlinear feedback control law is considered to increase the system’s damping ratio. This structure results in the simultaneous improvement of the steady-state accuracy and transient performance of time-delay nonholomic systems. Asymptotic stability of the proposed feedback control approach is derived using a Lyapunov–Krasovskii functional aimed at reaching a compromise between system’s transient performance and asymptotic stability. Simulation and analytical results are considered to highlight the robustness and superior performance of the proposed approach in controlling high-order-time-delay nonholonomic systems with nonlinear disturbances.

Keywords: Composite nonlinear feedback, nonholonomic systems, time-delay, nonlinear disturbances.

# Introduction

Though system stabilization is widely considered in the literature, reaching a tradeoff between stability, steady-state performance and transient response for highly nonlinear time-delay systems is still a challenging problem [1]. For instance, in stabilization problems, the transient performance should not be overlooked. There is agreement among researchers that the transient response of adaptive systems is generally not acceptable due to the large initial swings in their performance [2]. Hence, various control approaches were proposed to improve systems’ transient stability performance [3].

However, a trade-off between settling time and overshoot persists in most of existing nonlinear control designs [4]. The Composite Nonlinear Feedback (CNF) approach was recently proposed to overcome this problem and improve transient performance by combining linear and nonlinear feedback controllers without switching components [5]. The linear component is implemented to ensure a fast response whereas the nonlinear portion gradually changes the damping ratio as the system’s output converges to zero whilst reducing the overshoot produced via the linear component. The CNF method was first introduced in [6] to modify the transient performance of the tracking controller of second-order linear systems with input-saturation. A CNF-based Integral Sliding Mode Control (ISMC) approach was proposed in [7] for fast and accurate robust-tracker and model-follower design of linear uncertain systems subject to time-delays and external disturbances. In [8], a two-term CNF control technique is proposed for nonlinear time-delay systems with input saturations. Reference [9] developed a CNF-based finite-time robust tracker for chaotic systems with external disturbances, Lipschitz nonlinearities and time delays.

Nonholonomic systems are a special class of nonlinear systems with non-integrable constraints on their velocities [10]. In other words, systems with constraints on their velocity those are not derivable from position constraints. Characteristics commonly found in various mechanical systems such as surface vessels, space vehicles, wheeled robots, to name a few. Designing stabilizers or trackers for nonholonomic robotic systems is a challenging problem since this class of systems is not controllable (linearly) around the equilibria and does not guarantee the necessary smooth-feedback stability condition (Brockett theorem) [11]. Hence, smooth-feedback controllers cannot stabilize these systems. Some discontinuous control schemes such as Sliding Mode Control (SMC), hybrid control and time-varying feedback [12] were developed to control this class of dynamical systems. In practice, nonholonomic systems are also prone to time delays. Since certain time delays can be a potential source of instability, they should be taken into consideration at the control design stage [13-15]. Thus, given the complex nature of nonholonomic systems, their stabilization and tracking continues to be an active research topic [16].

In [14], a control approach is suggested to asymptotically stabilize the chained input-delay nonholonomic systems via input-state scaling static gain controller methods. The tracking problem of a chained-form nonholonomic system was also considered in [17] using the *K*-exponential control technique. System stabilization was considered using the Linear Matrix Inequality (LMI) method assuming the system is free of disturbances and time-delays. In [18], output-feedback stability of nonholonomic system with time-delay is addressed. One distinguished feature of [18] is that time delays exist in polynomial nonlinear growing circumstances. The considered system of [18] has low nonlinearity and is free of perturbations. A recursive Terminal Sliding Mode (TSM) strategy was proposed in [19] for the tracking problem of a chained-form nonholonomic system with disturbances. The design ensured that state trajectory is forced to converge to the origin in finite time; however, the proposed design did not take into consideration time delays. The global stabilization problem for nonholonomic systems in chained-form under input delays was investigated in [20]. The approach considered a specific transformation to convert the original system into a delay-free form. However, the approach was limited to constant and time invariant time delays. A control approach that ensures the global asymptotic stability for a class of time delay nonholonomic systems was proposed in [1]. The control design process entailed relaxing the powers of the nonlinear terms and adopting a new Lyapunov-Krasovskii functional. The design along with the performance assessment was focused on the asymptotic stability of the system and neglected its transient behavior. Hence, in this paper, we propose to further expand that work and design a CNF approach that is able to simultaneously improve the system’s transient behavior whilst guaranteeing asymptotic stability. Additionally, the simultaneous presence of time-varying time delays and nonlinear disturbances will be considered in this paper, contrary to [1], which only dealt with time delays.

The main contributions of this work are as follows:

* The construction of a new power-integrator-based Lyapunov–Krasovskii functional that takes into consideration system’s transient performance and time-delays.
* A CNF control approach to counteract the effect of time delays and ensure both robust stabilization and performance improvement of the system.
* An output control structure that guarantees both steady-state accuracy and improved transient performance despite the time delays and nonlinear disturbances.

The remainder of the paper is organized as follows. The problem under consideration is formulated in section 2. The design procedure for the CNF-based approach for nonholonomic systems with time delays and external disturbances is detailed in section 3. Simulation results illustrating the performance of the proposed approach are given in section 4. Finally, some conclusions are provided in section 5.

# Problem formulation

Given the following nonholonomic system [1]

|  |  |
| --- | --- |
|  | (1) |

where ,  denote the states and control inputs, respectively;  refer to the state’s time-delay;  represent the time-varying control coefficients;  represent the system perturbations;  and  are positive odd integers; and  are time-delay nonlinear functions. Furthermore, assume the following conditions:

* + - 1. For each , there exist coefficients  and  with [1].
      2. The delay term  fulfills  for positive constants  and .
      3. For each  there exist continuous non-negative functions  and  with

|  |  |
| --- | --- |
|  | (2) |
| where , , and  are defined as |
| [18]. | (3) |

Designing the control input  entails some transformations in the dynamical equations. When  for any finite time, the following scaling transformation is considered:

|  |  |
| --- | --- |
|  | (4) |
|  | (5) |

where  is a coefficient, and  and  are such that:

|  |  |
| --- | --- |
|  | (6) |
|  | (7) |
| , | (8) |
|  | (9) |

Differentiating Eq. (4), one gets

|  |  |
| --- | --- |
| , | (10) |

where substituting (1) into (10), one obtains

|  |  |
| --- | --- |
|  | (11) |

Now, using (7) and (9), and defining

|  |  |
| --- | --- |
| , | (12) |

one has

|  |  |
| --- | --- |
|  | (13) |

On the other hand, using (4) and (8), one obtains

|  |  |
| --- | --- |
| , | (14) |

where differentiating (14) and using (1), one finds

|  |  |
| --- | --- |
| . | (15) |

The positive constant  can be obtained from (7) as

|  |  |
| --- | --- |
| . | (16) |

Now, defining  and using (15) and (16), one achieves

|  |  |
| --- | --- |
| . | (17) |

Assume

|  |  |
| --- | --- |
|  | (18) |

such that the term  can be written as

|  |  |
| --- | --- |
|  | (19) |

where using (5), one obtains

|  |  |
| --- | --- |
|  | (20) |

Now, the transformations are introduced as

|  |  |
| --- | --- |
|  | (21) |
|  | (22) |

where ,  and  are some constant values which will be determined later. The Lyapunov candidate functions are chosen as

|  |  |
| --- | --- |
| , | (23) |

with , where  and  can be described by

|  |  |
| --- | --- |
| , | (24) |
|  | (25) |

where  and  are two positive constants.

In what follows, we consider the state-observer proposed in [21]:

|  |  |
| --- | --- |
| , | (26) |

where  is the constant design parameter and  is the observer variable. Similar to [21], the reduced-order observer is adopted as

|  |  |
| --- | --- |
|  | (27) |
|  | (28) |

where  and  are the estimations of  and , respectively.

**Lemma 1 [22].** For  and , one has

|  |  |
| --- | --- |
|  | (29) |
|  | (30) |
|  | (31) |

**Lemma 2 [1, 18].** For the real numbers  guaranteeing  and some nonnegative functions , it yields

|  |  |
| --- | --- |
|  | (32) |

**Lemma 3 [1, 18].** For two positive numbers  and , and a function , a function  exists so that

|  |  |
| --- | --- |
|  | (33) |

**Lemma 4 [18].** If the initial state  satisfies , there exist two constants  and  for  such that

|  |  |
| --- | --- |
|  | (34) |

**Lemma 5 [1].** The subsequent inequalities are satisfied:

|  |  |
| --- | --- |
|  | (35) |
|  | (36) |

where  and  are two positive constants.

**Lemma 6 [23].** The Leibniz rule for differentiating an integral is

|  |  |
| --- | --- |
| . | (37) |

**Lemma 7 [22].** Assume that *c* and *d* are two positive numbers and the function , we have:

|  |  |
| --- | --- |
| . | (38) |

# Proposed approach

The proposed approach aims at implementing the CNF control paradigm for high-order nonholonomic systems with time-delays. The linear feedback input is designed to generate a quick dynamic response with small damping ratio, whereas the nonlinear feedback is designed to improve the damping ratio as system states approach the origin. Thus, it results in simultaneous improvement in both steady-state accuracy and transient performance.

In what follows, we proceed to design the control laws *u*0(*t*) and *u*1(*t*). First, the control law *u*0(*t*) is designed and used to analyze the stability of the state *x*0. Then control law *u*1(*t*) is synthesized to ensure the asymptomatic stability and performance improvement of the other states in the presence of time delays and disturbances.

## Design of the control law *u*0(*t*)

The linear feedback controller  and the nonlinear control law  are defined as

|  |  |
| --- | --- |
|  | (39) |
|  | (40) |

where combining these two parts, one obtains

|  |  |
| --- | --- |
| . | (41) |

The nonlinear function  in (40) is defined as

|  |  |
| --- | --- |
| , | (42) |

where  is upper bound of external disturbance (),  is a positive scalar, and  is a positive continuous (uniformly bounded) function with

|  |  |
| --- | --- |
|  | (43) |

where . The selection process of  will be discussed in the next subsection.

Now, considering  in (1) and constructing the Lyapunov function as

|  |  |
| --- | --- |
| , | (44) |

the time-derivative of  is obtained as

|  |  |
| --- | --- |
|  | (45) |

where . Then, using  as the upper bound of external disturbance, Eq. (45) can be written as

|  |  |
| --- | --- |
| . | (46) |

The value of *Q* is positive; hence, the first term in the last function is negative. In order to have a negative derivation of the Lyapunov function, the nonlinear function  is chosen as (42).

## Design of the control law *u*1(*t*)

For designing the virtual controllers in the next section, the nonlinear functions  and  are defined as

|  |  |  |
| --- | --- | --- |
|  | (47) | |
|  | | (48) |

with

|  |  |
| --- | --- |
|  | (49) |
|  | (50) |

where  and  are positive values,  and  are positive uniform continuous bounded functions and  which satisfy

|  |  |
| --- | --- |
|  | (51) |
|  | (52) |

where  and  are two positive constants. The selection procedure of  and  will be discussed later.

**Theorem 1:** For the time-delay nonholonomic system (1), considering the CNF control law (41) and output-feedback controller as

|  |  |
| --- | --- |
| . | (53) |

Then, using the CNF virtual controllers , the control laws  and  guarantee that the system states are bounded for any initial condition  and .

**Proof:** Assume . The proofs of the stability analysis for the subsystems  and  are presented in the following procedure:

***Step I. Stability analysis and controller design for subsystem x0***

Substituting  in (1), one obtains

|  |  |
| --- | --- |
| . | (54) |

Construct the Lyapunov functional as

|  |  |
| --- | --- |
| , | (55) |

where differentiating (55) and using (54), one attains

|  |  |
| --- | --- |
| . | (56) |

Substitution (41) into (56), one can achieve

|  |  |
| --- | --- |
| . | (57) |

where . From (57), one obtains

|  |  |
| --- | --- |
|  | (58) |

where substituting the nonlinear function (42) into (58), one has

|  |  |
| --- | --- |
| . | (59) |

Considering the fact that

|  |  |
| --- | --- |
|  | (60) |

the last term of Eq. (59) is less than , and hence one obtains

|  |  |
| --- | --- |
| . | (61) |

Besides, there exist positive coefficients  and  so that for every  one gets

|  |  |
| --- | --- |
| . | (62) |

From (61) and (62), it follows that

|  |  |
| --- | --- |
|  | (63) |

Since , one can obtain ; then, it follows from (63) that

|  |  |
| --- | --- |
|  | (64) |

Now, notice that for , one obtains

|  |  |
| --- | --- |
|  | (65) |

where from (64) and (65), one attains

|  |  |
| --- | --- |
| . | (66) |

Furthermore, taking the limit of the last term of (63) as time goes to infinity follows that

|  |  |
| --- | --- |
|  | (67) |

where from (65) and (67), one can obtain

|  |  |
| --- | --- |
| . | (68) |

It is confirmed from (66) that  is (uniformly) bounded. Because  is a continuous signal, the term  in (68) is also (uniformly) continuous. Using Barbalat lemma [24] on (68) gives

|  |  |
| --- | --- |
| . | (69) |

Since  is positive, one obtains

|  |  |
| --- | --- |
| . | (70) |

***Step II. Stability analysis and controller design for the x-subsystem***

Since , one can obtain from (41) that  for any . Then, using (4) and (5), the *x*-subsystem (1) is transformed into -subsystem (13) and (20). In what follows, for simplification of the controller design, one can assume that , where  and *p* are even and odd integers, respectively. First, a state-feedback control law is designed assuming that all the states are measurable; then, a suitable state-observer is constructed to design an output feedback controller.

The Lyapunov-Krasovskii functional is constructed as

|  |  |
| --- | --- |
| , | (71) |

the time-derivative of (71) fulfills

|  |  |
| --- | --- |
|  | (72) |

where using the time-derivatives of (24) and (25), it follows that

|  |  |
| --- | --- |
|  | (73) |

From (13) and (21), one obtains

|  |  |
| --- | --- |
|  | (74) |

where using Assumption 2, one gets

|  |  |
| --- | --- |
|  | (75) |

From Lemma 4, one deduces that

|  |  |
| --- | --- |
|  | (76) |

According to Cauchy Lemma [25], one obtains

|  |  |
| --- | --- |
|  | (77) |

where from (76) and (77), one obtains

|  |  |
| --- | --- |
|  | (78) |

From (3), the following inequality is obtained for :

|  |  |
| --- | --- |
|  | (79) |

where . It follows from (78) and (79) that

|  |  |
| --- | --- |
|  | (80) |

Now, from Lemma 7 and considering , one attains

|  |  |
| --- | --- |
|  | (81) |

Eq. (81) can be simplified as

|  |  |
| --- | --- |
|  | (82) |

where using Lemma 1, one gets

|  |  |
| --- | --- |
|  | (83) |

Eq. (83) can be rewritten with the virtual controller  as

|  |  |
| --- | --- |
|  | (84) |

with  which is defined as

|  |  |
| --- | --- |
| , | (85) |

where , and if the condition  is satisfied, then  is obtained.

Similarly, for , to proceed the controller design, there exist a candidate Lyapunov-Krasovskii function  as (23) and a virtual control law  with

|  |  |
| --- | --- |
|  | (86) |

where ’s are positive constants. Then, choosing , the time-derivative of  is attained as

|  |  |
| --- | --- |
| . | (87) |

Since  is a functional of , it follows from (13) and (24) that

|  |  |
| --- | --- |
|  | (88) |

Now, using (25), (86)-(88) and Lemma 1, one obtains

|  |  |
| --- | --- |
|  | (89) |

or equivalently

|  |  |
| --- | --- |
|  | (90) |

Now, using Lemma 5, it follows that

|  |  |
| --- | --- |
|  | (91) |

where . Eq. (91) can be rewritten as

|  |  |
| --- | --- |
|  | (92) |

For obtaining the Lyapunov stability condition (), the virtual controller is deduced as

|  |  |
| --- | --- |
| , | (93) |

where substituting derivative of (24) (with respect to ) into (93), one obtains

|  |  |
| --- | --- |
| . | (94) |

From Eq. (94) and Lemma 1, one achieves

|  |  |
| --- | --- |
| , | (95) |

or equivalently

|  |  |
| --- | --- |
| , | (96) |

which results that substituting (96) into (92), one finds .

|  |  |
| --- | --- |
|  | (97) |

When *k=n,* the time-derivative of Lyapunov-Krasovskii functional is attained from (92) as

|  |  |
| --- | --- |
|  | (98) |

where taking , the Lyapunov stability condition  is ensured.

Now, from (5) and (96), the designed output-feedback controller is obtained as (53) and all the state trajectories of the system are globally bounded, i.e. . Then, from (20) and (21), and considering , one obtains: .

In what follows, the CNF-based virtual control function  is presented as

|  |  |
| --- | --- |
| , | (99) |

where . Substituting (99) into (84) follows that

|  |  |
| --- | --- |
|  | (100) |

Substituting the nonlinear function (47) into (100) and considering (60), one obtains

|  |  |
| --- | --- |
|  | (101) |

There exist positive constants , , ,  such that for every, it follows:

|  |  |
| --- | --- |
| , | (102) |
| , | (103) |

where from (23), (102) and (103), one can obtain

|  |  |
| --- | --- |
| . | (104) |

From (101) and (104), it follows that

|  |  |
| --- | --- |
|  | (105) |

Because  and  are both positive, then the expressions  and  are negative; hence

|  |  |
| --- | --- |
| . | (106) |

Now, notice that for , one gets

|  |  |
| --- | --- |
| , | (107) |

where from (106) and (107), one attains

|  |  |
| --- | --- |
| . | (108) |

On the other hand, taking the limit of the last term of (105) as time goes to infinity, one achieves

|  |  |
| --- | --- |
|  | (109) |

where from (107) and (109), one can find

|  |  |
| --- | --- |
| . | (110) |

It can be deduced from (110) that  and  are uniformly bounded. Since  and  are continuous, the terms  and  in (110) are also (uniformly) continuous. If Barbalat lemma is applied to (110), it gives

|  |  |
| --- | --- |
|  | (111) |

where since  and  are positive, it results that

|  |  |
| --- | --- |
| , | (112) |
| . | (113) |

The same procedure for the stability analysis can be considered for . □

## Procedure for selecting Ω

The main function of the nonlinear term is to accelerate the settling time and hence improve the system’s speed of response. When the norms of the system states are small, a significant amount is contributed to the linear control signal. The selection procedure of a suitable nonlinear function  is the central problem of the CNF control design. The function  is required to be selected such that:

1. Since  acts over the absolute value of  and should satisfy (107), then it follows that .
2. When the states  are far away from origin,  will have high value such that the term  will become small, hence making the contribution of the nonlinear portion insignificant.
3. When the states converge to zero,  converges to a low value such that  will be large, thus increasing the significance of the nonlinear portion of the control design.

The nonlinear portion  is not unique and one can define it in numerous procedures. In this article, the function  is described in exponential form as

|  |  |
| --- | --- |
| , | (114) |
| , | (115) |

where , ,  and  are some positive tuning coefficients. The function  reaches the maximum amount  when  increases and approaches to minimum amount (zero) while  converges to the origin.

The design procedure can be illustrated using the flowchart illustrated in Fig.1.

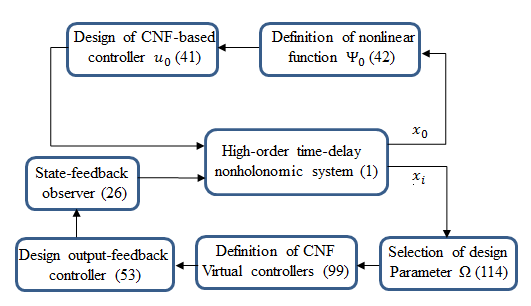


Fig.1. Flowchart of the proposed approach

# Simulation results

**Example.** Consider the following nonholonomic system with time delays and nonlinear disturbances [1]:

|  |  |
| --- | --- |
|  | (116) |

where  are system states;  and  are external disturbances;  and  are control inputs;  and  are time-varying delays. This example is simulated in MATLAB® Simulink® (2016b) and run on an Intel® Core i7- 6700K processor with 2 GB of memory.

The following coefficients were considered: , , , , , , , , , , , , , ,  and . The initial states are taken as , ,  and . (41), (53) and (99), yield the following control inputs:

|  |  |
| --- | --- |
|  | (117) |
|  | (118) |

with , where , , ,  and .

Two cases were considered in assessing the performance of the proposed control framework. Additionally, a comparison analysis to the approach proposed in [1] was also carried over.

*Case 1: Performance in the presence of time-varying time delays:*

In this case, the system is subjected to the following time-varying time delays:. The time histories of the states obtained in this case are ilustrated in Fig. 2. Note that the propose approach is able to properly mitigate the effect of time delays. Note also that the proposed approach is able to steer the states to the origin faster than the approach proposed in [1]. The control inputs for both approaches are depicted in Fig. 3. Note that the proposed controller requires less control effort compared to the approach proposed in [1]. The observer’s dynamics are highlighted in Fig. 4. As can be seen from Fig. 4, the proposed observer has faster low-frequency responses compared to the approach proposed in [1].



Fig. 2: Dynamics of the system’s states.



Fig. 3: Time histories of the control inputs.



Fig. 4: Time trajectories of observer’s variables.

To further assess the performance of both controller, we consider the Integral of Absolute-value of Error (IAE) as performance index:

|  |  |
| --- | --- |
|  | (119) |

where  is a signal in time domain. The obtained values of IAE () and settling time () are depicted in Table 1 for both the proposed approach and the one outlines in [1]. Note that the proposed approach yields smaller IAE and settling time values than the approach depicted in [1]. For instance, the improvements of IAE and settling time values for the state  using the proposed controller are 73.2% and 72.46%, respectively; whereas, the improvements of IAE values for the controller signals  and  are 35.86% and 13.9%, respectively.

Table 1: Performance indices (IAE and settling time values)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |
| Method in [1] | 1.2891 | 0.0559 | 0.8157 | 1.2891 | 3.2872 | 6.9 *s* | 1.4 *s* | 2.6 *s* |
| Proposed method | 0.3457 | 0.0638 | 0.3504 | 0.8268 | 2.8302 | 1.9 *s* | 0.8 *s* | 1.4 *s* |

*Case 2: Performance in the simultaneous presence of time-varying time delays and nonlinear disturbances:*

To further assess the robustness of the proposed approach, we subject the system to both time-varying delays and disturbances. Hence, we consider , , ,  and , ,  and . The obtained dynamics for the states, control signals and observer variables, in this case, are depicted Fig. 5 through Fig. 7, respectively. Note from Fig. 5 that the system states controlled with the proposed approach exhibit lower overshoot and settling time than the approach highlighted in [1]. Fig. 6 shows that the proposed control inputs are faster than the controllers of [1]. Fig. 7 illustrates that the proposed observers have lower frequency responses and smaller settling time compared to the approach proposed in [1]. Hence, the proposed approach is faster and more effective at controlling nonholonomic systems with time delays and disturbances that the approach proposed in [1].



Fig. 5: Time responses of system states.



Fig. 6: Time responses of controller signals.



Fig. 7: Time trajectories of observer variables.

A comparison between the IAE and settling time values obtained using the proposed approach compared to those obtained using the approach in [1] are illustrated in Table 2.

Table 2: The values of performance indices

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |
| Method in [1] | 1.2891 | 0.0705 | 2.0008 | 1.2891 | 6.4690 | 7 *s* | 2.5 *s* | 5.3 *s* |
| Proposed approach | 0.3457 | 0.0638 | 0.2987 | 0.8268 | 3.1612 | 1.2 *s* | 0.8 *s* | 1.2 *s* |

Note the reduction in IAE and settling time values when using the proposed control approach compared to the method in [1]. More specifically, the improvements of IAE values for the , , ,  and  are 73.18%, 9.5%, 85.07%, 35.86% and 51.13%, respectively, wheras the improvements of settling times for the states ,  and  using the proposed controller are 82.85%, 68% and 77.35%, respectively.

It is concluded from these simulations and analytical results that the proposed control approach exhibits robust performance in the simultaneous presence of time-varying delays and external disturbances.

# Conclusion

This paper proposed a control paradigm based on the composite nonlinear feedback control approach for nonholonomic systems with time-delays and external disturbances. To stabilize such systems and ensure the asymptotic convergence of the state trajectories to zero in the presence of external disturbances and time-varying time delays both linear and nonlinear feedback terms are synthesized. The linear term is designed to generate a quick dynamic response with small damping ratio, whereas the nonlinear feedback law is used to improve the damping ratio as system states approach the target reference. A robust stabilizer was synthesized to ensure the global asymptotic stability of nonholonomic systems by constructing a power-integrator-based Lyapunov–Krasovskii functional. Simulation results showed simultaneous improvement in both steady-state accuracy and transient performance. Comparison of IAE and settling times to the results of the method proposed in [1] showed also superior performance. Thus, the proposed approach is very effective at stabilizing highly complex nonholonomic and under-actuated systems. Extending the results to high-order nonholonomic systems with multiple time-varying input-delays will be the focus of our future work.

References

1. Liu Z-G, Wu Y-Q, Sun Z-Y. Output feedback control for a class of high-order nonholonomic systems with complicated nonlinearity and time-varying delay. Journal of the Franklin Institute. 2017, 354, 4289-310.

2. Sun W, Zhang Y, Huang Y, Gao H, Kaynak O. Transient-performance-guaranteed robust adaptive control and its application to precision motion control systems. IEEE Transactions on Industrial Electronics. 2016, 63, 6510-8.

3. Zhu D, Zou X, Zhou S, Dong W, Kang Y, Hu J. Feedforward current references control for DFIG-based wind turbine to improve transient control performance during grid faults. IEEE Transactions on Energy Conversion. 2017, 33, 670-81.

4. Aghababa MP. Sliding-mode control composite with disturbance observer for tracking control of mismatched uncertain nDOF nonlinear systems. IEEE/ASME Transactions on Mechatronics. 2017, 23, 482-90.

5. Lu T, Lan W. Composite nonlinear feedback control for strict-feedback nonlinear systems with input saturation. International Journal of Control. 2019, 92, 2170-7.

6. Lin Z, Pachter M, Banda S. Toward improvement of tracking performance nonlinear feedback for linear systems. International Journal of Control. 1998, 70, 1-11.

7. Majd VJ, Mobayen S. An ISM-based CNF tracking controller design for uncertain MIMO linear systems with multiple time-delays and external disturbances. Nonlinear Dynamics. 2015, 80, 591-613.

8. Singh S, Purwar S, Kulkarni A. Two term composite nonlinear feedback controller design for nonlinear time-delay systems. Transactions of the Institute of Measurement and Control. 2018, 40, 3424-32.

9. Mobayen S, Ma J. Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay. Chaos, Solitons & Fractals. 2018, 114, 46-54.

10. Gu J, Li W, Yang H. Distributed adaptive control for multiple nonholonomic systems with nonlinearly parameterized uncertainties. International Journal of Adaptive Control and Signal Processing. 2019, 33, 747-66.

11. Park BS, Yoo SJ. A low-complexity tracker design for uncertain nonholonomic wheeled mobile robots with time-varying input delay at nonlinear dynamic level. Nonlinear Dynamics. 2017, 89, 1705-17.

12. Janiak M, Tchoń K. Constrained motion planning of nonholonomic systems. Systems & Control Letters. 2011, 60, 625-31.

13. Hassan A, Torres-Perez A, Kaczmarczyk S, Picton P. The effect of time delay on control stability of an electromagnetic active tuned mass damper for vibration control. Journal of Physics: Conference Series. IOP Publishing2016. p. 012007.

14. Chen X, Zhang X, Zhang C, Chang L. Global asymptotic stabilization for input-delay chained nonholonomic systems via the static gain approach. Journal of the Franklin Institute. 2018, 355, 3895-910.

15. Jenabzadeh A, Safarinejadian B. Tracking control of nonholonomic mobile agents with external disturbances and input delay. ISA transactions. 2018, 76, 122-33.

16. Jenabzadeh A, Safarinejadian B, Binazadeh T. Distributed tracking control of multiple nonholonomic mobile agents with input delay. Transactions of the Institute of Measurement and Control. 2018, 0142331218771143.

17. Cao K-C. Global K-exponential trackers for nonholonomic chained-form systems based on LMI. International Journal of Systems Science. 2011, 42, 1981-92.

18. Wu Y-Q, Liu Z-G. Output feedback stabilization for time-delay nonholonomic systems with polynomial conditions. ISA transactions. 2015, 58, 1-10.

19. Mobayen S. Finite-time tracking control of chained-form nonholonomic systems with external disturbances based on recursive terminal sliding mode method. Nonlinear Dynamics. 2015, 80, 669-83.

20. Shang Y, Xie J. Global stabilization of nonholonomic chained form systems with input delay. Abstract and Applied Analysis. Hindawi2014.

21. Lei H, Lin W. Robust control of uncertain systems with polynomial nonlinearity by output feedback. International Journal of Robust and Nonlinear Control. 2009, 19, 692-723.

22. Shang Y, Yuan Y. Global Asymptotic Stabilization for a Class of High-Order Nonholonomic Systems with Time-Varying Delays. International Journal of Applied Mathematics. 2017, 47.

23. Sousa JVdC, de Oliveira EC. Leibniz type rule: ψ-Hilfer fractional operator. Communications in Nonlinear Science and Numerical Simulation. 2019, 77, 305-11.

24. Souahi A, Naifar O, Makhlouf AB, Hammami MA. Discussion on Barbalat Lemma extensions for conformable fractional integrals. International Journal of Control. 2019, 92, 234-41.

25. Wigren T. The Cauchy-Schwarz inequality: Proofs and applications in various spaces. 2015.

1. Corresponding Author, E-mail address: [mobayen@znu.ac.ir](mailto:mobayen@znu.ac.ir), Tel.:+98 24 33054219. [↑](#footnote-ref-1)