ARTICLE TEMPLATE

Adaptive security control for uncertain delayed semi-Markov jump systems subject to cyber attacks and actuator failures

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ABSTRACT
This note investigates the adaptive security control issue for uncertain delayed semi-Markov jump systems (DSMJSs) within the framework of sliding mode control (SMC), in which the DSMJSs are affected by generally unknown transition rates (GUTRs), actuator failures (AFs) and cyber attacks. By virtue of the strong approximation ability of neural network (NN), an adaptive NN-based SMC synthesis is carried out, which could not only force the state trajectories onto the proposed sliding surface but also ensure the DSMJSs operate as demanded in spite of the interference errors, structural uncertainty, hidden AFs, cyber attacks and GUTRs. Then, in view of the reachability of the proposed linear-type sliding mode surface (SMS), linear matrix inequalities (LMIs) and stochastic stability theory, a novel stochastically stable criterion for the resultant DSMJSs is obtained. At last, the single-link robot arm model is offered as an instance with simulation to illustrate the viability of the devisedstrategy.

KEYWORDS
Adaptive neural networks; sliding mode control; delayed semi-Markov jumping systems; generally unknown transition rates; actuator failures; cyber attacks

1. Introduction

Over the past decades, some steps towards adaptive nonlinear control for switched systems have been followed (e.g., the mode-dependent average dwell time switching in Cui, Ahn, and Xiang (2023), the arbitrary switching in Cui, Wu, and Xiang (2021) and the asynchronous switching in Li, Ahn, and Xiang (2018), etc). In comparison to these switching mechanisms, Markov jump system (MJS), as a typical sort of hybrid system, can modela wide range of physical plants with abrupt changes in structures and network interactions by virtue of a special stochastic jump mechanism (Li and Xiang, 2016; Liu, Chen, and Yu, 2023a; Sheng, Zhang, and Gao, 2014), where the switching signals are driven by a Markov chain. Distinct from the MJSs in which the transition time must rely on exponential distribution for the time-varying transition rates (TRs), semi-Markov jump systems (s-MJSs) are more suitable for describing the mutation situation of the actual systems, and a lot of efforts have thrown some light on the control approaches to s-MJSs, see Ouaret (2022); Zhang, Niu, Zhao, Zhao, and Yang (2021); Zong, Qi, and Karimi (2020). Furthermore, as a crucial role in the

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transition process between different system modes, TRs can be regarded as the main components for s-MJSs, and several researches within totally known TRs havebeen reported, see Zhang et al. (2021); Zhao, Niu, and Song (2023a); Zong et al. (2020). Nevertheless, the aforementioned works with known TRs may be too ideal. As a result, several noteworthy conclusions of control schemes for s-MJSs with partly unknown TRs were obtained (Liang, Zhang, Karimi, and Zhou, 2018; Zhang, Sun, Pan, and Lam, 2022). However, given the abrupt jumps of the system modes and occurrence of unknown uncertainties, it might be too restrictive to pay attention to partly unknown TRs in actual plant. Then, control study for s-MJSs with generally unknown transition rates (GUTRs) is more significant to model the relevant physical plants, and some remarkable control strategies have been reported in this direction (Jiang, Kao, Karimi, and Gao, 2018; Xu, Gao, and Qi, 2021). Meanwhile, limited by the construction of the system, the existence of time-delay in a dynamical system has been seen as a major factor of instability and/or subpar performance (Gu, Zhu, and Nouri, 2022; Sename, 2003). Thus, some steps towards both MJS and s-MJS with time-delay were pursued (Samidurai, Manivannan, Ahn, and Karimi, 2016; Yan, Tian, Li, Zhang, and Li, 2019; Zhu, Zhang, Sreeram, Shammakh, and Ahmad, 2016). Meanwhile, great attention has been attracted on control problems for the delayed s-MJSs (DSMJSs) subject to GUTRs recently, such as observer based anti-interference control (Xu et al., 2021) and sliding mode control (SMC) in Qi, Park, Cheng, and Kao (2017).

As a typical nonlinear control strategy, SMC has garnered significant attention in the light of its ability to handle disturbances and uncertainties (Liu, Wu, Wu, Luo, and Franquelo, 2019; Liu, Yu, and Lam, 2023b; Wu and Ho, 2010; Zhao, Liu, Jiang, and Gao, 2023b; Zhao, Yang, Xia, and Wang, 2016). In particular, in the light of the aforementioned superiorities, some results have been absorbed into analysis and synthesis of uncertain s-MJSs based upon SMC strategy (Song, Niu, Lam, and Zou, 2020; Wei, Park, Qiu, Wu, and Jung, 2017). In Qi, Zong, and Zheng (2020), an adaptive event-triggered SMC method was achieved to accomplish the desirable behavior for DSMJSs. Moreover, as a frequently occurring kind of disturbances to system operation, the failure issue may lead to inevitable parameter deviation andresultin unpredictable errors, which might cause serious obstacles to the operation of actuator. Consequently, meaningful fruits on actuator failures (AFs) were provided in some literature, see Song, Niu, and Zou (2018); Xing, Wen, Liu, Su, and Cai (2017); You, Yan, Sun, Zhang, and Li (2020); Zhu, Mobayen, Nemati, Zhang, and Wei (2023). Just recently, various attempts fors-MJSs impacted by AFs wereconducted (Jiang, Kao, Gao, and Yao, 2017; Li, She, Cheng, Shi, Peng, and Zhong, 2022). Furthermore, given the continuing advancement of networked embedded control technologies, the actuators are more susceptible to cyber attacks (Meng, Niu, Ding, and Zhao, 2018). In this case, attackers may take advantage of the flaws in communication protocols to tamper with data transmission through signal channels, which will lead to inevitable delays in s-MJS, and further degrade the system performance. For these reasons, some researches have been devoted to the issue of DSMJSs against actuator attacks (AAs) in recent years, see(Cao, Niu, and Zou, 2019; Wang and Ma, 2023). Nevertheless, the control methods against AAs in the above literature were not fully taken under the case of GUTRs and the norm-bounded assumption of the attack signals must be satisfied, which may bring conservatism for controller design to some extent and forms one impetus of this work.

Regarding an additional analytical aspect, in view of the approximation capability to unknown attack signals, neural networks (NNs) have received widespread researches, and some attention was paid toNN-based control methodsfor resisting the cyberattacks, see Cao et al. (2019). Additionally, the integralsliding mode surfaces (SMSs) and the corresponding sliding mode controller were established in most existing studies to cope with the issues of uncertain s-MJSs, see Song et al. (2020); Wei et al. (2017). Nevertheless, it is worth pointing out that fewer efforts have been dedicated to the DSMJSs with linear-type sliding surface design. Therefore, the topic combining adaptive NN and SMC to deal with the security control problem of DSMJSs still has much research space, which finally stimulates us to make small progress in this paper.

By employing an adaptive NN-based SMC method, this paper attemptsto address the security issue of uncertain DSMJSs with GUTRs,AFs and AAs. In brief, the main outcomes are listed below:

1) Differing from Liu et al. (2019), a simplified linear-type sliding mode surface (SMS) is established, from which an novel adaptive SMC framework for uncertain DSMJSs is introduced, and a new sufficient condition for the closed-loop system to be stochastically stable is derived from a set of linear matrix inequalities (LMIs), stochastic stability theory and the specified structure of the designed SMS;

2) In comparison to Wei et al. (2017), a novel NN-based sliding mode controller is synthesized to guarantee both the finite-time reachability of the proposed SMS and robustness performance despite unknown AAs and GUTRs, where the norm-bounded assumption for unknown nonlinear attacks in previous reported works such as Qi, Lv, Zong, and Ahn (2021); Song et al. (2018) will not be required.

3) The effectiveness of the proposed theoretical results has been verified by simulation compared with method in Qi et al. (2021), which shows that the proposed control approach could enhance the control accuracy with less energy consumption (EC).

Notation. Throughout the paper, for any matrix $G \in \mathcal{R}^{n\times n}$, G^{T} and G^{-1} denote its transpose and inverse respectively. $\mathrm{tr}\{G\}$ denotes the trace of G, the diagonal matrix is denoted by $\mathrm{diag}\{G_1, G_2, \cdots, G_m\}$ with diagonal matrices $G_1, G_2, \cdots, G_m$, and $\mathrm{sym}\{G\}$ is used to denote $G+G^{\mathrm{T}}$. $\mathcal{E}\{\cdot\}$ is used to denote the expectation operator. $|\cdot|$ denotes the absolute value of a real number, $\|\cdot\|_1$ and $\|\cdot\|$ denote the 1-norm and 2-norm of a matrix G. $\mathrm{sgn}\{\cdot\}$ denotes the sign function.

2. Problem description and preparations

The DSMJS considered in this paper is described as:

$$\begin{aligned}\dot{x}(t) = &\left(A_t(r_t) + \Delta A_t(r_t,t)\right)x(t) + \left(A_\tau(r_t) + \Delta A_\tau(r_t,t)\right)x(t-\tau)\\ &+ B(r_t)\left(\mu(t,u) + \Phi(x,t,\tau)\right),\end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathcal{R}^n$ denotes the state vector, τ is the system constant time-delay. $A_t(r_t) \in \mathcal{R}^{n\times n}$, $A_\tau(r_t) \in \mathcal{R}^{n\times n}$ and $B(r_t) \in \mathcal{R}^{n\times m}$ are fixed matrices. $\Delta A_t(r_t,t)$ and $\Delta A_\tau(r_t,t)$ are the model uncertainties of the system given by the following forms:

$$\Delta A_t(r_t,t) = \mathcal{M}(r_t)Y(t)\mathcal{N}(r_t),\ \Delta A_\tau(r_t,t) = \mathcal{M}(r_t)Y(t)\mathcal{N}_\tau(r_t),$$

where $\mathcal{M}(r_t)$, $\mathcal{N}(r_t)$ and $\mathcal{N}_\tau(r_t)$ are known matrices, and the time-dependent unknown parametric matrix is denoted by $Y(t)$. $\Phi(x,t,\tau)$ indicates the uncertain perturbations such as noise interference in communication channels injected by malicious attackers. The following mathematical model stands for the AFs: $\mu(t,u) = \Gamma(t)\,u(t)$, and $u(t)$ represents the control input. $\Gamma(t) \in \mathcal{R}^{m\times m}$ denotes an uncertain function, which means the damaged condition of the AFs with $\varGamma(0) = 0$, and the specific expression is defined as the following diagonal matrix:

$$\varGamma(t) = \operatorname{diag}\{\eth_1(t), \eth_2(t), \ldots, \eth_m(t)\}, \tag{2}$$

in which $0 \leqslant \underline{\eth}_j \leqslant \eth_j(t) \leqslant \bar{\eth}_j$, the known boundaries of $\eth_j(t)$ $(j = 1, 2, \ldots, m)$ are represented by $\underline{\eth}_j \in (0,1)$ and $\bar{\eth}_j \in (0,1)$. Meanwhile, we define $\varGamma_0 \triangleq \operatorname{diag}\{\eth^1{}_0, \eth^2{}_0, \ldots, \eth^m{}_0\}$ with $\eth^j{}_0 \triangleq (\underline{\eth}_j + \bar{\eth}_j)/2$, $\wp_j(t) \triangleq (\eth_j(t) - \eth^j{}_0)/\eth^j{}_0 \in [-\vartheta_j, \vartheta_j]$, in which $\vartheta_j = (\bar{\eth}_j - \underline{\eth}_j)/(\bar{\eth}_j + \underline{\eth}_j)$, $\varLambda(t) \triangleq \operatorname{diag}\{\wp_1(t), \wp_2(t), \ldots, \wp_m(t)\}$. $\varGamma(t)$ can be represented by $\varGamma(t) = \varGamma_0(I + \varLambda(t))$, in which γ is given as a threshold meeting $\|\varLambda(t)\| \leqslant \gamma$. Therefore, $\|\varGamma(t)\| = \|\varGamma_0(I + (\varLambda(t)))\| \leqslant \zeta$ can be obtained, where $\zeta > 0$ is a bounded scalar, thus one has $\frac{1}{\zeta}\|\varGamma(t)\| \leqslant 1$. $\{r_t, t \geq 0\}$ represents the semi-Markov process in $\mathcal{J} = \{1, 2, ..., J\}$, and the jump of r_t is subjected to the transition probability matrix $\Pi = [\eta_{ij}]_{J\times J}(i, j \in \mathcal{J})$ characterized with

$$\mathrm{P}_{ij} = \Pr(r_{t+\Delta t} = j | r_t = i) = \begin{cases} \eta_{ij}\Delta t + o(\Delta t), & \text{if} \quad i \neq j, \\ 1 + \eta_{ii}\Delta t + o(\Delta t), & \text{if} \quad i = j. \end{cases}$$

Herein, the TR is defined by $\eta_{ij} = \hat{\eta}_{ij} + \varDelta\eta_{ij}$, where η_{ij} stands for the TR from time t with mode i to time $t + \Delta t$ with mode j, in which $i \neq j$, $\eta_{ii} = -\sum_{j=1, j\neq i}^{J} \eta_{ij}$. $\hat{\eta}_{ij}$ stands for the determinable part and $\Delta\eta_{ij}$ refers to the corresponding uncertainty part with $|\Delta\eta_{ij}| \leqslant \beta_{ij}$. Then, the following two index sets are employed for assessing the unknown TRs:

$$\begin{aligned} {\Omega_k}^i &\triangleq \{j : \eta_{ij} \text{ can be available for } j \in \mathcal{J}\}, \\ {\Omega_{uk}}^i &\triangleq \{j : \eta_{ij} \text{ can not be available for } j \in \mathcal{J}\}. \end{aligned} \tag{3}$$

In order to explore the unknown TRs in different cases, for $\forall i \in \mathcal{J}$, the TRs are divided into four categories:

(1) $i \in {\Omega_k}^i$ and $j \in {\Omega_{uk}}^i \neq \oslash$;
(2) $i \in {\Omega_{uk}}^i$ and ${\Omega_k}^i \neq \oslash$;
(3) ${\Omega_{uk}}^i = \mathcal{J}$ and $j \in {\Omega_k}^j$ for some $j \in \mathcal{J}$;
(4) ${\Omega_{uk}}^i = \mathcal{J}$ and $j \notin {\Omega_k}^j$ for any $j \in \mathcal{J}$.

Let $r_t = i \in \mathcal{J}$, then $A_t(r_t)$, $\Delta A_t(r_t, t)$, $A_\tau(r_t)$, $\Delta A_\tau(r_t, t)$, $B(r_t)$, $\mathcal{M}(r_t)$, $\mathcal{N}(r_t)$ and $\mathcal{N}_\tau(r_t)$ can be simplified to A_i, $\Delta A_i(t)$, $A_{i\tau}$, $\Delta A_{i\tau}(t)$, $\mathcal{M}_i$, $\mathcal{N}_{it}$ and $\mathcal{N}_{i\tau}$. As a result, system (1) is written as:

$$\begin{aligned} \dot{x}(t) = &(A_{it} + \varDelta A_{it}(t))\, x(t) + (A_{i\tau} + \varDelta A_{i\tau}(t))\, x(t-\tau) \\ &+ B_i(\varGamma(t)\, u(t) + \varPhi(x, t, \tau)). \end{aligned} \tag{4}$$

In addition, based on the approximation capacity of NN, the AA signal $\varPhi(x, t, \tau)$ can be reconstructed as

$$\varPhi(x, t, \tau) = W^{\mathrm{T}}\varphi\left(\mathcal{H}^{\mathrm{T}}\bar{\mathbb{X}}\right) + \phi(x), \tag{5}$$

where $\bar{\mathbb{X}} = \left[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau), -1\right]^{\mathrm{T}}$ is the input signal to NN, and "-1" stands for the input offset. $\phi(x)$ represents the error vector, which fulfills $\|\phi(x)\| \leqslant \phi_0$ with $\phi_0 \geqslant 0$ being an arbitrary constant. $\mathcal{H} \in \mathcal{R}^{(2n+1)\times p}$ and $W \in \mathcal{R}^{p\times m}$ represent the optimization weight matrix of both input-to-hidden layer and hidden-to-output layer respectively, from which the hidden layer of NN is designed with p neurons, and the threshold of $\mathcal{H}$ is set as:

$$\varphi\left(x_l\right)=\frac{1}{1-e^{-\ell_l x_l}}, \quad l=1,2,\cdots,p, \tag{6}$$

with $\ell_l>0$. By reason of the unknown matrices $\mathcal{H}$ and W, the adaptive algorithm is developed while designing the NN-based SMC law. $\tilde{W}$ and $\tilde{\mathcal{H}}$ are denoted by $\tilde{W}=\hat{W}-W$ and $\tilde{\mathcal{H}}=\hat{\mathcal{H}}-\mathcal{H}$ respectively, and properties of the above NN mathematical model are given in the Lemmas as follows.

Remark 1. The type of AAs mentioned in this note can be divided into the false-data-injection (FDI) attacks (Li, Guo, Xia, and Yang, 2020). In such attacks, the false data can impede the actuator's functionality by injecting false information into the signal channel by hidden attackers, which further results in instability and/or breakdown of the DSMJSs. Furthermore, the FDI attacks may currently occur in practical engineering, e.g., cyber-physical systems, DC motor, power system and other engineering fields (An and Yang (2018); Cao et al. (2019); Liu et al. (2023a); Qi et al. (2021); Yang, Zhang, and Guo (2022); etc).

Lemma 2.1 (Yeşildirek and Lewis (1995)). *The attack signal estimation is written as* $\hat{\Phi}(x,t,\tau)=\hat{W}^{\mathrm{T}}\varphi(\hat{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}})$, *and the approximated error* $\tilde{\Phi}(x,t,\tau)=\hat{\Phi}(x,t,\tau)-\Phi(x,t,\tau)$ *is rebuilt as*

$$\tilde{\Phi}\left(x,t,\tau\right)=\tilde{W}^{\mathrm{T}}\left(\hat{\varphi}-\hat{\varphi}'\hat{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}\right)+\hat{W}^{\mathrm{T}}\hat{\varphi}'\tilde{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}+v\left(x\right), \tag{7}$$

where $v(x)$ *is the residual value with the representation of*

$$v\left(x\right)=\tilde{W}^{\mathrm{T}}\hat{\varphi}'\mathcal{H}^{\mathrm{T}}\bar{\mathbb{X}}+W^{\mathrm{T}}o\left(\tilde{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}\right)-h\left(x\right), \tag{8}$$

in which $o\left(\cdot\right)$ *is* $o\big(\tilde{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}\big)\rightarrow 0$ *with* $\tilde{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}\rightarrow 0$, $\hat{\varphi}'=\varphi'\big(\hat{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}\big)$ *and* $\hat{\varphi}=\varphi\big(\hat{\mathcal{H}}^{\mathrm{T}}\bar{\mathbb{X}}\big)$.

Lemma 2.2 (Fu (1996)). *The description of* $v\left(x\right)$ *satisfies the boundedness, which can be expressed as:*

$$\left\|v\left(x\right)\right\|<\alpha^{\mathrm{T}}\omega, \tag{9}$$

where $\alpha\in\mathcal{R}^4$ *is an uncertain vector,* ω *is a known vector with representation of* $\omega=\left(1,\left\|\bar{\mathbb{X}}\right\|,\left\|\bar{\mathbb{X}}\right\|\left\|\hat{W}\right\|_F,\left\|\bar{\mathbb{X}}\right\|\left\|\bar{\mathcal{H}}\right\|_F\right)^{\mathrm{T}}$.

Lemma 2.3 (Wang, Xie, and De Souza (1992)). *For any scalar* $\mathcal{F}>0$, *real vectors* d *and* e *with the required dimensions, the following inequality can be held:*

$$d^{\mathrm{T}}e+e^{\mathrm{T}}d\leqslant d^{T}\mathcal{F}^{-1}d+e^{\mathrm{T}}\mathcal{F}e. \tag{10}$$

Lemma 2.4 (Xiong and Lam (2009)). *Given any real scalar* δ *and matrix* $\mathcal{G}$, *it follows that*

$$\delta\left(\mathcal{G}+\mathcal{G}^{\mathrm{T}}\right)\leqslant\delta^2+\mathcal{G}T^{-1}\mathcal{G}^{\mathrm{T}}, \tag{11}$$

where matrix T meets $T > 0$.

Definition 2.5 (Boukas (2007))**.** Considering $\mathcal{V}\left(x\left(t\right),i\right)$ as a Lyapunov functional candidate, the infinitesimal operator $\mathcal{L}\mathcal{V}\left(x\left(t\right),i\right)$ can be defined as

$$\mathcal{L}\mathcal{V}\left(x\left(t\right),i\right)=\lim_{\Delta\to0^{+}}\frac{1}{\Delta}\left[\mathcal{E}\left\{\mathcal{V}\left(x\left(t+\Delta\right),r_{t+\Delta}\right)|x\left(t\right),r_{t}=i\right\}-\mathcal{V}\left(x\left(t\right),i\right)\right]. \tag{12}$$

3. Main results

3.1. Reachability analysis of linear-type SMS

This subsection is focused on the linear-type SMS design and its reachability assessment, where the SMS function is established as:

$$s\left(t\right)={B_{i}}^{\mathrm{T}}P_{i}x\left(t\right), \tag{13}$$

in which the parametric matrix $P_i > 0$ will be designed in Subsection 3.2.

Theorem 3.1. *If a SMC law based on the adaptive NN is constructed as:*

$$\begin{aligned}u\left(t\right)=&-\frac{1}{\zeta}\left({B_{i}}^{\mathrm{T}}P_{i}B_{i}\right)^{-1}\left({B_{i}}^{\mathrm{T}}P_{i}\right)\left(A_{it}x\left(t\right)+A_{i\tau}x\left(t-\tau\right)\right)\\&-\hat{W}^{T}\rho\left(\hat{\mathcal{H}}^{\mathrm{T}}\bar{G}\right)-\mathcal{F}_{i}\left(t\right),\end{aligned} \tag{14}$$

where

$$\begin{aligned}\mathcal{F}_{i}\left(t\right)=&\frac{1}{\zeta}\Big[\left({B_{i}}^{\mathrm{T}}P_{i}B_{i}\right)^{-1}\sigma+\hat{\alpha}^{\mathrm{T}}\omega+\left\|\left({B_{i}}^{\mathrm{T}}P_{i}B_{i}\right)^{-1}\left({B_{i}}^{\mathrm{T}}P_{i}\right)\right\|\\&\left(\left\|\mathcal{M}_{i}\right\|\left\|\mathcal{N}_{it}\right\|\left\|x\left(t\right)\right\|+\left\|\mathcal{M}_{i}\right\|\left\|\mathcal{N}_{i\tau}\right\|\left\|x\left(t-\tau\right)\right\|\right)\Big]\mathrm{sgn}(s(t)),\end{aligned} \tag{15}$$

in which σ is to be given later, and the adaptive rules are designed as:

$$\begin{aligned}\dot{\hat{\mathcal{W}}}&=\begin{cases}\hbar_{1}{}^{-1}\Upsilon\left(\hat{\rho}-\hat{\rho}'\mathcal{H}^{\mathrm{T}}\bar{G}\right)s^{\mathrm{T}}\left(t\right), & \text{if } s\left(t\right)\neq0,\\0, & \text{if } s\left(t\right)=0;\end{cases}\\\dot{\hat{\mathcal{H}}}&=\begin{cases}\hbar_{2}{}^{-1}\Upsilon\left(\bar{G}s^{\mathrm{T}}\left(t\right)\hat{\mathcal{W}}^{\mathrm{T}}\hat{\rho}'\right), & \text{if } s\left(t\right)\neq0,\\0, & \text{if } s\left(t\right)=0;\end{cases}\\\dot{\hat{\alpha}}&=\begin{cases}\hbar_{3}{}^{-1}\Upsilon\left\|s\left(t\right)\right\|\omega, & \text{if } s\left(t\right)\neq0,\\0, & \text{if } s\left(t\right)=0;\end{cases}\end{aligned} \tag{16}$$

in which Υ is defined as $\Upsilon=\max_{i\in\mathcal{J}}\left\|{B_{i}}^{\mathrm{T}}P_{i}B_{i}\right\|$, $\hbar_1$, $\hbar_2$, and $\hbar_3$ represent the coefficients of the adaptive rules respectively, σ is a positive constant which will be designed later. Then the signals $s\left(t\right)$, $\tilde{\mathcal{W}}$, $\tilde{\mathcal{H}}$ and $\tilde{\alpha}$ of the resultant system are bounded.

Proof. Construct the Lyapunov function as below:

$$\begin{aligned}\mathcal{V}_1\left(s\left(t\right),i\right)=&\frac{1}{2}s^{\mathrm{T}}\left(t\right)s\left(t\right)+\frac{1}{2}\mathrm{tr}\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\tilde{\mathcal{W}}\}\\&+\frac{1}{2}\mathrm{tr}\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\tilde{\mathcal{H}}\}+\frac{1}{2}\tilde{\alpha}^{\mathrm{T}}\hbar_3\tilde{\alpha}.\end{aligned}\tag{17}$$

By Definition 2.5, the infinitesimal operator $\mathcal{L}$ on $\mathcal{V}_1\left(s\left(t\right),i\right)$ gives

$$\begin{aligned}\mathcal{L}\mathcal{V}_1\left(s\left(t\right),i\right)=&\lim_{\varDelta\to0}\frac{1}{\varDelta}\left[\mathcal{E}\left\{\mathcal{V}_1\left(s\left(t+\varDelta\right),r_{t+\varDelta}\right)|s\left(t\right),r_t=i\right\}-\mathcal{V}_1\left(s\left(t\right),i\right)\right]\\
=&\lim_{\varDelta\to0}\frac{1}{\varDelta}\cdot\frac{1}{2}\Bigg[\Big[\sum_{j=1,j\neq i}^{N}\Pr\left\{r\left(t+\varDelta\right)=j|r\left(t\right)=i\right\}s^{\mathrm{T}}\left(t+\varDelta\right)s\left(t+\varDelta\right)\\
&+\sum_{j=1,j\neq i}^{N}\Pr\left\{r\left(t+\varDelta\right)=i|r\left(t\right)=i\right\}s^{\mathrm{T}}\left(t+\varDelta\right)s\left(t+\varDelta\right)\Big]\\
&-\frac{1}{2}s^{\mathrm{T}}\left(t\right)s\left(t\right)\Bigg]+\mathrm{tr}\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\dot{\tilde{\mathcal{W}}}\}+\mathrm{tr}\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\dot{\tilde{\mathcal{H}}}\}+\tilde{\alpha}^{\mathrm{T}}\hbar_3\dot{\tilde{\alpha}}\\
=&\lim_{\varDelta\to0}\frac{1}{\varDelta}\cdot\frac{1}{2}\Bigg[\Big[\sum_{j=1,j\neq i}^{N}\frac{\xi_{ij}\left(\mathbb{G}_i\left(h+\varDelta\right)-\mathbb{G}_i\left(h\right)\right)}{1-\mathbb{G}_i\left(h\right)}s^{\mathrm{T}}\left(t+\varDelta\right)s\left(t+\varDelta\right)\\
&+\frac{1-\mathbb{G}_i\left(h+\varDelta\right)}{1-\mathbb{G}_i\left(h\right)}s^{\mathrm{T}}\left(t+\varDelta\right)s\left(t+\varDelta\right)\Big]-\frac{1}{2}s^{\mathrm{T}}\left(t\right)s\left(t\right)\Bigg]\\
&+\mathrm{tr}\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\dot{\tilde{\mathcal{W}}}\}+\mathrm{tr}\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\dot{\tilde{\mathcal{H}}}\}+\tilde{\alpha}^{\mathrm{T}}\hbar_3\dot{\tilde{\alpha}},\end{aligned}\tag{18}$$

in which $\mathfrak{h}$ denotes the sojourn time, $\mathbb{G}_i\left(\mathfrak{h}\right)$ stands for the cumulative distribution function of $\mathfrak{h}$ if the subsystem stays at mode i, and ξ_{ij} represents the probability intensity from mode i to mode j. Regarding to the SMS function, the Taylor expansion of $s(t)$ is stated as

$$s\left(t+\varDelta\right)=s\left(t\right)+\dot{s}\left(t\right)\varDelta+o\left(\varDelta\right),\tag{19}$$

when $\varDelta\to0$. Taking the condition $\lim\limits_{\varDelta\to0}\frac{\mathbb{G}_i(\mathfrak{h}+\varDelta)-\mathbb{G}_i(\mathfrak{h})}{1-\mathbb{G}_i(\mathfrak{h})}=0$ and the characteristics of the cumulative distribution function into consideration, it follows that

$$\lim_{\varDelta\to0}\frac{\mathbb{G}_i\left(\mathfrak{h}+\varDelta\right)-\mathbb{G}_i\left(\mathfrak{h}\right)}{\varDelta\left(1-\mathbb{G}_i\left(\mathfrak{h}\right)\right)}=\eta_i\left(h\right),\ \lim_{\varDelta\to0}\frac{1-\mathbb{G}_i\left(\mathfrak{h}+\varDelta\right)}{1-\mathbb{G}_i\left(\mathfrak{h}\right)}=1,\tag{20}$$

where $\eta_i\left(\mathfrak{h}\right)$ represents the TR from mode i. Denote $\eta_{ij}\triangleq\xi_{ij}\eta_i\left(\mathfrak{h}\right)$ when $i\neq j$ and $\eta_{ii}\triangleq-\sum_{j=1,j\neq i}^{J}\eta_{ij}$. Based on (19), one has

$$\begin{aligned}\mathcal{L}\mathcal{V}_1\left(s\left(t\right),i\right)=&s^{\mathrm{T}}\left(t\right)\left(B_i{}^{\mathrm{T}}P_iB_i\right)\mu\left(t,u\right)\\
&+s^{\mathrm{T}}\left(t\right)\left(B_i{}^{\mathrm{T}}P_i\right)\left(\left(A_{it}+\varDelta A_{it}\left(t\right)\right)x\left(t\right)\right.\\
&\left.+\left(A_{i\tau}+\varDelta A_{i\tau}\left(t\right)\right)x\left(t-\tau\right)\right)+\mathrm{tr}\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\dot{\tilde{\mathcal{W}}}\}\\
&+\mathrm{tr}\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\dot{\tilde{\mathcal{H}}}\}+\tilde{\alpha}^{\mathrm{T}}\hbar_3\dot{\tilde{\alpha}}.\end{aligned}\tag{21}$$

By applying the controller in (14), it gives

$$
\begin{aligned}
\mathcal{L}\mathcal{V}_1\left(s\left(t\right),i\right)\leqslant & s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_i\right)A_{it}x\left(t\right)+s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_i\right)\varDelta A_{it}\left(t\right)x\left(t\right)\\
&+s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_i\right)A_{i\tau}x\left(t-\tau\right)+s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_i\right)\varDelta A_{i\tau}\left(t\right)x\left(t-\tau\right)\\
&+s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_iB_i\right)\varGamma\left(t\right)\big[-\frac{1}{\zeta}\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\left({B_i}^{\mathrm{T}}P_i\right)A_{it}x\left(t\right)\\
&-\frac{1}{\zeta}\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\left({B_i}^{\mathrm{T}}P_i\right)A_{i\tau}x\left(t-\tau\right)\\
&-\hat{\mathcal{W}}^{\mathrm{T}}\rho(\hat{\mathcal{H}}^{\mathrm{T}}\bar{G})-\big[-\frac{1}{\zeta}\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\left(\sigma+\hat{\alpha}^{\mathrm{T}}\omega\right)\\
&+\frac{1}{\zeta}\left\|\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\left({B_i}^{\mathrm{T}}P_i\right)\right\|\big(\left\|M_i\right\|\left\|\mathcal{N}_{it}\right\|\left\|x\left(t\right)\right\|\\
&+\left\|\mathcal{M}_i\right\|\left\|\mathcal{N}_{i\tau}\right\|\left\|x\left(t-\tau\right)\right\|\big)\big]\mathrm{sgn}(s(t))\big]\\
&+s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_iB_i\right)\varPhi\left(x,t,\tau\right)\\
&+\mathrm{tr}\big\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\dot{\hat{\mathcal{W}}}\big\}+\mathrm{tr}\big\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\dot{\hat{\mathcal{H}}}\big\}+\tilde{\alpha}^{\mathrm{T}}\hbar_3\dot{\hat{\alpha}}.
\end{aligned}
\tag{22}
$$

By the condition $\frac{1}{\zeta}\left\|\varGamma\left(t\right)\right\|\leqslant 1$ and Lemma 2.2, (22) is turned into

$$
\begin{aligned}
\mathcal{L}\mathcal{V}_1\left(s\left(t\right),i\right)\leqslant & \left\|s\left(t\right)\right\|_1\left\|\left({B_i}^{\mathrm{T}}P_i\right)\right\|\left(\left\|\mathcal{M}_i\right\|\left\|\mathcal{N}_{it}\right\|\left(\left\|x\left(t\right)\right\|_1-\left\|x\left(t\right)\right\|\right)\right.\\
&\left.+\left\|\mathcal{M}_i\right\|\left\|\mathcal{N}_{i\tau}\right\|\left(\left\|x\left(t-\tau\right)\right\|_1-\left\|x\left(t-\tau\right)\right\|\right)\right)\\
&+\left\|s\left(t\right)\right\|_1\left({B_i}^{\mathrm{T}}P_iB_i\right)\varGamma\left(t\right)\big[-\hat{\mathcal{W}}^{\mathrm{T}}\rho(\hat{\mathcal{H}}^{\mathrm{T}}\bar{G})\\
&-\big[\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\frac{\sigma}{\zeta}+\frac{\hat{\alpha}^{\mathrm{T}}\omega}{\zeta}\big]\mathrm{sgn}(s(t))\big]\\
&+\left\|s\left(t\right)\right\|_1\left({B_i}^{\mathrm{T}}P_iB_i\right)\varPhi\left(x,t,\tau\right)\\
&+\mathrm{tr}\big\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\dot{\hat{\mathcal{W}}}\big\}+\mathrm{tr}\big\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\dot{\hat{\mathcal{H}}}\big\}+\tilde{\alpha}^{\mathrm{T}}\hbar_3\dot{\hat{\alpha}}.
\end{aligned}
\tag{23}
$$

Since $\left\|\cdot\right\|\leqslant\left\|\cdot\right\|_1$, it follows that

$$
\begin{aligned}
\mathcal{L}\mathcal{V}_1\left(s\left(t\right),i\right)\leqslant & -\left\|s\left(t\right)\right\|\left({B_i}^{\mathrm{T}}P_iB_i\right)\dot{\hat{\mathcal{W}}}^{\mathrm{T}}\left(\hat{\rho}-\hat{\rho}'\mathcal{H}^{\mathrm{T}}\bar{G}\right)\\
&-\left\|s\left(t\right)\right\|\left({B_i}^{\mathrm{T}}P_iB_i\right)\hat{\mathcal{W}}^{\mathrm{T}}\hat{\rho}'\tilde{\mathcal{H}}^{\mathrm{T}}\bar{G}-\left\|s\left(t\right)\right\|\left({B_i}^{\mathrm{T}}P_iB_i\right)\alpha^{\mathrm{T}}\omega\\
&-\left\|s\left(t\right)\right\|\left({B_i}^{\mathrm{T}}P_iB_i\right)\sigma-\left\|s\left(t\right)\right\|\left({B_i}^{\mathrm{T}}P_iB_i\right)\hat{\alpha}^{\mathrm{T}}\omega\\
&+\mathrm{tr}\big\{\tilde{\mathcal{W}}^{\mathrm{T}}\hbar_1\dot{\hat{\mathcal{W}}}\big\}+\mathrm{tr}\big\{\tilde{\mathcal{H}}^{\mathrm{T}}\hbar_2\dot{\hat{\mathcal{H}}}\big\}+\tilde{\alpha}^{\mathrm{T}}\hbar_3\dot{\hat{\alpha}}\\
&\leqslant-\sigma\left\|s\left(t\right)\right\|\varUpsilon+\zeta\left\|s\left(t\right)\right\|\varUpsilon\left(\tilde{\alpha}^{\mathrm{T}}-\hat{\alpha}^{\mathrm{T}}-\alpha^{\mathrm{T}}\right)\omega\\
&\leqslant-\sigma\varUpsilon\left\|s\left(t\right)\right\|\leqslant 0.
\end{aligned}
\tag{24}
$$

According to inequality (24), it can be obtained that $s\left(t\right)$, $\tilde{\mathcal{W}}$, $\tilde{\mathcal{H}}$ and $\tilde{\alpha}$ are bounded. Therefore, the proof is completed. □

Remark 2. It can be seen that the boundedness of adaptive rules $\dot{\hat{\mathcal{W}}}$, $\dot{\hat{\mathcal{H}}}$ and $\dot{\hat{\alpha}}$ can also be derived drawing upon the above conclusion of Theorem 3.1, which also means that the estimation error $\tilde{\varPhi}\left(x,t,\tau\right)$ of AA has boundary. Subsequently, the following theorem is addressed to guarantee the SMS $s\left(t\right)=0$ in finite time with probability 1.

Theorem 3.2. *Consider the system (4), if the SMC law is synthesized as (14), (15) and (16) with the positive scalar σ selected to satisfy $\sigma \geqslant \left\| \tilde{\Phi}(x,t,\tau) \right\|$, the system trajectories can arrive on the devised SMS $s(t)=0$ in finite time almost surely.*

Proof. The Lyapunov candidate is selected as

$$\mathcal{V}_2(s(t),i)=\frac{1}{2}s^{\mathrm{T}}(t)\,s(t)\,. \tag{25}$$

The infinitesimal operator $\mathcal{L}$ on $\mathcal{V}_2(s(t),i)$ can be shown as

$$\begin{aligned}
\mathcal{L}\mathcal{V}_2(s(t),i)=&\lim_{\varDelta\to 0}\frac{1}{\varDelta}\left[\mathcal{E}\left\{\mathcal{V}_1(s(t+\varDelta),r_{t+\varDelta})\,|s(t),r_t=i\right\}-\mathcal{V}_1(s(t),i)\right]\\
=&\lim_{\varDelta\to 0}\frac{1}{\varDelta}\cdot\frac{1}{2}\Bigg[\Big[\sum_{j=1,j\neq i}^{N}\Pr\left\{r(t+\varDelta)=j|r(t)=i\right\}\\
&s^{\mathrm{T}}(t+\varDelta)\,s(t+\varDelta)+\sum_{j=1,j\neq i}^{N}\Pr\left\{r(t+\varDelta)=i|r(t)=i\right\}\Big]\\
&s^{\mathrm{T}}(t+\varDelta)\,s(t+\varDelta)-\frac{1}{2}s^{\mathrm{T}}(t)\,s(t)\Bigg]\\
=&\lim_{\varDelta\to 0}\frac{1}{\varDelta}\cdot\frac{1}{2}\Bigg[\Big[\sum_{j=1,j\neq i}^{N}\frac{\xi_{ij}\left(\mathbb{G}_i(\mathfrak{h}+\varDelta)-\mathbb{G}_i(\mathfrak{h})\right)}{1-\mathbb{G}_i(\mathfrak{h})}s^{\mathrm{T}}(t+\varDelta)\,s(t+\varDelta)\\
&+\frac{1-\mathbb{G}_i(\mathfrak{h}+\varDelta)}{1-\mathbb{G}_i(\mathfrak{h})}s^{\mathrm{T}}(t+\varDelta)\,s(t+\varDelta)\Big]-\frac{1}{2}s^{\mathrm{T}}(t)\,s(t)\Bigg].
\end{aligned} \tag{26}$$

By (19) and (20), it gives

$$\begin{aligned}
\mathcal{L}\mathcal{V}_2(s(t),i)=&s^{\mathrm{T}}(t)\left({B_i}^{\mathrm{T}}P_i\right)\big((A_{it}x(t)+\varDelta A_{it}(t))\,x(t)\\
&+(A_{i\tau}+\varDelta A_{i\tau}(t))\,x(t-\tau)\big)+s^{\mathrm{T}}(t)\left({B_i}^{\mathrm{T}}P_iB_i\right)\mu(t,u)\,.
\end{aligned} \tag{27}$$

Substituting (14) into (2), one has

$$\begin{aligned}
\mathcal{L}\mathcal{V}_2(s(t),i)=&s^{\mathrm{T}}(t)\left({B_i}^{\mathrm{T}}P_i\right)\big((A_{it}x(t)+\varDelta A_{it}(t))\,x(t)\\
&+(A_{i\tau}+\varDelta A_{i\tau}(t))\,x(t-\tau)\big)\\
&+s^{\mathrm{T}}(t)\left({B_i}^{\mathrm{T}}P_iB_i\right)\varGamma(t)\Big[-\frac{1}{\zeta}\left({B_i}^{\mathrm{T}}P_iB_i\right)\left({B_i}^{\mathrm{T}}P_i\right)\\
&(A_{it}x(t)+A_{i\tau}x(t-\tau))-\hat{W}^{T}\rho\left(\hat{\mathcal{H}}^{\mathrm{T}}\bar{G}\right)-\mathcal{F}_i(t)\Big].
\end{aligned} \tag{28}$$

Using (15), it gives

$$
\begin{aligned}
\mathcal{L}\mathcal{V}_2\left(s\left(t\right),i\right)=&s^{\mathrm{T}}\left(t\right)\left({B_i}^{\mathrm{T}}P_i\right)\left(\varDelta A_{it}\left(t\right)x\left(t\right)+\varDelta A_{i\tau}\left(t\right)x\left(t-\tau\right)\right)\\
&+s^{\mathrm{T}}\left(t\right){B_i}^{\mathrm{T}}P_iB_i\varGamma\left(t\right)\big[-\frac{1}{\zeta}\big\|\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\big\|\big\|{B_i}^{\mathrm{T}}P_i\big\|\\
&\cdot\left(\left\|M_i\right\|\left\|\mathcal{N}_{it}\right\|\left\|x\left(t\right)\right\|+\left\|M_i\right\|\left\|\mathcal{N}_{i\tau}\right\|\left\|x\left(t-\tau\right)\right\|\right)\\
&+\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\left(\sigma-\tilde{\varPhi}\left(x,t,\tau\right)\right)\big].
\end{aligned}
\tag{29}
$$

Since (23) and $\|\cdot\|\leqslant\|\cdot\|_1$, it follows that

$$
\begin{aligned}
\mathcal{L}\mathcal{V}_2\left(s\left(t\right),i\right)\leqslant&\left\|s\left(t\right)\right\|_1\left({B_i}^{\mathrm{T}}P_i\right)\left(\varDelta A_{it}\left(t\right)x\left(t\right)+\varDelta A_{i\tau}\left(t\right)x\left(t-\tau\right)\right)\\
&+\left\|s\left(t\right)\right\|_1{B_i}^{\mathrm{T}}P_iB_i\varGamma\left(t\right)\big[-\frac{1}{\zeta}\big\|\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\big\|\big\|{B_i}^{\mathrm{T}}P_i\big\|\\
&\cdot\left(\left\|M_i\right\|\left\|\mathcal{N}_{it}\right\|\left\|x\left(t\right)\right\|+\left\|M_i\right\|\left\|\mathcal{N}_{i\tau}\right\|\left\|x\left(t-\tau\right)\right\|\right)\\
&-\frac{1}{\zeta}\big\|{B_i}^{\mathrm{T}}P_iB_i\big\|\big\|{B_i}^{\mathrm{T}}P_i\big\|\big(\sigma-\tilde{\varPhi}\left(x,t,\tau\right)\big)\big]\\
\leqslant&\big\|s\left(t\right)\big\|\big\|{B_i}^{\mathrm{T}}P_iB_i\big\|\big\|\left({B_i}^{\mathrm{T}}P_iB_i\right)^{-1}\big\|\big(\sigma-\tilde{\varPhi}\left(x,t,\tau\right)\big)\\
\leqslant&-\sigma_1\left\|s\left(t\right)\right\|\leqslant0,
\end{aligned}
\tag{30}
$$

in which σ_1 represents a positive scalar. Further, we can obtain that

$$
\mathcal{L}\mathcal{V}_2\left(s\left(t\right),i\right)\leqslant-\sigma_1\sqrt{\mathcal{V}_2\left(s\left(t\right),i\right)},\quad\forall t\geqslant0.
\tag{31}
$$

By Itô's formula, it yields

$$
\mathcal{L}\left\|s\left(t\right)\right\|=\mathcal{L}\sqrt{\mathcal{V}_2\left(s\left(t\right),i\right)}\leqslant-\sigma_1/2,
\tag{32}
$$

and hence

$$
\mathcal{E}\left\|s\left(t\right)\right\|\leqslant\mathcal{E}\left\|s\left(0\right)\right\|-\left(\sigma_1/2\right)t.
\tag{33}
$$

It can be seen that $\mathcal{E}\left\|s\left(t\right)\right\|$ converges to zero in finite time, which concludes there is an instant $T=2m_0/\zeta$ meeting $\mathcal{E}\left\|s\left(t\right)\right\|=0$ almost surely for all $t>T$, and $m_0=\mathcal{E}\left\|s\left(0\right)\right\|<\infty$. Thus, the proof is completed. □

Remark 3. Differing from Niu, Lam, Wang, and Ho (2008) which only guaranteed the uniform boundedness of the sliding variable, the designed adaptive NN-based SMC strategy implemented can not only satisfy the boundedness of signals but also admit the trajectories onto the proposed SMS $s\left(t\right)=0$ in finite time almost surely, from which the assumption of norm boundedness for uncertain attacks is not required.

Remark 4. Adaptive NN is used to synthesize the novel SMC law (14) in this paper based on the approximation capability to uncertain attacks, and the control diagram of the devised scheme is displayed in Fig. 1. Furthermore, compared to the method in Wei et al. (2017) which only took the AFs into account, the proposed control scheme could ensure the closed-loop DSMJSs to be operated as demanded in spite of the simultaneous existence of actuator partial failures, AAs and GUTRs.

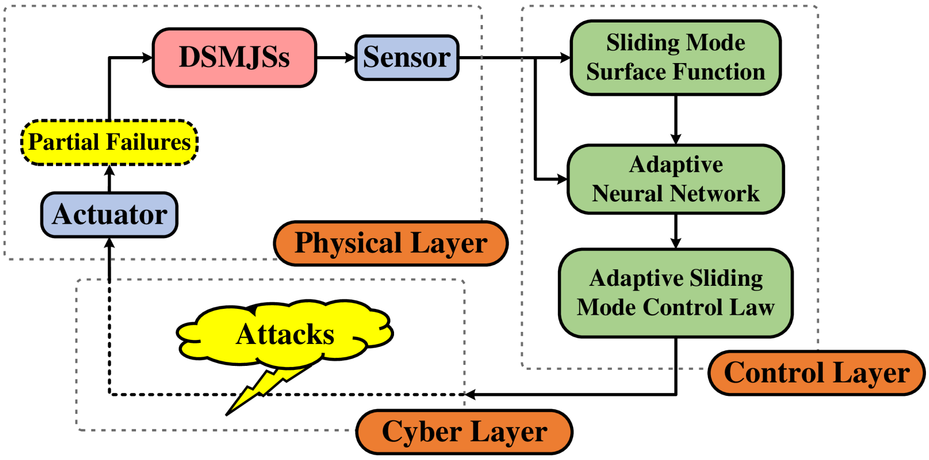

Figure 1. Block diagram of the control scheme

3.2. Stability analysis

In this section, a novel sufficient condition for the resultant DSMJSs to be stochastically stable on the sliding motion is presented in the light of the devised SMS.

Theorem 3.3. *If matrices $P_i > 0$, $Q > 0$, $\mathcal{U}_{g,i} > 0$, $\mathcal{Z}_{g,i} > 0$, $\mathcal{W}_i > 0$, $\mathcal{X}_{i,j_\alpha} > 0$ and a positive scalar ι exist such that the following requirements are fulfilled:*
Case 1: $i \in {\Omega_k}^i$ and $g \in {\Omega_{uk}}^i$,

$$\begin{bmatrix} \varXi_\varpi + \varXi_1 & P_i A_{i\tau} + \iota {N_i}^{\mathrm{T}} \mathcal{N}_{i\tau} & (P_i \mathcal{M}_i)^{\mathrm{T}} & \psi_1 \\ * & -Q + \iota {\mathcal{N}_{i\tau}}^{\mathrm{T}} \mathcal{N}_{i\tau} & 0 & 0 \\ * & * & -\iota I & 0 \\ * & * & * & -\mathcal{U}_{g,i} \end{bmatrix} < 0; \tag{34}$$

where

$$\begin{aligned} \varXi_\varpi &= \mathrm{sym}\left\{P_i\left(A_i - B_i K_i\right)\right\} + Q + {\iota_1}^{-1} {N_i}^{\mathrm{T}} N_i + P_i \mathcal{M}_i {\mathcal{M}_i}^{\mathrm{T}} P_i, \\ \varXi_1 &= \sum_{j \in \Omega_k^i} [\hat{\eta}_{ij}\left(P_j - P_g\right) + \frac{1}{4}\left(\beta_{ij}\right)^2 \mathcal{U}_{g,i}], \\ \psi_1 &= \left[\left(P_{j,1} - P_g\right), \left(P_{j,2} - P_g\right), \cdots, \left(P_{j_1} - P_g\right)\right]; \end{aligned} \tag{35}$$

Case 2: $i \in \Omega_{uk}^i$, $g \in {\Omega_{uk}}^i$ and $\Omega_k^i \neq \oslash$,

$$\begin{cases} \begin{bmatrix} \varXi_\varpi + \varXi_2 & P_i A_{i\tau} + \iota {N_i}^{\mathrm{T}} \mathcal{N}_{i\tau} & (P_i \mathcal{M}_i)^{\mathrm{T}} & \psi_2 \\ * & -Q + \iota {\mathcal{N}_{i\tau}}^{\mathrm{T}} \mathcal{N}_{i\tau} & 0 & 0 \\ * & * & -\iota I & 0 \\ * & * & * & -\mathcal{Z}_{g,i} \end{bmatrix} < 0, \\ P_i - P_g \geqslant 0; \end{cases} \tag{36}$$

where

$$\begin{aligned}
&\Xi_2 = \sum_{j\in\Omega_k^i} [\hat{\eta}_{ij}(P_j - P_g) + \frac{1}{4}(\beta_{ij})^2 \mathcal{Z}_{g,i}], \\
&\psi_2 = [(P_{j,1} - P_g), (P_{j,2} - P_g), \cdots, (P_{j_2} - P_g)];
\end{aligned} \tag{37}$$

Case 3: ${\Omega_k}^i = \mathcal{J}$, $g \in {\Omega_{uk}}^i$ *and* $j \in {\Omega_k}^j$ *for some* $j \neq i$,

$$\begin{bmatrix}
\Xi_\varpi + \Xi_3 & P_i A_{i\tau} + \iota {N_i}^{\mathrm{T}} \mathcal{N}_{i\tau} & (P_i \mathcal{M}_i)^{\mathrm{T}} & P_i - P_g \\
* & -Q + \iota {\mathcal{N}_{i\tau}}^{\mathrm{T}} \mathcal{N}_{i\tau} & 0 & 0 \\
* & * & -\iota I & 0 \\
* & * & * & -\mathcal{W}_i
\end{bmatrix} < 0; \tag{38}$$

where

$$\Xi_3 = \alpha_i \hat{\eta}_{jj}(P_i - P_g) + \frac{1}{4}(\beta_{jj})^2 \mathcal{W}_i; \tag{39}$$

Case 4: ${\Omega_{uk}}^i = \mathcal{J}$, $g \in {\Omega_{uk}}^i$, $j_\alpha \in {\Omega_k}^j$ *and* $j \notin {\Omega_k}^j$ *for any* $j \in \mathcal{J}$,

$$\begin{cases}
\begin{bmatrix}
\Xi_\varpi + \Xi_4 & P_i A_{i\tau} + \iota {N_i}^{\mathrm{T}} \mathcal{N}_{i\tau} & (P_i \mathcal{M}_i)^{\mathrm{T}} & P_{j_\alpha} - P_j \\
* & -Q + \iota {\mathcal{N}_{i\tau}}^{\mathrm{T}} \mathcal{N}_{i\tau} & 0 & 0 \\
* & * & -\iota I & 0 \\
* & * & * & -\mathcal{X}_{i,j_\alpha}
\end{bmatrix} < 0, \\
P_i - P_g \geqslant 0;
\end{cases} \tag{40}$$

where

$$\Xi_4 = \hat{\eta}_{jj_\alpha}(P_{j_\alpha} - P_g) + \frac{1}{4}(\beta_{jj_\alpha})^2 \mathcal{X}_{i,j_\alpha}. \tag{41}$$

Then the resultant DSMJS (4) will be stochastically stable.

Proof. Choose the Lyapunov function as

$$\mathcal{V}_3(x(t), i) = x^{\mathrm{T}}(t) P_i x(t) + \int_{t-\tau}^{t} x^{\mathrm{T}}(s) Q x(s)\, ds. \tag{42}$$

Then, it gives

$$\begin{aligned}
\mathcal{L}\mathcal{V}_3(x(t), i) = &\lim_{\Delta\to 0} \frac{1}{\Delta} \Big[\sum_{j=1, j\neq i}^{N} \Pr\{r(t+\Delta) = j | r(t) = i\} x^{\mathrm{T}}(t+\Delta) P_j x(t+\Delta) \\
&+ \Pr\{r(t+\Delta) = i | r(t) = i\} x^{\mathrm{T}}(t+\Delta) P_i x(t+\Delta) \\
&- x^{\mathrm{T}}(t) P_i x(t) \Big] + x^{\mathrm{T}}(t) Q x(t) - x^{\mathrm{T}}(t-\tau) Q x(t-\tau) \qquad (43) \\
= &\lim_{\Delta\to 0} \frac{1}{\Delta} \Big[\sum_{j=1, j\neq i}^{N} \frac{\xi_{ij}(\mathbb{G}_i(\mathfrak{h}+\Delta) - \mathbb{G}_i(\mathfrak{h}))}{1 - \mathbb{G}_i(\mathfrak{h})} x^{\mathrm{T}}(t+\Delta) P_j x(t+\Delta)
\end{aligned}$$

$$+\frac{1-\mathbb{G}_i(\mathfrak{h}+\Delta)}{1-\mathbb{G}_i(\mathfrak{h})}x^{\mathrm{T}}(t+\Delta)P_ix(t+\Delta)-x^{\mathrm{T}}(t)P_ix(t)\Big]$$
$$+x^{\mathrm{T}}(t)Qx(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau),$$

for the smaller Δ, the Taylor expansion of $x(t+\Delta)$ can be approximately set to

$$x(t+\Delta)=x(t)+\dot{x}(t)\Delta+o(\Delta), \tag{44}$$

when $\Delta\to 0$. Then the infinitesimal generator $\mathcal{L}\mathcal{V}_3(x(t),i)$ becomes

$$\begin{aligned}\mathcal{L}\mathcal{V}_3(x(t),i)=&x^{\mathrm{T}}(t)Qx(t)+x^{\mathrm{T}}(t)\cdot\mathrm{sym}\{P_i(A_i-B_iK_i)\}x(t)\\&+x^{\mathrm{T}}(t)P_iB_i(\Gamma_0(I+\Lambda(t))u(t)+\Phi(x,t,\tau)\\&+K_ix(t)+f(t))+\sum_{j=1}^{J}\eta_{ij}x^{\mathrm{T}}(t)P_jx(t)\\&+2x^{\mathrm{T}}(t)P_iA_{i\tau}x(t-\tau)+2x^{\mathrm{T}}(t)P_i\Delta A_{i\tau}x(t-\tau)\\&+2x^{\mathrm{T}}(t)P_i\Delta A_{it}x(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau).\end{aligned} \tag{45}$$

It is noteworthy that the reachability of SMS $s(t)=0$ has been satisfied. Therefore, (45) is turned into

$$\begin{aligned}\mathcal{L}\mathcal{V}_3(x(t),i)=&+2x^{\mathrm{T}}(t)P_iA_{i\tau}x(t-\tau)+2x^{\mathrm{T}}(t)P_i\Delta A_{i\tau}x(t-\tau)\\&+x^{\mathrm{T}}(t)\cdot\mathrm{sym}\{P_i(A_i-B_iK_i)\}x(t)\\&+\sum_{j=1}^{J}\eta_{ij}x^{\mathrm{T}}(t)P_jx(t)+x^{\mathrm{T}}(t)Qx(t)\\&+2x^{\mathrm{T}}(t)P_i\Delta A_ix(t)-x^{\mathrm{T}}(t-\tau)Qx(t-\tau).\end{aligned} \tag{46}$$

Additionally, one has

$$\begin{aligned}&2x^{\mathrm{T}}(t)P_i\Delta A_{it}(t)x(t)+\Delta A_{i\tau}(t)x(t-\tau)\\&\leqslant{\iota_{1i}}^{-1}x^{\mathrm{T}}(t)P_i\mathcal{M}_i{\mathcal{M}_i}^{\mathrm{T}}P_ix(t)+\iota_{1i}[\mathcal{N}_{it}x(t)\\&\quad+\mathcal{N}_{i\tau}x(t-\tau)]^{\mathrm{T}}\cdot[\mathcal{N}_{it}x(t)+\mathcal{N}_{i\tau}x(t-\tau)].\end{aligned} \tag{47}$$

Consequently, one can obtain

$$\mathcal{L}\mathcal{V}_3\leqslant\left[x^{\mathrm{T}}(t)\ \ ,x^{\mathrm{T}}(t-\tau)\right]^{\mathrm{T}}\mathcal{O}(x,i)\left[x(t)\ \ ,x(t-\tau)\right], \tag{48}$$

where

$$\mathcal{O}(x,i)=\begin{bmatrix}\Xi_{\varpi}+\sum_{j=1}^{J}\eta_{ij}P_j & P_iA_{i\tau}\\ * & -Q\end{bmatrix}. \tag{49}$$

Case 1: $i\in{\Omega_k}^i$.

Denote $\lambda_{i,k}\triangleq\sum_{i\in\Omega_k^i}\eta_{ij}$. Due to $\Omega_k^i\neq\oslash$, it gives that $\lambda_{i,k}<0$. Then $\sum_{j=1}^{J}\eta_{ij}P_j$

can be described as

$$\begin{aligned}\sum_{j=1}^{N}\eta_{ij}P_j &= \sum_{j\in{\Omega_k}^i}\eta_{ij}P_j+\sum_{j\in{\Omega_{uk}}^i}\eta_{ij}P_j\\ &= \sum_{j\in{\Omega_k}^i}\eta_{ij}P_j-\lambda_{i,k}\sum_{j\in{\Omega_{uk}}^i}\frac{\eta_{ij}}{-\lambda_{i,k}}P_j,\end{aligned} \tag{50}$$

in which $\sum_{j\in{\Omega_{uk}}^i}\frac{\eta_{ij}}{-\lambda_{i,k}}=1$ and $0<\frac{\eta_{ij}}{-\lambda_{i,k}}<1$, $j\in{\Omega_{uk}}^i$. Hence, for any $g\in{\Omega_{uk}}^i$, one has

$$\varXi_\varpi+\sum_{j\in\Omega_k^i}\eta_{ij}\left(P_j-P_g\right)=\sum_{g\in\Omega_{uk}^i}\frac{\eta_{ij}}{-\lambda_{i,k}}\Big[\varXi_\varpi+\sum_{j\in\Omega_k^i}\eta_{ij}\left(P_j-P_g\right)\Big]. \tag{51}$$

Evidently, one has

$$\sum_{j\in\Omega_k^i}\eta_{ij}\left(P_j-P_g\right)=\sum_{j\in\Omega_k^i}\hat{\eta}_{ij}\left(P_j-P_g\right)+\sum_{j\in\Omega_k^i}\Delta\eta_{ij}\left(P_j-P_g\right). \tag{52}$$

Then it yields that for any $\mathcal{U}_{g,i}$:

$$\begin{aligned}\sum_{j\in\Omega_k^i}\Delta\eta_{ij}\left(P_j-P_g\right) &= \sum_{j\in\Omega_k^i}\left[\frac{1}{2}\Delta\eta_{ij}\left(\left(P_j-P_g\right)+\left(P_j-P_g\right)\right)\right]\\ &\leqslant \sum_{j\in\Omega_k^i}\left[\frac{1}{4}\left(\beta_{ij}\right)^2\mathcal{U}_{g,i}+\left(P_j-P_g\right)\mathcal{U}_{g,i}{}^{-1}\left(P_j-P_g\right)^{\mathrm{T}}\right].\end{aligned} \tag{53}$$

By invoking the Schur complement and (35), $\mathcal{L}\mathcal{V}_3\left(x\left(t\right),i\right)<0$ is obtained, which illustrates that the stochastic stability of system (4) can be satisfied.

Case 2: $i\in\Omega_{uk}^i$ and $\Omega_k^i\neq\oslash$.

Denote $\lambda_{i,k}\triangleq\sum_{j\in\Omega_k^i}\eta_{ij}$. Since $\Omega_{uk}^i\neq\oslash$, then one has $\lambda_{i,k}>0$. And

$$\begin{aligned}\sum_{j=1}^{N}\eta_{ij}P_j &= \sum_{j\in\Omega_k^i}\eta_{ij}P_j+\eta_{ii}P_i+\sum_{j\in\Omega_{uk}^i}\eta_{ij}P_j\\ &= \sum_{j\in\Omega_k^i}\eta_{ij}P_j+\eta_{ii}P_i-\left(\eta_{ii}+\lambda_{i,k}\right)\sum_{j\in\Omega_{uk}^i,j\neq i}\frac{\eta_{ij}}{-\eta_{ii}-\lambda_{i,k}}P_j,\end{aligned} \tag{54}$$

in which $0\leqslant\eta_{ij}/\left(-\eta_{ii}-\lambda_{i,k}\right)\leqslant1$, $k\in\Omega_{uk}^i$ and $\sum_{j\in\Omega_{uk}^i,j\neq i}\frac{\eta_{ij}}{-\eta_{ii}-\lambda_{i,k}}=1$. For any $g\in\Omega_{uk}^i$, we have

$$\begin{aligned}\varXi_\varpi+\sum_{j=1}^{J}\eta_{ij}P_j=&\sum_{g\in\Omega_{uk}^i,j\neq i}\frac{\eta_{ig}}{-\eta_{ii}-\lambda_{i,k}}\Big[\varXi_\varpi+\mathrm{diag}\big\{\eta_{ii}\left(P_i-P_g\right)\\ &+\sum_{j\in\Omega_k^i}\eta_{ij}\left(P_j-P_g\right),\eta_{ii}\left(P_i-P_g\right)+\sum_{j\in\Omega_k^i}\eta_{ij}\left(P_j-P_g\right)\big\}\Big].\end{aligned} \tag{55}$$

Since $0 \leqslant \eta_{ij} \leqslant -\eta_{ii} - \lambda_{i,k}$, $\Xi_i + \sum_{j=1}^{J} \eta_{ij} P_j < 0$ is equivalent to

$$\Xi_{\varpi} + \eta_{ii}\left(P_i - P_g\right) + \sum_{j \in \Omega_k^i} \eta_{ij}\left(P_j - P_g\right) < 0. \tag{56}$$

By reason of $\eta_{ii} < 0$, then it holds if

$$\begin{cases} P_i - P_g \geqslant 0, \\ \Xi_{\varpi} + \sum_{j \in \Omega_k^i} \eta_{ij}\left(P_j - P_g\right) < 0. \end{cases} \tag{57}$$

Using the similar step as (53), for any $\mathcal{Z}_{i,g}$, one has

$$\begin{aligned} \sum_{j \in \Omega_k^i} \eta_{ij}\left(P_j - P_g\right) \leqslant & \sum_{j \in \Omega_k^i} \hat{\eta}_{ij}\left(P_j - P_g\right) \\ & + \sum_{j \in \Omega_k^i} \left[\frac{1}{4}\left(\beta_{ij}\right)^2 \mathcal{Z}_{g,i} + \left(P_j - P_g\right) \mathcal{Z}_{g,i}{}^{-1} \left(P_j - P_g\right)^{\mathrm{T}}\right]. \end{aligned} \tag{58}$$

In the light of Schur complement and (37), $\mathcal{L}\mathcal{V}_3\left(x\left(t\right), i\right) < 0$ also holds in this case.
Case 3: ${\Omega_k}^i = \mathcal{J}$ and $j \in {\Omega_k}^j$ for some $j \neq i$.

In such case, η_{ii} is estimated by $\alpha\eta_{jj}$. Denote $\lambda_{i,k} \triangleq \eta_{ii}$. Therefore, $\sum_{j=1} \eta_{ij} P_j$ is changed into

$$\sum_{j=1}^{J} \eta_{ij} P_j = \eta_{ii} P_i + \sum_{j \in {\Omega_{uk}}^i} \eta_{ij} P_j = \eta_{ii} P_i - \lambda_{i,k} \sum_{j \in {\Omega_{uk}}^i} \frac{\eta_{ij}}{-\lambda_{i,k}} P_j. \tag{59}$$

Noticing that $\sum_{j \in {\Omega_{uk}}^i} \eta_{ij} = -\eta_{ii} = -\lambda_{i,k} > 0$. Consequently, for any $g \in {\Omega_{uk}}^i$, it follows

$$\begin{aligned} \Xi_{\varpi} + \sum_{j=1}^{N} \eta_{ij} P_j = & \sum_{g \in {\Omega_{uk}}^i} \frac{\eta_{ig}}{-\lambda_{i,k}} \left[\Xi_{\varpi} + \eta_{ii}\left(P_i - P_g\right)\right] \\ & = \Xi_{\varpi} + \eta_{ii}\left(P_i - P_g\right) = \Xi_{\varpi} + \alpha_i \eta_{jj}\left(P_i - P_g\right). \end{aligned} \tag{60}$$

Along the line in (59) and (60), one has

$$\alpha_i \eta_{jj}\left(P_i - P_g\right) = \alpha_i \hat{\eta}_{jj}\left(P_i - P_g\right) + \alpha_i \Delta\eta_{jj}\left(P_i - P_g\right). \tag{61}$$

For any $\mathcal{W}_i > 0$, one has

$$\begin{aligned} \Delta\eta_{jj}\left(P_i - P_g\right) = & \left[\frac{1}{2}\Delta\eta_{jj}\left(P_i - P_g\right) + \frac{1}{2}\Delta\eta_{jj}\left(P_i - P_g\right)\right] \\ & \leqslant \left[\frac{\left(\beta_{jj}\right)^2}{4}\mathcal{W}_i + \left(P_i - P_g\right)\left(\mathcal{W}_i\right)^{-1}\left(P_i - P_g\right)^{\mathrm{T}}\right]. \end{aligned} \tag{62}$$

On the basis of (60), (61) and (62), one sees that the DSMJSs in (4) are stochastically stable based on Schur complement and (39).
Case 4: ${\Omega_{uk}}^i = \mathcal{J}$ and $j \notin {\Omega_k}^i$ for any $j \in \mathcal{J}$.

In such case, η_{ij_a} is estimated by η_{jj_a}. Also, we denote $\lambda_{i,k} \triangleq \eta_{ij_a}\,(j_a \neq i)$. Then, $\sum_{j=1}^{J} \eta_{ij} P_j$ is changed into

$$\begin{aligned}\sum_{j=1}^{J} \eta_{ij} P_j &= \eta_{ij_a} P_{j_a} + \eta_{ii} P_i + \sum_{j \in I_{i,uk}, j \neq i} \eta_{ij} P_j \\ &= \eta_{ij_a} P_{j_a} + \eta_{ii} P_i - (\eta_{ii} + \lambda_{i,k}) \sum_{j \in \Omega_{uk}{}^{i}, j \neq i} \frac{\eta_{ij} P_j}{-\eta_{ii} - \lambda_{i,k}},\end{aligned} \tag{63}$$

then, one has

$$\begin{aligned}\varXi_{\varpi} + \sum_{j=1}^{N} \eta_{ij} P_j = \sum_{g \in \varOmega_{uk}{}^{i}, j \neq i} \big[& \varXi_{\varpi} \\ & + \eta_{ii} \left(P_i - P_g\right) + \eta_{ij_a} \left(P_{j_a} - P_g\right) \big],\end{aligned} \tag{64}$$

since $0 \leqslant \eta_{ij} \leqslant -\eta_{ii} - \lambda_{i,k}$, then $\varXi_{\varpi} + \eta_{ii}\left(P_i - P_g\right) + \eta_{ij_a}\left(P_{j_a} - P_g\right) < 0$ is equivalent to

$$\varXi_{i,4} + \eta_{ii} \left(P_i - P_g\right) + \alpha_i \eta_{jj_a} \left(P_{j_a} - P_g\right) < 0. \tag{65}$$

It is obvious that $\eta_{ii} < 0$. Therefore, holds if

$$\begin{cases} P_i - P_g \geqslant 0, \\ \varXi_{\varpi} + \alpha_i \eta_{jj_a} \left(P_{j_a} - P_g\right) < 0. \end{cases} \tag{66}$$

For any $\mathcal{X}_{ij_\alpha} > 0$, it holds that

$$\begin{aligned}\eta_{jj_a} \left(P_{j_a} - P_g\right) &= \hat{\eta}_{jj_a} \left(P_{j_a} - P_g\right) + \varDelta \eta_{jj_a} \left(P_{j_a} - P_g\right) \\ &\leqslant \hat{\eta}_{jj_a} \left(P_{j_a} - P_g\right) \\ &\quad + \left[\frac{\left(\beta_{jj_a}\right)^2}{4} \mathcal{X}_{ij_a} + \left(P_{j_a} - P_g\right) \left(\mathcal{X}_{ij_a}\right)^{-1} \left(P_{j_a} - P_g\right)^{\mathrm{T}} \right].\end{aligned} \tag{67}$$

Similar to (61), $\mathcal{L}\mathcal{V}_3\left(x\left(t\right), i\right) < 0$ can hold, which accomplishes the proof. □

4. Simulation example

In this section, the model of single-link robot arm in Shen, Li, Cao, Wu, and Lu (2020) is introduced to evaluate the availability of the proposed method, which can be described as follows

$$\ddot{\partial}\left(t\right) = -\frac{M\left(r_t\right) gL}{J\left(r_t\right)} \sin\left(\partial\left(t\right)\right) - \frac{D}{J\left(r_t\right)} \dot{\partial}\left(t\right) + \frac{1}{J\left(r_t\right)} \mu\left(t, u\right),$$

in which $\partial\left(t\right)$, $M\left(r_t\right)$, L, $J\left(r_t\right)$, D and g mean the arm's angular position, mass of the payload, the length of the arm, the moment of inertia, the coefficient of viscous friction and the acceleration of gravity respectively, and the specific selection values are given as follows: $g = 9.80\mathrm{m/s^2}$, $L = 0.5\mathrm{m}$ and $D = 2\mathrm{N}\cdot\mathrm{s/m^2}$. In the actual operation of the robotarm, some variations of the payloads and working environment may cause random changes in $M(r_t)$ and $J(r_t)$. Then, setting $x_1(t) = \partial(t)$, $x_2(t) = \dot{\partial}(t)$, and the model of single-link robot arm with $r_t = i$ is considered as:

$$A_i = \begin{bmatrix} 0 & 1 \\ -\frac{M_i g L}{J_i} & -\frac{D}{J_i} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ \frac{1}{J_i} \end{bmatrix},$$

where the parameters M_i and J_i are given by: $M_1 = M_2 = M_3 = 1.5$, $J_1 = 1$, $J_2 = 1.5$ and $J_3 = 2$, which correspond to these three modes, respectively. The structural uncertainties $Y(t)$ and the AF model $\mu(t,u)$ are given as $Y(t) = 0.3\cos(100t)$, $\mu(t,u) = (0.5 + 0.3\sin(t))\,u(t)$. And the AA signals are chosen as:

$$\varPhi(x,t,\tau) = \begin{cases} 0.3 + 0.2\cos(100t), & 0.5\mathrm{s} \leqslant t < 4\mathrm{s}; \\ 3 + 0.5\sin(50t)\,(x_2(t) + x_2(t-\tau)), & 8\mathrm{s} \leqslant t < 10\mathrm{s}; \\ 0, & t \notin [0.5\mathrm{s}, 4\mathrm{s}) \cup [8\mathrm{s}, 10\mathrm{s}); \end{cases}$$

in which the system time-delay τ is set as $\tau = 0.1\mathrm{s}$. And the system matrices with three modes are displayed as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -7.35 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -4.9 & -1.33 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 1 \\ -3.8 & -1 \end{bmatrix};$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.67 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix};$$

$$A_{1\tau} = \begin{bmatrix} 0.9 & 0.9 \\ 0.4 & 1 \end{bmatrix}, A_{2\tau} = \begin{bmatrix} 0.8 & 0.7 \\ 0.7 & 0.7 \end{bmatrix}, A_{3\tau} = \begin{bmatrix} 0.7 & 0.6 \\ 0.6 & 0.5 \end{bmatrix};$$

$$K_1 = \begin{bmatrix} -0.2127 \\ 3.4535 \end{bmatrix}, K_2 = \begin{bmatrix} -1.9578 \\ 3.6754 \end{bmatrix}, K_3 = \begin{bmatrix} -1.2356 \\ -0.9631 \end{bmatrix};$$

$$\mathcal{M}_1 = \begin{bmatrix} -0.9 & 0.5 \\ 0.1 & -0.6 \end{bmatrix}, \mathcal{M}_2 = \begin{bmatrix} -0.5 & 0.3 \\ 0.1 & -0.5 \end{bmatrix}, \mathcal{M}_3 = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.2 \end{bmatrix};$$

$$\mathcal{N}_1 = \begin{bmatrix} 0.2 & -0.1 \\ 0 & -0.1 \end{bmatrix}, \mathcal{N}_2 = \begin{bmatrix} 0.1 & -0.1 \\ 0 & -0.1 \end{bmatrix}, \mathcal{N}_3 = \begin{bmatrix} -0.2 & 0.4 \\ 0 & -0.1 \end{bmatrix};$$

$$\mathcal{N}_{1\tau} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \mathcal{N}_{2\tau} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \mathcal{N}_{3\tau} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}.$$

Set the TR matrix as follows:

$$\Pi_1 = \begin{bmatrix} -0.45 + \varDelta\eta_{11} & ? & 0.27 + \varDelta\eta_{13} \\ ? & ? & 0.38 + \varDelta\eta_{23} \\ ? & ? & ? \end{bmatrix},$$

where the unknown element "?" implies that both $\hat{\eta}_{ij}$ and $\varDelta\eta_{ij}$ of the mentioned TR information are not available, $i, j \in \{1, 2, 3\}$. By virtue of Theorem 3.3, the corresponding matrices are solved:

$$P_1 = \begin{bmatrix} 1.5323 & 0.3790 \\ 0.3790 & 0.3955 \end{bmatrix}, P_2 = \begin{bmatrix} 1.6982 & 0.7321 \\ 0.7321 & 0.7443 \end{bmatrix}, P_3 = \begin{bmatrix} 1.7796 & 0.7818 \\ 0.7818 & 0.7879 \end{bmatrix};$$

$$Q_1 = \begin{bmatrix} 2.5755 & 2.3115 \\ 2.3115 & 2.3724 \end{bmatrix}, \mathcal{U}_{1,1} = \begin{bmatrix} 2.7836 & 0.1978 \\ 0.1978 & 2.5050 \end{bmatrix}, \mathcal{U}_{1,3} = \begin{bmatrix} 2.3848 & 0.0150 \\ 0.0150 & 2.3856 \end{bmatrix},$$

$$\mathcal{Z}_{2,3} = \begin{bmatrix} 2.4635 & 0.0816 \\ 0.0816 & 2.4968 \end{bmatrix}, \mathcal{W}_3 = \begin{bmatrix} 2.5410 & 0.0893 \\ 0.0893 & 2.5023 \end{bmatrix}.$$

In the simulation experiment, the NN has been designed with 20 neurons, and the gains $\hbar_i$ are chosen as $\hbar_1 = 0.1$, $\hbar_2 = 0.2$ and $\hbar_3 = 0.2$. Additionally, the initial condition for system state is given as $x(t) = [1, 0.5]^{\mathrm{T}}$, and the initial values of $\hat{\mathcal{W}}$, $\hat{\mathcal{H}}$ and $\hat{\alpha}$ are chosen as $\hat{\mathcal{W}}(0) = [0.1]_{20\times1}$, $\hat{\mathcal{H}}(0) = [0.1]_{5\times20}$ and $\hat{\alpha}(0) = [0.1, 0.2, 0.1, 0.1]^{\mathrm{T}}$ respectively. Herein, in order to reduce system chattering, $s(t) / (0.01 + \|s(t)\|)$ is considered to replace the term $\mathrm{sgn}(s(t))$. Meanwhile, to illustrate the validity of the proposed adaptive NN-based SMC strategy, the SMC methoddesigned in Qi et al. (2020) is utilized as a comparison, from which the corresponding matrices $\mathcal{K}_{com,i}$ are generated as follows:

$$\mathcal{K}_{com,1} = [-19.7826 - 0.8103], \mathcal{K}_{com,2} = [-21.3626 - 9.3536].$$

In view of the aforementioned settings, the simulation verification between the devised control strategy and the method in Qi et al. (2020) is performed below. The jump mode r_t, the state trajectories $x(t)$ and control signals $u(t)$, sliding variable $s(t)$ and norm curves of $\hat{\alpha}(t)$, $\hat{W}(t)$ and $\hat{\mathcal{H}}(t)$ are offered in Figs. 2-6. In detail, Fig. 1 shows the state response of the target system. Fig. 2 plots the curves of the control input. In Fig. 3 and Fig. 4, the norm curves of adaptive parameters are displayed. As seen from Fig. 1, although the state responses of the resultant DSMJSs are clearly influenced by AA signals during the time periods [0.5s, 4s] and [8s, 10s], the system will swiftly recover to its stable state under the proposed NN-based SMC law. As can be seen, the current control scheme could achieve better static and dynamic performance in contrast to the method in Qi et al. (2021).

Additionally, EC, integral absolute error (IAE), integral time multiplied absolute error (ITAE) and integral of squared error (ISE) are offered to analyze the two methods through the above quantitative indicators (Moawad, Elawady, and Sarhan, 2019; Mobayen and Majd, 2012), from which the detailed descriptions are dispalyed as fol-

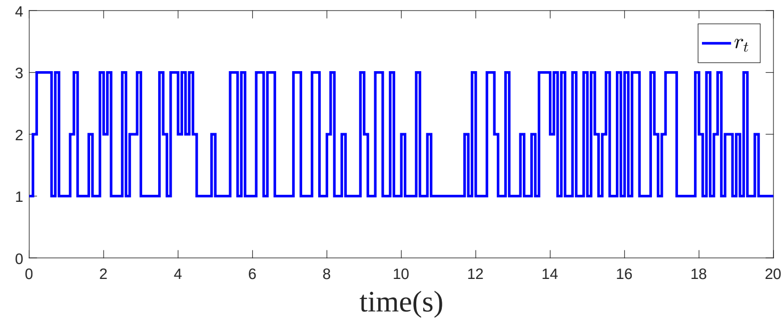

Figure 2. Jump modes r_t

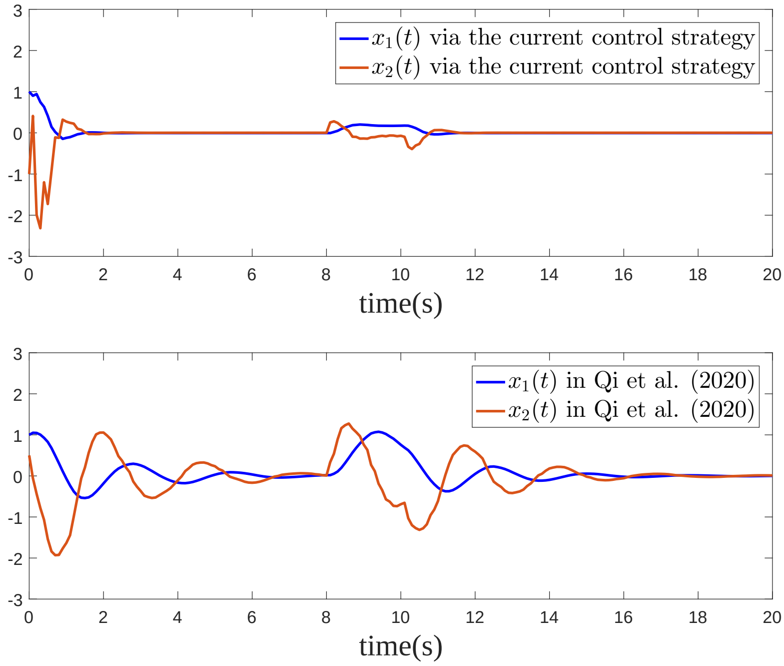

Figure 3. Responses of the DSMJSs via the devised method and the method in Qi et al. (2020)

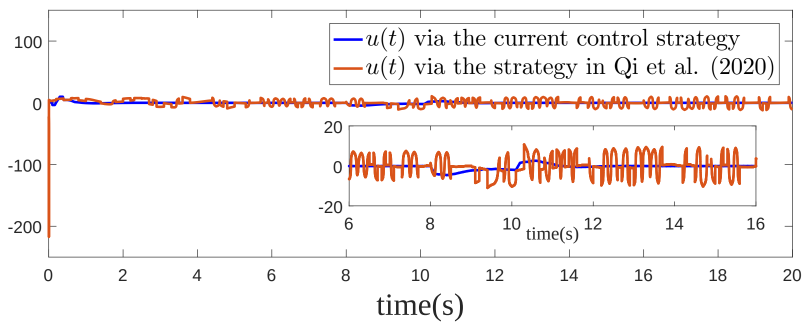

Figure 4. Evolutions of $u\,(t)$ via the devised method and the method in Qi et al. (2020)

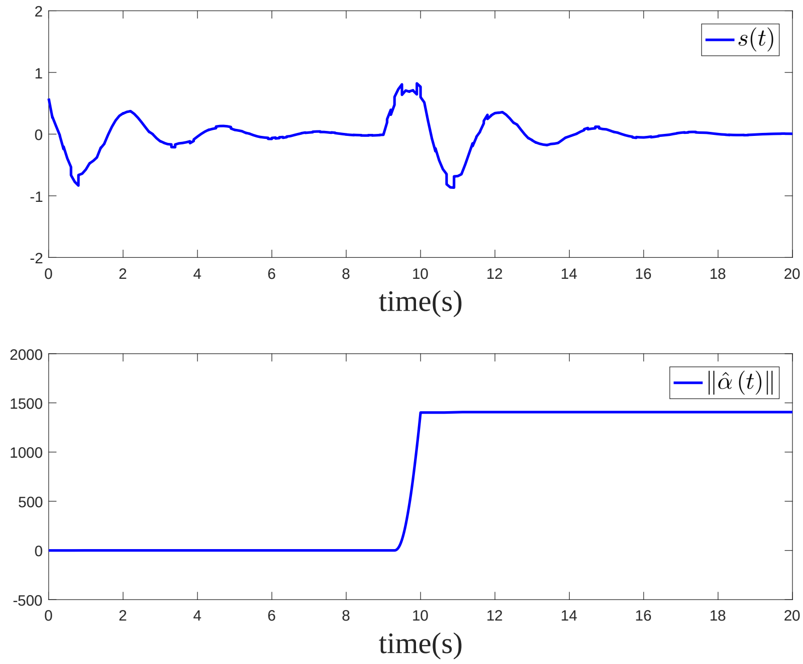

Figure 5. Plot of $s\left(t\right)$ and norm curve of $\hat{\alpha}\left(t\right)$

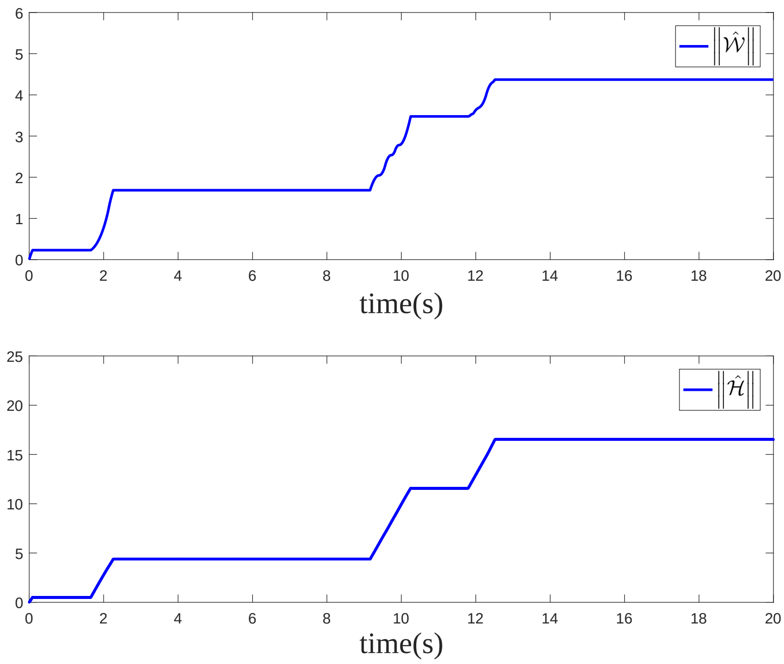

Figure 6. Norm curves of $\hat{W}\left(t\right)$ and $\hat{\mathcal{H}}\left(t\right)$

lows:

$$\mathrm{EC} = \int_0^{20\mathrm{s}} u^2(t)\,\mathrm{d}t, \quad \mathrm{IAE} = \int_0^{20\mathrm{s}} \|e(t)\|\,\mathrm{d}t,$$
$$\mathrm{ITAE} = \int_0^{20\mathrm{s}} t\,\|e(t)\|\,\mathrm{d}t, \quad \mathrm{ISE} = \int_0^{20\mathrm{s}} \|e(t)\|^2\,\mathrm{d}t,$$

where $e(t) = x_d(t) - x(t)$ refers to the state error, $x(t)$ stands for the actual state and $x_d(t) = 0$ denotes the ideal system state. Therefore, the results of the indicators are then displayed in Table 1, which demonstrates that the proposed control approach could enhance control accuracy with less EC.

Table 1. Comparisons results ofperformance indicators

Methods	Proposed method	Method in Qi et. al (2020)
EC	165.9948	5257.3375
IAE	1.7355	11.8778
ITAE	7.4881	97.0757
ISE	1.5486	13.4764

5. Conclusion

An adaptive NN-based SMC approach to address the security control problem for DSMJSs with GUTRs, AAs, and AFs has been proposed in this research. By virtue of strong approximation ability of NN, an adaptive NN-based sliding mode controller synthesis has been developed, which could not only force the state trajectories onto the devised SMS but also ensure the DSMJSs that operates as demanded. Then, a novelstochastic stability decision condition on the sliding motion has been derived under the premise of reachability of the proposed SMS. Finally, a single-link robot arm model has been adopted as an example with simulation comparison to demonstrate the feasibility of the prescribed control strategy.

In future research, the proposed control method will been considered to be applied for more general s-MJSs with time-varying delays, actuator attacks and sensor attacks, simultaneously. Besides, due to the limitations of physical devices, actuator saturation often inevitably appears in practical systems, which may reduce performance and even lead to instability of the system. Therefore, the security control for stochastic nonlinear systems with actuator saturation will also be the scope of our future work.

Disclosure statement

The authors have no relevant financial or non-financial interests to disclose.

Data availability statement

The data that supports the findings of this study is available from the corresponding author upon reasonable request.

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