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# **An improved U-control design for nonlinear systems represented by input/output differential models with a disturbance observer**

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#### **ABSTRACT**

This paper presents a new method to calculate the inversion of the controlled linear/nonlinear dynamic plants which are described by input–output differential equation models. This new U-model-based inverter (U-inverter), cancels both system dynamics and nonlinearities, can be used directly to facilitate the control system design, which the design of U-model-based control (U-control) systems is selected for demonstration in this study. The most important advantage of this U-inverter is that it does not require system state variables, only uses the input/output measurements from the controlled plants. A nonlinear disturbance observer is introduced into the U-control design framework to increase its robustness. The analysis explains the properties of this developed the disturbance observer-based U-control (DOBUC) method and its design procedures. Finally, a wind energy conversion system is simulated to illustrate the design of this DOBUC and the corresponding performance.

# <span id="page-1-1"></span>**ARTICLE HISTORY**

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# **KEYWORDS**

U-model based control (U-control); disturbance observer (DOB); dynamic inversion; wind energy conversion system (WECS)

# **1. Introduction**

<span id="page-1-11"></span><span id="page-1-10"></span>The discrepancies between the real-time controlled applications and their mathematical modelling are always one of the main problems in the control system design. These discrepancies include system external disturbances and internal modelling errors which can result in critically pernicious effects on engineering practical operations and research development. There are at least two ways to deal with these control design problems caused by mentioned discrepancies. The first one involves robust control which considers the worst-case situation in the control system design and uses feedback principium to suppress the uncertainties and disturbances (Ding et al., [2020;](#page-12-0) Shen et al., [2018;](#page-12-1) Wang et al., [2022\)](#page-12-2). However, this kind of methods may over-estimate the upper disturbances' bounds and therefore results in unsatisfactory system dynamic and control performance. The second approach is based on discrepancies estimation and compensation theory. Generally, this kind of composite control system (Zhang et al., [2018\)](#page-12-3) contains two design steps: design a baseline controller to meet the required control performance in the external loop and a disturbance attenuator to cancel or compensate the discrepancies in an inner loop. In the approach considered in this study, a disturbance observer (DOB) is used for discrepancies estimation between the real-time controlled plant and its mathematical modelling.

The accurate estimation of system uncertainties and disturbances is compulsory and required in DOB-based control (DOBC) design. The fundamental DOB structure in the frequency-domain was first proposed in 1980s (Ohishi et al., [1987\)](#page-12-4), which estimates the lumped system disturbances by using the dissimilarity between the control system input measurement and the theoretical input calculated by the controlled plant's dynamic inversion. Accordingly, the basic principle of this DOB (Ohishi et al., [1987\)](#page-12-4) is to amalgamate all uncertainties and system external disturbances into the control input channel. Remarkably, this frequency-domain DOB is applicable for linear control systems, or nonlinear control systems that consider nonlinearity as uncertainty. However, for some nonlinear control systems, the estimation performance by this linear frequencydomain DOB may not be satisfactory in practical operations. Generally, dynamic modelling of many practical nonlinear systems has been well understood and applied, the variables of which are also measurable with high accuracy (Chen et al., [2016\)](#page-12-5), therefore, making extensive assumptions about unknown elements subject to mild constraints, the cautiously reasonably using of such available information/assumptions could promote more efficient control system design.

<span id="page-1-12"></span><span id="page-1-8"></span><span id="page-1-7"></span><span id="page-1-6"></span><span id="page-1-5"></span><span id="page-1-4"></span><span id="page-1-3"></span><span id="page-1-2"></span>To deal with nonlinearities efficiently, nonlinear DOB (NDOB) was first proposed by (Chen et al., [2000\)](#page-12-6) based on the estimation of disturbance torque caused by a disturbance in robotic manipulators. In a work (Chen, [2003\)](#page-12-7), Lyapunov stability theory and mathematical analysis proved that the perturbation error estimated by the NDOB method will exponentially converge to zero. Based on these theoretical analysis and proof, the NDOB method has been widely used in academic research and many real-time applications, such as UAV attitude control design (Chen et al., [2020\)](#page-12-8), position control of seven-degree-offreedom nonlinear manipulators (Müller et al., [2020\)](#page-12-9), Quadrotor complex attitude tracking control (Ahmed et al., [2020\)](#page-12-10).

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<span id="page-2-5"></span>Another method, which develops the linear frequency-domain DOBC to be applied to a nonlinear system (Li, Zhu, Yang, et al., [2021\)](#page-12-11), involves nonlinear inversion calculated by the U-modelbased dynamic inversion method (Li et al., [2020\)](#page-12-12). Compared with (Chen et al., [2000\)](#page-12-6) both assume that the state variables are gauged, and the DOB in Li, Zhu, Yang, et al. [\(2021\)](#page-12-11) can amalgamate all the uncertainties and external system disturbances into a control input channel, which is more intuitional and can be used for system input compensation directly with less computation and simpler adjustment (Li, Zhu, Yang, et al., [2021\)](#page-12-11). However, the DOB design in Li, Zhu, Yang, et al. [\(2021\)](#page-12-11) only focuses on a fully actuated system and requires the high-order derivative of system output or state variables, which restricts its practical applications.

<span id="page-2-6"></span>Another component of the DOBC is a baseline controller that meets the required specifications of control system performance. Based on the principles of the U-model control (Ucontrol) (Zhang et al., [2020;](#page-12-13) Zhu et al., [2015\)](#page-12-14), the controlled plant can be recompensed to a unit constant or identity matrix through UM dynamic inversion method (Li et al., [2020\)](#page-12-12), therefore cancelling the system dynamic and nonlinearity simultaneously, providing a control system design method with no phase delay between system input and output. Therefore, the superiority of U-control has attracted research. For example, U-neuro-control (Zhu, Zhang, Zhang, et al., [2019\)](#page-12-15), U-internal model control (Hussain et al., [2019\)](#page-12-16), U-two-degree-of-freedom IMC (Li, Zhu, Narayan, et al., [2021\)](#page-12-17) and U-double-slidingmodel control (Zhu et al., [2022\)](#page-12-18). However, the approaches in Hussain et al. [\(2019\)](#page-12-16), Li, Zhu, Narayan, et al. [\(2021\)](#page-12-17), Zhang et al. [\(2020\)](#page-12-13), Zhu et al. [\(2015,](#page-12-14) [2022\)](#page-12-18) and Zhu, Zhang, Zhang, et al. [\(2019\)](#page-12-15) require system state variables to be known or measurable, in other words, if certain system state variables are unmeasurable or difficult to measure, these control methods cannot be applied or cost expensively. (Zhu, [2021\)](#page-12-19) applies an Extend State Observer (ESO) to observe state variables and uses the observed state variables to design a controller, but the limitations of ESO restrict its practical applications. Inspired by this significant research, the purpose of this study is to develop a novel Ucontrol design framework that only uses the input/output signal from the control system. Additionally, because of the high sensitivity of the U-inverter to the system modelling and system external disturbances, the proposed U-control design framework is combined with a nonlinear DOB (Ding et al., [2019\)](#page-12-20) which also only requires the input/output measurements from the control system to increase its robustness.

Summarily, the main contributions in this paper are: Firstly, a new U-control design framework has been proposed with the utility of input/output measurement from the control system, which also provides a robust dynamic inversion scheme and can cancel both system dynamics and nonlinearities. Secondly, this study provides a universal phase delay-free, fast-response and convenient control system design framework through the integration of U-control and NDOB, which is acceptable to all linear/nonlinear control systems. The rest of the organization of this paper is: Section 2 presents the basic theory and assumption linking to the follow-up technology study. Section 3 proposes a novel disturbance observer-based U-control (DOBUC) design framework with convergence analysis through mathematics. A simulation experiment of control of wind energy conversion

system (WECS) illustrates the feasibility and efficiency of the studies in Section 4. Finally, Section 5 concludes the whole findings in this study.

# <span id="page-2-3"></span>**2. Problem statement**

This study considers the following SISO input/output differential equation to describe a general nonlinear dynamic system

$$
y_1^{(n)} = f(Y_1) + g_0(Y_1)u + \ldots + g_m(Y_1)u^{(m)}, \ m \le n \qquad (1)
$$

where  $y_1$  is the system output, *u* is the system input,  $Y_1$  absorbs all the system output  $y_1$  and its derivatives except the highest order one  $y_1^{(n)}$ , that is

<span id="page-2-0"></span>
$$
Y_1 = (y_1, \dot{y}_1, \ddot{y}_1, \dots, y_1^{(n-1)})
$$
 (2)

<span id="page-2-9"></span>**Remark 2.1:** Model (1) is widely used to describe a kind of flat nonlinear system (Conte et al., [2007\)](#page-12-21). Generally,  $g_i(Y_1)$  =  $0, i \in \mathbb{N}^+$  applied in many practical applications, it means the high order derivatives of system input  $u^{(i)}$ ,  $i \in \mathbb{N}^+$  are not existed. When the system considers actuator,  $g_i(Y_1) = 0, i \ge 2$ , it means the control system only needs system input *u* and its derivative *u*˙.

<span id="page-2-10"></span><span id="page-2-4"></span><span id="page-2-2"></span>**Assumption 2.1 (BIBO):** *The system output y*<sup>1</sup> *and its related higher-order derivatives*  $y_1, y_1, \ldots, y_1^{(n-1)}$  can be bounded with a *proper control input u. That is, the bounded input and bounded output (BIBO).*

<span id="page-2-8"></span>**Assumption 2.2 (Invertibility):** *The following zero dynamic meets globally asymptotically stability:*

$$
g_0(Y_1)u + g_0(Y_1)u + \ldots + g_m(Y_1)u^{(m)} = 0 \qquad (3)
$$

<span id="page-2-7"></span>**Remark 2.2:** Assumption 2.1 has been well pre-requested by a lot of linear/nonlinear control systems with BIBO stability. Technically, this assumption would limit the field of applications with unstable zero dynamic plants. In this study, assumption 2.1 that 'the output y and its derivatives are bounded' is a reasonable assumption for practical BIBO systems. Surely in future studies, dealing with unstable dynamics is a challenging topic.

<span id="page-2-1"></span>Meanwhile, the zero dynamics of system (1) is in case of

$$
g_0(0)u + g_0(0)\dot{u} + \ldots + g_m(0)u^{(m)} = 0 \qquad (4)
$$

The dynamic system (4) is globally asymptotically stable when it satisfies Assumption 2.2, which means the control system described in (1) is minimum-phase/linear plants or stable zero dynamics/nonlinear plants. When using (1) to describe linear dynamic systems, it turns to:

$$
y_1^{(n)} + a_{n-1}y_1^{(n-1)} + \dots + a_1\dot{y}_1 + a_0y_1
$$
  
=  $b_m u^{(m)} + b_{m-1}u^{(m-1)} + \dots + b_0\dot{u} + b_0u, m \le n$  (5)

Then Assumption 2.2 turns to:

$$
b_{m}u^{(m)} + b_{m-1}u^{(m-1)} + \ldots + b_{0}\dot{u} + b_{0}u = 0 \qquad (6)
$$

which directly links to the property of stable zero dynamics for the control system (5). The next step is to use the U-control



<span id="page-3-0"></span>**Figure 1.** U-control system design framework.

method to design the baseline controller to cancel both the nonlinearity and dynamic of (1) and design a DOB to improve its robustness. It should be mentioned that all design of this new composite control system only uses the measurement from the control input *u* and the output *y* of the system (1), rather than requiring other information like high-order derivatives of system output  $y^{(n)}$  and/or system state variables.

# **3. New design DOBUC systems**

### *3.1 U-model-based control design*

The continues-time (CT) U-control system design framework is shown in Figure [1](#page-3-0) (Zhu, Zhang, Na, et al., [2019\)](#page-12-22). The U-control system separates the controlled plant inversion and baseline controller design in double feedback loops. The inner loop is controlled plant's dynamic inversion aiming to cancel system nonlinearities and dynamics (Zhu & Guo, [2002\)](#page-12-23). Accordingly, the output of the U-control system can be expressed as

$$
y = \frac{G_{c1}G_0^{-1}G_P}{1 + G_{c1}G_0^{-1}G_P}r + \frac{G_P}{1 + G_{c1}G_0^{-1}G_P}d\tag{7}
$$

where  $G_{c1}$  is called the invariant controller,  $G_P$  is the practical controlled plant and  $G_0^{-1}$  is its nominal inversion. *r* is the reference tracking signal. Model  $G_P = G_0$  can be attained when the control system is free of modelling errors and external disturbance, distinctively, the output of (7) can be simplified when  $G_P = G_0$ :

$$
y = \frac{G_{c1}}{1 + G_{c1}} r = Gr
$$
 (8)

where *G* describes the total gain of this control system, which can be easily designed by customer-defined damping ratio  $\zeta$  and undamped natural frequency ω*<sup>n</sup>* typically.

According to the U-M inversion algorithm (Li et al., [2020\)](#page-12-12), the inversion of system (1) is expressed as

$$
G_P^{-1} \Leftrightarrow u^{(m)} \in y_1^{(n)} - f(Y_1) - g_0(Y_1)u
$$
  
- ... - g\_m(Y\_1)u^{(m)} = 0, m \le n (9)

**Remark 3.1:** The implementation of system's (1) inversion requires system (1) be BIBO stable and invertible (that is, without unstable zero dynamic). When the highest-order derivatives of system input  $u^{(m)}$  is solved, the other *u* related derivatives can be obtained by integration of  $u^{(m)}$ .

#### *3.2 New U-inverter design*

Section 3.1 has explained the basic concept/configuration of the U-control system. Manifestly, the solving process of the controlled plant's inversion (9) requires the system input *u* and output *y* and their related high-order derivatives  $u^{(m)}$  and  $y^{(n)}$ . In this section, a new U-inverter design based on the input/output model (1) is presented. The dashed area in Figure [2](#page-4-0) shows the basic design framework of this new U-inverter. Where *r* is the reference input of the U-inverter (marked in dashed zone), *u* and  $y_1$  are the controlled plant input/output, respectively.

<span id="page-3-1"></span>Based on the U-control framework, the U-inverter is designed to convert the controlled plant into an identity matrix or a unit constant. When there are no modelling errors and external disturbance from the system,  $G_0^{-1}G_P = 1$ . In this case, the input of the U-inverter equals to the output of controlled plant, that is,  $r = v_1$ .

Let the controlled plant input *u* be

$$
u = k_1(r - y_1) \tag{10}
$$

<span id="page-3-2"></span>where  $k_1 > 0$  is a sufficient large constant. It follows from (10) that

$$
\begin{cases}\n u = k_1(r - y_1) \\
 \dot{u} = k_1(\dot{r} - \dot{y}_1) \\
 \vdots \\
 u^{(n)} = k_1(r^{(n)} - y_1^{(n)})\n\end{cases}
$$
\n(11)

Clearly, from Equations (10) and (11) and Figure [2,](#page-4-0) the proposed U-inverter only requires the signal information of system output  $y_1$  and reference  $r$ . Different from the general U-inverter, high-order derivatives of  $y_1$  are not required in the whole design procedure. From (11), it follows that

$$
y_1 = r - \frac{u}{k_1}
$$
  
\n
$$
\dot{y}_1 = \dot{r} - \frac{u}{k_1}
$$
  
\n
$$
\vdots
$$
  
\n
$$
y_1^{(n-1)} = r^{(n-1)} - \frac{u^{(n-1)}}{k_1}
$$
  
\n
$$
y_1^{(n)} = r^{(n)} - \frac{u^{(n)}}{k_1}
$$
  
\n(12)

From (1),

$$
y_1^{(n)} = f(y_1, \ddot{y}_1, \ddot{y}_1, \dots, y_1^{(n-1)})
$$



**Figure 2.** The design framework of new U-inverter.

<span id="page-4-0"></span>
$$
+ g_0(y_1, \dot{y}_1, \ddot{y}_1, \dots, y_1^{(n-1)})u + \dots
$$
  
\n
$$
+ g_m(y_1, \dot{y}_1, \ddot{y}_1, \dots, y_1^{(n-1)})u^{(m)}
$$
  
\n
$$
r^{(n)} - \frac{u^{(n)}}{k_1} = f\left(r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right)
$$
  
\n
$$
+ g_0\left(r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right)u
$$
  
\n
$$
+ \dots + g_m\left(r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right)u^{(m)}
$$
(13)

Add the same term  $f(R) + g_0(R)u + ... + g_m(R)u^{(m)}$  to each side of (13) with

$$
R = (r, \dot{r}, \ddot{r}, \dots, r^{(n-1)})
$$
 (14)

Then (13) turns to:

$$
r^{(n)} - \frac{u^{(n)}}{k_1}
$$
  
=  $f(R) + g_0(R)u + \dots + g_m(R)u^{(m)}$   
+  $f\left(r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right)$   
-  $f(r, \dot{r}, \dots, r^{(n-1)})$   
+  $g_0\left(r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right)u$   
-  $g_0(r, \dot{r}, \dots, r^{(n-1)})u$   
+  $\dots + g_m\left(r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right)u^{(m)}$   
-  $g_m(r, \dot{r}, \dots, r^{(n-1)})u^{(m)}$  (15)

Because the functions  $f(r, r, \ldots, r^{(n-1)})$  and  $g_i(r, r, \ldots, r^{(n-1)})$ *r*<sup>(*n*−1)</sup>), *i* ∈ N are both smooth, the following inequalities exist at least locally

$$
\left\| f\left(r - \frac{u}{k_1}, r - \frac{u}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1}\right) \right\|
$$
  
-  $f(r, r, \dots, r^{(n-1)}) \left\| \le \gamma_1 \left\| \left(\frac{u}{k_1}, \frac{u}{k_1}, \dots, \frac{u^{(n-1)}}{k_1}\right) \right\| \right\|$   
=  $\frac{\gamma_1}{k_1} \| (u, u, \dots, u^{(n-1)}) \|$  (16)

And

$$
\left\| g_i \left( r - \frac{u}{k_1}, \dot{r} - \frac{\dot{u}}{k_1}, \dots, r^{(n-1)} - \frac{u^{(n-1)}}{k_1} \right) \right\|
$$
  
-  $g_i(r, \dot{r}, \dots, r^{(n-1)}) \left\| \le \gamma_2 \left\| \left( \frac{u}{k_1}, \frac{\dot{u}}{k_1}, \dots, \frac{u^{(n-1)}}{k_1} \right) \right\| \right\|$   
=  $\frac{\gamma_2}{k_1} \| (u, \dot{u}, \dots, u^{(n-1)}) \|$  (17)

where  $\gamma_1$  and  $\gamma_2$  are Lipschitz constants. For convenient description, let

$$
U = ||(u, \dot{u}, \ddot{u}, \dots, u^{(n-1)})|| \tag{18}
$$

Move term  $f(R) + g_0(R)u + \ldots + g_m(R)u^{(m)}$  to the left side of Equation (15), organize it according to principles described in (16) and (17), then Equation (15) turns into

$$
\left| r^{(n)} - \frac{u^{(n)}}{k_1} - (f(R) + g_0(R)u + \dots + g_m(R)u^{(m)}) \right|
$$
  

$$
\leq \frac{\gamma_1}{k_1}U + \frac{\gamma_2}{k_1}U|u| + \frac{\gamma_2}{k_1}U|u| + \dots + \frac{\gamma_2}{k_1}U|u^{(m)}| \quad (19)
$$

From (19), it is clearly that when  $k_1 \rightarrow \infty$ , the right side of Equation (19) has

$$
\lim_{k_1 \to \infty} \frac{\gamma_1}{k_1} U + \frac{\gamma_2}{k_1} U|u| + \frac{\gamma_2}{k_1} U|u| + \ldots + \frac{\gamma_2}{k_1} U|u^{(m)}| = 0
$$
\n(20)

Therefore, the left side of Equation (19) correspondingly has

$$
\lim_{k_1 \to \infty} \left| r^{(n)} - \frac{u^{(n)}}{k_1} - (f(R) + g_0(R)u + \dots + g_m(R)u^{(m)}) \right|
$$
  
= 0

$$
\lim_{k_1 \to \infty} r^{(n)} - \frac{u^{(n)}}{k_1} = f(R) + g_0(R)u + \ldots + g_m(R)u^{(m)} \quad (21)
$$

Let (1) subtract (21), it comes

$$
y_1^{(n)} - \left(r^{(n)} - \frac{u^{(n)}}{k_1}\right)
$$
  
=  $f(Y_1) - f(R) + g_0(Y_1)u - g_0(R)u$   
+ ... +  $g_m(Y_1)u^{(m)} - g_m(R)u^{(m)}$   
=  $f(Y_1) - f(R) + (g_0(Y_1) - g_0(R))u + ...$   
+  $(g_m(Y_1) - g_m(R))u^{(m)}$  (22)

Since the functions  $f(r, r, \ldots, r^{(n-1)})$  and  $g_i(r, r, \ldots, r^{(n-1)})$ ,  $i \in N$  are smooth, the following inequalities exist at least locally

$$
||g_i(r, \dot{r}, \dots, r^{(n-1)}) - g_i(y_1, \dot{y}_1, \dots, y_1^{(n-1)})||
$$
  
\n
$$
\leq \gamma_3 ||(r - y_1, \dot{r} - \dot{y}_1, \dots, r^{(n-1)} - y_1^{(n-1)})||
$$
  
\n
$$
= \frac{\gamma_3}{k_1} ||(u, \dot{u}, \dots, u^{(n-1)})||
$$
\n(23)

where  $\gamma_3$  is Lipschitz constant. Move term  $f(Y_1) - f(R)$  to the left side of Equation (22), it has

$$
\left| y_1^{(n)} - \left( r^{(n)} - \frac{u^{(n)}}{k_1} \right) - (f(Y_1) - f(R)) \right|
$$
  
\n
$$
\leq \frac{\gamma_3}{k_1} U|u| + \frac{\gamma_3}{k_1} U|u| + \dots + \frac{\gamma_3}{k_1} U|u^{(m)}|
$$
  
\n
$$
= \frac{\gamma_3}{k_1} U(|u| + |u| + \dots + u^{(m)})
$$
(24)

When  $k_1 > 0$  tends to infinity, the right term in (24) has

$$
\lim_{k_1 \to \infty} \frac{\gamma_3}{k_1} U(|u| + |u| + \dots + u^{(m)}) = 0 \tag{25}
$$

Substituting (25) into (24), it comes

$$
\left| y_1^{(n)} - \left( r^{(n)} - \frac{u^{(n)}}{k_1} \right) - (f(Y_1) - f(R)) \right| = 0 \qquad (26)
$$

Based on (12),

$$
y_1^{(n)} - \left(r^{(n)} - \frac{u^{(n)}}{k_1}\right) = y_1^{(n)} - y_1^{(n)} = 0 \tag{27}
$$

Because the term  $y_1^{(n)} - \left(r^{(n)} - \frac{u^{(n)}}{k_1}\right)$  equals to 0, it can be concluded from (24) that

$$
|f(Y_1) - f(R)| = 0 \tag{28}
$$

Substituting (28) into (22), it is clear that the left side of Equation (22) is equal to 0 as well as the term  $f(Y_1) - f(R)$ , then it comes from (22) that

$$
(g_0(Y_1) - g_0(R))u + (g_1(Y_1) - g_1(R))u + ... + (g_m(Y_1) - g_m(R))u^{(m)} = 0
$$
\n(29)

Extend (29) by (11), then it becomes

$$
(g_0(Y_1) - g_0(R))u + (g_1(Y_1) - g_1(R))u + ...
$$
  
+ 
$$
(g_m(Y_1) - g_m(R))u^{(m)}
$$
  
= 
$$
k_1((g_0(Y_1) - g_0(R))(r - y_1) + (g_1(Y_1) - g_1(R))(r - y_1)
$$
  
+ ... + 
$$
(g_m(Y_1) - g_m(R))(r^{(m)} - y_1^{(m)})) = 0
$$
 (30)

Let the error  $e_1$  between the reference *r* and  $y_1$ , that is,  $e_1 = r$  $y_1$ . Because constant  $k_1 > 0$ , it follows (30) that

$$
(g_0(Y_1) - g_0(R))e_1 + (g_1(Y_1) - g_1(R))\dot{e}_1 + \dots
$$
  
+ 
$$
(g_m(Y_1) - g_m(R))e_1^{(m)} = 0
$$
 (31)

From Assumption 2.2, the system (31) is globally asymptotically stable. Accordingly, the error *e*<sup>1</sup> between the reference *r* and  $y_1$  approaches zero asymptotically, which implies that  $y_1$ converges to *r* with a large enough gain *k*. When  $y_1 = r$ , the system dynamic inversion implemented, which also means the cancelation of system dynamics and nonlinearities is achieved. Therefore, the structure in Figure [2](#page-4-0) can be used to design the U-inverter.

<span id="page-5-2"></span><span id="page-5-0"></span>**Remark 3.2:** From (25) to (31), the error between the reference and system output will converge to zero asymptotically when the parameter *k* adjusted to infinity. It seems this proposed Uinverter is similar to the high-gain feedback controller, but it is not the case. Generally, the high-gain controller requires system state variables (Krishnamurthy et al., [2003;](#page-12-24) Ma et al., [2018\)](#page-12-25), and the high-gain controller is designed to achieve desired system control performance, whereas the purpose of the proposed Uinverter is the cancelation of system dynamics and nonlinearity. Additionally, this U-inverter is only a part of the U-control system, which aims to construct an inverse model of the controlled plant. Therefore, the basic idea of the proposed U-inverter is different from the high-gain feedback controller. Any point should be mentioned is that the proposed U-inverter only requires the information of the controlled plant input and output. Kumar and Veerachary [\(2021\)](#page-12-26) do not require system state variables, but the frequency response, pole-zero locations are selected to design system controller.

<span id="page-5-1"></span>According to the U-control system design framework in Section 3.1, the updated U-control system design framework is showed in Figure [3,](#page-6-0) in where the dashed area shows this new Ucontroller, which contains both invariant controller *Gc*<sup>1</sup> and the proposed U-inverter. Clearly, only system output and controller output are required in the system design process.

#### *3.3 DOB design*

From Sections 3.1 and 3.2, a new U-controller design is proposed based on this new U-inverter. However, from Figure [3,](#page-6-0) the model in the U-controller required to be same with the controlled model, that is, the whole U-control system is designed without system internal uncertainties and external disturbances. In practice application, this 'perfect modelling' is hardly existed. Therefore, inspired by (Ding et al., [2019\)](#page-12-20), a nonlinear



<span id="page-6-0"></span>**Figure 3.** New U-control system design framework.



<span id="page-6-1"></span>**Figure 4.** The framework of nonlinear DOB.

input/output model-based DOB is introduced in the U-control system design, which also only requires the information of system input and output. The framework of NDOB in Ding et al. [\(2019\)](#page-12-20) is shown in Figure [4.](#page-6-1)

Set the nominal model of system (1) as follows for NDOB design:

$$
y_0^{(n)} = f(Y_0) + g_0(Y_0)v + \ldots + g_m(Y_0)v^{(m)}, \ m \le n \quad (32)
$$

where  $y_0$  and  $v$  are the observer system output and input, respectively,  $f(Y_0)$  and  $g_i(Y_0)$  are smooth function associated with

$$
Y_0 = (\gamma_0, \dot{\gamma}_0, \ddot{\gamma}_0, \dots, \gamma_0^{(n-1)})
$$

Let the input of nominal modelled system (32) be

$$
v = k_0(y_1 - y_0)
$$
 (33)

where  $y_1$  is the practice system output and  $k_0$  is a large constant. Thus, the disturbance *d* and other modelling uncertainties can be observed as  $\hat{d} = v - u_0$ .

It follows from (33) that

$$
\begin{cases}\n v = k_0(y_1 - y_1) \\
 \dot{v} = k_0(\dot{y}_1 - \dot{y}_0) \\
 \vdots \\
 v^{(n)} = k_0(y_1^{(n)} - y_0^{(n)})\n\end{cases}
$$
\n(34)

where  $k_0 > 0$  is a sufficient large constant, from (34), it follows that,

$$
y_0 = y_1 - \frac{v}{k_0}
$$
  
\n
$$
y_0 = \dot{y}_1 - \frac{v}{k_0}
$$
  
\n
$$
\vdots
$$
  
\n
$$
y_0^{(n-1)} = y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}
$$
  
\n
$$
y_0^{(n)} = y_1^{(n)} - \frac{v^{(n)}}{k_0}
$$
\n(35)

Then substituting (35) into (32), it comes

$$
y_1^{(n)} - \frac{v^{(n)}}{k_0}
$$
  
=  $f\left(y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right)$   
+  $g_0\left(y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right)v$ 

+...+ 
$$
g_m
$$
  $\left(y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right) v^{(m)}$  (36)

Add the same term  $f(Y_1) + g_0(Y_1)v + \ldots + g_m(Y_1)v^{(m)}$  to each side of Equation (36), then it becomes

$$
y_1^{(n)} = \frac{v^{(n)}}{k_0} + f(Y_1) + g_0(Y_1)v + \dots + g_m(Y_1)v^{(m)}
$$
  
+ 
$$
\left(f\left(y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right) - f(y_1, y_1, \dots, y_1^{(n-1)})\right)
$$
  
+ 
$$
\left(g_0\left(y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right) - g_0(y_1, y_1, \dots, y_1^{(n-1)})v + \dots + \left(g_m\left(y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right) - g_m(y_1, y_1, \dots, y_1^{(n-1)})v^{(m)}
$$
(37)

Because the functions  $f(r, r, \ldots, r^{(n-1)})$  and  $g_i(r, r, \ldots, r^{(n-1)})$ ,  $i \in N$  are smooth, the following inequalities exist at least locally

$$
\left\| f\left(y_1 - \frac{v}{k_0}, \dot{y}_1 - \frac{\dot{v}}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0}\right) - f(y_1, \dot{y}_1, \dots, y_1^{(n-1)}) \right\| \le \gamma_4 \left\| \left(\frac{v}{k_0}, \frac{\dot{v}}{k_0}, \dots, \frac{v^{(n-1)}}{k_0}\right) \right\|
$$
  
=  $\frac{\gamma_4}{k_0} ||(v, \dot{v}, \dots, v^{(n-1)})||$  (38)

And

$$
\left\| g_i \left( y_1 - \frac{v}{k_0}, y_1 - \frac{v}{k_0}, \dots, y_1^{(n-1)} - \frac{v^{(n-1)}}{k_0} \right) \right\|
$$
  
-  $g_i(y_1, y_1, \dots, y_1^{(n-1)}) \left\| \le y_5 \left\| \left( \frac{v}{k_0}, \frac{v}{k_0}, \dots, \frac{v^{(n-1)}}{k_0} \right) \right\| \right\|$   
=  $\frac{\gamma_5}{k_0} ||(v, v, \dots, v^{(n-1)})||$  (39)

where  $\gamma_4$  and  $\gamma_5$  are Lipschitz constants. Then move term  $f(Y_1) + g_0(Y_1)v + \ldots + g_m(Y_1)v^{(m)}$  to the left side of Equation (37), organize it according to principles described in (38) and (39), then Equation (37) can be turned into

$$
|y_1^{(n)} - (f(Y_1) + g_0(Y_1)v + \dots + g_m(Y_1)v^{(m)})|
$$
  
\n
$$
\leq \frac{v^{(n)}}{k_0} + \frac{\gamma_4}{k_0}H + \frac{\gamma_5}{k_0}H(|v| + |\dot{v}| + \dots + v^{(m)})
$$
 (40)

When  $k_0$  goes infinite, the right side of Equation  $(40)$  has

$$
\lim_{k_0 \to \infty} \frac{\nu^{(n)}}{k_0} + \frac{\gamma_4}{k_0} H + \frac{\gamma_5}{k_0} H(|\nu| + |\dot{\nu}| + \ldots + \nu^{(m)}) = 0 \quad (41)
$$

Therefore, the left side of Equation (40) turns into

$$
|y_1^{(n)} - (f(Y_1) + g_0(Y_1)v + \dots + g_m(Y_1)v^{(m)})| = 0
$$
  

$$
y_1^{(n)} = f(Y_1) + g_0(Y_1)v + \dots + g_m(Y_1)v^{(m)}
$$
 (42)

Combining (42) and (1), it gives

$$
y_1^{(n)} - y_1^{(n)} = f(Y_1) + g_0(Y_1)u + \dots + g_m(Y_1)u^{(m)}
$$
  
- 
$$
(f(Y_1) + g_0(Y_1)v + \dots + g_m(Y_1)v^{(m)})
$$
  
= 
$$
g_0(Y_1)(v - u) + g_1(Y_1)(v - u) + \dots
$$
  
+ 
$$
g_m(Y_1)(v^{(m)} - u^{(m)})
$$
 (43)

It is clear that the left side of Equation (43) equals to 0, let the error  $e_2$  be  $e_2 = v - u$ , it comes from (43) that

$$
g_0(Y_1)e_2 + g_1(Y_1)\dot{e}_2 + \ldots + g_m(Y_1)e_2^{(m)} = 0 \qquad (44)
$$

From Assumption 2.2, system (44) is globally asymptotically stable. Accordingly, the error between *u* and *v* approaches zero asymptotically. Therefore, the observed  $\hat{d}$  can converge to the lumped disturbance represented by the practical disturbance *d* and modelling uncertainties asymptotically.

# *3.4 DOBUC design*

The proposed U-control in 3.2 shows the efficient control design which only requires the system output, however, this efficiency strictly appears in the case of model matched. In practice, the modelling errors and system external noise will exist and restrict the proposed U-control performance. The input/output model-based DOB described in 3.3 presented its main advantages in observing the system disturbances with only system input/output information. This DOB can amalgamate all uncertainties and system disturbances into system input channel, which will make disturbance compensation be implemented easily. Accordingly, the combination of this nonlinear DOB and the proposed U-control makes it possible to reduce U-control's sensitivity and improve the robustness of the control system.

Based on the new U-control system design framework in 3.2 and DOB described in 3.3, this study proposed a new DOBUC design framework shown in Figure [5.](#page-8-0) Concretely, this DOBUC has three loops: DOB in the inner loop can estimate the disturbance caused by modelling errors and system noises, and then amalgamate them into the system input channel; the U-inverter in the inner loop will cancel both the system dynamics and nonlinearity, convert the controlled plant to an identity matrix or unit constant when combined with its dynamic inversion; *Gc*<sup>1</sup> in the outer loop is used to design the desired control performance. Manifestly, if the control system is perfect modelled and free of system external disturbance, the inner DOB will not be activated and the system will be converted into a simple linear closed-loop feedback control system with the dynamic cancelation from the U-inverter; when disturbance exists in system,



<span id="page-8-0"></span>**Figure 5.** New DOB-based U-control design framework.

the U-controller will compensate the disturbance estimated by activated DOB into the input channel. Finally, the framework of DOBUC is equivalent to the followed framework (Figure [6\)](#page-9-0):

Accordingly, the design procedures for the proposed DOBUC are:

(1) Design the U-inverter based on Section 3.2. The design parameter  $k_1$  should be selected large enough to satisfy  $(20)$ to implement an efficient U-inverter. It should be mentioned that the model inversion should exist and satisfy the Lipschitz continuity with globally uniformly:

$$
||G(x_1) - G(x_2)|| \leq \gamma_1 ||Gx_1 - x_2||, \quad \forall x_1, x_2 \in \mathbb{R}^n
$$
  

$$
||G^{-1}(x_1) - G^{-1}(x_2)|| \leq \gamma_2 ||G^{-1}x_1 - x_2||, \quad \forall x_1, x_2 \in \mathbb{R}^n
$$

- (2) Design the nonlinear DOB based on Section 3.3. The design parameter  $k_0$  should be large enough to satisfy  $(41)$ and guarantee that the disturbance estimation error can converge to a small region.
- (3) Design invariant controller *Gc*<sup>1</sup> with customer defined damping ratio ζ and undamped natural frequency ω*<sup>n</sup>* to achieve control performance specifications. Where  $G<sub>c1</sub>$  =  $\frac{G}{1-G}$  and  $G = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ .

**Remark 3.3:** Theoretically, the larger parameters  $k_0$ ,  $k_1$  can realise more efficient u-inverter and improved disturbance estimation performance. However, the system output noise level will also be increased by the increasing value of  $k_0$  and  $k_1$ . Accordingly, the parameters  $k_0$  and  $k_1$  should be adjusted from small to large until the control performance specifications are meted. It also should be noticed that  $k_1$  need to be larger than  $k_0$  to ensure the U-inverter converges faster than the nonlinear DOB because this DOB requires the information from U-controller's output to estimate the disturbance.

# **4. Experimental simulation**

Compared with the traditional fossil fuels, renewable energy has become popular in recent decades. In the situation of growing global energy demand while significantly reducing reliance on fossil-fuels, environmentally friendly wind energy received wide concertation (Ahmad et al., [2020\)](#page-12-27). According to the Global <span id="page-8-6"></span><span id="page-8-4"></span><span id="page-8-3"></span>Wind Energy Statistics 2020 (Global Wind Energy Council, [2021\)](#page-12-28), China accounted for 36.42% (237 029 MW) of the world's installed wind power capacity and the United States accounted for 16.20% (105 433 MW). In WECS, variable-speed wind turbines (VSWT) convert wind energy into electricity. Therefore, an efficient VSWT control strategy will exploit maximum power generation and improve the power quality (Wang et al., [2011\)](#page-12-29). The latest advanced control articles (de Siqueira & Peng, [2021;](#page-12-30) Ghaffarzadeh & Mehrizi-Sani, [2020;](#page-12-31) Milev et al., [2020\)](#page-12-32) mainly focus on solving the complex nonlinear dynamics and difficulty measuring speed of WECS in real-time applications. Paper (Li et al., [2020\)](#page-12-12) firstly applied U-control method on WECS, although it did not consider modelling errors and disturbances such as unmeasurable wind speed, the implementation of its complex nonlinear dynamics and good control performance are still worthy for further study.

#### *4.1 Modelling of WECS*

The aerodynamic torque of the turbine is:

<span id="page-8-5"></span><span id="page-8-2"></span>
$$
T_a = \frac{1}{2} \rho \pi R^3 C_q(\lambda, \beta) (\hat{v} - \xi)^2 \tag{45}
$$

where  $\rho$  and  $R$  are the air density and the length of the rotor blade;  $\hat{v}$  and  $\xi$  are the effective wind velocity estimate and practical measurement disturbance;  $C_q(\lambda, \beta)$  is the wind turbine power conversion efficiency coefficient, which depends on the tip-speed ratio  $\lambda$  and blade pitch angle  $\beta$ :

$$
C_q(\lambda, \beta) = \frac{0.22}{\lambda} \left( \frac{116}{m} - 0.4\beta - 5 \right) \exp(-12.5/m) \tag{46}
$$

With

$$
\frac{1}{m} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1}
$$
 (47)

The tip-speed ratio  $\lambda$  is defined as

$$
\lambda = \frac{R\omega_r}{\nu} \tag{48}
$$

<span id="page-8-1"></span>where  $\omega_r$  is the rotor angular speed. Therefore, the rotor power *Pa* captured by the wind turbine is the product of the aerodynamic torque and blade speed, which donated from (22)



<span id="page-9-0"></span>**Figure 6.** Equivalent U-control framework after NDOB compensation.

as

$$
P_a = \omega_r T_a = \frac{1}{2} \rho \pi R^2 C_p (\lambda, \beta) (\hat{v} - \xi)^3 \tag{49}
$$

With

$$
C_p(\lambda, \beta) = C_q(\lambda, \beta)\lambda \tag{50}
$$

According to the drive train scheme in Figure [7,](#page-10-0) the dynamics of rotor speed  $\omega_r$  and generator speed  $\omega_g$  are driven by the rotor torque  $T_a$ , the low-speed torque  $T_b$ , the high-speed torque *Ths* and the electromagnetic torque *Tem*:

$$
\begin{cases}\nJ_r \dot{\omega}_r = T_a - K_r \omega_r - T_{ls} \\
J_g \dot{\omega}_g = T_{hs} - K_g \omega_g - T_{em}\n\end{cases} \tag{51}
$$

where  $J_r$  and  $K_r$ ,  $J_g$  and  $K_g$  are inertia and external damping for rotor and generator, respectively. The conversion rate between  $\omega_r$  and  $\omega_g$  can be described by gearbox ratio  $n_g$  as

$$
n_g = \frac{\omega_g}{\omega_r} = \frac{T_{ls}}{T_{hs}}\tag{52}
$$

Invoking (52), it comes from the generator dynamic in (51):

$$
n_g^2 I_g \dot{\omega}_r = T_{ls} - n_g^2 K_g \omega_r - n_g T_{em}
$$
 (53)

Thus, the drive train model comes from the combination of (51) with (53),

$$
J_t \dot{\omega}_r = T_a - K_t \omega_r - T_g \tag{54}
$$

where

$$
\begin{cases}\nJ_t = J_r + n_g^2 J_g \\
K_t = K_r + n_g^2 K_g \\
T_g = n_g T_{em}\n\end{cases}
$$
\n(55)

Then, the system output is expressed as

$$
P_g = T_g \omega_r \tag{56}
$$

System (54) describes the relationship between system input and output. Substituting (56) into (54), the drive train model will be:

$$
J_t \left(\frac{\dot{P}_g}{T_g}\right) = T_a - K_t \frac{P_g}{T_g} - T_g
$$
  

$$
\left(\frac{\dot{P}_g T_g - P_g \dot{T}_g}{T_g^2}\right) = \frac{T_a}{J_t} - \frac{K_t}{J_t} \frac{P_g}{T_g} - \frac{1}{J_t} T_g
$$
  

$$
\dot{P}_g T_g - P_g \dot{T}_g = \frac{1}{J_t} T_a T_g^2 - \frac{K_t}{J_t} P_g T_g - \frac{1}{J_t} T_g^3 \tag{57}
$$

<span id="page-9-1"></span>



The description in (57) shows the modelling of WECS. Let the generator torque  $T_g$  and generator power  $P_g$  be control input *u* and output *y*, respectively, it has:

$$
\dot{y}u - y\dot{u} = \frac{1}{J_t}T_a u^2 - \frac{K_t}{J_t}yu - \frac{1}{J_t}u^3
$$
 (58)

Convert (58) into its U- realization:

$$
\begin{cases}\n\dot{y} = \lambda_0 f_0(\dot{u}) + \lambda_1 f_1(\dot{u}) \\
\lambda_0 = \frac{1}{J_t} T_a u - \frac{K_t}{J_t} y - \frac{1}{J_t} u^2, f_0(\dot{u}) = 1 \\
\lambda_1 = \frac{y}{u}, f_1(\dot{u}) = \dot{u}\n\end{cases} \tag{59}
$$

#### *4.2 Simulation results*

The related parameters in WECS modelling are provided in Table [1](#page-9-1) (Meng et al., [2013\)](#page-12-33).

The ratio of desired power  $P_d$  and ideal maximum power  $P_{amax}$  is  $n_p = 0.8$ . Therefore, the parameters in (59) are:

<span id="page-9-2"></span>
$$
\begin{cases}\nJ_t = J_r + n_g^2 J_g = 5.7998 \times 10^6 \\
K_t = K_r + n_g^2 K_g = 4.4131 \times 10^3 \\
\frac{K_t}{J_t} = 7.609 \times 10^{-4}\n\end{cases} (60)
$$

The turbulence intensity is 10% and the mean value of wind speed  $\nu$  in this experiment is  $9 \text{ m/s}$  (Meng et al., [2013\)](#page-12-33). This simulation experiment introduces the wind speed sensor error because of its inevitably disturbances. Accordingly, the speed sensor error is chosen as a 10 Hz white noise, with the value from −0.5 to 0.5. Figure [8](#page-10-1) shows the wind speed with and without sensor error.

The block framework of this control system is given in Figure [9.](#page-10-2)

According to the design framework presented in Section 3.4, the parameter in U-inverter is:  $k_1 = 1000$  and in nonlinear observer is  $k_2 = 500$ . To assure a fast-tracking performance without overshooting, the damping ratio and undamped natural frequency are chosen as  $\zeta = 1$  and  $\omega_n = 10$  for invariant



<span id="page-10-0"></span>**Figure 7.** Schematic diagram of drive train.



**Figure 8.** Wind speed measurement.

<span id="page-10-1"></span>

<span id="page-10-2"></span>**Figure 9.** Block framework of WECS with UDOBC.



(b) Tracking error

<span id="page-11-0"></span>**Figure 10.** Simulation results of WECS.

controller design:

$$
G_{c1} = \frac{1}{0.01s^2 + 0.2s}
$$

Assume the disturbance *d* is defined as:

$$
d = \begin{cases} 2\sin\left(\frac{t}{4}\right) & 0 \le t < 16\pi \\ 0.7\sin(t) & 16\pi \le t < 100 \end{cases}
$$

Additionally, modelling errors are considered in this experiment simulation with  $\Delta J_t = -0.9J_t$ ,  $\Delta K_t = 2K_t$  and  $\Delta T_a =$ 4*Ta*. Therefore, WECS (57) changed to:

$$
\dot{P}_g T_g - P_g \dot{T}_g = \frac{5T_a}{0.1f_t} T_g^2 - \frac{3K_t}{J_t} P_g T_g - \frac{1}{0.2f_t} T_g^3 \tag{61}
$$

The simulation results can be observed from Figure [10.](#page-11-0) From Figure [10\(](#page-11-0)a), the proposed DOBUC method has good tracking performance; the tracking error rate is less than 1% (compared to peak output power) from Figure [10\(](#page-11-0)b). Obviously, the NDOB shows efficacy in observation of both system disturbance and modelling errors from Figure  $10(c)$  $10(c)$  and the controller output is smooth without chattering from Figure [10\(](#page-11-0)d).



(c) Disturbance observer result



(d) System input total torque

# **5. Conclusions**

This study proposes a novel DOBUC framework for linear/nonlinear systems. It removes the restriction on the current state feedback U-control systems that request the state variables observable or measurable. The proposed U-inverter provides a robust dynamic inversion of the control plant, so that it makes the U-control inherit the robustness throughout the control system operation. It should be noted that this DOBUC requires the controlled input/output system being minimum-phase or stable zero dynamic in more general terms, which should take care in applications. Future study topics could be those expanding the proposed framework to MIMO systems and taking up bench tests of practical systems.

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#### **Disclosure statement**

No potential conflict of interest was reported by the author(s).

# **Data availability statement**

The data used to support the findings of this study are available from the corresponding author upon request.

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