Proceedings of the 3rd IASME/WSEAS Int. Conf. on FLUID DYNAMICS & AERODYNAMICS, Corfu, Greece, August 20-22, 2005 (pp84-89)

On Some Stabilisation Techniques of a Trapped Vortex

A. BOUFERROUK, S. I. CHERNYSHENKO
Aerodynamics and Flight Mechanics Research Group (AFM)
School of Engineering Sciences, University of Southampton, Southampton,
UNITED KINGDOM
a.bouferrouk@soton.ac.uk, scherny@soton.ac.uk

July 1, 2005

Abstract: In this study we investigate the effectiveness of some control techniques, both passive and active, for the stabilisation of a large-scale trapped vortex of a Lighthill's airfoil. The flow is two-dimensional, incompressible and inviscid solved using a discrete vortex method code. It was found that stabilisation improves the aerodynamic characteristics of the airfoil with active control achieving stabilisation with less energy input.

Key-Words: Trapped vortex, Lighthill's airfoil, Vortex Method, Stabilisation, Suction/blowing, Active control.

1 Introduction

Many aerodynamic flows are typified by the production of large-scale vortex structures. Usually, the nature of such is unsteady [1] and as such they move downstream behind the body and maintain a chain of vortices behind them, resulting in high drag and low lift. There is sometimes, however, the possibility of keeping these vortices close (or attached) to the body surface by means of a geometry modification and/or some type of control. Under such circumstances, the vortices are said to be captured or *trapped*. Research into trapped vortex flows began following Kasper's seminal efforts in designing a glider having a significant lift improvement at low speeds (without corresponding change in drag), which he attributed to a massive vortex residing over the upper surface of the wing [2].

A flow with a trapped vortex is a potentiallly useful technology. The projected benefits of trapped vortices include their use for drag reduction by postponing/preventing vortex shedding from aerodynamic vehicles, extending the post-stall performance, separation control, and even as a means for vehicle control. However, this is only possible if the trapped vortex remains stable at all times. The present study reports on some techniques, both passive and active, of stabilising a trapped vortex for enhancing aerodynamic flows.

2 Problem Formulation

2.1 The Simulation Model

The simulation model used for vortex trapping is a Lighthill's airfoil whose body shape was determined from a classical inverse problem. The inverse problem is that of determining the body shape for a given velocity distribution on its surface. For the Lighthill's airfoil the desired velocity distribution ensures no separation around the body surface except at a single point on its upper surface. Such point could be replaced by a cavity for vortex trapping [3], see Fig.1. There were no specific requirements on the particular shape of the cavity. A shear layer would originate at the upstream sharp edge **A** of the cavity and then becomes entrained inside it. This way, the Lighthill airfoil possesses a naturally desirable velocity distribution and is readily capable of trapping a vortex. Furthermore, the geometry is a thick airfoil and so it represents a good study model for future air transport as there is a tendency to favour thicker wings over streamlined shapes. This is due to better structural strength necessary for carrying larger loads in future large transport aircraft. This tendency is already seen for instance in Boeing's concept of a Blended-Wing-Body aircraft.

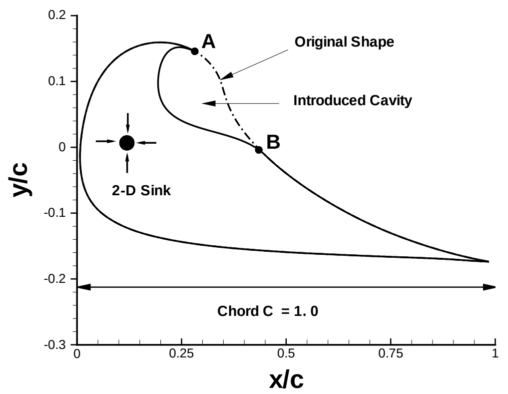


Figure 1: Lighthill's Airfoil Geometry

2.2 The Flow Model: The Vortex Method

The flow solution was obtained using the inviscid version of a Discrete (blob) Vortex Method (DVM) due to Spalart [4]. The basis of the DVM is to approximate a continuous field of vorticity ω by N blobs as

$$\omega(\mathbf{x}) = \sum_{j=1}^{N} \Gamma_j \delta_\sigma(|\mathbf{x} - \mathbf{x}_j|) \tag{1}$$

where δ_σ represents blob functions and Γ_j is the circulation of the jth vortex at the $\mathbf{x}_j$ position. The velocity is retrieved from vorticity using a Biot-Savart integral in the form [4]

$$\mathbf{u}(\mathbf{x}, t) = \int \mathbf{K}(\mathbf{x} - \mathbf{x}')\omega(\mathbf{x}', t)d\mathbf{x}' + \mathbf{U}_\infty \tag{2}$$

where $\mathbf{K} = \nabla \times G$, with G being the Poisson kernel, and $\mathbf{U}_\infty$ is the velocity at infinity. The circulation of each vortex is conserved so that $d\Gamma_j/dt = 0$ and the vortices move according to

$$\frac{d\mathbf{x}_j}{dt} = \mathbf{u}(\mathbf{x}_j, t) \tag{3}$$

The inviscid impermeable boundary condition on the body surface is satisfied by imposing zero mass flux between consecutive discrete wall points. Uisng a stream-function formulation this implies that between two wall points $\mathbf{x_k}$ and $\mathbf{x_{k+1}}$ we must have

$$\psi(\mathbf{x}_{k+1}) = \psi(\mathbf{x}_k) \tag{4}$$

where ψ is a stream function. Further details of the method are given in [4]. In terms of numerical parameters of the DVM it was found that a choice of 1300 blob vortices and a time step of 0.004 provides reasonable estimates. Also, the numerical scheme was initially found to be ill-posed. The application of a Tikhonov regularisation [5] technique made the problem well-posed. The aims of Tikhonov regularisation were two fold: first to extend the DVM solution to bodies with sharp edges characteristic of studies on trapped vortices, and second to reduce the numerical noise in the flow solution itself.

3 Problem Solution

3.1 Stabilisation by Steady Suction

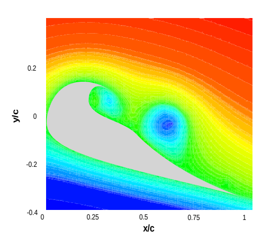
The original DVM was modified to simulate suction effects using a 2-D potential sink introduced inside the body, see Fig.1. Uniform suction was applied using the spacing between two consecutive wall points $\mathbf{x_k}$ and $\mathbf{x_{k+1}}$ as suction panels and suction was distributed along the whole cavity surface. For each panel ($\mathbf{x_k}$, $\mathbf{x_{k+1}}$), the suction flow rate Q_k is obtained as

$$\psi(\mathbf{x_{k+1}}, t) - \psi(\mathbf{x_k}, t) = Q_k \tag{5}$$

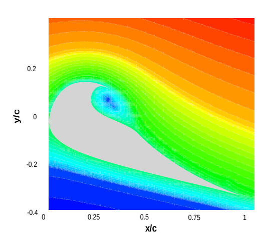
Therefore, for m panels the total suction rate is

$$Q_{total} = \sum_{k=1}^{m} Q_k. \tag{6}$$

The effects of suction can immediately be seen from examination of the streamline plot of Fig.2 in which a strong suction (Q_{total} = 0.02) clearly inhibits vortex shedding and results in a strong, coherent, and stable vortex which remained in a stationary position on the upper surface. In terms of forces (obtained as averaged values from sufficiently long runs), the effect of increasing suction on the drag and lift forces is illustrated in Fig.3, which clearly shows that with increasing suction the drag decreases and the lift increases. This enhanced performance is due to the gradual suppresion of vortex shedding with suction. The suction rate above which shedding ceases completely was about



(a) Q_{total} = 0.0



(b) Q_{total} = 0.02

Figure 2: Streamline Contours

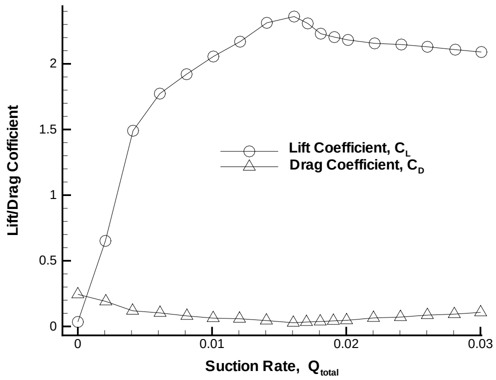


Figure 3: Drag and Lift Variations with Suction

0.016. Beyond this critical suction, however, the performance decreases. The increased drag is due to the so-called suction drag. Therefore, steady continuous suction improves the aerodynamic characteristics by stabilising the vortex but is clearly limited.

3.2 Stabilisation by Unsteady Suction

The main idea behind this approach was to cease suction completely once the trapped vorted was stabilised then switch it back on at a later time. The procedure relies on the intuitive fact that the self rotational behaviour of the trapped vortex enables it to remain stable for some time before its stability is lost. When suction was continuously switched between the value of Q_{total} = 0.02 and zero the trapped vortex was kept in the cavity with no shedding, and a 13% reduction in the required suction was achieved. Interestingly, compared to the case with continuous suction, the averaged lift coefficient remained the same but the averaged drag coefficient was reduced from 0.078 to 0.020 using unsteady suction. Unfortunately this method suffers from the impracticality of a real pump to provide suction in this way between two extreme suction values.

3.3 Stability of the Trapped Vortex

Although the results have clearly shown that a strong suction traps and witholds permanently a forming massive vortex structure inside the airfoil cavity, we seek a convincing mathematical argument for stability of the trapped vortex. From the work of Saffman [1], the equilibrium location of a stationary vortex is considered stable if the vortex returns to it after being subjected to a small perturbation. This means that the response of a system to a given perturbation does not develop an instability which destabilises the system, but one which decays with time. Such behaviour is true for a system described by a decaying exponential of the form

$$v(t) = a_0 + a_1 exp(\lambda t), \quad \lambda < 0. \qquad (7)$$

where $v(t)$ is any flow variable, a_0 and a_1 are constant coefficients. In order to establish such behaviour for the trapped vortex, it is sufficient to register in time any flow variable like velocity (at a given location) and to see whether it can be represented by a decaying exponential. At a suction rate of Q_{total} = 0.02, a sample of registered data for the normal velocity v is fitted with the exponential model (7) as depicted in Fig.4. Fig.4 clearly shows that the velocity behaviour asymptotes to a constant value, implying a stable steady state situation.

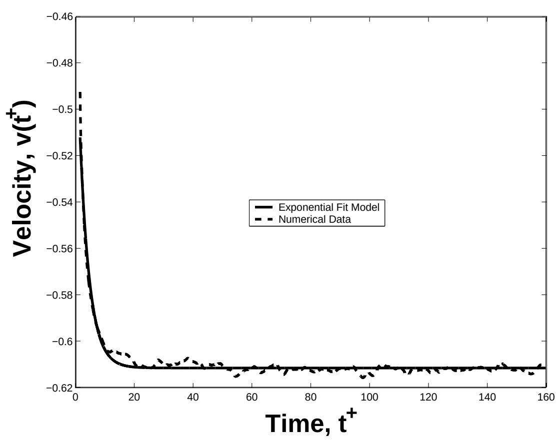


Figure 4: Stability Characterisation

3.4 Stabilisation by Active Linear Control

The aim here is to stabilise the trapped vortex and to delay vortex shedding for as long as possible and with as little suction as possible. We propose to use a feedback control law, based on an *artificial stabilising parameter* such that stability is maintained while slowly reducing the suction from a reference stable point. We consider a Single Input Single Output (SISO) standard linear (small perturbation) controller design with a constant gain parameter G (the stabilising parameter) in the form

$$Q_{total} = Q_0 + G.(V_t - V_{bias}) \qquad (8)$$

where V_t is the signal (or sensor variable) taken as the tangential velocity, V_{bias} is the constant biased velocity (averaged velocity of a flow run without control), Q_0 = 0.02, and Q_{total} is the control variable representing the total suction rate to be reduced. It is proposed to start from a stable state of the trapped vortex (with a strong suction) and then reduce suction very slowly while using various values of G such that the mean Q_{total} is reduced and stability is retained up to the onset of vortex shedding (the unstable system). The small perturbations for which the model (8) holds are modelled by the slow reduction in suction. For the Lighthill airfoil in question it was not possible to find any value of G which stabilises the trapped vortex below the minimum suction value of 0.0157, previously reported with passive suction alone. However, using a different cavity configuration, active linear control gave promising results that active control is capable of delaying vortex shedding with a reduced amount of suction (8%), see Fig.5, compared to the case of no active control. Please note that with the new cavity shape the minumum suction for stabilition is different from 0.0157; it is about 0.0087. The unstable system (vortex shedding) is exhibited by the the large fluctuations in drag. The value of G = -0.001 was the only one, so far, that gave encouraging results; other values were all destabilising. This is advantageous in two ways: drag reduction due to delay of vortex shedding and drag reduction due to redution in suction. In a more realistic turbulent flow, removal of fluid accelerates the loss of momentum with the consequence of increased friction drag. Reduction of suction also reduces the size and weight needed to install a suction pump. Future work will look into the effects of the position of the sensor variable V_t.

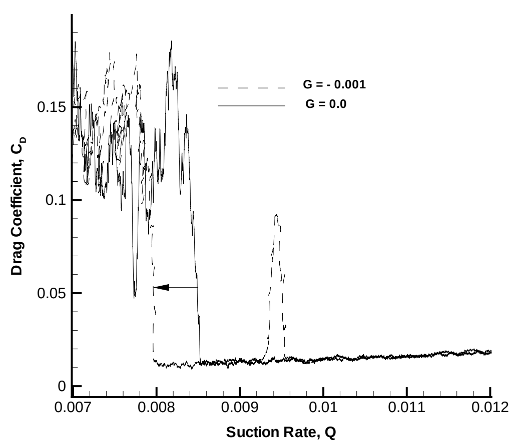


Figure 5: Linear Active Control Results

3.5 Stabilisation by Active Flux Control

The central aim of the current investigation is to actively stabilise the trapped vortex by imposing a flow control condition of a constant vorticity flux through the boundary of the vortex cell. The strategy relies on the simple idea that when a vortex leaves the cavity it carries with it its vorticity. Therefore, when the vortex crosses the control line the vorticity flux through it will peak. Controlling variations of this flux should, therefore, prevent vortex shedding. Instead of a continuous steady strong suction as in preceeding work, this approach uses dynamic flow rates of blowing and/or suction to satisfy the constant flux condition. It is hypothesised that such arrangement will achieve a stable trapped vortex with a reduced mean suction flow rate. In case of a continuous suction the perturbation no matter how small it might be is always accounted for by a large amount of suction. However, active blowing/suction is perceived to be able to recognise that a small perturbation requires only a small amount of suction to suppress it. Hence, in the mean a saving in energy is obtained.

The vorticity flux is calculated about a reference *control line* positioned at the boundary of the cell, see Figure . We only consider the contribution to this flux from the blob vortices within the cavity whose relative position to the line changes as a result of time integration. Initially, the flux was approximated by the algebraic summation the circulations of the blob vortices crossing the line per time step. With reference to Fig.6, a blob vortex which crosses from below the line to a position above it has a negative contribution to the sum, and vice versa. A vortex which does not cross the line in either direction has zero contribution to the flux. Thus, the flux F of N_P vortices is

$$F = \sum_{j=1}^{N_P} \Gamma_j \tag{9}$$

where Γ_j is the circulation of the jth vortex.

The desired amount of vorticity flux was obtained from the time averaged flux through the control line for which the trapped vortex was stably trapped with a strong suction of Q_{total} = 0.02. Active flux control ensures that the vorticity flux remains equal to this desired value at every subsequent time step using an appropriate amount of suction/blowing. To achieve this we employed a linear interpolation technique. We specify a range of flow rates such that at each time step the DVM code computes the predicted flux at each point in the range. Then, linear interpolation is invoked to solve for the appropriate value of flow rate which produces the desired vorticity flux. In simple terms, if the desired flux lies in the search range between two consecutive flow rates Q_k and Q_{k+1} the required flow rate, Q_{Req}, by interpolation is

$$Q_{Req} = Q_k + \frac{(Q_{k+1} - Q_k)}{(F_{k+1} - F_k)} (F_{Des} - F_k) \qquad (10)$$

where F_{Des} is the desired flux.

The current approach for flux evaluation was characterised by sudden large jumps in the predicted flux. This meant poor interpolation due to irregular behaviour of the predicted flux. To overcome this, we introduced a smoothing function f_s which effectively increases the core sizes of the blob vortices such that when they cross the line the variation in the predicted flux is smooth. The weighting f_s also means that the vorticity contained by the vortex cell is somewhat continuous due to enlarged core size. The function f_s was defined as

$$f_s = \frac{\left(\frac{d_V}{\sqrt{\sigma^2 + d_V^2}} + 1\right)}{2} \qquad (11)$$

where σ is a smoothing coefficient, and d_V is the vertical distance of a vortex from the control line. The function f_s is computed for both the old and new positions of a vortex during the prediction stage. The change in flux due to a change in a vortex position is taken as

$$\Delta F_j = \Gamma(j) \left(f_s(new) - f_s(old)\right) \qquad (12)$$

and so the predicted total flux becomes

$$F_{Pred} = \sum_{j=1}^{N_P} \Delta F_j \qquad (13)$$

Our first implementation of the active flux control was to conserve the desired flux of 0.0059 obtained from a stable steady state with Q_{total} = 0.02. Such value was successfully maintained for a very long simulation time using alternating blowing and suction, and the trapped vortex was stable. The overall mean flow rate was $< Q >$ = 0.0186. Hence, compared to a continuous suction of 0.02, stabilisation with active flux control achieves 7% reduction in flow rate. It was discovered that by increasing the desired flux, stabilisation with active control was achieved using reduced mean flow rates. The procedure was repeated until stabilisation could not be established beyond a critical value for

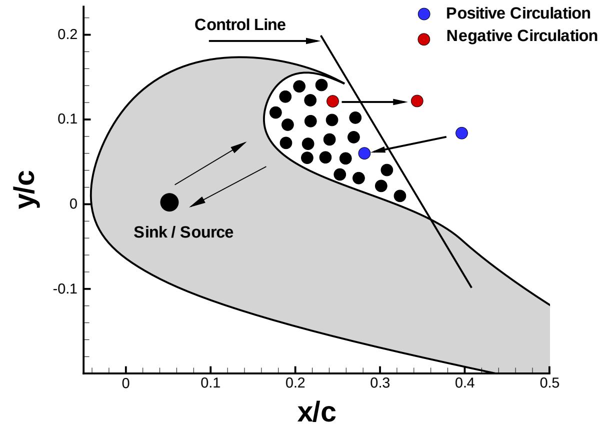


Figure 6: Implementation of Flux Control

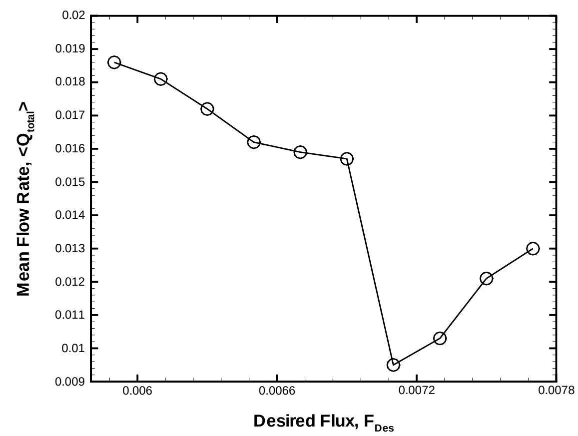


Figure 7: Variation of $< Q >$ with F_{Des}

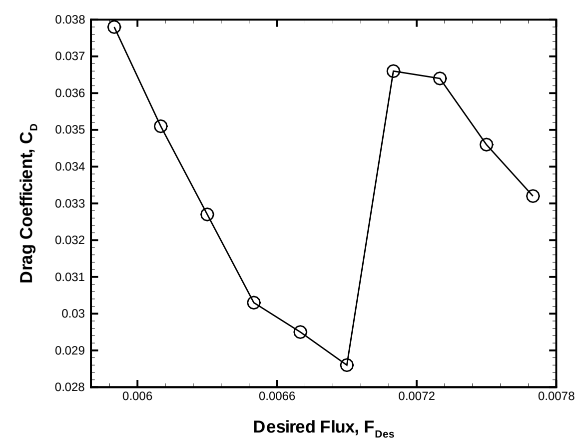


Figure 8: Variation of C_D with F_{Des}

the desired flux. The results are summarised in Fig.7. The corresponding changes in C_D are shown in Fig.8. The maximum desired flux with which the trapped vor-

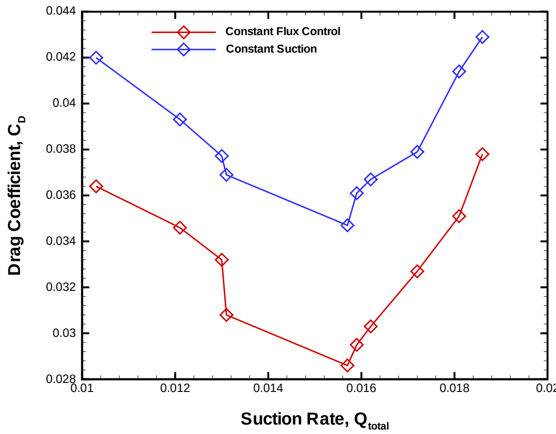


Figure 9: Variation of C_D with $< Q >$

tex was actively stabilised is 0.0069. Beyond this value vortex shedding occured. Please note that the overall mean flow rate at which vortex shedding was observed is about 0.0157. This is roughly the same value below which vortex shedding also occured with continuous suction alone. Therefore, the stability limit could not be extended to lower flow rates using active control. However, the drag values obtained with active flux control are lower compared to those computed with continuous suction alone for the same flow rate, see Fig.9.

4 Conclusion

A Lighthill's airfoil with a cavity and strong steady suction is capable of stabilising a large-scale vortex thereby enhancing its aerodynamic performance. However, this is only possible up to a critical suction rate. The alternative of unsteady suction provides stabilisation with a reduced suction rate but may be limited in practice by actuator performance. The trapped vortex stability, defined with respect to large-scale vortex shedding, was proved using a simple exponential decaying model. The use of a linear feedback controller based on a stabilising parameter G was effective in retaining the trapped vortex stability with a reduced suction rate compared to the passive suction schemes. Achievement of this, however, seems to be dependent on the cavity shape. Application of vorticity flux control concept is another viable way of stabilising a trapped vortex using dynamic flow rates of suction/blowing. Although stabilisation was not achieved with reduced flow rate compared to continuous suction alone, active flux control achieves stabilisation with decreased drag. This represents a potential saving in energy. There remain open questions on the optimisation of such stabilisation approaches and power balance requirements.

5 References

[1] Saffman, P. G., Sheffield, J. S., Flow over a Wing with an Attached Free Vortex, *Studies in Applied Mathematics.*, Vol. 57, 1977, pp.107-117.

[2] Cox, J., The Revolutionary Kasper Wing, *Soaring.*, Vol. 37, December 1973, pp.20-23.

[3] Buldakov, E. V., Chernyshenko, S.I. and Stepanov, G.Y., Inviscid Batchelor-model Flow Past an Airfoil with a Vortex Trapped in a Cavity, *Journal of Fluid Mechanics.*, Vol. 323, 1996, pp.367-376.

[4] Spalart, P. R., A Discrete Vortex Method for Separated Flows , *NASA Technical Memorandum 100068.*, 1988.

[5] Bouferrouk, A., Chernyshenko, S.I., Tikhonov Regularisation in Discrete Vortex Methods, *Computers and Fluids.*, Vol. 34, pp.275-281.