Endogenous Fluctuations and International Business Cycles

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Abstract

We introduce equilibrium indeterminacy into a two-country incomplete asset model with imperfect competition to analyze the role of self-fulfilling expectations or beliefs in explaining international business cycles. We show that when self-fulfilling beliefs are correlated with technology shocks, the model can account for the counter-cyclical behavior observed for the terms of trade and real net exports, while simultaneously generating higher volatilities relative to output, as in the data. However, the model cannot generate a positive correlation between the real exchange rate and relative consumption without a negative cross-country correlation for technology shocks, which is not supported by the data. We show that the inability to overcome the consumption-real exchange rate anomaly is common to a wide class of indeterminacy frameworks with an upward-sloping aggregate labor demand.

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1 Introduction

Since the pioneering work of Benhabib and Farmer (1994), there now exists a large literature exploring the role of equilibrium indeterminacy and self-fulfilling beliefs in explaining business cycle fluctuations.[1] While the endogenous-business-cycle approach has been successful in quantitatively explaining a number of features of closed-economy business cycles, the importance of endogenous fluctuations in understanding international business cycles remains largely unexplored. Previous studies have restricted their attention to explaining one specific feature of the open-economy data, namely the cross-country correlation between consumption and output.[2] Instead, this paper takes a broader look at international business cycle fluctuations. We find that indeterminacy and self-fulfilling fluctuations can explain some, but not all of the important aspects relating to international business cycles. This happens because the same transmission mechanism that is crucial for the model to generate counter-cyclical and volatile terms of trade and real net exports, as in the data, is fundamentally at odds with the observed negative correlation between relative consumption and the real exchange rate.

The model economy we consider is a two-country, two-good, incomplete-asset market economy with imperfect competition. Similar to the closed-economy studies of Farmer and Guo (1994) and Schmitt-Grohe (1997), among others, we assume increasing returns to scale technology. As a result, indeterminacy is generated via an upward-sloping aggregate labor demand schedule, which is a common feature of many indeterminacy models.[3] Under indeterminacy, the forecast errors become endogenous.[4] We consider two alternative assumptions. First, we assume that the forecast errors to the terms of trade are the only source of business cycle fluctuations (*autonomous beliefs*). Secondly, we assume that the forecast errors to the terms of trade are correlated with fundamental shocks (*correlated beliefs*).

Our main findings are summarized as follows. We first show that international business cycle fluctuations driven solely by autonomous beliefs cannot replicate any of the major features of the

[1] See, e.g., Farmer and Guo (1994), Schmitt-Grohé (1997, 2000), Benhabib and Wen (2004), Jaimovich (2007), Guo and Harrison (2010), Benhabib and Wang (2013), Dufourt et al. (2015), Pintus et al. (2016), and Pavlov and Weder (2017).

[2] See Guo and Sturzenegger (1998) and Xiao (2004).

[3] The upward-sloping aggregate labor demand schedule is common to many indeterminacy models because it arises under a wide set of modeling assumptions. Models with increasing returns to scale (e.g. Benhabib and Wen, 2004), models with positive externalities in production (e.g. Benhabib and Farmer, 1994), and models with firm entry under monopoly power (e.g. Jaimovich, 2007, and Pavlov and Weder, 2017) all feature an upward-sloping aggregate labor-demand schedule.

[4] In what follows we use the terms forecast errors, expectational errors, self-fulfilling expectations or beliefs interchangeably.

data. This finding is in stark contrast to Guo and Sturzenegger (1998) and Xiao (2004) who find that self-fulfilling expectations can help explain the positive cross-country correlations observed for consumption and output. However, both Guo and Sturzenegger (1998) and Xiao (2004) introduce indeterminacy into a two-country, one-good model, while we generate indeterminacy in a two-good framework, in order to look at a wider set of puzzles related to international relative prices and quantities.

In one-good models self-fulfilling expectations stimulate world demand and generate positive cross-country correlations for consumption and output, as in the data. However, in our two-good model revisions to the terms of trade forecasts are the source of endogenous fluctuations.[5] We show that a belief-induced depreciation of the terms of trade shifts the upward-sloping aggregate labor demand schedules in each country in opposite directions, raising domestic output and consumption at the expense of foreign output and consumption. Consequently, in two-goods models autonomous beliefs cannot on their own explain the data, since by causing a reallocation of output, they generate counterfactually negative cross-country correlations.

We next show that a number of the empirical irregularities of the data can be resolved by allowing the forecast or expectational errors to be correlated with technology shocks. Now, the indeterminacy model can generate counter-cyclical behavior for the terms of trade and real net exports, while at the same time, increasing significantly the volatility of international relative prices and cross-country trade flows. This improvement in volatility over the business cycle is not at the cost of reduced volatility of the other aggregate variables, whose standard deviations relative to output are also increased.

The improved performance of the model is due to the transmission mechanism of technology shocks, which is fundamentally altered under indeterminacy. In our model, technology shocks induce a change in beliefs by causing agents in both countries to revise their expectations. To explain the transmission mechanism, we show how to construct combined impulse responses that take into account the correlation of beliefs with fundamentals. We find that a very specific transmission of technology shocks, in which there is a negative response of employment to a positive technology shock and a delayed effect on output, best explains the data.[6] In particular, a positive domestic technology shock causes a belief-induced depreciation (increase) of the terms of trade and

[5] There is sizeable evidence to suggest that terms of trade shocks are an important source of business cycle fluctuations (see, e.g., Mendoza, 1995).

[6] This temporary contractionary transmission mechanism is not without empirical support. See, e.g., Basu et al. (2006) and Giuli and Tancioni (2017).

the delayed expansion generates the desired negative correlation between the terms of trade and output. Moreover, since exports are relatively higher than imports, real net exports are weakly counter-cyclical, as in the data. Finally, the depreciation of the terms of trade is sufficiently large relative to output that the model is able to generate volatile international relative prices.

Our approach is similar to Schmitt-Grohé (2000) and Benhabib and Wang (2013) in that we select the properties of the fundamental and forecast error shocks which best match the key moments of the data, but we specifically focus on international fluctuations. In this way, we give the indeterminacy model the best chance at matching the international business cycle facts. However, one main discrepancy with the data remains, the so-called consumption-real exchange rate anomaly or Backus-Smith (1993) puzzle. The model predicts a positive correlation between the real exchange rate and relative consumption, whereas in the data this correlation is negative. In our model, a belief-induced depreciation of the terms of trade generates a relatively stronger reduction in employment abroad than in the domestic economy. This increases the ratio of consumption across the two countries, thereby counterfactually implying a positive correlation between international relative prices and relative consumption. We show that this counterfactual mechanism is at the heart of all indeterminacy models that have an upward-sloping aggregate labor demand schedule. Consequently, the failures identified in this paper will hold for a wide-class of indeterminacy frameworks. We find that in order to resolve the Backus-Smith puzzle the model requires a strong negative cross-country correlation for technology shocks, which is not supported by other studies.[7]

In addition to the studies of Guo and Sturzenegger (1998) and Xiao (2004), the current paper is also related to several contributions within the indeterminacy literature. Recent studies have been successful in quantitatively explaining closed-economy business cycles using indeterminacy. For example, Jaimovich (2007) and Pavlov and Weder (2017) using one-sector models, and Guo and Harrison (2010) and Dufourt et al. (2015) using two-sector models, can broadly reproduce several key features of U.S. business cycles. This paper contributes to this literature by examining whether indeterminacy can also successfully replicate some of the most well-known properties of international business cycles. Similar to Pintus et al. (2016) and Pavlov and Weder (2017), we solve the model under indeterminacy using the Farmer-Khramov-Nicolò (2015) method.[8] However, they estimate their models with U.S. data using Bayesian techniques, whereas we use the method of

[7]See, e.g., Backus et al. (1992) and Heathcote and Perri (2004).

[8]A popular alternative to the Farmer-Khramov-Nicolò solution technique is the method of Lubik and Schorfheide (2003, 2004). As shown by Farmer et al. (2015), these two solution methods are equivalent. See also Bianchi and Nicolò (2017).

moments approach to try and resolve some well-known empirical puzzles in international business cycle theory.

The remainder of the paper is organized as follows. Section 2 outlines the model economy. Section 3 discusses the calibration of the model and the solution method employed. Sections 4 and 5 presents the main results and Section 6 discusses the transmission mechanism. Finally, Section 7 briefly concludes.

2 Model

We develop a two-country extension of the imperfect competition model studied by Farmer and Guo (1994) and Schmitt-Grohé (1997) for the closed economy. Following Wen (1998), we assume variable capacity utilization which significantly reduces the size of the steady-state markup needed to generate indeterminacy. Within each country there exists a representative agent, two final-good producers, and a continuum of intermediate-good producing firms. Intermediate firms operate under monopolistic competition and use domestic labor and capital as inputs to produce tradeable goods. The competitive final good producers use domestic and imported intermediate goods to produce non-tradeable consumption or investment goods, which are subsequently purchased by the domestic agent. However, final good producers are assumed to have a bias for domestically produced intermediate goods. While the law of one price is assumed to hold for all intermediate goods, with home bias, the real exchange rate deviates from purchasing power parity. The following presents the features of the model for the Home country on the understanding that the Foreign case can be analogously derived. All Foreign country variables are denoted by an asterisk.

2.1 Final good producers

In each country, there are two final goods, consumption and investment, which are produced with homogenous of degree one production functions using intermediate goods as the only inputs. The Home consumption final good C_t is produced by a competitive firm that uses $C_{H,t}$ and $C_{F,t}$ as inputs according to the following CES aggregation technology index:

$$C_t = \left[a^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the constant elasticity of substitution between aggregate Home and Foreign intermediate goods is $\theta > 0$ and the relative share of domestic and imported intermediate inputs used in the production process is $0 < a < 1$. The Home investment final good I_t is produced according to the following CES aggregation technology index:

$$I_t = \left[b^{\frac{1}{\rho}} I_{H,t}^{\frac{\rho-1}{\rho}} + (1-b)^{\frac{1}{\rho}} I_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{2}$$

where $\rho > 0$ and $0 < b < 1$. The inputs $C_{H,t}$, $C_{F,t}$, $I_{H,t}$, and $I_{F,t}$ are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$C_{H,t} = \left[\int_0^1 c_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad C_{F,t} = \left[\int_0^1 c_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}},$$
$$I_{H,t} = \left[\int_0^1 i_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad I_{F,t} = \left[\int_0^1 i_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}},$$

where the elasticity of substitution across domestic (imported) intermediate goods is $\kappa > 1$, and $c_t(j)$, $i_t(j)$, $c_t(j^*)$, $i_t(j^*)$ are the respective quantities of the domestic and imported type j and j^* intermediate goods. Intermediate firms sell their products to both consumption and investment final-good producers, where it is assumed that the law of one price holds. Cost minimization in final good production yields the demand conditions for Home and Foreign goods:

$$C_{H,t} = a \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, \qquad C_{F,t} = (1-a) \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t, \tag{3}$$

$$I_{H,t} = b \left(\frac{P^I_{H,t}}{P^I_t} \right)^{-\rho} I_t, \qquad I_{F,t} = (1-b) \left(\frac{P^I_{F,t}}{P^I_t} \right)^{-\rho} I_t, \tag{4}$$

and the corresponding aggregate price indices are given by:

$$P_t = \left[a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad P^I_t = \left[b (P^I_{H,t})^{1-\rho} + (1-b)(P^I_{F,t})^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{5}$$

where P_t is the consumer price index, P^I_t is the price of investment goods, and $P_{H,t}$, $P^I_{H,t}$, $P_{F,t}$, $P^I_{F,t}$ are the respective price indices of Home and Foreign intermediate goods.

2.2 Intermediate goods producers

All intermediate firms have access to the same technology. A Home firm of type j has a production technology given by

$$Y_t(j) = Z_t \left(u_t(j)K_t(j)\right)^{\alpha} L_t(j)^{\gamma} - \phi, \quad j \in [0,1] \tag{6}$$

where K_t and L_t represent capital and labor usage, respectively, Z_t is the exogenous level of technology or productivity, and the input share is $\alpha + \gamma \geq 1$. The rate of capacity utilization $u_t \in (0,1)$ is endogenously determined. Following Greenwood et al. (1988), it is assumed that the depreciation rate of capital δ_t is higher if it is used more intensively:

$$\delta_t = \frac{1}{\eta} u_t^{\eta}, \tag{7}$$

where $\eta > 1$. A fixed cost of production $\phi > 0$ is also included in the production technology (6). Therefore, regardless of how much output Y_t is produced, a proportion ϕ of the intermediate good is used up in each period. As in Schmitt-Grohé (1997), allowing for a fixed production cost enables the model to generate zero profits without imposing any restrictions on the size of the steady-state markup.[9] Given competitive prices of labor and capital, cost-minimization yields:

$$w_t = \gamma mc_t(j) Z_t \left(u_t(j)K_t(j)\right)^{\alpha} L_t(j)^{\gamma-1}, \tag{8}$$

$$rr_t + \delta_t = \alpha mc_t(j) Z_t u_t^{\alpha}(j) K_t(j)^{\alpha-1} L_t(j)^{\gamma}, \tag{9}$$

$$u_t^{\eta} = \alpha mc_t(j) Z_t u_t^{\alpha}(j) K_t(j)^{\alpha-1} L_t(j)^{\gamma}, \tag{10}$$

where mc_t is real marginal cost, w_t is the real wage, and $rr_t + \delta_t$ is the user cost of capital.

Given that the total demand for firm j's output can be expressed as:

$$Y_t(j) = \left(\frac{p_t(j)}{P_{H,t}}\right)^{-\kappa} \left[C_{H,t} + C^*_{H,t}\right] + \left(\frac{p_t(j)}{P^I_{H,t}}\right)^{-\kappa} \left[I_{H,t} + I^*_{H,t}\right],$$

it follows from the firm's profit maximization problem that the optimal price-setting rule is:

$$p_t(j) = \chi mc_t(j) P_t, \tag{11}$$

[9] As discussed by Rotemberg and Woodford (1996), Schmitt-Grohé (1997), and Jaimovich (2007), positive profits are not observed in the U.S. economy despite the presence of market power.

where $\chi \equiv \frac{\kappa}{\kappa-1}$ is the markup.

2.3 Representative agent

The representative agent has an expected utility function of the form:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right),$$

where C_t and L_t are consumption and work effort, respectively, and the discount factor is $0 < \beta < 1$. Following Greenwood et al. (1988), we assume that the period utility function is given by:

$$U(C_t, L_t) = \frac{1}{1-\sigma}\left[\left(C_t - \frac{\psi}{1+\nu} L_t^{1+\nu}\right)^{1-\sigma} - 1\right],$$

where $\sigma > 0$ is the relative risk aversion in consumption, $\nu \geq 0$ is the inverse of the Frisch elasticity of labor supply, and $\psi > 0$.

The representative agent during period t supplies labor and capital to intermediate-good producing firms, receiving real income from wages w_t, a rental return on capital rr_t, and nominal profits from the ownership of domestic intermediate firms Π_t. The agent then uses these resources to purchase the two final goods, dividing purchases between consumption C_t and investment I_t. The purchase of an investment good forms next period's capital according to the law of motion

$$K_{t+1} = (1 - \delta_t) K_t + I_t. \tag{12}$$

The asset market structure is assumed to be incomplete. The Foreign agent is able to trade two non-state contingent bonds $B^*_{H,t}$ and $B^*_{F,t}$, whereas the Home agent can only purchase domestic bonds $B_{H,t}$. All bonds are denominated in units of the domestic aggregate consumption index. For the Foreign agent, there is a transaction cost Ψ of adjusting the internationally traded bond $B^*_{H,t}$, where it is assumed that Ψ is a positive and differentiable function.[10] This transaction cost, which is paid to financial firms, captures the costs of adjusting bond holdings and is sufficient to ensure that bond holdings are stationary.[11] Consequently, the period budget constraints of the

[10] Following Benigno (2009), we assume that $\Psi = 1$ when bond holdings are at their steady-state level and Ψ is positive, differentiable, and strictly decreasing in a neighborhood of the steady state.

[11] For an in-depth discussion of the stationary problem of incomplete market, open-economy models, see Schmitt-Grohé and Uribe (2003) and Ghironi (2006).

Home and Foreign agent can be expressed in real terms as:

$$\frac{B_{H,t}}{r_t} + C_t + \frac{P_t^I}{P_t} I_t \leq B_{H,t-1} + \int_0^1 w_t L_t(j)dj + \int_0^1 (rr_t + \delta_t(j))K_t(j)dj + \int_0^1 \Pi_t(j)dj + R_t, \quad (13)$$

$$\frac{B^*_{H,t}}{Q_t r_t} \frac{1}{\Psi(B^*_{H,t})} + \frac{B^*_{F,t}}{r^*_t} + C^*_t + \frac{P_t^{*I}}{P^*_t} I^*_t \leq \frac{B^*_{H,t-1}}{Q_t} + B^*_{F,t-1} + \int_0^1 w^*_t L^*_t(j^*)dj^* + \int_0^1 (rr^*_t + \delta^*_t(j^*))K^*_t(j^*)dj^* + \int_0^1 \Pi^*_t(j^*)dj^* + R^*_t, \quad (14)$$

where R_t and R^*_t denote rebates from financial firms, r_t and r^*_t are the Home and Foreign (gross) real interest rates, and Q_t is the CPI-based real exchange rate.

The Home agent's maximization problem yields:

$$U_c(C_t, L_t) = \left(C_t - \frac{\psi L_t^{1+\nu}}{1+\nu} \right)^{-\sigma} = \lambda_t, \quad (15)$$

$$-\frac{U_L(C_t, L_t)}{U_c(C_t, L_t)} = \psi L_t^{\nu} = w_t, \quad (16)$$

$$\lambda_t \frac{P_t^I}{P_t} = \beta E_t \lambda_{t+1} \left[rr_{t+1} + \delta_{t+1} + (1 - \delta_{t+1}) \frac{P_{t+1}^I}{P_{t+1}} \right], \quad (17)$$

$$\beta r_t E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] = 1, \quad (18)$$

where λ_t denotes the shadow price of wealth. Analogous conditions to (15)-(18) apply for the Foreign agent, where the following interest-rate parity condition can be derived:

$$r_t = \frac{r^*_t}{\Psi(B^*_{H,t})} E_t \left[\frac{Q_{t+1}}{Q_t} \right]. \quad (19)$$

Optimizing behavior implies that the budget constraints (13) and (14) hold with equality in each period and the appropriate transversality conditions are satisfied.

2.4 Market clearing and equilibrium

We now focus on a symmetric equilibrium where all firms in Home and Foreign set the same price in each period t, rent the same amount of capital, and employ the same amount of labor. Consequently, $p_t(j) = P_{H,t} = P^I_{H,t}$ and the index j can be dropped from all variables. Market clearing in the Home goods market requires:

$$Y_t = C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t}, \tag{20}$$

and assuming that the Foreign non-state contingent bond is in zero net supply, bond market clearing requires:

$$B_{H,t} + B^*_{H,t} = 0, \quad B^*_{F,t} = 0. \tag{21}$$

The aggregate resource constraint is given by:[12]

$$C_t + \frac{P^I_t}{P_t} I_t + \frac{B_{H,t}}{r_t} = B_{H,t-1} + \frac{P_{H,t}}{P_t} Y_t, \tag{22}$$

where

$$\frac{P^I_t}{P_t} = \frac{\left[b + (1-b)T_t^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[a + (1-a)T_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}, \qquad \frac{P_{H,t}}{P_t} = \left[a + (1-a)T_t^{1-\theta}\right]^{\frac{1}{\theta-1}} \tag{23}$$

follow from the aggregate price indices (5). The terms of trade T_t can be expressed as:

$$T_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{\left[a + (1-a)T_t^{\theta-1}\right]^{\frac{1}{\theta-1}}}{\left[a + (1-a)T_t^{1-\theta}\right]^{\frac{1}{\theta-1}}} Q_t. \tag{24}$$

In what follows, we call an increase (decrease) in the terms of trade, or the real exchange rate, a depreciation (appreciation). Finally, we measure net exports as the difference between exports and imports, divided by total output (all evaluated at steady state prices):[13]

$$NX_t = \frac{C^*_{H,t} + I^*_{H,t} - \overline{T}\left(C_{F,t} + I_{F,t}\right)}{C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t}}. \tag{25}$$

Equilibrium. An equilibrium for the world economy consists of a set of real prices r_t, r^*_t, w_t, w^*_t, rr_t, rr^*_t, δ_t, δ^*_t, mc_t, mc^*_t, λ_t, λ^*_t; a set of relative prices $\frac{P_{H,t}}{P_t}$, $\frac{P^*_{F,t}}{P^*_t}$, $\frac{P^I_t}{P_t}$, $\frac{P^{*I}_t}{P^*_t}$, Q_t, T_t; a collection of allocations for the Home and Foreign agent C_t, C^*_t, I_t, I^*_t, L_t, L^*_t, K_t, K^*_t, u_t, u^*_t, $B_{H,t}$, $B^*_{H,t}$, $B^*_{F,t}$; and a collection of allocations for Home and Foreign final and intermediate good producers Y_t, Y^*_t, $C_{H,t}$, $C_{F,t}$, $C^*_{H,t}$, $C^*_{F,t}$, $I_{H,t}$, $I_{F,t}$, $I^*_{H,t}$, $I^*_{F,t}$, NX_t satisfying (i) the optimality conditions of each agent; (ii) the optimality conditions of final and intermediate good producing firms; (iii) all markets clear; and (iv) the aggregate resource constraints of both countries.

[12]By Walras' Law, the aggregate resource constraint of the Foreign country is redundant.
[13]Thus, our measure of net exports is unaffected by fluctuations in relative prices.

3 Numerical solution and calibration

3.1 The solution method under indeterminacy

To solve the indeterminacy model, we log-linearize the equilibrium conditions around a symmetric, deterministic steady state, where bond holdings are zero and the steady-state terms of trade is equal to 1.[14] Let $\boldsymbol{s}_t = \left[\widehat{K}_{t+1}, \widehat{K}^*_{t+1}, \tilde{B}_{H,t}, \widehat{T}_t, \widehat{C}_t, \widehat{C}^*_t, E_t\widehat{T}_{t+1}, E_t\widehat{C}_{t+1}, E_t\widehat{C}^*_{t+1}, \widehat{Z}_t, \widehat{Z}^*_t\right]'$ denote the vector of endogenous variables expressed in terms of percentage deviations from their steady state values.[15] The linearized system can be written as:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi}\boldsymbol{\varepsilon}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t, \tag{26}$$

where $\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}$, and $\boldsymbol{\Pi}$ are matrices of structural parameters, $\boldsymbol{\varepsilon}_t = [\varepsilon_t, \varepsilon^*_t]'$ is the vector of fundamental or exogenous technology shocks, and $\boldsymbol{\eta}_t = \left[\eta^T_t, \eta^C_t, \eta^{C^*}_t\right]'$ is the vector of non-fundamental or endogenous shocks, which collects the one-step ahead forecast errors for the expectational variables of the system. The log of technology in both countries is assumed to follow an AR(1) process with zero mean. We assume that $E_{t-1}(\boldsymbol{\varepsilon}_t) = 0$ and $E_{t-1}(\boldsymbol{\eta}_t) = 0$.

If marginal costs are assumed to be decreasing in output (i.e., $\alpha + \gamma > 1$), then the system (26) may not have a unique solution. With our chosen value of increasing returns to scale (see Section 3.2 below), the number of non-predetermined variables exceeds the number of unstable roots by one, and thus we have one degree of indeterminacy.

The model is solved using the Farmer-Khramov-Nicolò (2015) solution method, whereby we redefine one expectational error $\eta_{f,t}$ of vector $\boldsymbol{\eta}_t$ as a new fundamental disturbance.[16] In this way the number of non-predetermined variables is decreased by one. This transformation enables us to treat the indeterminacy model as determinate and we use the popular algorithm of Uhlig (1999) to solve the model. Importantly, Farmer et al. (2015) show that the choice of which expectational error to redefine as a new fundamental shock is irrelevant. They demonstrate that the same solution can be obtained under alternative specifications of $\eta_{f,t}$. We choose the forecast error of the terms of trade as the new fundamental, $\eta_{f,t} = \eta^T_t \equiv \widehat{T}_t - E_{t-1}\widehat{T}_t$, and show that our results are robust to the choice of expectational error. We refer to the forecast error η^T_t as a self-fulfilling expectation

[14] In the steady state, the degree of increasing returns to scale can be expressed as the ratio between average and marginal costs, which is equal to the markup: i.e., $\frac{(\alpha+\gamma)(\overline{Y}+\phi)}{\overline{Y}} = \chi$. Consequently, for a steady state to exist, the steady-state markup cannot be lower than the degree of diminishing marginal cost i.e., $\chi \equiv \frac{\kappa}{\kappa-1} \geq \alpha + \gamma$.

[15] For bond holdings $\tilde{B}_{H,t}$, we take the linear deviation relative to steady-state Home consumption.

[16] Pintus et al. (2016) and Pavlov and Weder (2017) adopt a similar solution method.

or belief.

An equilibrium is characterized by $\theta^* \in \boldsymbol{\Theta}$, where $\boldsymbol{\Theta}$ is a parameter space which includes the parameters of the structural equations, the variance covariance matrix of the original fundamental shocks, and the variance and covariances of the new fundamental shock with the original set of fundamentals:

$$\boldsymbol{\Theta} \equiv \left\{ \boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Omega}_{\varepsilon\varepsilon}, \boldsymbol{\omega}_{\eta\varepsilon}, \sigma_\eta^2 \right\}, \tag{27}$$

where $\boldsymbol{\Omega}_{\varepsilon\varepsilon} \equiv E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)$, $\boldsymbol{\omega}_{\eta\varepsilon} \equiv \left[E\left(\varepsilon_t \eta_t^T\right), E\left(\varepsilon_t^* \eta_t^T\right)\right] = E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right)$, and $\sigma_\eta^2 \equiv E\left[\left(\eta_t^T\right)^2\right]$. By specifying a new fundamental shock together with $\boldsymbol{\omega}_{\eta\varepsilon}$ and σ_η^2 we select a unique rational expectations equilibrium. The covariance of η_t^T with $\boldsymbol{\varepsilon}_t$ represents the response of beliefs to the original set of fundamentals, which amplify or attenuate the effects of technological shocks in the economy (Dufourt et al., 2015).

Farmer et al. (2015) demonstrate that this representation of equilibrium under indeterminacy can be alternatively characterized in terms of a linear forecasting rule that expresses the forecast errors as a function of fundamentals and sunspot shocks. This alternative solution methodology has been proposed in the seminal contributions of Lubik and Schorfheide (2003, 2004). As shown by Farmer et al. (2015), the two representations of equilibrium indeterminacy are entirely equivalent, because for each indeterminate equilibrium $\theta^* \in \boldsymbol{\Theta}$ there exists a unique linear forecasting rule that implements equilibrium θ^*, and vice versa.

The equivalence between the two solution methods enables us to compute the parameters of a linear forecasting rule à la Lubik and Schorfheide, in order to illustrate the relationship between fundamental and sunspot disturbances. For our purposes, we specify the linear forecasting rule as follows:

$$\eta_t^T = [\beta_1, \quad \beta_2] \cdot \boldsymbol{\varepsilon}_t + \zeta_t = \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t + \zeta_t, \tag{28}$$

where the residual ζ_t can be interpreted as a "pure"sunspot shock uncorrelated with fundamentals: $E\left(\zeta_t\right) = 0$, $E\left(\zeta_t^2\right) \equiv \sigma_\zeta^2 > 0$, and $E\left(\zeta_t \boldsymbol{\varepsilon}_t\right) = \mathbf{0}$.

To aid our understanding of the indeterminacy model we consider two alternative assumptions. (i) *Autonomous beliefs*: shocks to the forecast error of the terms of trade η_t^T are the only source of business cycle fluctuations ($\boldsymbol{\Omega}_{\varepsilon\varepsilon} = \mathbf{0}$ and $\boldsymbol{\omega}_{\eta\varepsilon} = \mathbf{0}$); (ii) *Correlated beliefs*: the forecast error η_t^T is correlated with fundamentals, thus both $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$ and $\boldsymbol{\omega}_{\eta\varepsilon}$ are not restricted to be zero.[17] In this case,

[17] In the indeterminacy literature (e.g., Dufourt et al., 2015), the forecast error is assumed to be perfectly correlated with fundamentals, i.e. $\sigma_\zeta = 0$ in (28). While this assumption can be imposed by placing appropriate restrictions on the covariance vector $\boldsymbol{\omega}_{\eta\varepsilon}$, we choose to leave it unrestricted so as not to lose any degree of freedom in our

we can use the equivalence between the Farmer-Khramov-Nicolò and the Lubik and Schorfheide solution methods to recover $\boldsymbol{\beta}$ and σ_ζ^2 pertaining to equation (28).[18] Multiplying equation (28) by $\boldsymbol{\varepsilon}_t'$ and taking expectations yields:

$$\boldsymbol{\beta} = E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right) E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)^{-1} = \boldsymbol{\omega}_{\eta\varepsilon} \boldsymbol{\Omega}_{\varepsilon\varepsilon}^{-1}. \tag{29}$$

To compute the variance of the pure sunspot shock, first note:

$$\sigma_\zeta^2 = E\left(\zeta_t \zeta_t'\right) = E\left(\left[\eta_t^T - \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t\right]\left[\eta_t^T - \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t\right]'\right),$$

and since $E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right) = \boldsymbol{\beta} E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)$ it follows that:

$$\sigma_\zeta^2 = E\left[\left(\eta_t^T\right)^2\right] - \boldsymbol{\beta} E\left(\boldsymbol{\varepsilon}_t \eta_t^T\right) = \sigma_\eta^2 - \boldsymbol{\beta} \boldsymbol{\omega}_{\eta\varepsilon}'. \tag{30}$$

Next, we describe how we calibrate the structural parameters of matrices $\boldsymbol{\Gamma}_0$, $\boldsymbol{\Gamma}_1$, and $\boldsymbol{\Psi}$ in Section 3.2 below. Since the alternative assumptions of autonomous and correlated beliefs imply different strategies for the calibration of the stochastic processes, we discuss the calibration of σ_η^2, $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$, and $\boldsymbol{\omega}_{\eta\varepsilon}$ separately in Sections 4 and 5 below.

3.2 Parameterization

The baseline parameter values used to compute the indeterminate equilibrium are summarized in Table 1. The U.S. is assumed to be the Home country and the rest of the world represents the Foreign country. As is standard in the literature, we set the time interval to be a quarter, the discount factor $\beta = 0.99$, and the steady-state depreciation rate $\overline{\delta} = 0.025$ (which implies $\eta \simeq 1.4$). The labor share in production is set equal to 0.7 and we set the inverse elasticity of labor supply $\nu = 0$ (i.e., indivisible labor) to help generate indeterminacy for a small degree of returns to scale, a standard assumption of the indeterminacy literature. The preference parameter ψ is set so that in the steady state the agent in each country allocates one-third of their time to market activities.

calibration strategy.

[18] Notice that under a linear forecasting rule the equilibrium is characterized by an alternative parameter space $\tilde{\boldsymbol{\Theta}}$ whereby $\boldsymbol{\beta}$ and σ_ζ^2 replace $\boldsymbol{\omega}_{\eta\varepsilon}$ and σ_η^2 in (27):

$$\tilde{\boldsymbol{\Theta}} \equiv \left\{\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Omega}_{\varepsilon\varepsilon}, \boldsymbol{\beta}, \sigma_\zeta^2\right\}.$$

Alternatively, a researcher may want to consider a linear transformation of equation (28) and adjust the parameter space accordingly.

Table 1: Baseline parameter values

β	0.99	Discount factor
$\overline{\delta}$	0.025	Steady state depreciation rate of capital
ν	0	Inverse elasticity of labor supply
σ	2	Inverse of the intertemporal substitution elasticity of consumption
θ	1	Elasticity of substitution between home & foreign consumption goods
ρ	1	Elasticity of substitution between home & foreign investment goods
a	0.88	Home bias in consumption goods
b	0.88	Home bias in investment goods
ω	0.001	Bond adjustment cost
$\overline{L}$	1/3	Steady state hours worked
S_L	0.7	Labor share in production
χ	1.2	Steady state markup
α	0.36	Elasticity of output with respect to capital
γ	0.84	Elasticity of output with respect to labor

In the IRBC literature, the risk aversion parameter typically chosen lies between $1 \leq \sigma \leq 2$. Following Stockman and Tesar (1995), we set $\sigma = 2$. In line with Benigno and Thoenissen (2008), we set the bond adjustment cost $\omega = 0.001$ and the steady-state terms of trade equal to 1. We set $a = b = 0.88$ to ensure that the ratio of imports to GDP is equal to 0.12, consistent with the U.S. economy.

Empirical studies offer no clear conclusion on the magnitude of the trade price elasticities, θ and ρ. We initially set $\theta = \rho = 1$ broadly consistent with the empirical estimates of Heathcote and Perri (2002).[19] However, the robustness of the numerical results are examined for variations in these parameters. In particular, we consider a low trade elasticity parameterization $\theta = \rho = 0.5$ roughly consistent with the estimates of Anderton et al. (2004) and Corsetti et al. (2008).

A key issue is to generate indeterminacy with empirically plausible values for the steady-state markup χ. Since intermediate firms use only capital and labor in the production process (6), this implies that the markup is value added. As discussed by Jaimovich (2007), value-added markups are estimated for the U.S. economy to lie between 1.2 to 1.4. We set the steady-state markup $\chi = 1.2$, consistent with the lower range of these empirical estimates.[20] The numerical analysis suggests that under the baseline parameterization there are many values of α and γ that generate indeterminacy for empirically plausible values of the steady-state markup. For simplicity, we follow

[19] Heathcote and Perri (2002) estimate the trade elasticity for the U.S. to be approximately 0.9.

[20] A sensitivity analysis was conducted using a higher value for the steady-state markup $\chi = 1.3$ with little significant change in the results found.

Hornstein (1993) and set $\alpha + \gamma = \chi = 1.2$, which implies that profits are zero in every period.

4 Autonomous beliefs

4.1 The international business cycle facts

The estimated moments for the data, given in column 2 of Table 2, are for the period 1973(1) − 2007(4) and are taken from Gao et al. (2014), except for the moments for real net exports and first-order autocorrelations, which we compute using data from the Quarterly National Accounts of the OECD.[21]

To understand the role of self-fulfilling beliefs, column 3 of Table 2 reports simulation results for the determinacy version of the model, where marginal costs are assumed to be constant (i.e., $\alpha+\gamma = 1$), expectational shocks do not exist, and technology shocks are assumed to follow an AR(1) process with zero mean.[22] The autocorrelation parameters are set equal to $\upsilon = \upsilon^{*} = 0.96$. The standard deviations and cross-correlation of Home and Foreign technology shocks are calibrated so as to match the standard deviation of U.S. output and the cross-correlation of U.S. and Foreign output.

To evaluate the ability of the indeterminacy model to explain international fluctuations, we compare its predictions with respect to a number of well-known stylized facts. In the data, the terms of trade and the real exchange rate are more volatile than output, whereas real net exports is significantly less volatile than output (*volatility anomalies*). Both the terms of trade and real net exports are counter-cyclical over the cycle (*output-correlation puzzles*). The data suggests that the cross-country correlation of output is greater than the cross-country correlation of consumption (*the cross-country correlation anomaly*). Finally, in the data the correlation between consumption and the real exchange rate is negative (*the Backus-Smith puzzle*). All these stylized facts have posed a challenge to international macro models (see, e.g., Thoenissen, 2010). By comparison of columns 2 and 3 of Table 2, the determinacy version of the model fails to generate any of these key features of the data.

[21] All series are logged, except real net exports, and Hodrick-Prescott (HP) filtered with a smoothing parameter of 1600. We adopt the HP filter to ensure comparability of our results with the existing literature. The statistics in Gao et al. (2014) are computed where the U.S. is taken as the Home country and the Foreign country is the aggregate of Canada, Japan, and 19 European countries.

[22] The parameter values used in the simulations are the same as in Table 1 of Section 3.2 above with the exception that $\alpha + \gamma = 1$. The capacity utilization rate is assumed to be constant.

Table 2: Main results: Second moments of alternative model versions

	Data†	Determinacy	Indeterminacy: Autonomous Beliefs	Indeterminacy: Correlated Beliefs
Standard deviations‡				
Consumption	0.62	0.91	0.81	0.93
Investment	2.92	1.38	2.24	2.43
Employment	0.68	0.80	0.91	1.04
Terms of Trade	1.77	0.59	0.73	1.53
Real Exchange Rate	2.38	0.45	0.56	1.17
Real Net Exports	0.38*	0.05	0.17	0.65
First-order autocorrelations				
Output	0.87*	0.72	0.73	0.76
Real Exchange Rate	0.82*	0.75	0.74	0.72
Real Net Exports	0.85*	0.95	0.73	0.71
Correlations with output				
Consumption	0.82	0.99	1.00	0.99
Investment	0.94	0.96	1.00	0.63
Employment	0.85	1.00	1.00	0.99
Terms of Trade	-0.16	0.45	0.99	-0.40
Real Net Exports	-0.47*	0.20	-0.97	-0.22
Cross-country correlations				
Output	0.58	0.58	-1.00	0.45
Consumption	0.43	0.77	-1.00	0.48
Investment	0.41	0.07	-1.00	-0.50
Employment	0.45	0.70	-1.00	0.48
Correlation with the real exchange rate				
Relative Consumption	-0.17	0.97	0.99	0.42
Shock processes				
s.d. of ε_t (σ_ε)		0.32	-	0.29
s.d. of ε_t^* (σ_{ε^*})		0.32	-	0.29
s.d. (σ_η)		-	0.83	1.80
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		0.32	-	0.00
cross-correlation $\rho_{\eta,\varepsilon}$		-	-	0.96
cross-correlation $\rho_{\eta,\varepsilon^*}$		-	-	-0.11

Notes:

† The estimated sample moments for the data are taken from Gao et al. (2014), except for values denoted by $*$ which are from the authors' own calculations.

‡ The standard deviations of all variables are divided by the standard deviation of output, except for the standard deviation of real net exports which is expressed in absolute terms.

4.2 Results

Column 4 of Table 2 reports the results under autonomous beliefs. Here, the forecast error is assumed to be the only source of business cycle fluctuations, and the standard deviations and correlations with technology shocks are set equal to zero: $\boldsymbol{\Omega}_{\varepsilon\varepsilon} = \mathbf{0}$ and $\boldsymbol{\omega}_{\eta\varepsilon} = \mathbf{0}$. Since we choose $\eta_{f,t} = \eta_t^T$, under autonomous beliefs equation (28) is simply:

$$\widehat{T}_t - E_{t-1}\widehat{T}_t = \zeta_t.$$

In this scenario we treat the standard deviation σ_ζ as a free parameter and we calibrate it so as to match the standard deviation of U.S. output in all our experiments. For example, in the baseline parametrization we set $\sigma_\zeta = 0.832$ in order to produce a standard deviation of output of 1.49.

By comparing columns 2 and 4 of Table 2, one observes that the model is unable to resolve any major empirical irregularity of the data in relation to international relative prices or quantities. The terms of trade and the real exchange rate are less volatile than output and the model fails to generate sufficient volatility for real net exports. The terms of trade and output are predicted to move in the same direction leading to a counterfactual positive correlation. The model generates cross-country correlations which are equal to -1 and the correlation between the real exchange rate and relative consumption is positive and close to 1, such that the Backus-Smith puzzle arises. While the model can generate counter-cyclical real net exports, the negative correlation generated between net exports and output is very close to -1, which is much stronger than the data (-0.47).

An important element in understanding how self-fulfilling beliefs are transmitted relates to the labor market. The log-linearized Home and Foreign aggregate labor demands can be expressed as:

$$\widehat{w}_t = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t + \left[\frac{\eta\gamma}{\eta-\alpha} - 1\right]\widehat{L}_t - \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t, \tag{31}$$

$$\widehat{w}_t^* = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t^* + \left[\frac{\eta\gamma}{\eta-\alpha} - 1\right]\widehat{L}_t^* + \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t^*, \tag{32}$$

where in our parameterization $\eta - \alpha > 0$, $\frac{\eta\gamma}{\eta-\alpha} - 1 > 0$, and $\widehat{Z}_t = \widehat{Z}_t^* = 0$ under autonomous beliefs. With decreasing marginal costs, the source of indeterminacy arises from an upward-sloping aggregate labor demand schedule, which is steeper than the horizontal aggregate labor supply schedule (arising from an infinite elasticity of labor supply parameterization). The labor market of each country is depicted in Figure 1.

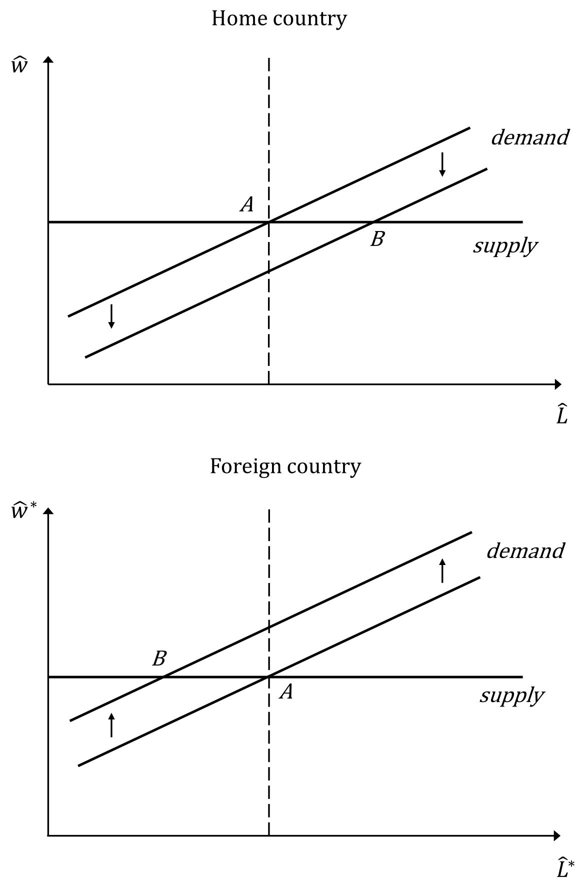

Figure 1: The transmission of pure sunspot shock in the two country model

To understand the poor performance of the indeterminacy model under autonomous beliefs, consider a positive revision to the terms of trade forecast, which results in a depreciation (increase) of the terms of trade $\widehat{T}_t$. The impulse response functions are depicted in Figure 2 and the underlying transmission mechanism is illustrated in Figure 1. After a terms of trade depreciation, the upward-sloping Home aggregate labor demand schedule shifts down (Equation 31 and Figure 1) increasing Home employment, which raises Home output and consumption. Consequently, belief-induced fluctuations counterfactually generate a positive correlation between the terms of trade and output. As the demand for imports increases in the Home country due to higher consumption, real net exports decrease. For the Foreign country, the Foreign aggregate labor demand schedule shifts up in Figure 1, and as a result, Foreign employment decreases, reducing Foreign output and consumption. This explains the perfect negative cross-country correlations generated under autonomous beliefs. Furthermore, while the data suggests that cross-country consumption

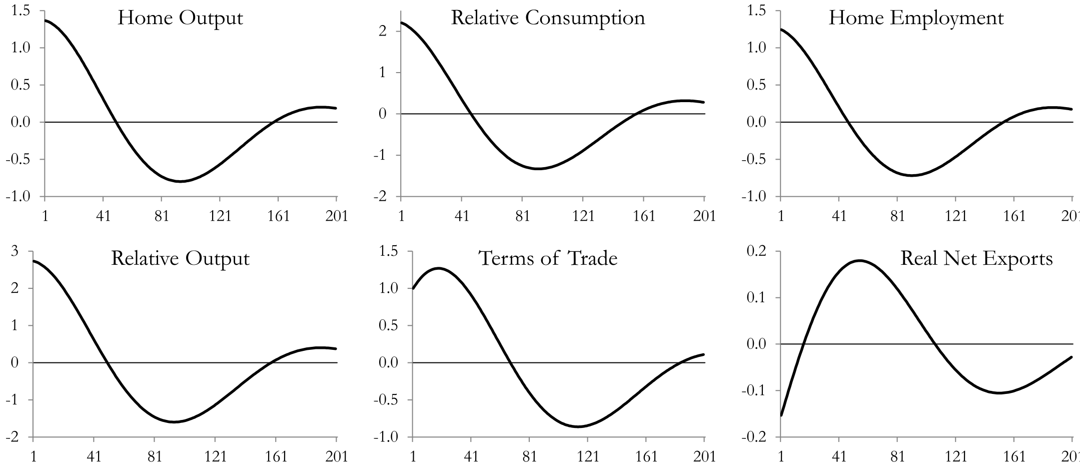

Figure 2: Impulse responses under a positive 1% shock to the terms of trade forecast. Vertical axes: % deviation from the steady state; Horizontal axes: quarters.

increases in response to an appreciation of international relative prices, self-fulfilling expectations induce a counterfactual positive correlation between relative consumption and the terms of trade. Overall, the indeterminacy model under autonomous beliefs cannot replicate the observed behavior for international relative prices and quantities nor solve the Backus-Smith puzzle.

The above mechanism is in stark contrast to the two-country, one-good models of Guo and Sturzenegger (1998) and Xiao (2004), where self-fulfilling expectations result in positive cross-country correlations for consumption and output. Due to the absence of international relative prices in these models, belief-induced fluctuations stimulate consumption and output in both counties. In our two-good model, self-fulfilling beliefs are global extrinsic shocks that affect the terms of trade, inducing an output reallocation between the two countries. Consequently, cross-country correlations for consumption and output are negative.

The above analysis shows that the inability of the autonomous beliefs model to replicate the stylized facts stems from the labor market. The transmission of pure (uncorrelated) sunspot shocks is at odds with the data because the upward-sloping Home and Foreign labor demands, which are steeper than supply, move in opposite directions (Figure 1). However, since an upward-sloping aggregate labor demand is at the core of traditional indeterminacy models, our results will extend to all two-good, open-economy models with self-fulfilling beliefs as the only source of fluctuations.

5 Correlated beliefs

The quantitative results from the previous section showed that self-fulfilling beliefs alone cannot replicate the basic international business cycle facts. However, when the forecast error of the terms of trade is correlated with productivity shocks the analysis differs significantly. Inspection of the aggregate labor demand equations (31) and (32) suggest that the indeterminacy model should perform better under positively correlated shocks. In this case, a Home technology shock $\widehat{Z}_t$ causes a revision of expectations (Equation 28) and therefore a belief-induced change in the terms of trade $\widehat{T}_t$. Since both a positive $\widehat{Z}_t$ and a positive $\widehat{T}_t$ affect the aggregate labor demand schedule (Figure 1) in opposite directions, the response of domestic employment and output will not be as strong as under autonomous beliefs. Indeed, both domestic employment and output could actually fall provided the shocks to Home technology and the expectational error are sufficiently positively correlated to generate an upward shift of the Home labor demand schedule (31), and consequently, the correlation between the terms of trade and output would become negative, as in the data. Furthermore, if a temporary domestic contraction results in low Home imports then the model would also generate countercyclical real net exports. Finally, if the model can induce a large enough adjustment in the terms of trade relative to output then the model could potentially generate sizeable volatility improvements for international relative prices and quantities helping to resolve the volatility anomalies.

5.1 Shock processes

To test the above conjecture we introduce technology shocks and leave the covariances between the fundamental shocks and the forecast error η_t^T unrestricted, and therefore, the matrix $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$ and the vector $\boldsymbol{\omega}_{\eta\varepsilon}$ are not assumed to be zero. As a result, we have six free parameters: the standard deviations of the technology shocks and forecast error (σ_ε, σ_{ε^*}, and σ_η), and the cross correlations between the shocks ($\rho_{\varepsilon,\varepsilon^*}$, $\rho_{\eta,\varepsilon}$, and $\rho_{\eta,\varepsilon^*}$). The vector $\boldsymbol{\omega}_{\eta\varepsilon}$ of the covariances between η_t^T and the technology shocks can be interpreted as a coordination mechanism for revising expectations, which amplify (or attenuate) the effects of technological shocks in the economy.

In line with the IRBC literature, we assume that the stochastic processes for productivity are quite persistent and we set the Home and Foreign autocorrelation parameters equal to $\upsilon = \upsilon^* = 0.96$. Similar to Schmitt-Grohé (2000) and Benhabib and Wang (2013), the standard deviations and cross-correlations of the stochastic processes are calibrated using a method of moments approach, where we include all the moments that define the main stylized facts of international business cycle fluctuations in the objective function. Thus, we explicitly look for the shock properties that maximize the model's ability to match the data, as we want to give the indeterminacy model the best chance at matching the international business cycles facts. Specifically, we calibrate the volatility and cross-correlations of the shocks so as to minimize the distance between selected model moments and data moments.[23] Consistent with the empirical evidence of Backus et al. (1992) and Heathcote and Perri (2004), the cross-country correlation of technology shocks is restricted to be non-negative. We check that the covariance matrix of the shocks that minimizes the objective function is positive semi-definite.[24]

The objective function is computed as the sum of the squared differences between HP-filtered model moments and data moments, with the identity matrix as the weighting matrix. The following eight moments are included in the objective function: the standard deviations of output, the terms of trade, and net exports; the correlations with output of the terms of trade and net exports; the cross-country correlations of output and consumption; and the correlation of the real exchange rate with relative consumption. Therefore, the number of moment conditions exceeds the number of parameters to be estimated by two.

The calibrated standard deviations and shock cross-correlations that maximize the model's ability to replicate the data are summarized in the bottom panel of Table 2, which confirm our previous conjecture: in order to match the stylized facts revisions to the terms of trade forecasts must be positively correlated with Home productivity shocks. We also find that the correlation of beliefs with Home productivity shocks must be near one, and the standard deviation of beliefs must be relatively high.[25] The high values of $\rho_{\eta,\varepsilon}$ and σ_η drive the coefficient β_1 in equation (28) above unity, and the relative low value of $\rho_{\eta,\varepsilon^*}$ drives the coefficient β_2 near zero. In fact, in the baseline scenario the implied vector $\boldsymbol{\beta}$ is $[5.95,\ -0.71]$. These results indicate that domestic productivity shocks, amplified by self-fulfilling beliefs (revisions to the terms of trade forecasts), have a stronger effect on the business cycle than foreign productivity shocks.

[23] Model moments are computed using frequency domain techniques as described in Uhlig (1999).

[24] In a small number of cases the estimated covariance matrix is not positive semi-definite. In these cases, we replace the estimated covariance matrix with its closest positive semi-definite matrix.

[25] In our model, we find that a one percent shock to the forecast error has a relatively modest impact on the variables compared to technology shocks. Consequently, the estimation procedure selects a relatively higher standard deviation for the forecast error in order to match the selected moments.

5.2 Results

For the parameter values given in Table 1, and the shock processes given in Table 2, the final column of Table 2 summarizes the simulation results when self-fulfilling expectations are correlated with technology shocks. Under correlated beliefs, the quantitative performance of the indeterminacy model improves significantly in terms of replicating the data. Now, both the terms of trade and the real exchange rate are more volatile than output generating over 86% of the observed standard deviation of the terms of trade. The volatilities of international relative prices have increased by a factor of 2 in comparison to autonomous beliefs, and by a factor of over 2.5 relative to the determinacy baseline model.[26] Furthermore, the model can also simultaneously generate sufficient volatility for real net exports. Remarkably, under correlated beliefs the indeterminacy model results in a standard deviation for real net exports nearly 4 times larger than under autonomous beliefs and 13 times larger than the determinacy baseline model.

In terms of output correlations, the indeterminacy model with correlated beliefs correctly predicts that both real net exports and the terms of trade are counter-cyclical. In stark contrast to autonomous beliefs, the nearly perfect negative correlation between net exports and output no longer arises. While the indeterminacy model struggles to generate cross-country output correlations higher than cross-country consumption correlations, it does much better compared to the determinacy baseline model. The main discrepancy between the model and the data relates to the correlation between the real exchange rate and relative consumption. Although the model generates a significantly lower positive correlation than under autonomous beliefs (0.42 vs. 0.99) and the determinacy baseline model (0.42 vs. 0.97), this correlation remains counterfactual with the data where a negative correlation is observed (-0.17).

5.3 Robustness

We test the sensitivity of our results in two ways. First, we show that our results are robust to the choice of expectational error. Table 3 summarizes the simulation results for the indeterminacy model when the forecast error of Home consumption is selected (instead of the terms of trade) as the new fundamental: $\eta_{f,t} = \widehat{C}_t - E_{t-1}\widehat{C}_t$. Comparing Tables 2 and 3, our results are robust to the choice of forecast error, as proved by Farmer et al.(2015). Table 4 considers the sensitivity of our

[26] Under correlated beliefs, the model still generates only half the volatility for the real exchange rate relative to the data. This is unsurprising since the real exchange rate in our model is a linear transformation of the terms of trade (due to the assumption of the law of one price and the absence of non-traded goods). See Corsetti et al. (2008) for further discussion.

Table 3: Simulated results under an alternative choice of forecast error (Home consumption)

	Data	Autonomous Beliefs	Correlated Beliefs
Standard deviations			
Consumption	0.62	0.81	0.93
Investment	2.92	2.24	2.43
Employment	0.68	0.91	1.04
Terms of Trade	1.77	0.73	1.54
Real Exchange Rate	2.38	0.56	1.17
Real Net Exports	0.38	0.17	0.65
First-order autocorrelations			
Output	0.87	0.73	0.76
Real Exchange Rate	0.82	0.74	0.72
Real Net Exports	0.85	0.73	0.71
Correlations with output			
Consumption	0.82	1.00	0.99
Investment	0.94	1.00	0.63
Employment	0.85	1.00	0.99
Terms of Trade	-0.16	0.99	-0.40
Real Net Exports	-0.47	-0.97	-0.22
Cross-country correlations			
Output	0.58	-1.00	0.45
Consumption	0.43	-1.00	0.48
Investment	0.41	-1.00	-0.50
Employment	0.45	-1.00	0.48
Correlation with the real exchange rate			
Relative Consumption	-0.17	0.99	0.42
Shock processes			
s.d. of ε_t (σ_ε)		-	0.29
s.d. of ε_t^* (σ_{ε^*})		-	0.29
s.d. (σ_η)		0.92	0.96
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		-	0.00
cross-correlation $\rho_{\eta,\varepsilon}$		-	-0.81
cross-correlation $\rho_{\eta,\varepsilon^*}$		-	-0.24

Notes: See Table 2 above.

results to variations in the trade elasticity parameters θ and ρ, where we consider either $\theta = \rho = 0.5$ or $\theta = \rho = 1.24$.[27] By inspection of columns 3 and 4 of Table 4, the autonomous beliefs model still

[27] We set $\theta = \rho = 1.24$ as this is the highest value for the trade elasticities that generate indeterminacy with $\chi = 1.2$.

Table 4: Simulated results for variations in the trade elasticity

		Autonomous Beliefs		Correlated Beliefs	
	Data	$\theta = 0.5$	$\theta = 1.24^\star$	$\theta = 0.5$	$\theta = 1.24^\star$
Standard deviations					
Consumption	0.62	0.81	0.81	0.92	0.92
Investment	2.92	2.72	2.00	2.47	2.61
Employment	0.68	0.91	0.91	1.03	1.03
Terms of Trade	1.77	0.73	0.73	1.52	1.50
Real Exchange Rate	2.38	0.55	0.56	1.15	1.14
Real Net Exports	0.38	0.32	0.10	0.60	0.72
First-order autocorrelations					
Output	0.87	0.74	0.72	0.74	0.77
Real Exchange Rate	0.82	0.75	0.73	0.73	0.71
Real Net Exports	0.85	0.74	0.75	0.73	0.71
Correlations with output					
Consumption	0.82	1.00	1.00	0.99	0.99
Investment	0.94	1.00	1.00	0.52	0.64
Employment	0.85	1.00	1.00	0.99	0.99
Terms of Trade	-0.16	0.98	0.99	-0.34	-0.39
Real Net Exports	-0.47	-1.00	-0.92	-0.09	-0.25
Cross-country correlations					
Output	0.58	-1.00	-1.00	0.37	0.47
Consumption	0.43	-1.00	-1.00	0.42	0.50
Investment	0.41	-1.00	-1.00	-0.69	-0.43
Employment	0.45	-1.00	-1.00	0.41	0.49
Correlation with the real exchange rate					
Relative Consumption	-0.17	0.98	0.99	0.43	0.41
Shock processes					
s.d. of ε_t (σ_ε)		-	-	0.28	0.28
s.d. of ε_t^* (σ_{ε^*})		-	-	0.28	0.29
s.d. (σ_η)		0.81	0.84	1.74	1.77
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		-	-	0.00	0.00
cross-correlation $\rho_{\eta,\varepsilon}$		-	-	0.95	0.96
cross-correlation $\rho_{\eta,\varepsilon^*}$		-	-	-0.09	-0.13

Notes: See Table 2 above.
$\star$ For all variations in the trade price elasticities we set $\theta = \rho$. We set $\theta = 1.24$ as this is the highest value for which indeterminacy is possible.

performs poorly under different values for the trade elasticities. For the correlated beliefs model, most moments remain unaffected to the choice of trade elasticities and the general conclusions of the previous subsection continue to hold. The major discrepancy between the model and the data remains the Backus-Smith puzzle, i.e., the positive correlation between the real exchange rate and relative consumption. In the next section we will show that this anomaly can only be solved by allowing the correlation between the Home and Foreign technology shocks to be negative.

6 Inspecting the mechanism

6.1 Indeterminacy and the propagation of technology shocks

With the notable exception of the Backus-Smith puzzle, our results show that when self-fulfilling beliefs are correlated with productivity shocks the indeterminacy model can solve several international relative price and quantity puzzles. Since the correlation is key, conventional impulse responses cannot adequately represent the transmission of productivity shocks. Thus, we now illustrate how to derive combined impulse responses that are applicable when expectational errors are correlated with fundamentals.

Letting $\overline{\Phi}_{\varepsilon}^{X}$ denote the impulse response of variable X to an uncorrelated Home technology shock and $\overline{\Phi}_{\eta_T}^{X}$ denote the impulse response of variable X to a shock to the terms of trade forecast η^T, then the combined impulse response Φ_{ε}^{X} to a positive productivity shock is given by:[28]

$$\Phi_{\varepsilon}^{X} = \overline{\Phi}_{\varepsilon}^{X} + E_t \left(\eta_t^T | \varepsilon_t = 1 \right) \overline{\Phi}_{\eta_T}^{X} = \overline{\Phi}_{\varepsilon}^{X} + \beta_1 \overline{\Phi}_{\eta_T}^{X}, \tag{33}$$

where β_1 is the first element of vector $\boldsymbol{\beta}$ given in (29). For simplicity we have abstracted from Foreign technology shocks.[29] Figure 3 depicts selected impulse response functions, which combine the effect of Home technology shocks with the revision of expectations.

When shocks to the terms of trade are positively correlated with Home technology shocks, a positive Home technology shock results in a belief-induced increase (depreciation) in the terms of trade $\widehat{T}_t$. From inspection of (31), the rise in $\widehat{Z}_t$ and $\widehat{T}_t$ shift the Home aggregate labor demand schedule in opposite directions. The top-right panel of Figure 3 suggests that the increase in employment caused by an increase in $\widehat{T}_t$ is more than offset by the fall in employment caused by the rise in $\widehat{Z}_t$. Consequently, the Home aggregate labor demand schedule in Figure 1 shifts

[28] $\overline{\Phi}_{\varepsilon}^{X}$ and $\overline{\Phi}_{\eta_T}^{X}$ are obtained using the Farmer-Khramov-Nicolò (2015) solution method under the assumption that all shocks are uncorrelated.

[29] We ignore the cross-country correlation of Home and Foreign technology shocks, since in our simulated results $\rho_{\varepsilon,\varepsilon^*} = 0.001$.

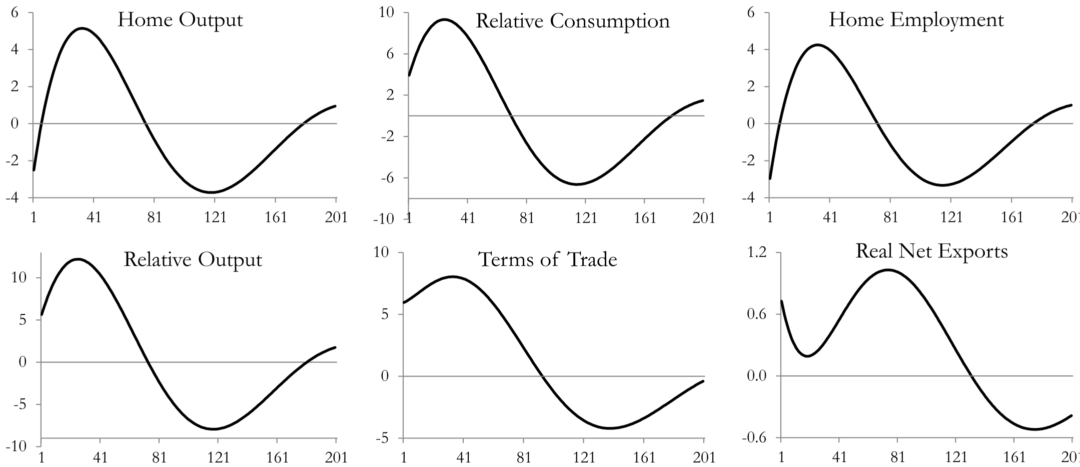

Figure 3: Impulse responses under a positive 1% productivity shock in the Home country. Vertical axes: % deviation from the steady state; Horizontal axes: quarters.

upwards causing an overall fall in Home employment and thus Home output. However, as shown in Figure 3, the negative effect on these variables is small and temporary. This finding that technology improvements can have temporary contractionary effects is supported by a number of empirical studies (see, for example, Galí, 1999; Francis and Ramey, 2005; Pesavento and Rossi, 2005; Basu et al., 2006, Fernald, 2007; Giuli and Tancioni, 2017) and is crucial for the model to solve the output-correlation puzzles.[30] Furthermore, since the change in the terms of trade is large relative to domestic output, international relative prices are now more volatile than output, as in the data. This is in stark contrast to the model with autonomous beliefs, which predicts a near perfect negative correlation between real net exports and output, and insufficient volatility. Recall that when the international business cycle is driven only by sunspot shocks, domestic output and consumption are stimulated and imports rise more than exports such that real net exports fall. Under correlated beliefs, the deterioration in the terms of trade and the temporary fall in output implies that imports fall more than exports, thereby generating a weak negative correlation between real net exports and output. Moreover, the delayed effect on output is key for generating sufficient volatility for real net exports.

Finally, the perfect negative cross-country correlations generated under autonomous beliefs no longer arises with correlated beliefs. Under autonomous beliefs, a belief-induced increase in $\widehat{T}_t$ stimulates Home output. In the Foreign country, the rise in $\widehat{T}_t$ causes the Foreign aggregate

[30] For example, Giuli and Tancioni (2017) show that the short-term response of both hours and investment to a positive technology shock is negative and the contraction is significant over approximately four to five quarters.

labor demand curve in Figure 1 to shift up, and the resulting fall in Foreign employment and output generates counterfactual negative cross-country correlations. However, when self-fulfilling expectations and Home technology shocks are sufficiently positively correlated, the increase in $\widehat{Z}_t$ more than offsets the belief-induced rise in $\widehat{T}_t$. Now the aggregate labor demand schedules in both countries shift upwards, resulting in positive cross-country correlations for employment and output.

6.2 Indeterminacy and the Backus-Smith puzzle

The main discrepancy between the model of Section 5 and the data is that the model generates a positive correlation between the real exchange rate and relative consumption, whereas in the data this relationship is negative. The transmission mechanism generated from an upward-sloping aggregate labor demand, which is key to replicating several features of the data, is also the obstacle for resolving the Backus-Smith puzzle, provided Home and Foreign technology shocks are not permitted to be negatively correlated.

To understand why correlated self-fulfilling expectations and technology shocks fail to resolve the Backus-Smith puzzle, we concentrate on the transmission of Home technology shocks, which have a more marked effect on the revision to the terms of trade forecasts than Foreign technology shocks.[31] First, recall that a positive Home technology shock causes a belief-induced increase (deterioration) in the terms of trade, and therefore an increase (depreciation) in the real exchange rate. Consequently, in order to solve the Backus-Smith puzzle the response of Foreign consumption must be above the response of Home consumption for relative consumption to fall, thereby generating a negative correlation with the real exchange rate. However, this cannot happen in our calibrated model despite the delayed effect on output of technology shocks. In Figure 1, the upward shift of the Foreign aggregate labor demand schedule caused by the belief-induced increase in $\widehat{T}_t$ is always greater than that of the Home country, since the rise in $\widehat{T}_t$ partially offsets the upward shift of the Home aggregate labor demand schedule due to $\widehat{Z}_t$. Because Foreign employment is relatively lower than Home employment, the response of Foreign consumption is below Home consumption.

However, the above analysis suggests that by allowing for a negative correlation between the Home and Foreign technology shocks, the indeterminacy model could generate a response for Foreign consumption greater than Home consumption. In this case, the upward shift of the Foreign

[31] With correlated beliefs, the vector $\boldsymbol{\beta}$ (Equation 28) controls how expectations are affected by technology shocks. In our calibration, β_2 is close to zero.

Table 5: Second moments under correlated beliefs: Unrestricted cross-country correlations for the productivity shocks $\rho_{\varepsilon,\varepsilon^*}$

			Sensitivity analysis	
	Data	Baseline	$\theta = 0.5$	$\theta = 1.24$
Standard deviations				
Consumption	0.62	0.91	0.91	0.90
Investment	2.92	2.43	1.87	2.74
Employment	0.68	1.02	1.01	1.02
Terms of Trade	1.77	1.65	1.77	1.56
Real Exchange Rate	2.38	1.26	1.34	1.19
Real Net Exports	0.38	0.66	0.38	0.77
First-order autocorrelations				
Output	0.87	0.78	0.75	0.78
Real Exchange Rate	0.82	0.71	0.71	0.70
Real Net Exports	0.85	0.71	0.72	0.71
Correlations with output				
Consumption	0.82	0.99	0.99	0.99
Investment	0.94	0.88	0.94	0.79
Employment	0.85	0.99	0.99	0.99
Terms of Trade	-0.16	-0.19	-0.17	-0.23
Real Net Exports	-0.47	-0.44	-0.46	-0.39
Cross-country correlations				
Output	0.58	0.40	0.50	0.49
Consumption	0.43	0.42	0.50	0.50
Investment	0.41	-0.56	-0.16	-0.57
Employment	0.45	0.42	0.50	0.49
Correlation with the real exchange rate				
Relative Consumption	-0.17	-0.12	-0.19	-0.08
Shock processes				
s.d. of ε_t (σ_ε)		0.29	0.30	0.28
s.d. of ε_t^* (σ_{ε^*})		0.22	0.25	0.21
s.d. (σ_η)		1.97	2.07	1.87
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		-0.84	-0.79	-0.76
cross-correlation $\rho_{\eta,\varepsilon}$		0.95	0.95	0.95
cross-correlation $\rho_{\eta,\varepsilon^*}$		-0.97	-0.93	-0.92

Notes: See Tables 2 and 4 above.

aggregate labor demand schedule in Figure 1 caused by the belief-induced increase in $\widehat{T}_t$ is now offset by a fall in $\widehat{Z}_t^*$. To verify this conjecture, we re-estimate the shock properties of the indeterminacy model without restricting the cross-country correlations for the productivity shocks to be non-negative. Table 5 summarizes the second moments and shock processes estimated for this exercise. Indeed, we find that our method of moments approach selects a negative correlation between Home-Foreign technology shocks, as we have hypothesized. By inspection, the indeterminacy baseline can indeed generate a negative correlation between the real exchange rate and relative consumption (-0.12) almost matching the data (-0.17). This finding is robust to alternative calibrations for the trade elasticities and is not at the expense of any of the other international puzzles.

7 Conclusion

We have analyzed whether equilibrium indeterminacy and self-fulfilling belief-driven fluctuations can explain the major features of international business cycles. We have found that when self-fulfilling beliefs are correlated with technology shocks, the indeterminacy model with can solve the volatility and output-correlation puzzles, and generate significantly improved statistics for the cross-correlation anomaly than the determinate benchmark. However, despite giving the indeterminacy model the best chance at matching the data, it generates a consumption-real exchange rate anomaly. Consequently, our analysis suggests that endogenous fluctuations cannot provide a satisfactory account of the most well-known features of the open-economy data. This conclusion is is stark contrast to the closed-economy literature, where indeterminacy models have been shown to generate business-cycle predictions consistent with U.S. data.

In our model, indeterminacy arises from increasing returns to scale which induces the aggregate labor demand schedule to become upward-sloping in the labor market, a common feature of many indeterminacy models.[32] We have shown that this labor market feature implies a counterfactual positive correlation between international relative prices and relative consumption, preventing endogenous fluctuations from overcoming the Backus-Smith puzzle. Consequently, the inability to solve the consumption-real exchange rate anomaly is not just a specific feature of our model, but of all indeterminacy models with an upward-sloping aggregate labor-demand schedule.

We have shown that the Backus-Smith puzzle can only be solved by allowing the cross-country

[32] Recent studies have shown that endogenous budget constraints can generate indeterminacy without requiring an upward-sloping labor demand schedule. See, e.g., Benhabib and Wang (2013) and Liu and Wang (2014).

correlation for technology shocks to be negative, but one obvious problem with this strategy is the absence of empirical evidence for this negative correlation. Alternatively, there could be other strategies, which may be worth exploring. For example, our analysis shows that in order to solve the Backus-Smith puzzle the transmission mechanism of at least one shock must induce a negative co-movement between the real exchange rate and relative consumption. We speculate that this may be possible in indeterminacy models that permit two self-fulfilling beliefs (i.e., indeterminacy of degree two). In this case, the cross-country correlations for consumption and output will now depend on how these non-fundamental shocks are related. Consequently, depending on the properties of the two self-fulfilling beliefs, it may be possible to generate a negative correlation between international relative prices and relative consumption. We leave this topic for future research.

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