U-model based LMI Robust Controller Design

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Abstract: In this study, an LMI (Linear Matrix Inequality) based H_∞ robust controller design approach is proposed to improve the performance of the designed U-pole placement control systems. Unlike the classical design procedures, the control-oriented U-model based nonlinear control systems cancel the nonlinearity of the nonlinear models. Therefore, the closed loop transfer function of U-pole placement control system can be regarded as a linear block. The solvability and sub-optimality of discrete-time H_∞ robust control are converted to find feasible solutions for LMIs. Once the internal parameters changed, the LMI based H_∞ controllers have a higher level of robustness compared with traditional controllers. A nonlinear dynamic model is selected to test the performance of the LMI robust controller to demonstrate the proposed approach effective.

Key Words: Nonlinear control systems, U-model, LMI, H_∞ robust control, pole placement control.

1 Introduction

Control problems arising in a wide variety of engineering fields are characterised by essential nonlinearities. In this case, LMI based H_∞ robust controller design approach generally cannot be directly applied because the dynamic behaviour of nonlinear plants cannot be easily determined according to the expressions cannot be written in the form of state space. It is obviously that applying pole placement to nonlinear plants is to synthesise a control system in such a way that the nonlinearities of the nonlinear plant should be removed and the resultant closed loop system behaves linearly.

It must be noted that the main difficulty in the design of nonlinear control systems is the lack of a general modelling framework which allows the synthesis of a simple control law. In some instances linearizing structures have been used but these suffer from 'local applicability' (Isidori, 1995) and therefore, are not very attractive. In order to simplify the control law synthesis part in nonlinear modelling, a new control-oriented model termed as the U-Model has recently been suggested (Zhu and Guo, 2002). The U-Model has a more general appeal as compared to other nonlinear models (NARMAX model and Hammerstein model). Additionally, this model is control-oriented in nature which makes the control synthesis part easier. Specifically, the control law based on the U-model exhibits a polynomial structure in the current input term.

Based on the U-Model, pole placement controllers (Zhu and Guo, 2002) for nonlinear plants with known parameters have been proposed. Some previous works (Zhu et al, 1991; Zhu and Warwick, 1991) discussed how to design the effective controller for nonlinear dynamic plants. Other works (Muhammad and Butt, 2011; Ali et al, 2010; Du et al, 2012; Chang et al, 2011) focused on the research of different methods of control system design enhanced by U-Model. The parameters of nonlinear plants in these studies are all regarded as given without considering the uncertainties. Therefore, the initial robustness analysis (Peng et al., 2013) of U-Model based controllers has been proposed to provide a basis procedure. Motivated by some previous theoretical results, in this study the LMI based robust controller is designed to improve the system performance of U-pole placement control system against the uncertainties. The uncertainty of the nonlinear plant is taken into consideration and the internal parameter changes of nonlinear plant are selected to test the robust performance of U-model robust control system.

The main contents of this paper are divided into four sections. In section 2 the proposed approach of a U-model based pole placement controller is introduced to represent the fundamental methodologies. In section 3 the basic idea on LMI based robust controller design approach is introduced for enlarging the control system robustness against uncertainties. In section 4 a step by step procedure of proposed LMI robust control system design is listing. In section 5 a Hammerstein model is selected to demonstrate the robustness analysis and the corresponding simulation results are presented with graphical illustrations. In section 6 a summary of the paper is presented.

2 U-pole placement controller design

The U-pole placement controller design proposed (Zhu and Guo, 2002) will be presented in this section as the fundamental methodology.

Consider single input and single output (SISO) nonlinear dynamic plants with a NARMAX (nonlinear auto-regressive moving average with exogenous inputs) representation of the form

$$y(t) = f\left(y(t-1),\ldots,y(t-n), u(t-1),\ldots,u(t-n), e(t),\ldots,e(t-n)\right) \quad (2.1)$$

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where $y(t)$ and $u(t)$ are the output and input signals of the plant respectively at discrete time instant t, n is the plant order, $f(\cdot)$ is a nonlinear function, and the modelling error term $e(t)$. The control oriented model can be obtained by expanding the nonlinear function $f(.)$ as a polynomial with respect to $u(t\text{-}1)$ as follows

$$y(t) = \sum_{j=0}^{M} \lambda_j(t) u^j(t-1) \quad (2.2)$$

where M is the degree of model input $u(t-1)$, the time varying parameter vector $\lambda(t) = [\lambda_0(t) \quad \cdots \quad] \in R^{M+1}$ is a function of past inputs, outputs ($u(t\text{-}2)$, …, $u(t\text{-}n)$, $y(t\text{-}1)$, …, $y(t\text{-}n)$), and errors ($e(t)$, …, $e(t\text{-}n)$). By this arrangement, the control oriented model can be treated as a pure power series of input $u(t\text{-}1)$ with associated time-varying parameters $\lambda_j(t)$.

Fig. 1 shows the block diagram of the U-model based pole placement control system. In the U-pole placement design, the U-model is firstly transferred from the nonlinear model. With the polynomial equation of U-model as a root solver, the Newton-Raphson algorithm can be used to find the controller output.

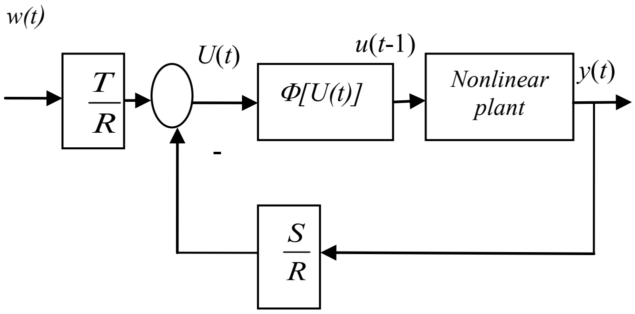

Fig. 1: Block diagram of U-pole placement control system

A standard reference (Astrom and Wittenmark, 1995) is used to develop following formulations for designing pole placement controller. Consider the U-model of (2.2), a general controller can be described by

$$RU(t) = Tw(t) - Sy(t) \quad (2.3)$$

where $w(t)$ is the reference for output target and R, S, and T are the polynomials of the forward shift operator.

The control law of (2.3) represents a negative feedback with transfer function $-S/R$ and a feedforward with transfer function T/R. It thus has two degree of freedom. A block diagram of the closed loop control system is shown in Fig. 1. The output $y(t)$ can be linked to the reference $w(t)$ as

$$y(t) = \frac{T}{R+S} w(t) = \frac{T}{A_c} w(t) \quad (2.4)$$

where polynomial A_c is the closed loop characteristic equation. The polynomials R, S, and T can be resolved by a Diophantine equation to make control output equals the desire output, which means that the steady state error equal to zero at the control output. The polynomial T is specified with $T = A_c(1)$ from equation (2.4). The key idea of the design is to specify the desired closed loop characteristic polynomial A_c, then resolve. The signal $U(t)$ can be obtained by (2.3) as long as polynomials R, S, and T are determined. With $U(t)$ a root solver, Newton-Raphson algorithm (Yan, 1999), can be used to find the controller output $u(t\text{-}1)$.

The identification error and stability of the controller of the U-pole placement control system have been discussed in (Zhu and Guo, 2002). An enhanced Newton-Raphson algorithm is proposed to guarantee the stability of the controller in a minimum phase system (Zhu et al, 1999).

Due to the U-model framework, the nonlinearity of the nonlinear model is cancelled. The closed loop of U-pole placement control system behaves similarly to that of a linear system. The equivalent block diagram of U-model based pole placement closed loop system is shown in Fig. 2.

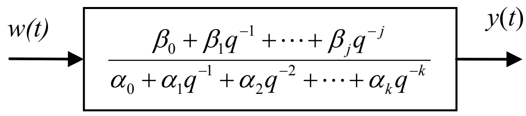

Fig. 2: Equivalent block diagram of U-pole placement control system

The LMI based robust controller is designed to enhance the performance of the closed loop system as shown in Fig. 3. When internal parameters are changed, the robustness of the closed loop system will be guaranteed in order to meet the specified requirements.

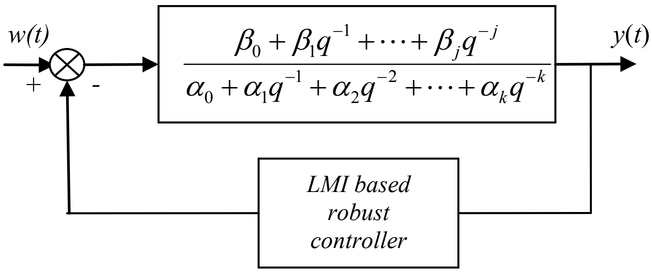

Fig. 3: Block diagram of U-pole placement control system with LMI based robust controller

3 Basic ideas on robust controller design

In the robustness analysis of control systems, the definition of uncertainty is very significant (Calafiore and Dabbene, 2002). To design an effective control system, a complex dynamic plant should be described as a relative simple model. The model uncertainty always exists in the control systems. Besides the uncertain of the simplified model expression, the uncertainties are caused by the environmental change, components aging, parameters drift and unknown errors. This uncertainty is quite different from the external factors such as external disturbance and measurement noise. In this section, the disturbance of internal parameter variation is picked up as the first concerned uncertain factor. The robustness analysis includes two aspects, one is the robust stability, and the other one is robust performance which means make control system not only has stability robustness, but also satisfy some performance constrains.

The H$_\infty$ state space model adopted for U-pole placement control system is shown in Fig. 3 and can be expressed as (Yu, 2002):

$$\begin{aligned} x(k+1) &= Ax(k)+B_1w(k)+B_2u(k) \\ z_{\inf}(k) &= C_{\inf}x(t)+D_{\inf 1}w(k)+D_{\inf 2}u(k) \\ y(k) &= C_ox(k)+D_{o1}w(k)+D_{o2}u(k) \end{aligned} \quad (3.1)$$

where $u(k), w(k), y(k), z_{\inf}(k)$ is the discrete-time state variable, $u(k), w(k), y(k), z_{\inf}(k)$ are the discrete-time robust controller output, system disturbance input, system output, and disturbance output respectively. By the way, A is the state matrix, B_1, B_2 is the disturbance input matrix and regulated input matrix respectively, $C_{\inf}, C_o$ are the H$_\infty$ output matrix and system output matrix. Matrices D ($D_{\inf 1}, D_{\inf 2}, D_{o1}, D_{o2}$) with different subscribes are real matrices with proper dimension for the system.

As mentioned by (3.1), a H$_\infty$ output feedback controller $K(z)$ should be designed forcing the closed loop system to have the performance of asymptotic stability. The state space expression for $K(z)$ can be present as:

$$\begin{aligned} x_k(k+1) &= A_kx_k(k)+B_ku(k) \\ u(k) &= C_kx_k(k)+D_ky(k) \end{aligned} \quad (3.2)$$

where x_k is the state variable and A_k, B_k, C_k, D_k are unknown H$_\infty$ output feedback controller matrices. Combining (3.1) with (3.2), the closed loop system can be expressed as

$$\begin{aligned} x_{ct}(k+1) &= A_{ct}x_{ct}(k)+B_{ct}w(k) \\ z_{\inf ct}(k) &= C_{\inf ct}x_{ct}(k)+D_{\inf ct}w(k) \end{aligned} \quad (3.3)$$

where

$$x_{ct}(k) = \begin{bmatrix} x(k) \\ x_k(k) \end{bmatrix},$$

$$A_{ct} = \begin{bmatrix} A+B_2D_kC_o & B_2C_k \\ B_kC_o & B_kD_{o1} \end{bmatrix}, \; B_{ct} = \begin{bmatrix} B_1+B_2D_kD_{o1} \\ B_kD_{o1} \end{bmatrix}$$

$$C_{\inf ct} = \begin{bmatrix} C_{\inf}+D_{\inf 2}D_kC_o & D_{\inf 2}C_k \end{bmatrix},$$

$$D_{\inf ct} = D_{\inf 1}+D_{\inf 2}D_kD_{o1}$$

The H$_\infty$ output feedback controller should be designed to take H$_\infty$ performance ($\|T_{cl\infty}(z)\|_\infty < \gamma_1$) into consideration, where $\|T_{cl\infty}(z)\|_\infty$ is the H$_\infty$ norm of the transfer function from w to $z_{\inf ct}$, and γ_1 is the upper bound of $\|T_{cl\infty}(z)\|_\infty$. Such output feedback controller ought to be designed to make the system to have an acceptable H$_\infty$ norm form w to $z_{\inf ct}$ to keep the system robustness.

In order to enhance the performance of the U-pole placement control system, LMIs is applied to discuss the solvability of discrete-time H$_\infty$ robust control system design problems. With the theorem of γ-suboptimal controller for discrete-time plants (Gahinet and Apkarian, 1994), consider a proper discrete-time plant realization (3.1), and assume that

(a) (A, B_2, C_0) is stabilisable and detectable

(b) $D_{02} = 0$

Let W_{12} and W_{21} denotes bases of null spaces of $(I-D_{\inf 2}^{+}D_{\inf 2})B_2^T$ and $(I-D_{o1}D_{01}^{+})C_0$, where $D_{\inf 2}^{+}$ and D_{01}^{+} are respectively for the Moore-Penrose pseudoinverse of matrix $D_{\inf 2}$ and D_{o1}.

The discrete-time γ-suboptimal H$_\infty$ problem is solvable if and only if there exist symmetric matrices *R, S* satisfying the following LMI system (Gahinet and Apkarian, 1994):

$$\begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} ARA^T-R & ARC_{\inf}^T & B_1 \\ C_{\inf}RA^T & -\gamma_1 I+C_{\inf}RC_{\inf}^T & D_{\inf 1} \\ B_1^T & D_{\inf 1}^T & -\gamma_1 I \end{bmatrix} \begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix} < 0 \quad (3.4)$$

$$\begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A^TSA-S & A^TSB_1 & C_{\inf}^T \\ B_1^TSA & -\gamma_1 I+B_1^TSB_1 & D_{\inf 1} \\ C_{\inf} & D_{\inf 1}^T & -\gamma_1 I \end{bmatrix} \begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix} < 0 \quad (3.5)$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0 \quad (3.6)$$

where N_R and N_S respectively denotes bases of the null spaces of $(B_2^T, D_{\inf 2}^T)$ and (C_2, D_{01}).

Lemma (Zhai et al, 2001) the following statements are equivalent:

(*i*) A is Schur stable and $\|C(zI-A)^{-1}B+D\|_\infty < \gamma_1$

(*ii*) The desired H$_\infty$ controller exists if and only if there are matrices P and K positive definite solution P to the LMI:

$$\begin{bmatrix} -P & PA_{cl} & PB_{cl} & 0 \\ A_{cl}^TP & -P & 0 & C_{cl}^T \\ B_{cl}^TP & 0 & -\gamma I & D_{cl}^T \\ 0 & C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (3.7)$$

where $K = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix}$. The LMI (3.7) is a BMI with respect to *P* and *K*. The controller matrices A_k, B_k, C_k, D_k can be obtained by solving bilinear matrix inequalities (3.7) mentioned in the lemma.

4 The proposed robust control system design procedure

A step-by-step procedure for the H_∞ robust controller design of U-model pole placement control system can be specified as following:

Step 1. Design the controller for the selected nonlinear model converted into U-model framework (2.2) to reach the requirement of desired closed loop system characteristic equation (Zhu and Guo, 2002).

Step 2. Design a LMI based H_∞ robust controller to achieve a higher performance for the desired closed loop system with U-model framework in step 1.

Step 3. Determine the variation of the internal parameter for the selected nonlinear model and test the system response of the U-pole placement control system with these uncertainties.

Step 4. Apply the designed robust controller in step 2 to the U-pole placement control system with the same uncertainties and simulate the performance of the U robust control system.

5 Case studies

A Hammerstein model is selected for the robust stability test. The closed loop characteristic equation is specified with

$$A_c = q^2 - 1.3205q + 0.4966 \quad (4.1)$$

Therefore the closed loop poles are a complex conjugate pair of -0.6603 ± j0.2463 . This design specification corresponds to a natural frequency of 1 rad/sec and a damping ratio of 0.7. To achieve zero steady state error, specify

$$T = A_c(1) = 1 - 1.3205 + 0.4966 = 0.1761 \quad (4.2)$$

For the polynomials *R* and *S*, specify

$$\begin{aligned} R &= q^2 + r_1 q + r_2 \\ S &= s_0 q + s_1 \end{aligned} \quad (4.3)$$

Substitute the specifications of (4.1) and (4.3) into Diophantine equation of (3.5), the coefficients in polynomials *R* and *S* can be expressed with

$$\begin{aligned} r_2 + s_1 &= 0.4966 \\ r_1 + s_0 &= -1.3205 \end{aligned} \quad (4.4)$$

To guarantee the computation convergence of the sequence $U(t)$, that is to keep the difference equation with stable dynamic, let $r_1 = -0.9 \quad r_2 = 0.009$. This assignment corresponds the characteristic equation of $U(t)$ as $(q - 0.89)(q - 0.01) = 0$. Then the coefficients in polynomial *S* can be determined from the Diophantine equation of (4.4)

$$s_0 = -0.4205 \quad s_1 = 0.4876$$

Substitute the coefficients of the polynomials *R* and *S* into controller of (2.3), gives rise to

$$U(t+1) = 0.9U(t) - 0.009U(t-1)$$
$$+ 0.1761w(t-1) + 0.4205y(t) - 0.4876y(t-1) \quad (4.5)$$

Therefore the controller output $u(t)$ can be determined.

Consider the following Hammerstein model

$$\begin{aligned} y(t) &= 0.5y(t-1) + x(t-1) + 0.1x(t-2) \\ x(t) &= 1 + u(t) - u^2(t) + 0.2u^3(t) \end{aligned} \quad (4.6)$$

The corresponding control oriented model is obtained from formulation (2.2)

$$y(t) = \lambda_0(t) + \lambda_1(t)u(t-1) + \lambda_2(t)u^2(t-1) + \lambda_3(t)u^3(t-1)$$

where

$$\lambda_0(t) = 0.5y(t-1) + 1 + 0.1x(t-2) \quad \lambda_1(t) = 1$$
$$\lambda_2(t) = -1 \quad \lambda_3(t) = 0.2$$

The system response under the proposed pole placement control has been discussed in (Zhu and Guo, 2002). It can be seen from simulation result that the resultant closed loop system behaves similar to that of a linear system, which is due to cancellation of the nonlinearity by the proposed control-oriented model and controller design approach.

However, if the internal parameter of the nonlinear model is changed, the controller performance will not be same standard and that is the purpose of using a robust controller which is going to be studied in the simulations.

In the simulation of this paper, the LMI based H-infinity output feedback controller is tested to improve the system performance of the designed U-pole placement control system. To the selected Hammerstein model, the variation of the internal parameter is the change of $\lambda_j(t)$. The characteristic equation of the LMI based H_∞ robust controller (step 2) can be expressed as:

$$A_{rc}(z) = z^2 + 0.4084 + 0.1452 \quad (4.7)$$

The controller is going to be applied for all cases in the U-model system simulations.

Case I: For the selected model with uncertainty, that is the internal parameter $\lambda_j(t)$ is changed to

$$\lambda_0(t) = 1.1y(t-1) + 3 - x(t-2) \quad (4.8)$$

The plant output of the U-model pole placement control system before and after robust controller applied are shown in Fig. 4. The output of the U-model pole placement control system without plant uncertainty is also shown in Fig. 4.

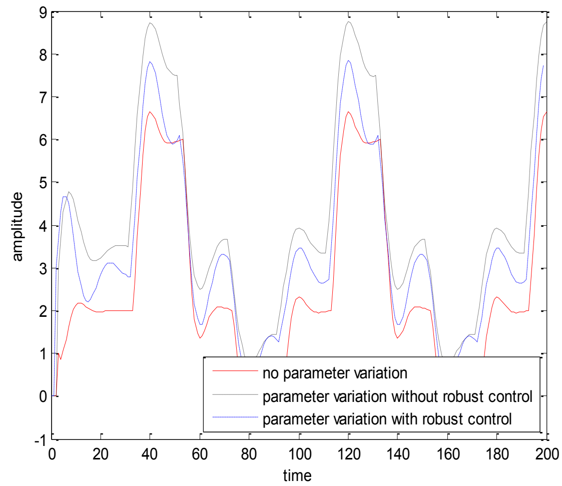

Fig. 4: System output after internal parameter changed – case I

Deriving from the simulation result, the system performance after internal parameter variations have been improved by the designed robust controller. The amplitude of the output decreased from 8.8 to 7.6 compared with the case without robust controller.

Case II: In the other different case, that is the internal parameter $\lambda_j(t)$ is changed to

$$\lambda_0(t) = -0.2y(t-1) + 1 - 0.1x(t-2) \quad (4.9)$$

The closed loop system becomes unstable and the output of the system without robust control is shown in Fig. 5.

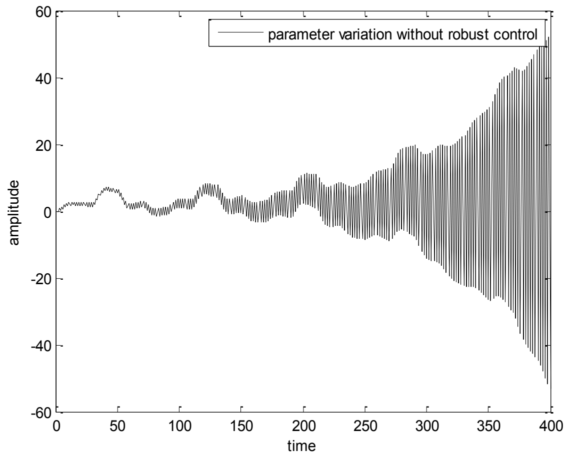

Fig. 5: System output after internal parameter changed – case II (No robust controller)

While the LMI based robust controller is applied to the closed loop system, acceptable simulation results can be achieved. Fig. 6 shows the system output response.

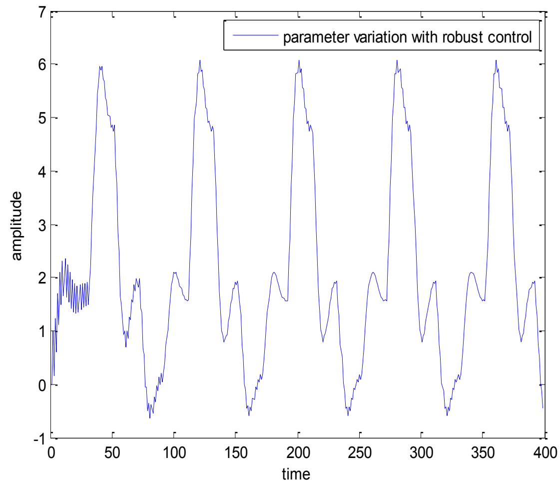

Fig. 6: System output after internal parameter changed – case II (With robust controller)

It can be inspected from the figure that the system can be stabilised with the help of the robust controller. In another aspect, the variation of the internal parameter exceeds the stability margin (Peng et al, 2013) of the U-model pole placement control system itself. However, with the help of the LMI based robust controller, the stability margin of the system is enlarged and such internal parameter variation can be guaranteed with a stable performance.

The simulation results for both cases show that the robustness of the robust controller design for U-model based pole placement control system is effective. The U-model controller can keep the system to be stable within a certain range of the parameter uncertainty. However, if the parameter of the nonlinear model is changed far away from the original one, the performance of the controller cannot be guaranteed. At this time, the LMI based robust controller can help to main its stability in a relatively large range of uncertainty.

6 Conclusions

A general control oriented U-model and the corresponding pole placement controller design for the dynamic nonlinear plants have been introduced to be the fundamental methodologies. With the modularisation the procedure of nonlinear control system design can be conducted as linear control system design. The LMI based output feedback H-infinity robust controller design can be implemented for nonlinear models within U-model framework. The simulation results show that robust control system design is effective and efficient against nonlinear system uncertainties.

Further studies on the developed methodology, such as different robust controller design approaches for general nonlinear systems within U-model framework, expansion to other types of controllers, and so on, will be conducted to provide a comprehensive prospectus in designing nonlinear control systems by using linear control system design techniques.

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