RESEARCH ARTICLE

Gathering and Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification

Indunil Sikurajapathi, Department of Computer Science and Creative Technologies, University of the West of England, Bristol, UK. Email: indunil.sikurajapathi@uwe.ac.uk.

Karen L. Henderson, Department of Computer Science and Creative Technologies, University of the West of England, Bristol, UK. Email: karen.henderson@uwe.ac.uk.

D. Rhys Gwynllyw, Department of Computer Science and Creative Technologies, University of the West of England, Bristol, UK. Email: rhys.gwynllyw@uwe.ac.uk.

Abstract

This article gives an overview of the interactive book called '*Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)*' which has been produced as a result of the Common Student Errors Project (CSE Project) set at the University of the West of England, (UWE Bristol). The process of creating this CSE Book is discussed in this article, namely, through the systematic collection and compilation of CSEs, and classification of them taxonomically according to a taxonomy presented in existing literature by examining first year Engineering Mathematics students' rough answer scripts, and e-Assessment-stored data. We believe that the CSEs presented in the CSE book would be useful for mathematics teachers when providing feedback to students to correct CSEs. Further, institutions can utilise it in the future development of teaching and support resources to ensure that these CSEs will be addressed to help students to acquire better understanding of mathematics. Moreover, mathematics learners can try these questions online by using the respective hyperlinks given in the CSE Book. If any of the identified CSEs are entered in the solution, then enhanced feedback is provided to correct their misconceptions instantly. Currently, the CSE Book is freely available at UWE Bristol's Repository.

Keywords: Mathematical Common Student Errors, Dewis e-Assessment system, Taxonomy

1. Introduction and Background

1.1 Common Student Errors

Students arrive at an incorrect answer when answering a mathematical question due to variety of reasons. The reasons can be listed as random errors, calculation errors or misreading the questions. These errors lead to incorrect answers or loss of accuracy marks. Many of these errors are made by just a few students. However, some of these errors are commonly made by a considerable number of students. These commonly made errors are sometimes referred to as common errors (Rushton, 2014).

Researchers express different opinions about the difference between errors and misconceptions in the literature. For Confrey (1990), the reasons for both errors and misconceptions are the rules and beliefs that students hold. They argue that the difference between errors and misconceptions is that misconceptions are attached to particular theoretical positions. However, Nesher (1987) uses the term misconceptions to describe systematic errors without reference to a theoretical position.

Rees and Barr (1984) use the term '*mal-rule'* to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. The term '*bug*' is used by

VanLehn (1982) to refer to a systematic error resulting from wrong steps in the calculation procedure. A *Borrow Across-Zero bug* is a systematic error caused by a student having trouble with borrowing, especially in the presence of zeros (VanLehn, 1982). For example, a student answering 98 when asked to calculate $305 - 117$ would be considered as a *Borrow Across - Zero bug*. In the aforementioned calculation, the student skips the step where the zero changed to nine during borrowing across zero (VanLehn, 1982).

Research has been conducted to identify misconceptions in different areas of mathematics. For example, Brown and Burton (1978) investigated bugs (misconceptions) in high school algebra problems, and Swan (1990) focused on the misconceptions that occur in four operations (addition, subtraction, multiplication and division), and in the interpretation of graphs.

Some Mathematics Education research has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning (Booth et al., 2014; Confrey, 1990; Fischbein, 1989; Nesher, 1987; Brown and Burton, 1978). After having investigated bugs (misconceptions) in high school algebra problems, Brown and Burton (1978) discussed possible arithmetic bugs which might lead to some specific algebraic bugs. Booth et al., (2014) conducted a study to assess algebraic misconceptions that algebra students make at school. They concluded that students who make specific persistent errors due to underlying misconceptions in arithmetic may need additional intervention since misconceptions are not corrected through typical instruction. They conclude that these additional interventions can be carried out by targeting individual misconceptions or by improving conceptual understanding throughout the algebra course. The findings of Brown and Burton (1978) and then the findings of Booth et al. (2014) hold the same conclusions, that the arithmetic misconceptions held by students affect their algebraic thinking. Further, Booth et al. (2014) state that these arithmetic misconceptions can obstruct their performance and learning of algebra.

There has been recent research into theorising student errors supported by empirical studies in the topics of natural number bias (Obersteiner et al., 2013), visual saliency (Kirshner and Awtry, 2004) and over-generalisation (Knuth et al., 2006). Rushton (2014) conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, as referenced by examiner reports, and errors were catalogued into themes and subthemes. More recently, Ford et al. (2018) developed a taxonomy of errors made by undergraduate mathematics students. In their study they gathered errors by firstly recalling the most obvious errors that occur and secondly by analysing students' exam scripts to categorise them in a taxonomical manner.

1.2 Assessments, e-Assessments and Feedback in Higher Education

Assessment plays a vital role in higher education. It determines the extent of students' skill and knowledge in order to ensure that they have achieved the desired learning outcomes (Stödberg, 2012). Assessment is considered an integral parts of students' learning. Not only does it promote student learning but it also allows them to receive support in order to improve their learning (JISC, 2010). Preparation and marking of traditional paper-based assessments is an expensive and long process and it also requires a significant amount of time and effort by teachers. To mitigate this situation, the use of information technologies to conduct assessment has significantly risen in higher education (Stödberg, 2012; Rolim and Isaias, 2019).

Over the past years, several e-Assessment systems, such as STACK (Sangwin, 2004), Dewis (Gwynllyw and Henderson, 2009), Math e.g. (Greenhow and Kamavi, 2012), and Numbas (Foster at al., 2012) have been developed at several universities in the UK. Easy accessibility and advantages of e-Assessment systems have led mathematics departments in many universities to conduct formative and summative assessments in the form of e-Assessments (Sangwin, 2013).

Properly performing e-Assessments are hugely beneficial for both teachers and students. Some benefits of using e-Assessment are its capability to provide instant and tailored feedback, that it can be accessed in different geographical locations at any time, and that students can undertake online tests several times to improve their learning (Sikurajapathi, Henderson and Gwynllyw, 2020; Gwynllyw and Henderson, 2009).

Dermo (2009) and Gikandi et al (2011) posit that high quality and accurate feedback delivered in a timely manner plays an important role in students' learning. In addition, by reviewing and studying this feedback, students can identify their weakness as well as their strengths in order to achieve continuous improvement in their learning. Gill and Greenhow (2008) conducted a study to find out the effectiveness of e-Assessment feedback and found that students improve their performance by engaging with the feedback provided in e-Assessments. Therefore, Greenhow (2015) suggests that e-Assessments which select questions based on pedagogic principles should be promoted as a learning tool due to its capability of providing effective feedback.

E-Assessments cannot act very flexibly like a human marker when faced with ill-posed or unanticipated student responses (Greenhow 2015). Detecting CSEs on traditional paper-based assignments compared to e-Assessments is more straightforward since the human marker has access to the students' intermediate workings and thus can spot when a CSE has been made. E-Assessment systems cannot easily point out CSEs on student answers since typically few intermediate working steps are submitted. Also, each student attempts a different but equivalent version of the question due to the use of random parameters (Walker et al, 2015).

In their paper, Walker et al (2015) states that an e-Assessment would act more like a human marker, if it could detect and report CSEs, and provide effective and tailored feedback instantly by correcting students' misconceptions. Sikurajapathi, Henderson and Gwynllyw, (2021) developed a method to detect CSEs and to provide tailored feedback in Engineering Mathematics e-Assessment questions. Sikurajapathi, Henderson and Gwynllyw, (2021) then conducted a questionnaire to find out the effectiveness of addressing CSEs in e-assessments through enhanced feedback. The questionnaire findings reveal that the majority of participants had positive feelings towards the CSE enhanced feedback. Students appreciated that the CSE enhanced feedback helped them to correct their misunderstandings and to improve their engineering mathematics learning. The highly positive perception of the enhanced feedback suggests that students find the CSE enhanced feedback valuable and that it helped them to correct conceptual understanding while improving their learning (Sikurajapathi, Henderson and Gwynllyw, 2021).

1.3 Dewis e-Assessment System

Dewis is a fully algorithmic open-source e-Assessment system, which was primarily designed and developed for numerate e-Assessments by a team of Mathematicians, Statisticians and Software Engineers at UWE Bristol (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Dewis supports different question input types such as numerical inputs, matrices, vectors, algebraic expressions, multiple-choice, multiple-selection, graphical input, and computer programs. It has a lossless data collection feature and a number of student-friendly features, such as shutdown recovery and pre-processing checks on student input.

Over the past decade, Dewis has been used very successfully to facilitate both formative and summative e-Assessments across a number of modules, delivered to students in a wide range of fields, e.g. Business, Computer Science, Nursing, Software Engineering, Engineering, Mathematics and Statistics. One aim of the CSE project is to enhance the full potential of Dewis, by developing and using additional features allowing Dewis to detect CSEs and to provide instant tailored feedback.

1.4 The Common Student Errors Project at UWE Bristol

The CSE project at UWE began in 2017 with an aim of developing a technique to detect CSEs and to provide tailored feedback in Dewis e-Assessment questions, used in a first year Engineering Mathematics module (CSE Project at UWE, 2019; Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021). We started the project with the aim of answering the following research questions:

- What CSEs do first year Engineering Mathematics students make in e-Assessment questions?
- How to detect CSEs and improve Dewis feedback to address these CSEs?

There are several benefits to answering these research questions. Even though this research has been done in a particular context using the Dewis e-Assessment system, the research outcomes contribute to the knowledge to inform more general practice in assessment and learning. For example, the collection of mathematical CSEs collected during this research is not only beneficial for first year Engineering mathematics students and lecturers, but also it is equally beneficial for secondary, and first year university level mathematics students and teachers. The CSE collection presented in Sikurajapathi, Henderson and Gwynllyw (2022) can be used to correct students' mathematical misconceptions either in hand-written assessments or e-assessment questions.

Further, this CSE detecting technique will be beneficial to several disciplines and organisations that either use Dewis or any other e-assessment system which has features to give dynamic feedback based on a student answer. The new knowledge raised from this research can be used in any e-assessment system so that it emulates a human marker to provide instant enhanced feedback highlighting possible CSEs. This will help students to correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE project into other e-assessment systems will be beneficial to generations to come (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022).

The CSE Project involves five stages (Stage One: Data (CSEs) Collection; Stage Two: CSE code Development; Stage Three: CSE code Trial Phase; Stage Four: Students' Perceptions on CSE Feedback and Stage Five: Impact of CSE Project). Detailed information about these five stages and other findings can be found in CSE Project at UWE Bristol (2019), Sikurajapathi, Henderson and Gwynllyw (2020) and Sikurajapathi, Henderson and Gwynllyw (2021).

2. Creating Collection of Taxonomically Classified Mathematical Common Student Errors in e-Assessments (CSE Book)

2.1. Gathering Mathematical CSEs in in e-Assessments Questions

The main aim of the CSE Project at UWE Bristol was to identify CSEs made in First Year Engineering Mathematics e-Assessment questions. The CSEs presented in the CSE Book were collected by examining the 2017-2018 and 2018-2019 e-examination data on the Dewis e-Assessment system and from students' rough work scripts. These e-examinations were run using the Dewis e-Assessment system and were held under controlled conditions. The e-examinations were held in two sessions (morning and afternoon) to mitigate logistic issues. In each session, all of the students received the same, fixed parameter questions. During the e-examination, students were given booklets to use for their rough work. These booklets were used by students to work through the mathematical questions before submitting their final answers on Dewis.

All of the CSEs that students made are documented in the CSE Book, regardless of whether they are mal-rules, bugs, slips, misconceptions, systematic errors etc. The reason for this is that all of these CSEs can be useful for educators, institutions, assessment makers, and most importantly for mathematics learners. Altogether 65 CSEs were identified in the following different topics areas of Engineering Mathematics: Algebra, Unit-step functions, Wave forms, Trigonometric functions, Differentiation, Implicit differentiation, Partial differentiation, Mean Value Theorem, Complex numbers, Geometric series, Maclaurin Expansion, Centre of Mass, Integration by parts, Volume of revolution and Dimensions.

This CSE Book (Sikurajapathi, Henderson and Gwynllyw, 2022) can be freely access at UWE Bristol's Repository on Public URL: https://uwe-repository.worktribe.com/output/9303961

2.2. Compiling Mathematical Common Student Errors in e-Assessment Questions with Taxonomical Classification

All of the CSEs found in the course of the CSE project are documented in a systematic order in the CSE book together with their mathematical taxonomy coding. Here we adapted the general taxonomy proposed by Ford et al. (2018) to select and categorise only those CSEs which are relevant to e-assessment.

The theoretical study of classification, including its bases, principles, procedures and rules is called a taxonomy (Ford et al., 2018; Simpson, 1961, p.11). The entities in a successful taxonomy can be verifiable by observation and will offer both an appropriate and suitable class for each entity (Ford et al., 2018; Bailey, 1994, p.3). The taxonomy of cognitive mechanisms and the phenomenological taxonomy can be considered as the two main styles that can be used to categorise mathematical errors (Senders and Moray, 1991, Ford et al., 2018).

The taxonomy introduced by Ford et al. (2018) was developed to categorise the errors which undergraduate mathematics students make. Ford et al. (2018) identified six main error categories by firstly recalling obvious mathematical errors that occur among mathematics undergraduates and secondly by analysing a selection of students' paper-based exam scripts from first year undergraduate mathematics courses. These main categories were named as Errors of slips of action (S), Errors of understanding (U), Errors in choice of method (CM), Errors in the use of a method (UM), Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C). Here we sought to use the same Main Categories, Codes and Errors given in the taxonomy introduced by Ford et al. (2018) to categorise mathematical CSEs in the e-Assessment questions.

The CSEs that we have found during the CSE project only fall into four of the error categories (S, U, CM and UM) from the Ford et al. (2018) taxonomy. Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C) were not found among the CSEs made by the Engineering Mathematics students, due to the nature of the questions asked and the nature of the system used to deliver the questions. None of the e-Assessment questions delivered by Dewis involve mathematical theorems and proofs and hence Errors related to proof (P) were not viable in this CSE collection. Further, the e-examination did not contain questions that required student's communication of their mathematical solutions, correct use of notation or labelling and qualitative judgements on clarity of expression. Therefore, errors in student's communication of their mathematical solutions (C) were not found in this CSE collection. Further, a few of the CSEs found fall into two categories due to the mix of misconceptions made by the students as they arrived at their incorrect answer.

Under the category Errors of slip of action (S), three main errors, namely copying error, careless errors on simple calculations, and incorrect algebraic manipulation were identified. A total of 13 out of 65 CSEs were found to fall into the Errors of slip of action category (S).

Seven main errors were identified under the Errors of understanding (U) category, such as confusing different mathematical structures, incorrect argument, lack of consideration of potential indeterminate forms, proposed solution is not viable, definition/method/theorem not recalled correctly, partial solution given and Incorrect assumptions. In total 43 CSEs are in the Errors of understanding category.

Only one main error was found in each of the Errors in choice of method (CM) and Errors in use of method (UM) categories. Five CSEs were grouped into the main error of applying an inappropriate formula/method/theorem in CM. There were 9 CSEs which fell into Error in use of an appropriate definition/method/theorem in the UM category. Table 1 shows how we categorised the CSEs we found related to e-Assessment questions into Main Categories, relevant Codes and Errors using the taxonomy introduced in Ford et al. (2018) together with examples from an e-Assessment context.

Table 1: Taxonomy of Mathematical Common Student Errors in e-Assessments

Main Category	**Code**	**Error**	**Examples**
Slip of action	S1	Copying error	Incorrect copying of the question
			Mistake copying/ submitting answer into e-assessment
			Incorrect interpretation of the question
	S2	Careless errors on simple calculations	Overlooking negative signs
			Omission of denominator
	S3	Incorrect algebraic manipulation	Incorrect division of two complex numbers
			Sum of product is split as a product of two sums
			Incorrect handling of powers

Errors of understanding	U1	Confusing different mathematical structures	Confusing the structure of completing the square and the quadratic equation
			Stating that a unit step function is a number
	U2	Incorrect argument	Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives
			Taking the integration of the product of two functions as the product of individual integrals
	U3	Lack of consideration of potential indeterminate forms	Taking the square of a negative number to be negative
	U4	Proposed solution is not viable	Angle is not within the given range
	U5	Definition/method/ theorem not recalled correctly	Method of completing the square is not recalled correctly
			Definition of waveform properties not recalled correctly
			Method of differentiating a standard function is not recalled correctly
			Method of solving trigonometry equation is not recalled correctly
			Chain rule is not recalled correctly
			Method of Partial differentiation not recalled correctly
			Method of differentiating implicit functions is not recalled correctly
			Mean value theorem is not recalled correctly
			Method of calculating the argument of a complex number is not recalled correctly
			Binomial theorem is incorrectly followed
			Definition of Centre of Mass is not recalled correctly

			Method of finding the principal value of the argument of a complex number is not recalled correctly
			Method of integrating not recalled correctly
			Definition of volume of revolution is not recalled correctly
	U6	Partial solution given	Correct workings but unfinished solution
	U7	Incorrect assumptions	Incorrect assumptions on the mean value theorem
			Taking dimension of velocity is $[v] = [MT^{-1}]$
Errors in choice of method	CM1	Applying an inappropriate formula/ method/ theorem	Uses a method which is not relevant in the situation
			Uses a formula which is not relevant in the situation
Errors in use of method	UM2	Error in use of an appropriate definition/ method/ theorem	Error in the use of the chain rule
			Error in use of partial differentiation method
			Incorrect units applied
			Method finding the volume of revolution is incorrectly followed

2.3. Guide to the CSE Recording Template

Each CSE found to date has been recorded using the template as shown in Table 2 below. The template contains seven areas and each area and its contents are described in detail below.

① The link to the online Dewis e-assessment question is available here. The reader may access the online question by clicking the Question hyper-link. By attempting the question and answering with a relevant CSE response, it is possible to see how Dewis detects the CSE and provides instant tailored feedback to address the CSE made in the solution.

② In this area, a screenshot of the Dewis question is given.

③ The correct solution to the question is presented in brief here.

④ The taxonomy code of the CSE, which is presented in ⑤ **,** is given here.

⑤ A sample of the CSE and the incorrect answer(s) that led from it is presented here. At the top of this area, the CSE error is summarised by a statement which is presented in red text. Then the detailed steps of the exact way the CSE is made and the solution as written by students in their rough work booklets is presented. We use tilde (~) on the CSE answer to differentiate it from the correct answer. For example, in Table 2, the CSE answer for this question is denoted as, $\tilde{f}(2) = 55$ in red text.

⑥ In this section, the number of CSE answers made, the total incorrect answers made in the question and the CSE percentage for each year are presented as No. of CSEs /No. incorrect answers **(CSE %)**. For example, in Table 2, in the 2017-18 exam, this particular CSE was made by 35 out of the 86 students who gave an incorrect answer to this question; therefore, the CSE percentage is **41%**. This data is presented in this area as 35/86 (**41%**). Similarly, the data for 2018-19 is presented as 32/100 (**32%**).

⑦ The exam year that data was collected from is presented here. Table 2 shows that 35/86 (**41%**) and 32/100 (**32%**) presented in ⑥ relate to the years 2017-18 and 2018-19 presented in ⑦ respectively.

Table 2: CSE Recording Template

<table>
<tr><td colspan="4">Question 1</td></tr>
<tr><td colspan="4">The function $f(t) = 7u(t+5) - 3u(t-4)$
where $u(t)$ represents the unit step function.
Calculate the value of $f(2)$.
Enter $f(2)$: ______ 2</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(t) = 7u(t+5) - 3u(t-4)$
$f(2) = 7u(2+5) - 3u(2-4)$
$= 7u(7) - 3u(-2)$
$= 7 \times 1 - 3 \times 0$
$f(2) = 7$
3</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U1 4</td></tr>
<tr><td colspan="4">Answer was derived by assuming $u = 1$ and not a function.
$f(t) = 7u(t+5) - 3u(t-4)$
$f(2) = 7u(7) - 3u(-2)$
$\tilde{f}(2) = 7(7)u - 3(-2)u$
$\tilde{f}(2) = 49u + 6u$
$\tilde{f}(2) = 55u$ since $u = 1$
$\tilde{f}(2) = 55$
5</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>35/86 (41%)
32/100 (32%) 6</td><td>Date collected</td><td>2017-18
2018-19 7</td></tr>
</table>

3. Common Student Error Examples

In this section we present examples of CSEs in each taxonomical category (Slip of action, Errors of understanding, Errors in choice of method, and Errors in use of method). These and the rest of the CSEs we found in the CSE Project can be found in UWE Bristol's Repository (Sikurajapathi, Henderson and Gwynllyw, 2022).

3.1. Common Student Errors due to Slip of Action

Table 3 shows a CSE related to a question in Algebra (Completing the Square) (see Section 2.1.1. Sikurajapathi, Henderson and Gwynllyw, (2022)). Students' answer scripts indicated that even though students had solved the question correctly, they submitted incorrect answers for b which corresponded to the negative of the correct value of b. Therefore, this CSE can be considered as copying error when submitting answer into e-assessment. In 2017-2018, 28 students, out of the 56 who answered this question incorrectly (50%) made this CSE. In 2018-2019, 33 students from 57 who answered this question incorrectly (58%) made the same mistake.

Table 3: CSE in Algebra (Completing the Square) Question due to Slip of action in algebra

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">The expression
$t^2 - 12t + 40$
can be expressed in the form:
$a(t-b)^2 + c$
where a, b and c are constants.
Calculate the values of these constants - note that all these solutions are integers:
Enter the value of a [ ]
Enter the value of b [ ]
Enter the value of c [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$\begin{aligned} t^2 - 12t + 40 &= (t-6)^2 - 36 + 40 \\ &= (t-6)^2 + 4 \end{aligned}$$ $$a = 1, \quad b = 6 \text{ and } c = 4$$</td></tr>
<tr><td>CSE 1 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>S1</td></tr>
<tr><td colspan="4">Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b.
$$\begin{aligned} t^2 - 12t + 40 &= (t-6)^2 - 36 + 40 \\ &= (t-6)^2 + 4 \end{aligned}$$ $$\tilde{b} = -6 \text{ and } c = 4$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>28/56 (50%)
33/57 (58%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

3.2 Common Student Errors due to Errors of Understanding

Table 4 shows a CSE related to a question on complex numbers (rectangular form) (see Section 3.3.1. Sikurajapathi, Henderson and Gwynllyw, (2022)) Students' answer scripts indicated that the square of a negative number was taken to be negative. Therefore, this CSE can be considered as lack of consideration of potential indeterminate forms. In 2017-2018, 40 students, out of the 57 who answered this question incorrectly **(70%)** triggered this CSE.

Table 4: CSE in Complex Number (Rectangular Form) Question due to Error of understanding

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Find the modulus $|z|$ of the complex number $z = -2 + 5j$, correct to two decimal places.

Enter $|z|$ correct to 2 decimal places:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = -2 + 5j$
$|z| = \sqrt{(-2)^2 + 5^2}$
$= \sqrt{4 + 25}$
$= \sqrt{29}$
$|z| = 5.39$</td></tr>
<tr><td>CSE 1 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>U3</td></tr>
<tr><td colspan="4">Taking $(-n)^2 = -n^2$

$z = -2 + 5j$
$|z| = \sqrt{(-2)^2 + 5^2}$
$|\widetilde{z}| = \sqrt{-4 + 25}$
$= \sqrt{21}$
$|\widetilde{z}| = 4.58$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>40/57(70%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

3.3. Common Student Errors due to Errors in Choice of Method

Table 5 shows a CSE related to a question on infinite geometric series (see Section 4.1.2. Sikurajapathi, Henderson and Gwynllyw, (2022)). Students' answer scripts indicated that 34 students out of 67 who answered this question incorrect (51%) just summed the first four terms instead of using the formula to find the sum of the infinite series. Therefore, this CSE can be considered as applying an inappropriate formula in Error in Choice of Method.

Table 5: CSE in Infinite Geometric Series Question due to Errors in Choice of Method

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Consider the following geometic series, S, where:
$S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots.$
Write down the first term, a and the common ratio, r in the boxes below.
Enter a: [ ]
Enter r: [ ]
Hence calculate the sum, S and enter your result in the box below.
Enter S (to three decimal places) here: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">The first term $a = 2$. The common ratio $r = 0.7$
The sum of an infinite series (S) exists, provided $|r| < 1$
$$S = \frac{a}{1-r} = 6.667$$</td></tr>
<tr><td>CSE 1 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>CM1</td></tr>
<tr><td colspan="4">Finding the sum of first four terms instead of the sum of the infinite series.
$$\tilde{S} = \frac{a(1-r^n)}{1-r}$$
$$\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$$
$$\tilde{S} = 5.066$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>34/67(51%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

3.4. Common Student Errors due to Errors in Use of Method

Table 6 shows a CSE related to differentiating $f(x) = cos^4(3x)$ (See Section 5.1.2. Sikurajapathi, Henderson and Gwynllyw, (2022)). 22 students out of 73 **(30%)** incorrectly answered that the differentiation of $f(x)$ is $-12\sin^3(3x)$ due to an error in the use of the Chain Rule. Therefore, this CSE can be considered as an error in use of an appropriate method.

Table 6: CSE in Differentiation (Chain Rule) Question due to Errors in Use of method

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$.
Select ▾
Hence find $\dfrac{df}{dx}$ as a function of x.
Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(x) = cos^4(3x)$
$f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$
$f'(x) = -12\sin(3x)\cos^3(3x)$</td></tr>
<tr><td>CSE 2 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>UM2</td></tr>
<tr><td colspan="4">$\textit{Taking } \frac{d}{dx}(cos^n(ax)) = -a \times sin^{n-1}(ax) \times a = -a^2 sin^{n-1}(ax)$
$f(x) = cos^4(3x)$
$\tilde{f}'(x) = -4 \times \sin^3(3x) \times 3$
$\tilde{f}'(x) = -12\sin^3(3x)$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>22/73(30%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

4. Discussion, conclusion and future work

This article presents an overview of the CSE Book created by collecting and compiling CSEs systematically by examining First Year Engineering Mathematics students' rough answer scripts, and Dewis e-Assessment-stored data. All of the CSEs found in this process have been categorised taxonomically. One of the special features of this book is that it provides hyperlinks to each question on the Dewis e-Assessment system in order to facilitate the reader to try these questions online. If any of the identified CSEs are submitted as answers, then enhanced feedback will be provided, which aims to correct any misconceptions in a timely manner.

The information in this book may be used to inform teachers so that they can provide students with a better understanding of the mathematical skills and knowledge while teaching the subject. It may also be useful for institutions as they can utilise it in the future development of teaching materials to ensure that these CSEs will be addressed. Further, the content of this book can be used to develop support materials and resources to address CSEs which will help students to acquire better understanding of mathematics. In addition, students who learn mathematics at university level or in secondary school can refer to this booklet to address their misconceptions and can try the Dewis questions several times. Since, in each attempt, Dewis produces questions with random parameters, student can use this facility to correct their misconceptions by practicing the same question but with different parameters.

We anticipate that this book will be useful to identify and address some misconceptions that students have in mathematics. We plan to continue with this research and will update the book if we find new CSEs in the future. Currently, the CSE Book is freely available at UWE Bristol's Repository.

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