Complex dynamics of cellular automata emerging in chaotic rules

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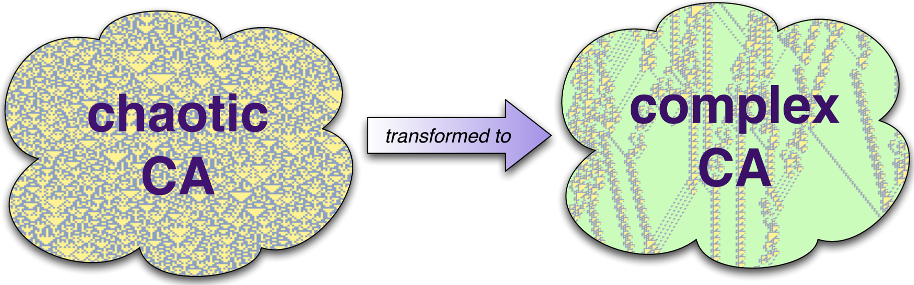
The 3rd Int. Conference on Complex Systems and Applications
Le Havré University, Normandy, France
June 29 – July 1, 2009

Abstract

We show novel techniques of analysing complex dynamics of cellular automata (CA) with chaotic behaviour. CA are well known computational substrates for studying emergent collective behaviour, complexity, randomness and interaction between order and disorder. A number of attempts have been made to classify CA functions on their spatio-temporal dynamics and to predict behaviour of any given function. Examples include mechanical computation, λ and Z-parameters, mean field theory, differential equations and number conserving features. We propose to classify CA based on their behaviour when they act in a historical mode, i.e. as CA with memory. We demonstrate that cell-state transition rules enriched with memory quickly transform a chaotic system converging to a complex global behaviour from almost any initial condition. Thus just in few steps we can select chaotic rules without exhaustive computational experiments or recurring to additional parameters. We provide analysis of well-known chaotic functions in one-dimensional CA, and decompose dynamics of the automata using majority memory.

Objective and goal

In this talk we will display a simple tool to extract complex systems from a family of chaotic discrete dynamical system.
We will employ a technique named **memory** based rule analysis of using past history of a system to construct its present state and intent predict its future.



MEMORY: depend on the state and history of the system

Cellular automata

Cellular automata (CA) are discrete dynamical systems evolving on an infinite regular lattice.

Definition

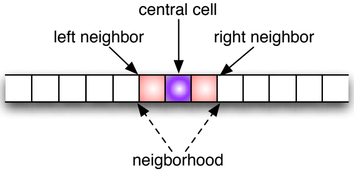
A *CA* is a 4-tuple $A =< \Sigma, u, \varphi, c_0 >$ evolving in d-dimensional latice, where $d \in \mathbb{Z}^+$. Such that:

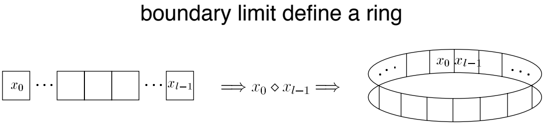
- Σ represents the alphabet
- u the local connection, where, $u = \{x_{0,1,\ldots,n-1:d} | x \in \Sigma\}$, therefore, u is a neigborhood
- φ the local function, such that, $\varphi : \Sigma^u \rightarrow \Sigma$
- c_0 the initial condition, such that, $c_0 \in \Sigma^{\mathcal{Z}}$

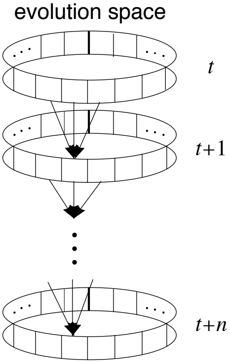
Also, the local function induces a global transition between configurations:

$$\Phi_\varphi : \Sigma^{\mathcal{Z}} \rightarrow \Sigma^{\mathcal{Z}}.$$

Dynamics in one dimension







Elemental CA (ECA) is defined as follow:

- $\Sigma = \{0, 1\}$
- $u = \{x_1, x_0, x_{-1}\}$ such that $x \in \Sigma$
- the local function $\varphi : \Sigma^3 \rightarrow \Sigma$
- c_0 the initial condition is the first ring with $t = 0$

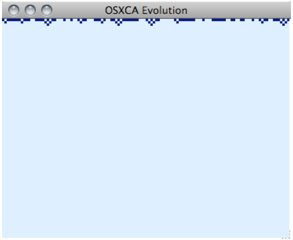
Wolfram's classification

Wolfram defines his classification in simple rules [Wolfram86], known as ECA. Also, this classification is extended to *n*-dimension.

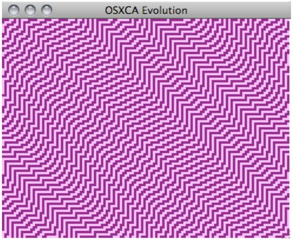
Classes

- A CA is class I, if there is a stable state $x_i \in \Sigma$, such that all finite configurations evolve to the *homogeneous configuration*.
- A CA is class II, if there is a stable state $x_i \in \Sigma$, such that any finite configuration become periodic.
- A CA is class III, if there is a stable state, such that for some pair of finite configurations c_i and c_j with the stable state, is decidable if c_i evolve to c_j.
- Class IV includes all CA also *called complex CA*.

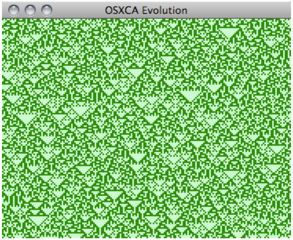
Wolfram's classes



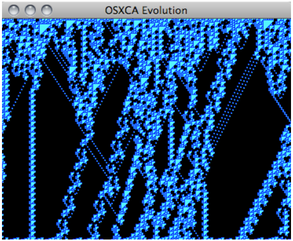
Rule 32



Rule 15



Rule 90

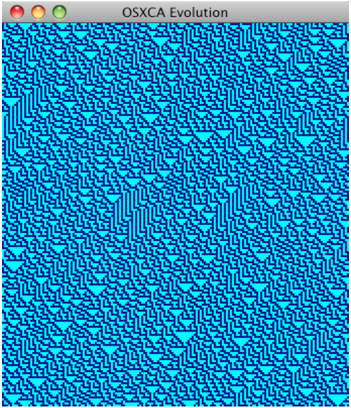


Rule 110

Figure: Behavior classes in ECA: *uniform*, *periodic*, *chaotic* and *complex*.

The case of study: ECA Rule 86 and 101

$$\varphi_{R86} = \begin{cases} 1 & \text{if} \quad 110, 100, 010, 001 \\ 0 & \text{if} \quad 111, 101, 011, 000 \end{cases} \qquad \varphi_{R101} = \begin{cases} 1 & \text{if} \quad 110, 101, 010, 000 \\ 0 & \text{if} \quad 111, 100, 011, 001 \end{cases}$$



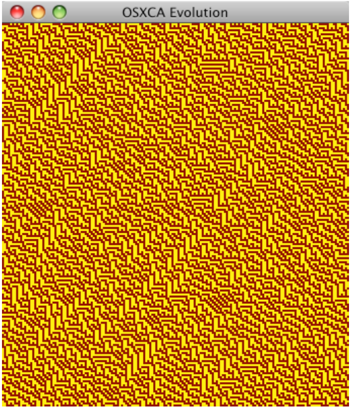


Figure: Chaotic ECA evolution rules 86 and 101 respectively. Initial density start with a 50% on a ring of 172 cells to 190 generations.

Mean field analysis

Mean field theory is a proven technique for discovering statistical properties of CA without analyzing evolution spaces of individual rules. In this way, it was proposed to explain Wolfram's classes by probability theory, resulting in a classification based on mean field theory curve:

- class I: monotonic, entirely on one side of diagonal;
- class II: horizontal tangency, never reaches diagonal;
- class III: no tangencies, curve crosses diagonal.
- class IV: horizontal plus diagonal tangency, no crossing;

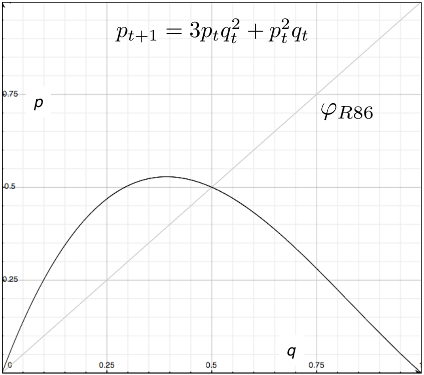
Thus for one dimension we have:

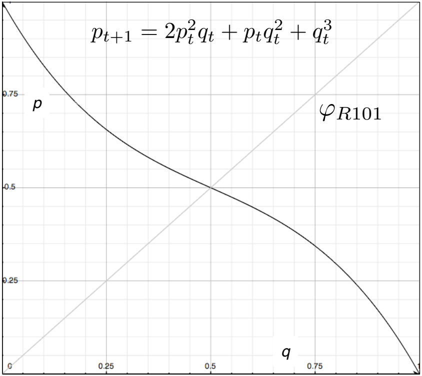
$$p_{t+1} = \sum_{j=0}^{k^{2r+1}-1} \varphi_j(X) p_t^v (1-p_t)^{n-v} \tag{1}$$

such that j is a number of relations from their neighborhoods and X the combination of cells $x_{i-r}, \ldots, x_i, \ldots, x_{i+r}$. n represents the number of cells in neighborhood, v indicates how often state one occurs in Moore's neighborhood, $n-v$ shows how often state zero occurs in the neighborhood, p_t is a probability of cell being in state one, q_t is a probability of cell being in state zero (therefore $q = 1 - p$).

Mean field polynomial for φ_{R86} and φ_{R101}

Initially φ_{R86} has produces states zero and one equiprobably. There is an equilibrium of states in Φ. Initially φ_{R86} has produces states 0 and 1 equiprobably. There is an equilibrium of states in Φ. On the other hand, φ_{R86} determines a surjective correspondence and therefore all the configuration has at least one ancestor and no Garden of Eden configurations.
For φ_{R101} has the same probability as φ_{R86} to produce states one and zero. However φ_{R101} is not a surjective rule and therefore has the Garden of Eden configurations, i.e., not all configurations have ancestors.





ECA with memory

Conventional CA are ahistoric (memoryless): i.e., the new state of a cell depends on the neighborhood configuration solely at the preceding time step of φ. CA with *memory* can be considered as an extension of the standard framework of CA where every cell x_i is allowed to remember some period of its previous evolution.

Thus to implement a memory we design a memory function ϕ, as follow:

$$\phi(x_i^{t-\tau}, \ldots, x_i^{t-1}, x_i^t) \rightarrow s_i \tag{2}$$

such that $\tau < t$ determines the degree of memory backwards and each cell $s_i \in \Sigma$ being a state function of the series of states of the cell x_i with memory up to time-step.

Finally to execute the evolution we apply the original rule as follows:

$$\varphi(\ldots, s_{i-1}^t, s_i^t, s_{i+1}^t, \ldots) \rightarrow x_i^{t+1}.$$

Thus in CA with memory, while the mapping φ remains unaltered, historic memory of all past iterations is retained by featuring each cell as a summary of its past states from ϕ. Therefore cells *canalize* memory to the map φ.

ECA with memory

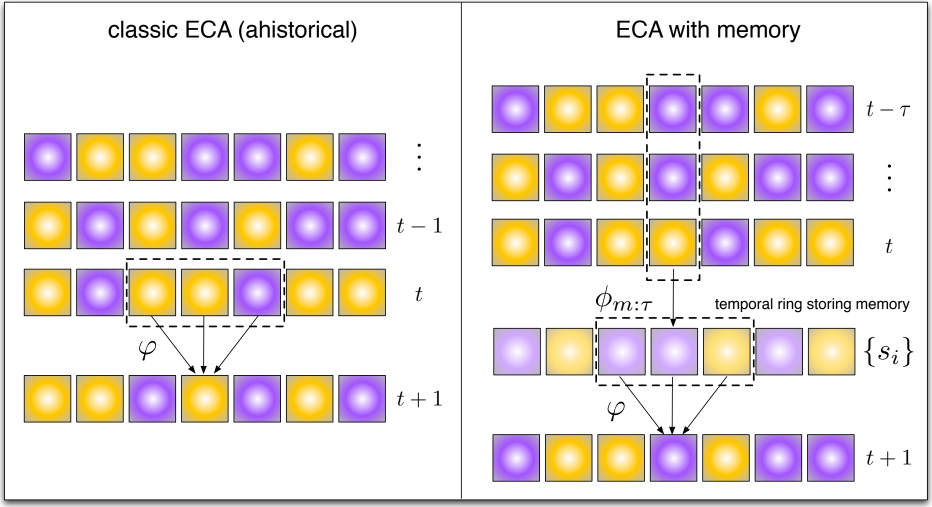


Figure: Dynamic of MEMORY working on ECA.

ECA with memory

Firstly we should consider a kind of memory, in this case the **majority memory** ϕ_{maj} and then a value for τ. This value represent the number of cells backward to consider in the memory. Therefore a way to represent functions with memory and one ECA associated is proposed as follow:

$$\phi_{CAm:\tau} \tag{3}$$

such that *ca* represents the decimal notation of an specific ECA and *m* a kind of memory given. This way the majority memory working in ECA rule 86 checking tree cells on its history is denoted simply as $\phi_{R86maj:3}$. Implementing the majority memory ϕ_{maj} we can select some ECA and experimentally look what is the effect.

Complex dynamics emerging in φ_{R86} with majority memory

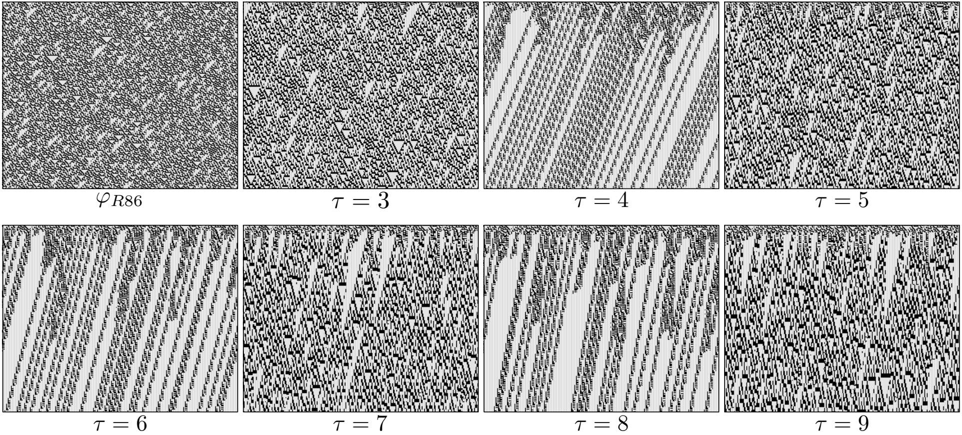


Figure: Majority memory ϕ_{maj} working in φ_{R86} with τ values of 3 to 9 respectively, evolving with the same random initial condition. Also a filter is used to get a better view.

Complex dynamics emerging in φ_{R101} with majority memory

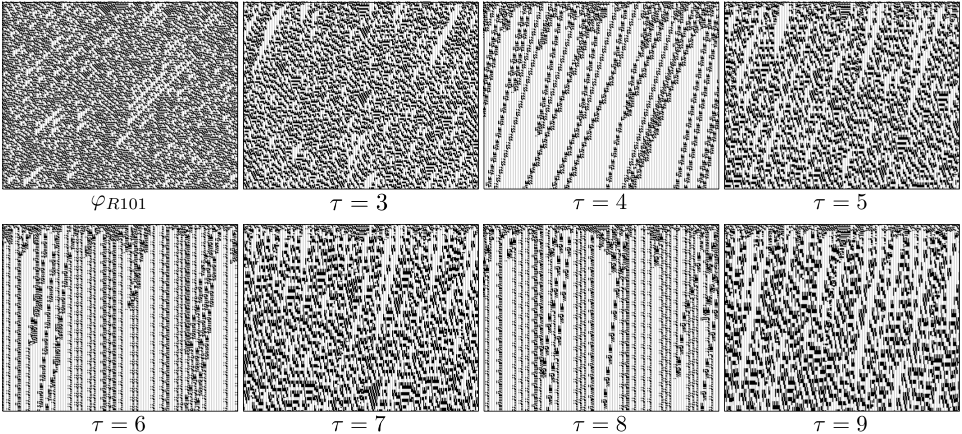
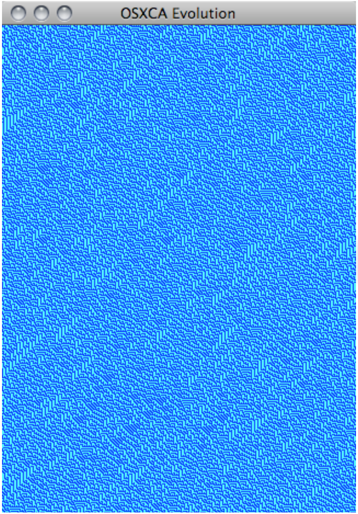


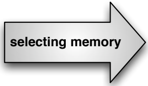
Figure: Majority memory ϕ_{maj} working in φ_{R101} with τ values of 3 to 9 respectively. Also a filter is used to get a better view.

Complex dynamics emerging since chaos

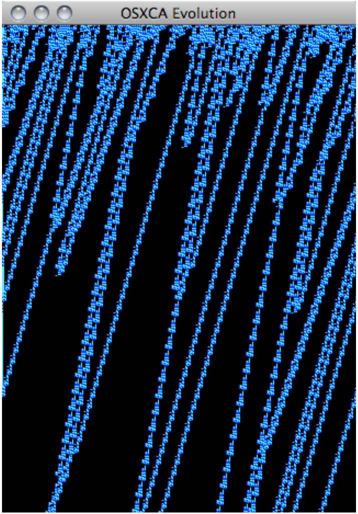
φ_{R101}



classic (ahistoric)



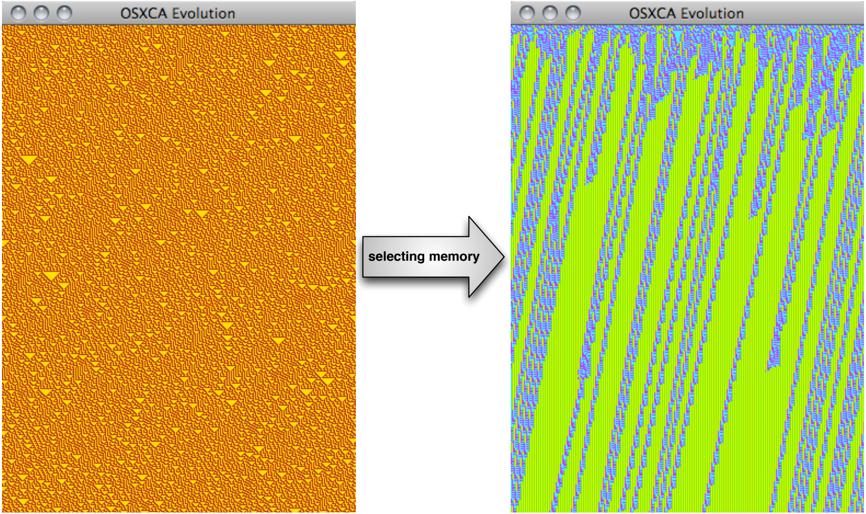
$\phi_{R101maj:4}$



with memory

Complex dynamics emerging since chaos

φ_{R86} $\qquad$ $\phi_{R86maj:8}$



classic (ahistoric) $\qquad$ with memory

Complex dynamics emerging since chaos

Implementing *memory* on ECA with OSXCA systems. Free software available since:

`http://uncomp.uwe.ac.uk/genaro/OSXCASystems.html`

some simulations ...

Self-organization by structure formation in $\phi_{R101maj:4}$

Patterns as particles and non-trivial behavior emerging in these new ECA with memory ϕ_{R86maj} and $\phi_{R101maj}$, naturally conduce to known problems as self-organization.

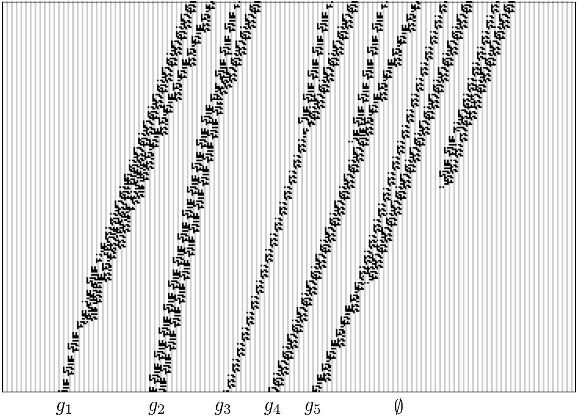


Figure: Self-organization by particle collisions to form the set $\mathcal{G}_{\phi_{R101maj:4}}$, also evolution is filtered to get a better view.

Implementing basic functions (unconventional computing)

Also we can use the particles codification to represent solutions of some basic functions. Of course, thinking how a complex systems could be organized and controlled to get a construction, as computation.

Let consider the new ECA rule $\phi_{R86maj:8}$. Because we want to implement a simple function as addToHead working on two strings $w_1 = A_1, \ldots, A_n$ and $w_2 = B_1, \ldots, B_m$, such that, $n, m \geq 1$. For example, if $w_1 = AAA$, $w_2 = BBB$ and $w_3 = w_1 w_2$ then the **addToHead(**$|w_2|$**)** will yield: $w_3 = w_2 w_1$. As the next diagram shows.

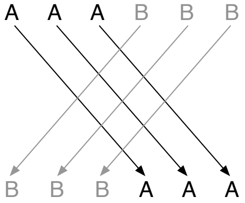


Figure: Schematic diagram adding the string w_2 to head of the list w_3.

Implementing basic functions (unconventional computing)

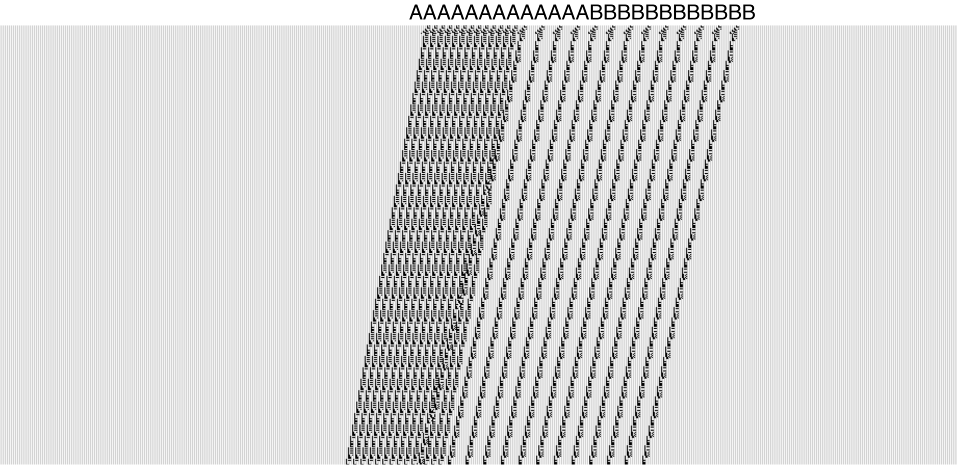


Figure: A simple substitution system processing the word $A^{12}B^{12}$ to $B^{12}A^{12}$ with ECA $\phi_{R86maj:8}$ synchronizing particle reactions.

Implementing basic functions (unconventional computing)

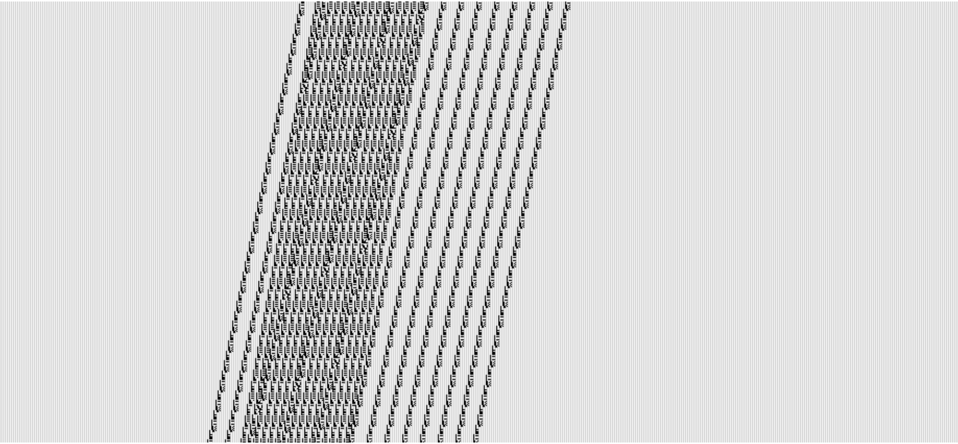


Figure: middle state transition with multiple collisions ...

Implementing basic functions (unconventional computing)

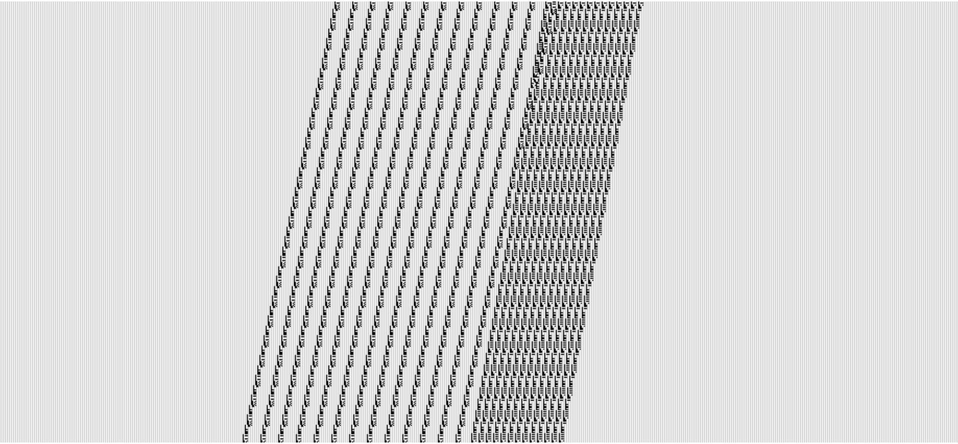


Figure: The final production is reached to 6,888 generations with synchronization of soliton reactions and coding particles.

Final remarks and future work

Conclusions

1. We have demonstrated that *memory* in ECA offers a new approach to discover complex dynamics based on particles and non-trivial reactions across them.
2. We have enriched some chaotic ECA rules with *majority memory* and demonstrated that by applying certain filtering procedures we can extract rich dynamics of travelling localizations, or particles.
3. This way the *memory* can be applied on any CA or dynamical system.

Next stage

- Done a systematic analysis in ECA and other orders of CA, including more dimensions.
- Proof formally such results and inherent implications of types of memory.
- Look universal computing devices since ECA with memory.

Thank you for your attention!

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