Understanding the influence of slope on the threshold of coarse grain motion: revisiting critical stream power

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Abstract

This paper investigates the slope-dependent variation in critical mean bed shear stress for coarse grain motion, and evaluates stream power per unit bed area as an alternative threshold parameter. Explanations for observed slope-dependency and existing approaches for predicting the critical stream power per unit bed area are reviewed. An analysis of secondary bed-load transport datasets is used to examine the strength of associations between stream power per unit bed area, mean bed shear stress and mean velocity, with bed-load transport rate. Data from an original flume study are combined with secondary data from similar flume experiments to investigate the effect of slope on both critical stream power per unit bed area and critical mean bed shear stress. Results suggest that stream power per unit bed area is most closely correlated with bed-load transport rate, and also that critical stream power per unit bed area is less variable with slope than critical mean bed shear stress. Alternative solutions to approximating critical stream power are explored. These include: (1) modifying existing expressions for critical stream power to account for higher critical mean bed shear stresses at higher slopes, and (2) applying a constant dimensionless critical stream power criterion.

Keywords: critical; threshold; initiation of motion; stream power; shear stress; Shields; sediment transport; erosion; river

1. Introduction

Despite more than 150 years of research into the mechanics of sediment motion in open channels, both the threshold for the initiation of sediment transport and the prediction of transport rates remain active, and somewhat inconclusive, subjects of research (Simons and Senturk, 1992). Historically, two parameters have dominated definitions of the flow responsible for the initiation of grain motion: near-bed velocity (notably following the work of Hjulström, 1935), and bed shear stress (following the work of du Boys, 1879). While most pioneering researchers interested in the threshold of bed-material entrainment recognised the physical importance of a critical near-bed velocity, the difficulties in defining a constant reference height above the bed at which "near-bed" velocity could be measured quickly resulted in alternative measures of bed shear stress becoming the more popular approach. Owing again to the practicalities of measurement and application, bed shear stress was most commonly represented by a mean value, averaged over the width of the channel, so that:

$$\tau = \rho_w \cdot g \cdot d \cdot S \qquad (1)$$

where τ is the mean bed shear stress in kg/m s^2; ρ_w is the density of water in kg/m^3; g is the gravitational acceleration in m/s^2; d is the mean flow depth in m; and S is the bed, water surface, or energy gradient (where S, or Tan β, is assumed to be equivalent to Sin β, where β is the slope angle and is small enough to allow this small angle approximation). Shields (1936) recognised a joint dependence of critical mean bed shear stress for the initiation of motion on particle size and bed roughness; and also, that this could be expressed as a function of the grain size:

$$\tau_{ci} = \theta_{ci} \cdot \left[D_i \cdot (\rho_s - \rho_w) \cdot g \right] \qquad (2)$$

in relation to the shear velocity and the thickness of the laminar sub-layer using the grain Reynolds number:

$$R_* = \frac{u_* \cdot d}{\nu} \tag{3}$$

where τ_c is the critical mean bed shear stress in kg/m s^2; θ_{ci} is a dimensionless shear stress criterion for a specified grain size and varies with R_*; D_i is the diameter of the specified grain being entrained in m; ρ_s is the density of the sediment material in kg/m^3; R_* is the dimensionless grain Reynolds number; u_* is the shear velocity in m/s; and ν is the kinematic viscosity of the water in m^2/s. Shields (1936) demonstrated that the θ_{ci} of near-uniform grains varies with R_* and hypothesized that θ_{ci} attains a constant value of about 0.06 above $R_* = 489$.

Shields' application of mean bed shear stress to the problem of incipient motion has since formed the foundation for the majority of subsequent studies into the subject. For example, notable work on the influences of hiding (Andrews and Parker, 1987), and proportion of fines content (Wilcock and Crowe, 2003) favours mean bed shear stress (and Shield's approach) as the parameter associated with the initiation of bed material motion. Nevertheless, despite its popularity, there have been several studies which reveal considerable scatter around the relationship between Shields' criterion and the grain Reynolds number (Buffington and Montgomery, 1997; Shvidchenko and Pender, 2000; Lamb et al., 2008), and which suggest a possible dependence upon other factors, notably slope.

As an alternative to mean bed shear stress, stream power per unit bed area has been described as a conceptually, pragmatically, and empirically attractive means of predicting sediment transport rate (Bagnold, 1966; Gomez and Church, 1989; Ferguson, 2005). Yet, despite this, both Petit et al. (2005) and Ferguson (2005) identified that practitioners and academic researchers have paid "a lack of attention to the specification of the (stream power) threshold" (Ferguson, 2005: 34). Following Bagnold's original work, little sustained research has aimed to define the threshold stream power necessary for sediment transport other than some empirical studies performed in coarse bed streams (Costa, 1983; Williams, 1983; Petit et al., 2005) and the theoretical treatment by Ferguson (2005).

The purpose of this paper is to improve understanding of how and why critical mean bed shear stress varies with channel slope, and evaluate stream power per unit bed area as a more consistent parameter for predicting the initiation of bed material motion. This paper first reviews explanations for a slope dependency in critical stress, and existing approaches for predicting the critical stream power per unit bed area. Available bed-load transport datasets are used to examine the strength of association between stream power per unit bed area, mean bed shear stress and mean velocity with bed-load transport rate. Results from a new flume study are then combined with data from similar flume experiments to investigate the effect that slope has on critical stream power and mean bed shear stress. The results suggest that stream power per unit bed area is most closely correlated with bed-load transport rate, and also that critical stream power per unit bed area is less variable with slope than critical mean bed shear stress. Alternative solutions to approximating critical stream power are then explored. These include: (1) modifying existing expressions for critical stream power to account for higher critical shear stresses at higher slopes, and (2) applying a constant dimensionless critical stream power criterion.

2. Review

2.1 Variability in critical mean bed shear stress

Existing datasets indicate that, for a given grain size and mean bed shear stress, there is at least a threefold range in θ_{ci} (Buffington and Montgomery, 1997). This variation is detrimental to sediment transport studies because uncertainties in the estimation of θ_{ci} may lead to large errors in computed transport rate as entrainment is generally considered to be a nonlinear function of flow strength (Bagnold, 1966; Wilcock and Crowe, 2003; Gomez, 2006). A number of different causes for the variation in θ_{ci} have been identified. Some studies have identified that critical mean bed shear stress increases as a result of bed surface structures and channel morphology (Church et al., 1998). Others have demonstrated that the choice of measurement method can have a significant impact on the resultant θ_{ci} (Buffington and Montgomery, 1997), but in addition, a number of studies have highlighted that variation in channel gradient has an influence over the mean bed shear stress at which sediment is entrained (Ashida and Bayazit, 1973; Bathurst et al., 1987; Graf, 1991; Shvidchenko and Pender, 2000; Shvidchenko et al., 2001; Mueller et al., 2005; Lamb et al., 2008). It is the findings of this final group of studies that form the focus of this paper.

Using a threshold for the initiation of motion based on the probability for sediment entrainment, Shvidchenko and Pender (2000) employed flume data to study the effect of channel slope on the incipient motion of uniform bed material. In a subsequent paper, Shvidchenko et al. (2001) performed similar experiments with graded bed material. Both sets of experiments demonstrated that higher mean bed shear stresses were necessary to reach a critical transport rate at higher slopes. Investigating the same problem using field data, Mueller et al. (2005) examined variations in the mean bed shear stress at the threshold of motion for 45 gravel-bed streams and rivers in the western United States and Canada. Applying a reference sediment transport threshold in a manner similar to that applied by Shvidchenko et al. (2001), they focused on differences in θ_{ci} associated with changes in channel gradient and relative submergence, and again found that values of θ_{ci} increased systematically with channel gradient.

Numerous other studies have highlighted the elevated critical mean bed shear stress values in steep channels that are generally found toward the headwaters of natural streams (Ashida and Bayazit, 1973; Bathurst et al., 1983; Bathurst et al., 1987; Petit et al., 2005). A number of factors have been attributed to causing the positive correlation between high channel slopes and higher θ_{ci} values (Lamb et al., 2008). Stabilising bed structures that result from the interlocking of bed particles are undoubtedly responsible for increasing the threshold of motion toward steeper stream headwaters (Church et al., 1998). Similarly, hiding effects are also more active in steeper, headwater streams because of the increased size of the largest particles on the bed acting to shield the remaining grains from the force of the water. Also, increased channel form roughness in steeper streams is thought to reduce the shear stress available for sediment transport because of greater fluid drag on the channel boundary (Petit et al., 2005). Finally, Wittler and Abt (1995) claimed that the apparent relationship between slope and critical shear stress is due to inaccurate representation of the weight of the water when the flow in rivers is turbulent and aerated at high slopes. Under such conditions, fluid density is lower than generally represented in shear stress calculations. However, Lamb et al. (2008) suggest that other factors, including slope's influence on relative roughness and flow resistance, are responsible for the correlation between channel slope and critical shear stress.

2.2 The role of stream power per unit bed area in sediment transport

Stream power per unit bed area was defined by Bagnold (1966) using:

$$\omega = \frac{\rho_w \cdot g \cdot Q \cdot S}{w} = \tau \cdot U \qquad (4)$$

where ω is stream power per unit bed area in N/m s; Q is the total discharge in m^3/s; w is the width of the flow in m; and U is the depth-averaged velocity in m/s. In this form, ω quantifies the rate of loss of potential energy as water in a river flows downslope. Bagnold therefore argued that it should represent the rate of energy potentially available to perform geomorphic work, with the river acting as a sediment transporting machine, of varying efficiency. Most importantly, Bagnold suggested that the rate of work done in transporting sediment is equal to the available power beyond a threshold value multiplied by the efficiency with which energy is used in transporting sediment:

$$i_b = e_b(\omega - \omega_c) \qquad (5)$$

where i_b is the rate of work done in transporting sediment in N/m s; e_b is the efficiency of the river as a sediment transporting machine; and ω_c is the stream power per unit bed area associated with the initiation of motion in N/m s. This line of reasoning has a long provenance (Clifford, 2008): Seddon (1896) first formalised a relation between the rate of energy expenditure, the debris-carrying capacity of the stream and the channel morphology, and his research was followed by a number of other researchers (Shaler, 1899; Gilbert, 1914; Cook, 1935; Rubey, 1938).

Unlike near-bed velocity and mean bed shear stress, stream power can be approximated from gross channel properties (width and slope), combined with the discharge provided by the catchment. Channel width and average channel slope may be obtained from remotely sensed data, and discharge can be estimated through a combination of known flow gauge data and drainage basin characteristics, even for entire catchments (Barker et al., 2008). Thus, stream power has a considerable practical advantage over locally variable parameters such as velocity and mean bed shear stress which require direct measurements of channel flow properties.

Bagnold's (1966) stream power criterion generally performs strongly in comparative tests using empirical data. Gomez and Church (1989), for example, found that, although no formula predicted sediment transport rates consistently well, formulae based upon stream power were the most appropriate as stream power has a more straightforward correlation with sediment transport than any other parameter. Notwithstanding this predictive success, stream power has not been universally popular in sediment transport studies, and there is some confusion over its derivation and application. In Bagnold's (1966) paper, gravitational acceleration (g) is included in his expression for stream power (Eq. 4), whereas in his later papers Bagnold (1980) omitted g in order to achieve dimensional similarity. Because sediment transport rate is commonly given as a mass of sediment over time per unit channel width (kg/m s), removal of g enables stream power per unit bed area to be expressed in similar units. In this paper, because the theoretically correct units for stream power per unit bed area are N/m s (or W/m^2), stream power is compared against sediment transport rate reported in terms of weight of sediment over time per unit channel width (N/m s or W/m^2) rather than mass of sediment over time.

2.3 Existing approximations of critical stream power

Bagnold (1980) recognised that the necessary threshold value for stream power is not directly measurable in natural rivers. Instead, he suggested it must be predicted using a modal bed material grain size (D_{mod}) and channel flow variables. Based on Eq. 4, he derived critical power using $\omega_c = \tau_c \cdot U_c$, where U_c is the depth-averaged velocity at the threshold of motion. Bagnold defined τ_c using Shields' expression in Eq. 2, assuming θ_c to have a constant value of 0.04. He then defined U_c based on τ_c and a logarithmic flow resistance equation:

$$U_c = 5.75 \cdot \log\left(\frac{12 \cdot d_c}{D_b}\right) \cdot \sqrt{\frac{\tau_c}{\rho_w}} \qquad (6)$$

As a result, in combination with Eq. 2, Bagnold (1980) expressed critical stream power per unit bed area as:

$$\omega_c = \tau_c \cdot \left\{5.75 \cdot \log\left(\frac{12 \cdot d_c}{D}\right) \cdot \sqrt{\frac{\tau_c}{\rho_w}}\right\} = 2860.5^* \cdot D^{1.5} \cdot \log\left(\frac{12 \cdot d_c}{D}\right) \qquad (7)$$

**Bagnold actually gave 290 instead of 2860.5 as the coefficient in his 1980 paper. Like Ferguson (2005), we assume that Bagnold divided stream power by gravitational acceleration to achieve dimensional similitude with sediment transport rate by mass.*

where d_c is the depth of flow at the threshold of motion; θ_c is assumed to have a value of 0.04; ρ_s is assumed to have a value of 2600 kg/m^3; ρ_w is assumed to have a value of 1000 kg/m^3; and g is assumed to have a value of 9.81 m/s^2. Bagnold did not differentiate between the grain diameter used to represent bed material roughness (D_b - Eq. 6) and the grain diameter representative of the bed-load entrained (D_i - Eq. 2). Instead, he applied the modal bed material diameter (D_{mod}) to both.

A number of limitations with Bagnold’s (1980) expression for critical stream power (Eq. 7) have been identified. The first, and perhaps most significant, is that it is too complex for practical application given that it requires the flow depth at the threshold of motion (Petit et al., 2005). This requires not only knowledge of local flow properties, but also application of an iterative procedure to determine the critical flow depth in question. This limitation is especially relevant, as one of the key advantages of using stream power per unit bed area in sediment transport applications is its independence from local flow properties.

Partly as a result of this limitation, Petit et al. (2005) set out to determine a relationship for the stream power per unit bed area required to initiate bed-load movement in three types of rivers in the Belgian Ardenne region. The river types were determined based on an arbitrary classification into large (catchment area > 500 km^2), medium (40 km^2 < catchment area < 500 km^2), and small/headwater streams (catchment area < 40 km^2). Through the application of tracer pebbles in 14 streams and rivers with slopes ranging from 0.001 to 0.071, they investigated the relationship between grain size and critical stream power within a variety of rivers.

The empirical relationships collected by Petit et al. (2005) were in the form: $\omega_c = a \cdot D_i^{\,b}$ and, as can be observed in their Table 1, the constants a and b generally fall within 1,000-10,000 and 1.3-1.7, respectively (when D_i is in m rather than mm). The general tendency for the exponent of grain size b to fall around an average value of 1.5 in these empirical datasets is supported well by theoretical examinations of critical threshold in the literature: critical mean bed shear stress (τ_c) is generally considered to be related linearly to D_i^1 (Shields, 1936) and critical velocity near the bed (u_{0c}) is generally considered to be linearly related to $D_i^{1/2}$ based on the “sixth power law” ($u_{0c}^{\,6} \propto D^3$) where the velocity required to entrain a particle to the power of 6 is linearly related to the volume of that sediment particle (Vanoni, 1975). Based on $\omega_c = \tau_c \cdot U_c$, critical stream power per unit bed area should thus be linearly proportional to $D_i^{1.5}$.

Petit et al.'s (2005) data showed considerable variation in the empirical values for critical stream power per unit bed area, both between rivers, but also between sites on the same river. They claimed that the differences are due to the increased influence of bedform resistance in smaller, steeper rivers, based on the argument that, where form roughness is low in comparison to grain roughness, a large part of the river's energy is used up in overcoming the resistance of bedforms, with little remaining to perform work on the bed material: higher critical stream powers thus occur in the steeper, smaller rivers with higher form roughness. In the middle-order streams, where form roughness was less significant, they observed lower critical stream powers. Petit et al. (2005) therefore argued that Bagnold's (1980) expression for critical power is limited because it does not account for the effect of bed-form resistance in its derivation. This argument is considered further in section 5.5, but what is clear at this point is that because of the between and within site variation in grain size-critical stream power relationships this type of approach produces expressions that are applicable only to the conditions under which they were derived. Therefore, whilst useful in investigating the factors influencing critical stream power, this type of relationship should not be applied universally as a means of predicting critical stream power per unit bed area.

The findings of Petit et al. (2005), inspired Ferguson (2005) to re-visit and revise Bagnold's (1980) expression for critical stream power, noting that, given $\omega_c = \tau_c \cdot U_c$, critical stream power should be the product of a critical mean bed shear stress and the mean velocity associated with that shear stress through resistance laws. In summary, the changes suggested by Ferguson (2005) included:

(i) *A differentiation between the grain sizes that are entrained by the flow and the grain size representative of the bed roughness*. The grain size entrained by the flow (D_i) is important in controlling the critical mean bed shear stress (Eq. 2), whereas the bed material roughness grain size (D_b) affects the calculation of the mean velocity associated with a given mean bed shear stress (Eq. 6). Bagnold (1980) did not discriminate between these two different grain sizes within his critical stream power formula despite the fact that they are generally dissimilar in natural streams. Flow resistance is normally dominated by the more coarse grains in the bed, whereas transport is generally dominated by the finer grains. Ferguson therefore amended Eq. 7 to incorporate a distinction between the grain size entrained and the grain size responsible for bed roughness.

(ii) *A suggestion for an alternative resistance formula*. As demonstrated above, Bagnold (1980) used a logarithmic flow resistance law to derive the mean velocity associated with a given critical shear stress. For generality, Ferguson (2005) derived two versions of his critical stream power formula ─ one applying the logarithmic flow resistance law used by Bagnold, and a second using a Manning-Strickler flow resistance law. Ferguson (2005) observed no significant difference between the results of his two formulae.

(iii) *Recognition of the influence of relative size effects*. It is well recognised in the literature that critical mean bed shear stress depends on the relative size of the grain in question against the size of the grains in the surrounding bed. These "relative size effects" were made popular in geomorphology following the work of Parker et al. (1982). Since then, a number of functions quantifying the hiding effect given to smaller particles and the protruding effect given to larger particle have been specified. In general they take the form:

$$\frac{\theta_{ci}}{\theta_{cb}} = \left(\frac{D_i}{D_b}\right)^{-h} \qquad (8)$$

where θ_{cb} is the dimensionless critical shear stress criterion for a grain size representative of the bed; and h is a hiding factor which has values between 0 (no hiding or protrusion – critical shear stress is linearly related to grain size) and 1 (maximum hiding and protrusion – critical shear stress is equal for all grain sizes). Because Bagnold did not include any term to compensate for relative size effects, Ferguson (2005) incorporated a function similar to that in Eq. 8 into his critical power expression.

(iv) *Elimination of the dependence on depth*. As identified earlier, perhaps the most critical flaw in Bagnold's expression for critical stream power is its dependence on the depth of flow at the threshold of motion. Ferguson suggested a relatively simple means by which the depth term could be removed from Bagnold's (1980) critical power expression. By manipulating Eq. 1 so that it is in terms of d, Ferguson used the following expression to replace the depth term:

$$d_c = \frac{\tau_{ci}}{\rho_w \cdot g \cdot S} \tag{9}$$

As a result of these changes, Ferguson produced simplified versions of the following expressions for critical stream power per unit bed area:

$$\omega_{ci} = \tau_{ci} \cdot \left[5.75 \cdot \log\left(\frac{12 \cdot \left(\tau_{ci} / \left(\rho_w \cdot g \cdot S \right) \right)}{D_b} \right) \cdot \sqrt{\frac{\tau_{ci}}{\rho_w}} \right] \tag{10}$$

when applying the logarithmic flow resistance law or

$$\omega_{ci} = \tau_{ci} \cdot \left[8.2 \cdot \left(\frac{\left(\tau_{ci} / \left(\rho_w \cdot g \cdot S \right) \right)}{D_b} \right)^{1/6} \cdot \sqrt{\frac{\tau_{ci}}{\rho_w}} \right] \tag{11}$$

when applying the Manning-Strickler flow resistance law, where

$$\tau_{ci} = \left[\theta_{cb} \cdot \left(\frac{D_i}{D_b} \right)^{-h} \right] \cdot \left(\rho_s - \rho_w \right) g \cdot D_i \tag{12}$$

Based on these equations, Ferguson produced a theoretical graph (Figure 1 in Ferguson, 2005) of predicted critical stream power against entrained grain size (D_i), grain size representative of the bed (D_b), and slope (S). This figure illustrated that Eqs. 10 -12 imply an increase in critical stream power with increases in both D_i and D_b, as expected. However, the figure also demonstrated that, assuming all other factors remain equal, both equations predict lower critical stream powers at higher slopes ─ a result that is less obvious. In fact, this contradicts the results of the tracer experiments performed by Petit et al. (2005), who found that critical stream powers were higher in steeper, albeit smaller and “rougher”, streams. Based on these findings Ferguson (2005) attempted to argue theoretically that, contrary to Petit et al.’s (2005) findings, critical stream power is unaffected by form resistance. These arguments are explored further in section 5.5.

3. Datasets and methods

3.1 Correlations between hydraulic parameters and bed-load transport rate from published datasets

Hydraulic, sedimentological and sediment transport measurements were obtained for all known and available bed-load transport studies. These included data from 133 different river or flume datasets described in a selection of agency reports, academic journal papers, theses, and files provided by researchers through personal communication (Yang, 1979; Gomez and Church, 1988; Bravo-Espinosa, 1999; Wilcock et al., 2001; King et al., 2004; Ryan et al., 2005). These datasets are summarised in Table 1. The resultant dataset is designed to be as expansive and inclusive as possible, spanning a wide range of flow dimensions, experimental designs, channel gradients and bed material sizes. The integrity was accepted as given in the source publication unless obvious errors were observed, in which case the data were rejected.

This early stage of data analysis did not attempt to formally test the accuracy of any particular critical threshold relation, but merely sought to verify Gomez and Church’s (1989) claim that stream power per unit bed area offers the most suitable correlation with sediment transport. As a result, a one-tailed Spearman’s Rank correlation was selected as a suitable means with which to carry out this analysis - it does not assume the nature of the relationship between the two variables, other than an increase in one variable should lead to an increase in the other. The hydraulic parameters investigated were: mean velocity, mean bed shear stress and stream power per unit bed area.

Table 1

3.2 Investigation of the impact of slope on critical entrainment threshold

Given the previously observed dependence of critical mean bed shear stress on slope and the apparent contradiction between the empirical findings of Petit et al. (2005) and the theoretical expressions derived by Ferguson (2005), a flume-based experimental procedure was designed to evaluate the impact of slope on both critical mean bed shear stress and critical stream power per unit bed area. Additional data were obtained from existing flume datasets where slope had been treated as a controlled variable. These included datasets from the studies of Johnson (1943), Shvidchenko and Pender (2000), and Shvidchenko et al. (2001).

The original experiments described herein were conducted in a 10 m -long, 0.3 m -wide by 0.45 m -deep tilting flume with glass walls. The pump of the flume is capable of producing a flow up to 0.025 m^3/s, and the slope of the flume can be set up to 0.025. The flow regime can be manipulated using a tailgate at the outlet end of the flume. Discharge was measured using averaged velocity and depth measurements. Flow depth was measured using a moving point gauge, and depth-averaged velocity was calculated based on point measurements taken at various heights above the bed. Observations of particle entrainment were made from a mobile bed section, situated halfway along the flume, which measured 0.5 m long and 0.3 m wide, taking up the entire width of the flume. Three different sediment mixes were used during the experiments, the compositions of which are given in Fig. 1 below. Each of the sediment mixtures consisted of 20% sand, with the remaining 80% composed of gravel spanning three Φ classes. The distributions of each of the mixtures from “1” to “3” were incrementally finer than the previous mixture by half a Φ class. All of the grains, other than the sand, were coloured to aid sediment transport observations. The remainder of the flume bed was composed of a fixed layer of sediment that approximated a roughness similar to that of the active section.

Figure 1

Prior to each experimental run, the appropriate bed material was mixed, laid within the active flume section to a depth of ~0.03 m, and levelled. Then the experimental slope was set, the tailgate was raised, and the flow was started at a very low discharge to fill the flume. Experimental runs were carried out at five slopes for each of the sediment mixtures (0.0071, 0.0100, 0.0125, 0.0143, 0.0167). For each slope/bed-material combination, a low initial discharge was chosen at which no sediment transport was observed; and then a series of incrementally larger flows were applied until the bed was broken up or the maximum discharge was reached. Discharges varied from 0.004 to 0.025 m^3/ s. Care was taken to ensure that uniform flow was maintained throughout the experiments. Because of transient increases in sediment transport rate following changes in flow intensity (Shvidchenko and Pender, 2000: Figure 4), a 10-minute period was allowed to pass before any sediment transport observations were made after discharge and slope were varied.

Sediment transport intensity was measured using a methodology similar to that of Shvidchenko and Pender (2000), defining sediment transport intensity as the relative number of particles moving in unit time: $I = m/(NT)$, where I is the intensity of sediment transport; m is the number of particle displacements during the time interval T out of the total number of surface particles observed N. In this study, the number of particle displacements was recorded using high-definition video equipment so that the sediment transport intensity could later be measured. Because Shvidchenko and Pender (2000) demonstrated that sediment transport intensity (I) has a 1:1 relationship with Einstein's (1942) dimensionless bed-load transport parameter (q_{b*}), I can be expressed in terms of q_{b*}. Einstein's dimensionless bed-load transport parameter is given by the expression

$$q_{b*} = \frac{q_b}{g \cdot (\rho_s - \rho_w) \cdot \sqrt{\frac{\rho_s - \rho_w}{\rho_w} \cdot g \cdot D_i^3}} \qquad (13)$$

where q_b is the unit width sediment transport rate (submerged weight) in N/m s. A number of other recent studies have used a different form of dimensionless transport rate ($W_* = q_{b*} / \tau_*^{3/2}$), as defined by Parker et al. (1982), but the Einstein bed-load parameter can be most readily interpreted in terms of the probability of bed particle entrainment (the proportion of mobilised particles relative to immobile particles in the bed surface).

In this study, a reference transport method relating incipient motion of bed material to a small, practically measurable, sediment transport rate was applied. This method provides a clear, quantitative and reproducible definition of a "critical" threshold that is otherwise difficult to define. A reference value of q_{b*} = 0.0001 was defined as "critical" in this study. This value is close to the practical lower limit of sediment transport rate that can be reliably measured in open channel experiments. It has visually been defined as occasional particle movement at some locations (Van Rijn, 1989).

Table 2

In order to both improve understanding of how and why critical mean bed shear stress varies with channel slope, and evaluate stream power per unit bed area as a more consistent parameter for predicting the initiation of bed material motion, the data from the flume study are presented in three different forms:

1. the effect of slope on critical stream power per unit bed area is presented to investigate the contradiction between Ferguson's (2005) hypothesis that critical stream power should decrease with slope and Petit et al.'s (2005) claims that critical stream power increases with slope (section 4.2.1);
2. the effect of slope on the relationship between mean bed shear stress and mean velocity is presented to test Ferguson's (2005) justification for critical stream power being inversely proportional to slope (section 4.2.2);
3. the effect of slope on critical mean bed shear stress is presented to test the assumption of both Bagnold's (1980) and Ferguson's (2005) critical stream power expressions that critical mean bed shear stress is independent of slope in fully turbulent flow (section 4.2.3).

4. Results and analysis

4.1 Correlations between hydraulic parameters and bed-load transport rate from published datasets

The mean Spearman's Rank correlation coefficients between sediment transport rate and mean velocity, mean bed shear stress, and stream power per unit bed area across all 133 datasets were 0.83, 0.77, and 0.85 respectively. Whilst the difference between these coefficients is small, it does support Gomez and Church's (1989) claim that Bagnold's (1966) stream power is the most appropriate parameter for representing bed-load transport capacity. Furthermore, correlations for both mean velocity and mean bed shear stress with sediment transport are very poor in certain datasets, despite stream power per unit bed area having a strong relationship with sediment transport rate in the same datasets (Fig. 2). This occurs when mean bed shear stress and velocity are poorly correlated, and the explanation for this is explored in section 5.2.

Figure 2

4.2 Investigation of the impact of slope on critical entrainment threshold

4.2.1 The effect of slope on critical stream power

As described in section 2.3, Ferguson's (2005) expression for critical stream power implies that an increase in slope should result in a decrease in critical stream power, assuming all other factors are equal. Figure 3 demonstrates that this is not the case for either the new flume experiments performed in this study or for the ancillary results obtained from other studies: there is no clear relationship between the "critical" stream power at which $q_{b*} = 0.0001$ and slope. Although there is a decrease in the "critical" stream power at extremely high slopes within Shvidchenko and Pender's (2000) results, this occurs with very steep slopes approaching the angle of repose for the bed material, which increases bed mobility independently of flow conditions because of the redistributed effect of gravitation. However, slopes this steep are exceptionally rare in natural systems; and other than these extreme cases in Shvidchenko and Pender's (2000) data, no relationship was found between slope and critical stream power. These results thus appear to contradict the interpretations suggested by Ferguson's Fig. 1 and also raise concerns over the validity of Eqs. 10-12. In view of this, further analysis was undertaken, the results of which are detailed below.

Figures 3A and 3B

4.2.2 The effect of slope on the mean bed shear stress−mean velocity relationship

Ferguson's (2005) justification for critical stream power being inversely proportional to slope is based upon the idea that, for a given critical mean bed shear stress, the associated velocity will have an inverse relationship to slope because of the effects of relative roughness. This relationship between mean bed shear stress, slope, and velocity is as predicted by widely accepted flow resistance equations. Figure 4 demonstrates that, within the assimilated flume data, this is the case. Using an analysis similar to that applied by Bathurst (1985), Fig. 4A shows that at elevated slopes the mean velocity at a given mean bed shear stress is lower than it is at more gentle slopes. Further, the two flow resistance formulations applied by Ferguson both generally predict velocities within the analysed data to a reasonable degree of accuracy (Fig. 4B). The poor accuracy observed for certain data points is considered to be a result of the backwater effects present within some of the flume studies.

Figures 4A and 4B

4.2.3 The effect of slope on critical mean bed shear stress

Because sections 4.2.1 and 4.2.2 have identified that the velocity for a given mean bed shear stress is inversely proportional to slope but that critical stream power is not dependent on slope, it is prudent to test Ferguson's (2005) assumption that critical mean bed shear stress is independent of slope in fully turbulent flow.

Fig. 5 demonstrates that, in the flume study data considered here, there is a strong relationship between critical mean bed shear stress and slope. For each of the datasets studied, at higher slopes the mean bed shear stress necessary to meet the critical threshold of sediment transport is increased (Fig. 5A). Fig. 5B demonstrates the impact that slope has on θ_{ci} within the flume data analysed in this study. A clearly distinguishable relationship exists between slope and the critical Shields' parameter, with a power relation of the form

$$\theta_{ci} = 0.19 \cdot S^{0.28} \quad (14)$$

providing the best fit ($R^2 = 0.75$).

Although a power law provides the best fit to the empirical data observed within this study, it is likely that, at extremely low slopes, the critical Shields' parameter will become asymptotic to a constant value (R. I. Ferguson, University of Durham, personal communication, 2009). This is due to the improbability of near-zero critical mean bed shear stresses.

Figure 5A and 5B

A potential explanation for the observed impact of slope on the critical Shields' parameter is the dependence of θ_{ci} on grain Reynolds number (R_*) already recognised by Shields (1936). As R_* is partially dependent on slope (higher slopes increase R_*), it could be assumed that the observed increases in θ_{ci} with slope are merely a consequence of the relationship recognised by the Shields diagram. However, Fig. 6 clearly demonstrates that this is not the case. Not only is the dependence of θ_{ci} on slope present when R_* is greater than the value at which Shields considered θ_{ci} to be constant, but even below this value, there is a clear dependence of θ_{ci} on slope that is independent from its relationship with R_*.

Figure 6

5. Discussion

5.1 Influence of slope on critical mean bed shear stress

Section 2.1 identified several arguments that could be used to explain the positive relationship between slope and critical mean bed shear stress observed in Fig. 5B, including: the prominence of stabilising bed structures and hiding effects in steep headwater streams; increased channel form roughness in steep headwater streams; and flow aeration at high slopes. None of these, however, completely account for the effect of slope. The experimental data analysed within this study used well-sorted, unimodal sediment in flumes without any notable form roughness elements; yet critical shear stress was still found to be positively related to slope. Furthermore, Mueller et al. (2005) found that critical shear stress values increase systematically with slope even in flows where form roughness is consistently low.

This finding is supported by the work of Lamb et al. (2008) who found that the effect of slope on bed shear stress is not caused by increased form drag (the magnitude of the effect is the same in both field and flume experiments). Despite recognising the validity of Wittler and Abt's (1995) suggestion that flow aeration at high slopes results in reduced mobility due to a reduction in the density of the water-air mixture, Lamb et al. (2008) concluded that this also could not fully explain the observed slope dependence of critical shear stress because aeration only occurs at very high slopes whilst slope impacts critical shear stress across a broad range. Instead, Lamb et al. (2008) suggest that slope's influence on relative roughness and flow resistance is responsible for the correlation between channel slope and critical shear stress.

Slope and relative roughness are strongly positively associated, as is evident theoretically by combining Eqs. 2 and 9 (to give $S \propto D/d$), and empirically in Bathurst's (2002) Fig. 3. Flow resistance is typically found to increase as slope, and consequently relative roughness (D_b/d), increase (Bathurst, 2002). As identified by Reid and Laronne (1995), the primary effect of the increased flow resistance at high slopes is to shift the position of a bed-load rating curve toward higher mean bed shear stresses, a pattern which is evident in the flume data analysed here (Fig. 5). A number of authors have suggested that this trend is due to the increase in relative roughness at higher slopes causing a decrease in local flow velocity around bed particles (Ashida and Bayazit, 1973; Graf, 1991; Shvidchenko and Pender, 2000). This is supported by the results of Chiew and Parker (1994) who, in a sealed duct, showed that when relative roughness is held constant critical shear stress actually decreases with increasing channel slope due to the increased gravitational component in the downstream direction. This increase of friction resistance in steeper, shallower flows is due to the increased effect of the wake eddies from bed particles on the overall flow resistance (Shvidchenko and Pender, 2000). As a result of this increased flow resistance at higher slopes, there is a lower flow velocity. Shvidchenko and Pender (2000), like Rubey (1938) and Brooks (1958), assumed this was responsible for a lower transport rate. Similarly, using their 1-D force-balance model, Lamb et al. (2008) demonstrated that local flow velocities decrease at higher slopes because of variations in the vertical structure of mixing and large-scale turbulent motions as a result of changes in relative roughness.

The dependence of critical mean bed shear stress on slope (and relative roughness) can be understood by appreciating the limitations of mean bed shear stress as a parameter representing the forces acting on bed-load. Section 4.1 provided evidence that, compared with stream power per unit bed area, mean bed shear stress is relatively poorly correlated with bed-load transport rate. Indeed, the extensive work of Rubey (1938) identified that, whilst mean bed shear stress is indeed an important driver behind the entrainment of particles, mean velocity also plays an important role. Rubey favoured near-bed velocity as having the greatest discriminating power as it reflected the relationship between mean velocity, the velocity gradient, depth, and slope. Similarly, Brooks (1958) observed that in flumes with flows of the same mean bed shear stress, velocities, transport rates, and bed-forms varied. Therefore, as mean velocity can vary independently of mean bed shear stress, and mean velocity is also an important driver behind the entrainment of particles, mean bed shear stress alone cannot predict the variation observed experimentally.

It is not only slope that influences relative roughness and consequently, velocity. Increases in relative roughness independently from slope have also been demonstrated to increase Shields' dimensionless critical shear stress criterion (Mueller et al., 2005); and critical mean shear stresses have been demonstrated as being lower in narrow streams as a result of the reduced velocity (Carling, 1983). Therefore, the reduced velocity is responsible for elevating the critical mean bed shear stress values in channels with higher slopes. Yet the most common means of identifying the critical threshold of motion (those based on Shields' criterion) do not account for variations in velocity, concentrating instead on mean bed shear stress.

5.2 Importance of both mean velocity and mean bed shear stress in mobilising sediment

Section 4.1 identified that in datasets where mean bed shear stress and mean velocity are poorly correlated, both are very poorly associated with bed-load transport despite stream power per unit bed area having a strong relationship with sediment transport rate in the same datasets (Fig. 2). This finding is closely linked to the idea explored in section 5.1 above, i.e. that it is the reduced velocity resulting from elevated relative roughness that is responsible for increasing the critical shear stress values in channels with higher slopes. Both of these findings suggest that both mean bed shear stress and mean velocity are important in influencing sediment motion.

Despite many researchers recognising the importance of both near bed velocity and shear stress in the transport of bed-load, almost all give attention to either one or the other, with the vast majority of contemporary studies focusing on mean bed shear stress. The justification for doing so seems to result from the general covariance that exists between τ and u_0. However, whilst it is true that in any particular channel conditions:

$$\tau \propto u_0^{\,2} \tag{15}$$

the relationship between mean bed shear stress and near bed velocity may vary *between* channel conditions as a result of differences in roughness. Results from this study show that critical mean bed shear stress varies with mean velocity (as a result of variation in slope); moreover, others have shown that the critical velocity required to entrain sediment varies with shear stress (Sundborg, 1956; Sundborg, 1967; both cited in Richards, 2004). Neither of these findings would be possible if the relationship between mean bed shear stress and velocity were independent of channel conditions. Therefore, the assumption that, by accounting for shear stress, velocity is also accounted for, is invalid.

5.3 Revision of existing expressions for critical stream power per unit bed area

The above empirical analysis and exploration of the literature demonstrates that Shields' dimensionless shear stress criterion (θ_{ci}) alone cannot predict the threshold of sediment motion to a consistent degree of accuracy, even within flows considered to be fully turbulent ($R_* > 500$). The dependence of the threshold of motion on flow velocity means that critical mean shear stress is strongly dependent on channel slope and relative roughness. Therefore, application of Bagnold's (1980) expression (Eq. 7) or Ferguson's expressions (Eqs. 10-12) for critical power with the assumption that θ_{ci} is constant will result in potential error. Ferguson (2005) himself recognised the presence of evidence to suggest that θ_{ci} was higher in steep streams and, therefore, was aware of a potential limitation of his expressions. This also accounts for Bagnold predicting critical stream power to be positively related to relative roughness and for Ferguson predicting that critical stream power per unit bed area is inversely related to channel slope. Instead, whilst the velocity associated with a critical mean shear stress is inversely related to channel slope, critical mean shear stress itself is positively related to slope. Therefore critical stream power appears to remain relatively constant with slope. In recognition of this, it is proposed that Bagnold's and Ferguson's expressions for critical stream power should be modified to take into account the variability of θ_{ci}.

This is possible by substituting the following expression in place of Eq. 2 into Eqs. 7, 10 and 11:

$$\tau_{ci} = \left(0.19 \cdot S^{0.28}\right)\left(\rho_s - \rho_w\right) g \cdot D_i \qquad (16)$$

where Eq. 16 is based upon the empirical relationship between θ_{ci} and S observed in Eq. 14.

5.4 Alternative expression for critical stream power per unit bed area

The findings of this study support Shvidchenko and Pender's (2000) argument that the Shields' curve is an inappropriate means of universally evaluating the threshold of motion. However, it is proposed that their chosen solution, to calibrate Shields' dimensionless critical shear stress criterion against slope as has been applied in section 5.3 above, is not ideal, as a dimensionless criterion that does not vary with slope or relative submergence is more appropriate. This solution would yield a revised dimensionless critical stream power.

As described in section 3.2, Einstein (1942) proposed that sediment transport rate could be given in dimensionless terms by applying Eq. 13. Because the units for unit width sediment transport rate in submerged weight (N/m s) are the same as those applied for stream power, it is relatively simple to follow the same procedure as Einstein to generate a dimensionless form of critical stream power using the expression

$$\omega_{c*} = \frac{\omega_c}{g \cdot (\rho_s - \rho_w) \sqrt{\frac{\rho_s - \rho_w}{\rho_w} \cdot g \cdot D_i^3}} \tag{17}$$

where the flume data analysed in this study had a mean ω_{c*} of 0.1. Eq. 17 predicts critical stream power to be proportional to $D_i^{1.5}$. This order of relationship is supported by the findings of section 2.3 where it was identified that the critical stream power relationships described by Petit et al.'s (2005) empirical datasets all predict ω_c to also be proportional to approximately $D_i^{1.5}$.

Using a dimensionless critical stream power criterion to identify the threshold of motion is both conceptually and practically attractive. Applying expressions of the type originally proposed by Bagnold (1980) and later modified by Ferguson (2005) requires a critical mean bed shear stress to be identified (which is dependent on slope), a mean velocity appropriate for the chosen critical shear stress to be calculated, and the critical stream power per unit bed area to be determined from their product. Instead, a critical stream power should be attainable independently from local variations in velocity and shear stress, dependent instead only on grain size. Therefore, like the stream power parameter in general, critical stream power seems to offer a more practical alternative to other flow parameters.

However, further work is necessary to test the general applicability of a constant dimensionless critical stream power. It is currently unknown whether increases in critical mean shear stress as a result of higher slope or relative roughness are proportionately balanced by decreases in the associated mean velocity. One potential area of inconsistency comes as a result of wide variations in form roughness. As cited earlier, based on a series of marker pebble experiments in streams within the Belgian Ardenne, Petit et al. (2005) suggested that critical stream powers are higher in smaller, steeper streams because of greater bedform resistance. This argument is explored in the following section.

5.5 The effect of form resistance on critical stream power per unit bed area

Petit et al. (2005) argued that the higher critical stream powers observed in the steeper, smaller rivers with higher form roughness is a result of additional energy losses in overcoming form resistance. Ferguson's (2005) paper was written partly in response to Petit et al.'s findings. Using the Manning roughness equation, Ferguson (2005) attempted to demonstrate analytically that, contrary to Petit et al.'s arguments, the reduction in critical velocity resulting from form roughness always balances the associated increase in critical shear stress, so that critical stream power remains invariant.

However, an in-depth examination of his argument reveals that his conclusions may not necessarily be true. Ferguson (2005) described two theoretical channels, identical to each other apart from one having only grain roughness (n'), and one with both grain and a significant amount of form resistance ($n' + n'' = n$). He correctly described how *for a given discharge*, the mean velocity in the channel with n' roughness (U') will be a factor (f) greater than the mean velocity in the channel with n roughness (U), and that the average depth in the channel with n' roughness (d') will be the same factor (f) smaller than the average depth in the channel with n roughness (d). Using this fact combined with Manning's roughness equation:

$$n = \frac{d^{2/3} \cdot S^{1/2}}{U} \qquad (18)$$

Ferguson properly identified that under these conditions, *for a given discharge*, the Manning's n in the channel with just grain roughness (n') is a factor ($f^{5/3}$) greater than the Manning's n in the channel with grain and form roughness (n). However, when Ferguson later considered the problem of relating the higher *critical* shear stresses and lower *critical* velocities associated with channels with significant form roughness, an inconsistency arose. Because the critical shear stress (and therefore, using Eq. 9, the associated depth) in the channel with n' roughness (τ_c' and d_c') may be a factor (f) lower than the critical shear stress and associated depth in the channel with n roughness (τ_c and d_c), Ferguson claimed that the lower velocity in the channel with n roughness can be calculated based on a Manning's n value that is higher than that in the channel with n' roughness by the factor $f^{5/3}$. The relationship between changes in depth and changes in Manning's n was realised on the assumption that any increase in depth must be balanced by an equal decrease in velocity where discharge remains constant. Therefore, Ferguson found that the critical velocity in the channel with n roughness is the same factor lower than the critical velocity in the channel with n' roughness as the critical shear stress (and associated depth) is higher. However, in reality, the critical shear stress in a channel with n roughness may not occur at the same discharge as the channel with n' roughness. Therefore, a change in form roughness may result in the critical shear stress increasing by a different factor to the velocity decrease so that the critical stream power varies.

Therefore, in regard to Petit et al.'s (2005) findings, it is possible that an increase in form roughness may have indeed resulted in higher critical stream powers. However, as noted by Ferguson (2005), a number of other factors also increase critical stream powers in the headwater streams, exaggerating the influence that form roughness itself may have had. Whilst Petit et al. claimed that hiding effects are similar in all river types as the D_i/D_{50} ratios are relatively close to 1, the range in bed material size in headwater streams is generally considerably greater so that the larger grain sizes offer a more considerable hiding effect than in larger rivers. Furthermore, the proportion of fines within headwater streams is usually low in comparison with stream beds lower down in the catchment. Because Wilcock (2001) identified that gravel transport rates increase significantly with the proportion of fines within the bed, this trend may also result in higher critical stream powers in smaller, steeper streams. Imbrication between bed particles is also more common in smaller, steeper streams; and this may also act to stabilise the bed, resulting in higher critical stream powers in the headwaters. Ferguson also highlighted that the trendlines for several of the rivers in Petit et al.'s dataset are fitted to composite sets of data, combining results of tracer experiments in several different reaches with different bed materials. Merging data from reaches with the same slope but different beds would result in a composite curve that is steeper than the individual composite curves, predicting higher than expected values of critical shear stress.

6. Empirical evaluation of dimensionless critical stream power per unit bed area

The flume data from the large collection of sediment transport datasets referred to in section 3.1 was used to test the proposed dimensionless critical power relation (Eq. 17). All flume data used to derive the dimensionless critical stream power value of 0.1 was removed from the validation. As with the analysis of the critical threshold of motion earlier, a reference value of Einstein's dimensionless transport parameter of 0.0001 was used to identify the critical stream power for each dataset. This was only possible for a selection of the datasets as many did not include values low enough for the power at the reference transport rate to be identified. It was not possible to test the expressions based on Eq. 16 against this data as they require a slope value and slope was not held constant within these flume datasets.

Figure 7 illustrates that application of a dimensionless critical stream power value of 0.1 in Eq. 17 predicts the critical stream power observed in the flume studies extremely well. Not only are the predicted and observed values strongly associated (r^2 coefficient = 0.99), but the values also fall along a 1:1 proportionality line.

Figure 7

7. Conclusion

Although stream power per unit bed area is generally more strongly associated with sediment transport, mean bed shear stress has been the parameter most commonly applied in the prediction of a critical transport threshold. A combination of newly gathered critical stream power data and existing data from previous flume studies demonstrates that critical stream power is relatively invariant with slope, but that critical mean bed shear stress is strongly positively related to slope. The positive relationship between critical shear stress and slope is explained as a result of higher relative roughness at high slopes causing increased resistance so that the velocity for a given shear stress is reduced. Because velocity is important in influencing sediment transport in combination with mean bed shear stress, when resistance is increased, a higher shear stress is necessary to reach the critical threshold. Based on these findings, solutions to approximating critical stream power. include: (i) modifying Ferguson's existing expressions for critical stream power to account for higher critical shear stresses at higher slopes; and (ii) applying a dimensionless critical stream power criterion based on the conclusion that critical stream power is less variable than critical shear stress. An empirical evaluation of the dimensionless critical stream power criterion demonstrates its efficacy in predicting critical stream powers with unimodal flume data, but further research is now needed to examine its constancy or otherwise under a wider range of grain size, relative roughness and flow and transport stages.

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Figure Captions

Fig. 1. Grain size distributions for experimental sediment mixtures.

Fig. 2. Examples of a sediment transport dataset where (A) mean velocity and (B) mean bed shear stress are poorly correlated with sediment transport rate compared with (C) stream power per unit bed area - Johnson's (1943) laboratory investigations on bed-load transportation, series II, taken from the Gomez and Church (1988) collection of data;.

Fig. 3. The influence of slope on critical stream power per unit bed area. (A) Dimensionless bed-load parameter q_{b*} increasing as a function of stream power at various slopes for each dataset. The line at a dimensionless transport rate of 0.0001 identifies the point at which transport rates meet the level assigned as being "critical." The key gives the dataset, sediment mixture, and slope for each of the experimental runs; (B) Critical dimensionless stream power identified from (A) plotted against slope. The solid line describes the mean value that best approximates the flume data observed in this study.

Fig. 4. The effect of slope on the relationship between mean bed shear stress and mean velocity. (A) Slope versus resistance function for all analysed flume data; (B) Mean velocity predicted using the flow resistance equations applied by Ferguson (2005) against the measured velocity.

Fig. 5. The influence of slope on critical mean bed shear stress. (A) Dimensionless bed-load parameter q_{b*} increasing as a function of mean bed shear stress at various slopes for each dataset. The line at a dimensionless transport rate of 0.0001 identifies the point at which transport rates meet the level assigned as being "critical." The key gives the dataset, sediment mixture, and slope for each of the experimental runs; (B) Critical Shield's dimensionless shear stress identified from (A) plotted against slope. The solid line describes the power relationship that best approximates the flume data observed in this study.

Fig. 6. The influence of slope over the Shields' diagram. Each series of points represents the critical Shields' values from a range of slopes used for each sediment mixture within each flume dataset.

Fig. 7. Predicted critical stream power per unit bed area values based upon a dimensionless critical stream power criterion of 0.1 compared against observed critical stream power values for a selection of flume datasets.

Table 1
Summary of collated sediment transport data used in exploratory analysis

Author	***Year***	***Title/description***	***Data type***	***No. of datasets***
Yang	1979	Unit stream power equations for total load	Flume and field	40
Gomez and Church	1988	Catalogue of equilibrium bed-load transport data for coarse sand and gravel-bed channels	Flume and field	22
Bravo-Espinosa	1999	Prediction of bed-load discharge for alluvial channels – PhD Thesis	Field	14
Wilcock et al.	2001	Experimental study of the transport of mixed sand and gravel	Flume	5
King et al.	2004	Sediment transport data and related information for selected coarse-bed streams and rivers in Idaho	Field	33
Ryan et al.	2005	Coarse sediment transport in mountain streams in Colorado and Wyoming, USA	Field	19

Table 2
Summary of datasets used to test theoretical expressions for critical stream power

Data source	***Range of bed sediment types (D_{50} in m)***	***Range of slopes***	***Range of discharges (m^3/s)***
This study	Graded; D_{50}: 0.006 (Mix 3) – 0.0115 (Mix 1)	0.0071 - 0.0167	0.004 - 0.025
Johnson, 1943 – cited in Gomez and Church, 1988	Graded; D_{50}: 0.0014 – 0.0044	0.0015 - 0.0100	0.002 - 0.077
Shvidchenko and Pender, 2000	Uniform; D_{50}: 0.0015 (U1) – 0.012 (U8)	0.0019 - 0.0287	0.000 - 0.029
Shvidchenko et al., 2001	Graded; D_{50}: 0.0026 – 0.0064	0.0041 - 0.0141	0.003 - 0.140