

THE MEMORY-CONSERVATION MODEL OF MEMRISTANCE

Ella M. Gale¹

¹Unconventional Computing Group, University of the West of England, Bristol, Avon, UK

The memristor, a device which relates charge, q , and magnetic flux, ϕ , was postulated to exist by Chua in 1971 [1] via the relation: $d\phi = M(q(t)) dq$, where $M(q(t))$ is the memristance, which is a charge-dependent. This theory wasn't related to an experimental device (despite several such devices having been fabricated) until 2008 [2] when Strukov et al put forward a phenomenological model of the form: $M(q(t)) = R_{off} (1 - R_{off} R_{on} \beta q(t))$ where R_{off} is the resistance of the TiO_2 , R_{on} is the resistance of the auto-doped $TiO_{(2-x)}$ and $\beta = \mu_v / D^2$, where β is here called the material parameter. This model expressed the expected hysteretic behaviour of the device but included no magnetic flux. Because of this, and because of the lack of experimental measurements of a magnetic flux which fit the memristor equations, questions have been raised about whether the Strukov memristor is a real Chua memristor at all [3] and even if the Chua memristor might only be a theoretical curiosity with nothing to do with resistive switching memories or ReRAM.

Here it is demonstrated that the relevant charge to put into Chua's equations is not the electronic charge q_e . but the vacancy charge q_v . Using electro-dynamical theory, we can then calculate the flux of the vacancy charge flow, J . J is given by:

$$J = (q_v \mu_v L) / (D E F)$$

Where, μ_v , is the vacancy ion mobility, L is the average electric field and $D E F$ is the volume of TiO_2 in the memristor. The titanium dioxide layer is D thick, crossed electrodes of width E and F , the boundary between TiO_2 and $TiO_{(2-x)}$ is w and moves over time as a function of the total charge of oxygen vacancies. We can calculate associated magnetic field at as experienced at a point p and time t , $B(p)$, where $p = \{x, y, z\}$, located outside of the memristor:

$$B(p) = (\mu_0 / 4\pi) A L \mu_v q_v \{0, -x z P_y, x y P_z\}.$$

Where A is the area we calculate B over (ie the side of the memristor) and P_y and P_z are complicated terms which only include the dimensions of the titanium dioxide layer, (D , E and F), in the memristor, these terms are dependant on q_v , as the boundary between TiO_2 and $TiO_{(2-x)}$, w , changes with q_v (and time as q_v is time-dependent). Note that the form of the vector arises from the x direction is taken to be the direction of vacancy current flow (ie. w and D are measured on the x axis).

This magnetic field gives rise to the following magnetic flux passing through a surface i - j (again the side of the memristor, note the flux is zero in the plane perpendicular to the vacancy current flow):

$$\phi = (\mu_0 / 4\pi) i j L \mu_v P_k q_v.$$

From comparison with Chua's definitions, we get the following equation for memristance (note we have explicitly put in the time dependence here):

$$M(q_v(t)) = (\mu_0/4\pi) i j L \mu_v P_k(q_v(t)) .$$

which $\mu_v P_k(q_v(t))$ is also called, β , the material parameter. The existence of this equation for memristance, which relates charge and flux, and which is only dependent on one state variable (q_v), in the Strukov memristor shows that it is a true Chua memristor.

The calculated flux for Strukov memristor is 2.44×10^{-29} Wb (approximately 100 000 times smaller than the conducting electron's magnetic flux) and its tiny value explains the lack of experimental measurements.

To understand the measurable I - V curves, we need to look at the memristor as a two layer system and include both the effects of the true Chua memristance between the vacancy charge (called the magnetic subsystem as it contains the magnetic flux) and flux as well as the measurable effects of this ionic motion on the electronic current (the electronic subsystem).

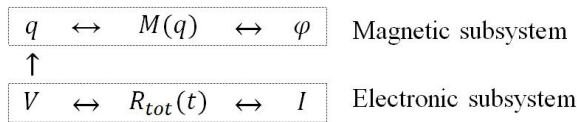


Figure 1: The two layer model of memristance [4]. The magnetic subsystem deals with the Chua memristance that arises as a result of the vacancies, the electronic subsystem deals with the effect of vacancy motion on the conducting electrons and the two systems are related as both the conducting electrons and the oxygen vacancies are responding to the applied voltage.

First we need to calculate the memory function, $M_e(t) = C M(q(t))$ where C is an experimentally determined parameter related to the different mobilities for electrons and vacancies and is the Chua memristance as experienced by the conducting electronic current, this covers resistance change in the $TiO_{(2-x)}$ part of the memristor. We then need to include an expression for the time varying resistance of the TiO_2 , which is called the conservation function (as it keeps space conserved in this model) and it is given by:

$$R_{con}(t) = [(D - w(t)) \rho_{TiO2}] / (E F) .$$

The total time varying resistance of the device, $R_{tot}(t)$, is then given by:
 $R_{tot}(t) = M(q_v(t)) + R_{con}(t)$. $R_{tot}(t)$, the memory function and conservation function are all memristances in that their resistance varies with time as a result of a vacancy flow, but only the Chua memristance directly relates charge and magnetic flux.

Further details are given in [4].

[1] L. O. Chua, IEEE Trans. Circuit Theory, **18**, 507-519, (1971).
 [2] D. B. Strukov et al, Nature, **453**, 80-83 (2008).
 [3] B. Mouttet, ISCAS 2010, <http://www.slideshare.net/blaisemouttet/mythical-memristor> (downloaded 13/03/12)
 [4] E. Gale, The Missing Memristor Flux Found, [arXiv:1106.3170v1](https://arxiv.org/abs/1106.3170v1) [cond-mat.mtrl-sci] (submitted)