Collection of Taxonomically Classified Mathematical Common Student Errors in E-Assessments (CSE Book)

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Content

1 Introduction

This book presents Common Student Errors (CSEs) that have been made by first year students taking Engineering Mathematics e-examinations at the University of the West of England, Bristol (UWE, Bristol). The CSEs presented in this document were collected from 2017-2019 as part of the Common Student Errors Project (CSE Project at UWE) conducted at UWE, Bristol. This research has been conducted by Indunil Sikurajapathi for her Doctoral studies under the guidance of Dr Karen Henderson and Dr Rhys Gwynllyw.

This book is organised as follows. In Section 1.1 and Section 1.2 we present a brief background of Common Student Errors (CSEs) and the Dewis e-Assessment System. Then we explain how we conducted the CSE Project at UWE, Bristol in Section 1.3 and the present the CSEs Collection found during the project in Section 1.4. The taxonomy coding utilised to classify the CSEs in e-Assessments found in this project is discussed in detail in Section 1.5. After that we provide a Guide to the CSE recording template in Section 1.6 which is used in the subsequent Sections 2-23 to present all of the 65 CSEs found to date during the CSE Project at UWE, Bristol.

One of the special features of this book is that it provides hyperlinks to each question on the Dewis e-Assessment System in order to facilitate the reader to try these questions online. If any of the identified CSEs are submitted as answers, then enhanced feedback will be provided, which aims to correct any misconceptions in a timely manner.

The information in this book may be used to inform teachers so that they can provide students with a better understanding of the mathematical skills and knowledge while teaching the subject. It may also be useful for institutions as they can utilise it in the future development of teaching materials to ensure that these CSEs will be addressed. Further, the content of this book can be used to develop support materials and resources to address CSEs which will help students to acquire better understanding of mathematics. In addition, students who learn mathematics at university level or in secondary school can refer to this booklet to address their misconceptions and can try the Dewis questions several times. Since, in each attempt, Dewis produces questions with random parameters, student can use this facility to correct their misconceptions by practicing the same question but with different parameters.

We anticipate that this book will be useful to identify and address some misconceptions that students have in mathematics. We plan to continue with this research and will update the book if we find new CSEs in the future.

1.1 Common Student Errors (CSEs)

Students arrive at an incorrect answer when answering a mathematical question due to variety of reasons. The reasons can be listed as random errors, calculation errors or misreading the questions. These errors lead to incorrect answers or loss of accuracy marks. Many of these errors are made by just a few students. However, some of these errors are commonly made by a considerable number of students. These commonly made errors are sometimes referred to as common errors (Rushton, 2014).

Researchers express different opinions about the difference between errors and misconceptions in the literature. For Confrey (1990), the reasons for both errors and misconceptions are the rules and beliefs that students hold. They argue that the difference between errors and misconceptions is that misconceptions are attached to particular theoretical positions. However, Nesher (1987) uses the term misconceptions to describe systematic errors without reference to a theoretical position.

Rees and Barr (1984) use the term '*mal-rule*' to refer to an understandable but incorrect implementation of a process resulting from a student's misconception. For example, a classic *mal-rule* students make is to answer $a^2 + b^2$ when asked to expand $(a + b)^2$. The term '*bug*' is used by VanLehn (1982) to refer to a systematic error resulting from wrong steps in the calculation procedure. A *Borrow Across-Zero bug* is a systematic error caused by a student having trouble with borrowing, especially in the presence of zeros (VanLehn, 1982). For example, a student answering 98 when asked to calculate $305 - 117$ would be considered as a *Borrow Across - Zero bug*. In the aforementioned calculation, the student skips the step where the zero changed to nine during borrowing across zero (VanLehn, 1982).

Research has been conducted to identify misconceptions in different areas of mathematics. For example, Brown and Burton (1978) investigated bugs (misconceptions) in high school algebra problems, and Swan (1990) focused on the misconceptions that occur in four operations (addition, subtraction, multiplication and division), and in the interpretation of graphs.

Some Mathematics Education research has explored possible causes and effects of certain mathematical misconceptions and the impact that they have on students' future learning (Booth et al., 2014; Confrey, 1990; Fischbein, 1989; Nesher, 1987; Brown and Burton, 1978). After having investigated bugs (misconceptions) in high school algebra problems, Brown and Burton (1978) discussed possible arithmetic bugs which might lead to some specific algebraic bugs. Booth et al., (2014) conducted a study to assess algebraic misconceptions that algebra students make at school. They concluded that students who make specific persistent errors due to underlying misconceptions in arithmetic may need additional intervention since misconceptions are not corrected through typical instruction. They conclude that these additional interventions can be carried out by targeting individual misconceptions or by improving conceptual understanding throughout the algebra course. The findings of Brown and Burton (1978) and then the findings of Booth et al. (2014) hold the same conclusions, that the arithmetic misconceptions held by students affect their algebraic thinking. Further, Booth et al. (2014) state that these arithmetic misconceptions can obstruct their performance and learning of algebra.

There has been recent research into theorising student errors supported by empirical studies in the topics of natural number bias (Obersteiner et al., 2013), visual saliency (Kirshner and Awtry, 2004) and over-generalisation (Knuth et al., 2006). Rushton (2014) conducted a study of common errors in Mathematics made in certain General Certificate of Secondary Education mathematics papers taken by candidates in England, including an internationally available version, as referenced by examiner reports, and errors were catalogued into themes and subthemes. More recently, Ford et al. (2018) developed a taxonomy of errors made by undergraduate mathematics students. In their study they gathered errors by firstly recalling the most obvious errors that occur and secondly by analysing students' exam scripts to categorise them in a taxonomical manner.

1.2 Dewis e-Assessment System

Dewis is a fully algorithmic open-source e-Assessment system, which was primarily designed and developed for numerate e-Assessments by a team of Mathematicians, Statisticians and Software Engineers at UWE Bristol (Gwynllyw and Henderson, 2009; Gwynllyw and Henderson, 2012). Dewis supports different question input types such as numerical inputs, matrices, vectors, algebraic expressions, multiple-choice, multiple-selection, graphical input, and computer programs. It has a lossless data collection feature and a number of student-friendly features, such as shutdown recovery and pre-processing checks on student input.

Over the past decade, Dewis has been used very successfully to facilitate both formative and summative e-Assessments across a number of modules, delivered to students in a wide range of fields, e.g. Business, Computer Science, Nursing, Software Engineering, Engineering, Mathematics and Statistics. One aim of the CSE project is to enhance the full potential of Dewis, by developing and using additional features allowing Dewis to detect CSEs and to provide instant tailored feedback.

1.3 The Common Student Errors Project at UWE, Bristol

The CSE project at UWE began in 2017 with an aim of developing a technique to detect CSEs and to provide tailored feedback in Dewis e-Assessment questions, used in a first year Engineering Mathematics module (CSE Project at UWE, 2019; Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021). We started the project with the aim of answering the following research questions:

- What CSEs do first year Engineering Mathematics students make in e-Assessment questions?
- How to detect CSEs and improve Dewis feedback to address these CSEs?

There are several benefits to answering these research questions. Even though this research has been done in a particular context using the Dewis e-Assessment system, the research outcomes contribute to the knowledge to inform more general practice in assessment and learning. For example, the collection of mathematical CSEs collected during this research is not only beneficial for first year Engineering mathematics students and lecturers, but also it is equally beneficial for secondary, and first year university level mathematics students and teachers. The CSE collection presented in Sikurajapathi, Henderson and Gwynllyw (2022) can be used to correct students' mathematical misconceptions either in hand-written assessments or e-assessment questions.

Further, this CSE detecting technique will be beneficial to several disciplines and organisations that either use Dewis or any other e-assessment system which has features to give dynamic feedback based on a student answer. The new knowledge raised from this research can be used in any e-assessment system so that it emulates a human marker to provide instant enhanced feedback highlighting possible CSEs. This will help students to correct their mathematical misconceptions. Also, teachers can use the findings to identify areas in which more help is needed in student learning. Integrating the research outcomes from the CSE project into other e-assessment systems will be beneficial to generations to come (Sikurajapathi, Henderson and Gwynllyw, 2020; Sikurajapathi, Henderson and Gwynllyw, 2021; Sikurajapathi, Henderson and Gwynllyw, 2022).

The CSE Project involves five stages (Stage One: Data (CSEs) Collection; Stage Two: CSE code Development; Stage Three: CSE code Trial Phase; Stage Four: Students' Perceptions on CSE Feedback and Stage Five: Impact of CSE Project). Detailed information about these five stages and other findings can be found in CSE Project at UWE Bristol (2019), Sikurajapathi, Henderson and Gwynllyw (2020) and Sikurajapathi, Henderson and Gwynllyw (2021).

In this book, we only focus on Stage One: Data (CSEs) Collection, which provides an answer to the question 'What CSEs do first year Engineering Mathematics students make?

1.4 Common Student Error Collection

The CSEs presented in this booklet were collected by examining the 2017-2018 and 2018-2019 e-examination data on the Dewis e-Assessment system and from students' rough work scripts. These e-examinations were run using the Dewis e-Assessment system and were held under controlled conditions. The e-examinations were held in two sessions (morning and afternoon) to mitigate logistic issues. In each session, all of the students received the same, fixed parameter questions. During the e-examination, students were given booklets to use for their rough work. These booklets were used by students to work through the mathematical questions before submitting their final answers on Dewis.
Altogether 65 CSEs were identified in the following different topics areas of Engineering Mathematics:

- Algebra
- Unit-step function
- Wave forms

- Trigonometric functions
- Differentiation
- Implicit differentiation
- Partial differentiation
- Mean Value Theorem
- Complex numbers
- Geometric series
- Maclaurin Expansion
- Centre of Mass
- Integration by parts
- Volume of revolution
- Dimensions

1.5 Taxonomy of Mathematical Common Student Errors in e-Assessments

All of the CSEs found in the course of the CSE project are documented in a systematic order in the CSE book together with their mathematical taxonomy coding. Here we used the taxonomy coding described in Ford et al. (2018) as a guideline.

The theoretical study of classification, including its bases, principles, procedures and rules is called a taxonomy (Ford et al., 2018; Simpson, 1961, p.11). The entities in a successful taxonomy can be verifiable by observation and will offer both an appropriate and suitable class for each entity (Ford et al., 2018; Bailey, 1994, p.3). The taxonomy of cognitive mechanisms and the phenomenological taxonomy can be considered as the two main styles that can be used to categorise mathematical errors (Senders and Moray, 1991, Ford et al., 2018).

The taxonomy introduced by Ford et al. (2018) was developed to categorise the errors which undergraduate mathematics students make. Ford et al. (2018) identified six main error categories by firstly recalling obvious mathematical errors that occur among mathematics undergraduates and secondly by analysing a selection of students' paper-based exam scripts from first year undergraduate mathematics courses. These main categories were named as Errors of slips of action (S), Errors of understanding (U), Errors in choice of method (CM), Errors in the use of a method (UM), Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C).

The CSEs that we have found during the CSE project only fall into four of the error categories (S, U, CM and UM) from the Ford et al. (2018) taxonomy. Errors related to proof (P), and Errors in student's communication of their mathematical solutions (C) were not found among the CSEs made by the Engineering Mathematics students, due to the nature of the questions asked and the nature of the system used to deliver the questions. None of the e-Assessment questions delivered by Dewis involve mathematical theorems and proofs and hence Errors related to proof (P) were not viable in this CSE collection. Further, the e-examination did not contain questions that required student's communication of their mathematical solutions, correct use of notation or labelling and qualitative judgements on clarity of expression. Therefore, errors in student's communication of their mathematical solutions (C) were not found in this CSE collection. Further, a few of the CSEs found fall into two categories due to the mix of misconceptions made by the students as they arrived at their incorrect answer.

Under the category Errors of slip of action (S), three main errors, namely copying error, careless errors on simple calculations, and incorrect algebraic manipulation were identified. A total of 13 out of 65 CSEs were found to fall into the Errors of slip of action category (S).

Seven main errors were identified under the Errors of understanding (U) category, such as confusing different mathematical structures, incorrect argument, lack of consideration of potential indeterminate forms, proposed solution is not viable, definition/method/theorem not recalled correctly, partial solution given and Incorrect assumptions. In total 45 CSEs are in the Errors of understanding category.

Only one main error was found in each of the Errors in choice of method (CM) and Errors in use of method (UM) categories. Three CSEs were grouped into the main error of applying an inappropriate formula/method/theorem in CM. There were 9 CSEs which fell into Error in use of an appropriate definition/method/theorem in the UM category. All the codes, errors and examples that we found in this CSE collection process are shown in Table 1.

Table 1: Taxonomy of Mathematical Common Student Errors in e-Assessments

Main Category	Code	Error	Examples
Slip of action	S1	Copying error	Incorrect copying of the question
			Mistake copying/ submitting answer into e-assessment
			Incorrect interpretation of the question
	S2	Careless errors on simple calculations	Overlooking negative signs
			Omission of denominator
	S3	Incorrect algebraic manipulation	Incorrect division of two complex numbers
			Sum of product is split as a product of two sums
			Incorrect handling of powers
Errors of understanding	U1	Confusing different mathematical structures	Confusing the structure of completing the square and the quadratic equation
			Stating that a unit step function is a number

	U2	Incorrect argument	Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives
			Taking the integration of the product of two functions as the product of individual integrals
	U3	Lack of consideration of potential indeterminate forms	Taking the square of a negative number to be negative
	U4	Proposed solution is not viable	Angle is not within the given range
	U5	Definition/method/theorem not recalled correctly	Method of completing the square is not recalled correctly
			Definition of waveform properties not recalled correctly
			Method of differentiating a standard function is not recalled correctly
			Method of solving trigonometry equation is not recalled correctly
			Chain rule is not recalled correctly
			Method of Partial differentiation not recalled correctly
			Method of differentiating implicit functions is not recalled correctly
			Mean value theorem is not recalled correctly
			Method of calculating the argument of a complex number is not recalled correctly
			Binomial theorem is incorrectly followed
			Definition of Centre of Mass is not recalled correctly
			Method of finding the principal value of the argument of a complex number is not recalled correctly
			Method of integrating not recalled correctly
			Definition of volume of revolution is not recalled correctly
	U6	Partial solution given	Correct workings but unfinished solution
	U7	Incorrect assumptions	Incorrect assumptions on the mean value theorem

			Taking dimension of velocity is $[v] = [MT^{-1}]$
Errors in choice of method	CM1	Applying an inappropriate formula/method/theorem	Uses a method which is not relevant in the situation
			Uses a formula which is not relevant in the situation
Errors in use of method	UM2	Error in use of an appropriate definition/method/theorem	Error in the use of the chain rule
			Error in use of partial differentiation method
			Incorrect units applied
			Method finding the volume of revolution is incorrectly followed

1.6 Guide to the CSE recording template

Each CSE found to date has been recorded using the template as shown in Figure 1. The template contains seven areas and each area and its contents are described in detail below.

① The link to the online Dewis e-assessment question is available here. The reader may access the online question by clicking the Question hyper-link. By attempting the question and answering with a relevant CSE response, it is possible to see how Dewis detects the CSE and provides instant tailored feedback to address the CSE made in the solution.

② In this area, a screenshot of the Dewis question is given.

③ The correct solution to the question is presented in brief here.

④ The taxonomy code of the CSE, which is presented in ⑤**,** is given here.

⑤ A sample of the CSE and the incorrect answer(s) that led from it is presented here. At the top of this area, the CSE error is summarised by a statement which is presented in red text. Then the detailed steps of the exact way the CSE is made and the solution as written by students in their rough work booklets is presented. We used tilde (~) on the CSE answer to differentiate it from the correct answer. For example, in Figure 1, the CSE answer for this question is denoted as, $\tilde{b} = -6$, in red text.

⑥ In this section, the number of CSE answers made, the total incorrect answers made in the question and the CSE percentage for each year are presented as No. of CSEs /No. incorrect answers **(CSE %)**. For example, in Figure 1, in 2017-18 exam, this particular CSE was made by 28 out of the 56 students who gave an incorrect answer to this question; therefore the CSE percentage is **50%**. This data is presented in this area as 28/56 **(50%)**. Similarly, the data for 2018-19 is presented as 33/57 (**58%**).

⑦ The exam year that data was collected from is presented here. Figure 1 shows that 28/56 **(50%) and** 33/57 (**58%**) presented in ⑥ relate to the years 2017-18 and 2018-19 presented in ⑦ respectively.

Question ①

The expression

$t^2 - 12t + 40$

can be expressed in the form:

$a(t-b)^2 + c$

where a, b and c are constants. ②

Calculate the values of these constants - note that all these solutions are integers:

Enter the value of a [ ]
Enter the value of b [ ]
Enter the value of c [ ]

Correct Solution

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$

$$= (t-6)^2 + 4$$

$$a = 1, b = 6 \text{ and } c = 4$$

③

CSE 1 related to this question	**CSE Taxonomy Code:**	S1 ④

Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b.

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$

$$= (t-6)^2 + 4$$

$$\tilde{b} = -6 \text{ and } c = 4$$

⑤

No. of CSEs /No. incorrect answers **(CSE %)**	28/56 (**50%**) 33/57 (**58%**) ⑥	**Date collected**	**2017-18** **2018-19** ⑦

Figure 1: CSE Template

2 Common Student Errors due to Slip of Action

2.1 Copying Error

2.1.1 Algebra (Completing the Square)

Mistake copying/ submitting answer into e-assessment

Question
The expression $t^2 - 12t + 40$ can be expressed in the form: $a(t-b)^2 + c$ where a, b and c are constants. Calculate the values of these constants - note that all these solutions are integers: Enter the value of a [ ] Enter the value of b [ ] Enter the value of c [ ]
Correct Solution
$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $a = 1, b = 6$ and $c = 4$

CSE 1 related to this question	**CSE Taxonomy Code**:	**S1**
Give answer $\tilde{b}$ which corresponds to the negative of the correct value of b. $t^2 - 12t + 40 = (t-6)^2 - 36 + 40$ $= (t-6)^2 + 4$ $\tilde{b} = -6$ and $c = 4$		
No. of CSEs /No. incorrect answers **(CSE %)**	28/56 (**50%**) 33/57 (**58%**)	**Date collected**	**2017-18** **2018-19**

2.1.2 Binomial Series

Incorrect copying of the question

Question		
Use the binomial theorem to expand $(2x+3y)^5$ and enter the fourth term (that is the full term involving x^2y^3) as a function of x and y :		
Correct Solution		
When n is a positive integrer, the binomial theorem states that $$(ax+by)^n = \binom{n}{0}(ax)^n + \binom{n}{1}(ax)^{n-1}(by)^1 + \binom{n}{2}(ax)^{n-2}(by)^2 + \cdots + \binom{n}{n}(by)^n$$ So in this case, the fourth term is $$T_4 = \binom{5}{3}(2x)^{5-3}(3y)^{5-2}$$ $$= \frac{5!}{3! \times 2!}(2x)^2(3y)^3$$ $$= 1080x^2y^3$$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**S1**
Submitting terms up, and including, the fourth term $$(2x+3y)^5 = \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4(3y)^1 + \binom{5}{2}(2x)^3(3y)^2 + \binom{5}{3}(2x)^2(3y)^3 + \binom{5}{4}(2x)^1(3y)^4 + \binom{5}{5}(3y)^5$$ $$\widetilde{T_4} = 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3$$		

No. of CSEs /No. incorrect answers **(CSE %)**	9/165**(5%)**	**Date collected**	**2018-19**

2.1.3 Maclaurin Expansion

Incorrect interpretation of the question

Question			
Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$, of $f(x) = e^{4x}$ up to and including the cubic term. Give the values of a_1 and a_2 below. Enter your answer for a_1 (to three decimal places) here: ☐ Enter your answer for a_2 (to three decimal places) here: ☐ Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$ Enter your approximate value for $e^{0.8}$ (to three decimal places) here: ☐			
Correct Solution			
Using MacLaurin expansion, $$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2!} + \frac{(4x)^3}{3!} \dots$$ $$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$ $$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$ $$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$			
CSE 3 related to this question		**CSE Taxonomy Code**:	**S1**
Submitted a_2 value for a_1 and a_3 value for a_2 $$\widetilde{a_1} = \frac{4^2}{2!} = 8.000$$ $$\widetilde{a_2} = \frac{4^3}{3!} = 10.667$$			
No. of CSEs /No. incorrect answers **(CSE %)**	5/53**(9%)**	**Date collected**	**2018-19**

2.1.4 Dimensions

Incorrect interpretation of question

Question

The quantity $K = \rho^2 r^{-1} v^{1.5}$

where ρ represents density, r represents distance, v represents velocity.

The dimensions of K are given by $[M^a L^b T^c]$. Calculate the values of a, b and c.

Enter a:

Enter b:

Enter c:

Correct Solution

$$K = \rho^2 r^{-1} v^{1.5}$$

$$[K] = [ML^{-3}]^2 [L]^{-1} [LT^{-1}]^{1.5}$$

$$= [M]^2 [L\ ]^{-6-1+1.5} [T]^{-1.5}$$

$$= [M]^2 [L]^{-5.5} [T]^{-1.5}$$

$$a = 2, b = -5.5, c = -1.5$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**S1**

Submitted

a = the power of ρ in the question

b =the power of r in the question

c =the power of v in the question

$$K = \rho^2 r^{-1} v^{1.5}$$

$$a = 2, \tilde{b} = -1, \tilde{c} = 1.5$$

No. of CSEs /No. incorrect answers **(CSE %)**	9/46 **(20%)**	**Date collected**	**2018-19**

2.1.5 Dimensions

Copying error

Question			
The quantity $K = \rho^2 r^{-1} v^{1.5}$ where ρ represents density, r represents distance, v represents velocity. The dimensions of K are given by $[M^a L^b T^c]$. Calculate the values of a, b and c. Enter a: ☐ Enter b: ☐ Enter c: ☐			
Correct Solution			
$K = \rho^2 r^{-1} v^{1.5}$ $[K] = [ML^{-3}]^2[L]^{-1}[LT^{-1}]^{1.5}$ $= [M]^2[L]^{-6-1+1.5}[T]^{-1.5}$ $= [M]^2[L]^{-5.5}[T]^{-1.5}$ $a = 2, b = -5.5, c = -1.5$			
CSE 3 related to this question	**CSE Taxonomy Code**:	**S1**	
Omitted the powers of ρ, r and v $[K] = [ML^{-3}][L][LT^{-1}]$ $= [M][L]^{-3+1+1}[T]^{-1}$ $= [M]^1[L]^{-1}[T]^{-1}$ $\tilde{a} = 1, \tilde{b} = -1, \tilde{c} = -1$			
No. of CSEs /No. incorrect answers **(CSE %)**	8/20 **(40%)**	**Date collected**	**2018-19**

2.2 Careless Errors on Simple Calculations

2.2.1 Complex Numbers (Argument of $z = a - bj$)

Overlooking negative sign

Question			
Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \le \pi$. Express your answers correct to three decimal places. Enter r: [ ] Enter θ: [ ]			
Correct Solution			
$z = 2 - 3j$ $r = \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{13}$ $r = 5.39$ z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -0.983$			
CSE 1 related to this question	**CSE Taxonomy Code**:	S2	
Taking $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ *when* $z = a - bj$ *(missing the negative sign)* $\tilde{\theta} = \tan^{-1}\left(\frac{3}{2}\right)$ $= 0.983$			
No. of CSEs /No. incorrect answers **(CSE %)**	19/88**(22%)**	**Date collected**	**2018-19**

2.2.2 Complex Numbers (Argument of $z = a - bj$)

Overlooking negative sign and incorrect units applied

Question			
Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$. Express your answers correct to three decimal places. Enter r: ☐ Enter θ: ☐			
Correct Solution			
$z = 2 - 3j$ $r = \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{13}$ $r = 5.39$ z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -0.983$			
CSE 4 related to this question	**CSE Taxonomy Code**:	**S2, UM2**	
Calculating $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ *in degrees (and missing a negative sign) when* $z = a - bj$ $\tilde{\theta} = \tan^{-1}\left(\frac{3}{2}\right)$ $= 56.31^o$			
No. of CSEs /No. incorrect answers **(CSE %)**	11/88**(13%)**	**Date collected**	**2018-19**

2.2.3 Series (Maclaurin Expansion)

Omission of denominator / correct working and unfinished solution is given

Question			
Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, of $f(x) = e^{4x}$ up to and including the cubic term. Give the values of a_1 and a_2 below. Enter your answer for a_1 (to three decimal places) here: [ ] Enter your answer for a_2 (to three decimal places) here: [ ] Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$ Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]			
Correct Solution			
Using MacLaurin expansion, $$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \ldots$$ $$P_3(x) = a_0 + a_1x + a_2x^2 + a_3x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$ $$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$ $$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$			
CSE 2 related to this question	**CSE Taxonomy Code**:	**S2, U6**	
Missing division by 2! $$\widetilde{a_2} = 4^2 = 16.000$$			
No. of CSEs /No. incorrect answers **(CSE %)**	11/59(**19%**) 16/80(**20%**)	**Date collected**	**2017-18** **2018-19**

2.2.4 Integration ($\int x \cos(ax)\, dx$)

Overlooking negative sign

Question
Evaluate the following: $$\int x\cos(3x)dx$$ as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**S2**

$$Taking \int sin(3x) = \frac{cos(3x)}{3} \ (missing\ a\ negative\ sign)$$

$$I = \int xcos(3x)dx = x\left(\frac{sin(3x)}{3}\right) - \int \frac{sin(3x)}{3}dx$$

$$\tilde{I} = \frac{sin(3x)}{3} - \frac{cos(3x)}{9} + c$$

No. of CSEs /No. incorrect answers **(CSE %)**	13/143 **(9%)**	**Date collected**	**2017-18**

2.3 Incorrect Algebraic Manipulation

2.3.1 Complex Numbers (Real and Imaginary parts of $z = \frac{a+bj}{c+dj}$)

Incorrect division of two complex numbers

Question			
Find the real and imaginary parts of $z = \dfrac{2-7j}{3+5j}$ correct to three decimal places. Enter $Re(z)$: [ ] Enter $Im(z)$: [ ]			
Correct Solution			
$z = \frac{2-7j}{3+5j}$ $z = \frac{(2-7j)(3-5j)}{(3+5j)(3-5j)}$ $z = \frac{-29-31j}{34}$ $Re(z) = \frac{-29}{34} = -0.853, \quad Im(z) = \frac{-31}{34} = -0.912$			
CSE 1 related to this question		**CSE Taxonomy Code**:	**S3**
Consider $z = \frac{a+bj}{c+dj}$ $Re(z) = \frac{a}{c}, Im(z) = \frac{b}{d}$ $\widetilde{Re(z)} = \frac{2}{3} = 0.667, \quad \widetilde{Im(z)} = \frac{-7}{5} = -1.4$			
No. of CSEs /No. incorrect answers **(CSE %)**	9/109 **(8%)** 9/95**(9%)**	**Date collected**	**2017-18** **2018-19**

2.3.2 Centre of Mass

Sum of product is split as a product of two sums

Question		
Masses of 4 kg, 6 kg and 10 kg are located at points with co-ordinates (-2,5) , (1,-4) and (3,1) respectively. Find the co-ordinates of their Centre of Mass, $(\overline{x}, \overline{y})$, correct to one decimal place. Enter $\overline{x}$: Enter $\overline{y}$:		
Correct Solution		
$\bar{x} = \frac{\sum_{i=1}^{3} m_i\, x_i}{\sum_{i=1}^{3} m_i} = \frac{4\times(-2)+6\times 1+10\times 3}{(4+6+10)} = 1.4$ $\bar{y} = \frac{\sum_{i=1}^{3} m_i\, y_i}{\sum_{i=1}^{3} m_i} = \frac{4\times 5+6\times(-4)+10\times 1}{(4+6+10)} = 0.3$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**S3**
Taking $\sum_{i=1}^{n} m_i x_i = \sum_{i=1}^{n} m_i \sum_{i=1}^{n} x_i$ *and* $\sum_{i=1}^{n} m_i y_i = \sum_{i=1}^{n} m_i \sum_{i=1}^{n} y_i$ $\tilde{\bar{x}} = \frac{(4+6+10)\times(-2+1+3)}{(4+6+10)} = 2$ $\tilde{\bar{y}} = \frac{(4+6+10)\times(5-4+1\ )}{(4+6+10)} = 2$		
No. of CSEs /No. incorrect answers **(CSE %)**	7/28 **(25%)** 12/64**(19%)**	**Date collected** — **2017-18** **2018-19**

2.3.3 Integration Applications (Volume of Revolution)

Incorrect handling of powers

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$.

Give your answer correct to two decimal places.

The volume of the solid is: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">The volume of revolution of y over the range $a < x < b$ given by
$$V = \pi \int_a^b y^2 dx$$
For given data,
$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$
$$= \pi \int_1^4 0.64\, x^3 dx$$
$$= 128.18$$</td></tr>
<tr><td colspan="2">CSE 4 related to this question</td><td>CSE Taxonomy Code:</td><td>S3</td></tr>
<tr><td colspan="4">Taking $(x^p)^q = x^{p^q}$
$$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$
$$\tilde{V} = \pi \int_1^4 0.8^2\, x^{1.5^2} dx$$
$$\tilde{V} = \pi \int_1^4 0.64\, x^{2.25} dx$$
$$\tilde{V} = 0.64\,\pi \left[\frac{x^{3.25}}{3.25}\right]_1^4 = 55.38$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>7/107 (7%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

2.3.4 Integration Applications (Volume of Revolution)

Incorrect handling of powers

Question			
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is: [ ]			
Correct Solution			
The volume of revolution of y over the range $a < x < b$ given by $V = \pi \int_a^b y^2 dx$ For given data, $V = \pi \int_1^4 (0.8x^{1.5})^2 dx$ $= \pi \int_1^4 0.64\, x^3 dx$ $= 128.18$			
CSE 6 related to this question	**CSE Taxonomy Code**:	**S3**	
Taking $(x^p)^q = x^{p+q}$ $V = \pi \int_1^4 (0.8x^{1.5})^2 dx$ $\tilde{V} = \pi \int_1^4 0.8^2\, x^{1.5+2} dx$ $\tilde{V} = 0.64\, \pi \left[\frac{x^{4.5}}{4.5}\right]_1^4$ $= 228.32$			
No. of CSEs /No. incorrect answers **(CSE %)**	6/135 **(4%)**	**Date collected**	**2018-19**

3 Common Student Errors due to Errors of Understanding

3.1 Confusing Different Mathematical Structures

3.1.1 Algebra (Completing the Square)

Confusing the structure of completing the square and the quadratic equation

Question
The expression $t^2 - 12t + 40$ can be expressed in the form: $a(t-b)^2 + c$ where a, b and c are constants. Calculate the values of these constants - note that all these solutions are integers: Enter the value of a Enter the value of b Enter the value of c
Correct Solution
$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$ $$= (t-6)^2 + 4$$ $$a = 1, b = 6 \text{ and } c = 4$$

CSE 3 related to this question	**CSE Taxonomy Code:**	**U1**
Incorrectly assign b and c to be the t and constant coefficients of the original expression. $t^2 - 12t + 40$ $\tilde{b} = 12$ $\tilde{c} = 40$		

No. of CSEs /No. incorrect answers **(CSE %)**	4/26 (**15%**) 2/24 (**8%**)	**Date collected**	**2017-18** **2018-19**

3.1.2 Functions (Unit-Step Function)

Stating that a unit step function is a number

Question

The function $f(t) = 7u(t+5) - 3u(t-4)$

where $u(t)$ represents the unit step function.

Calculate the value of $f(2)$.

Enter $f(2)$: [ ]

Correct Solution

$$f(t) = 7u(t+5) - 3u(t-4)$$
$$f(2) = 7u(2+5) - 3u(2-4)$$
$$= 7u(7) - 3u(-2)$$
$$= 7 \times 1 - 3 \times 0$$
$$f(2) = 7$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**U1**

Answer was derived by assuming $u = 1$ and not a function.

$$f(t) = 7u(t+5) - 3u(t-4)$$
$$f(2) = 7u(7) - 3u(-2)$$
$$\tilde{f}(2) = 7(7)u - 3(-2)u$$
$$\tilde{f}(2) = 49u + 6u$$
$$\tilde{f}(2) = 55u \quad \text{since } u = 1$$
$$\tilde{f}(2) = 55$$

No. of CSEs /No. incorrect answers **(CSE %)**	35/86(**41%**) 32/100(**32%**)	**Date collected**	**2017-18** **2018-19**

3.2 Incorrect Argument

3.2.1 Differentiation ($e^{ax}\,sin(bx)$)

Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives

Question		
Select the most appropriate method to use in order to find the derivative of $f(x) = e^{2x}\sin(5x)$. Select ▾ Hence find $\dfrac{df}{dx}$ as a function of x. Enter the answer as a function of x:		
Correct Solution		
Use the product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ $u(x) = e^{2x}$ and $v(x) = \sin(5x)$ $u'(x) = 2e^{2x}$ and $v'(x) = 5\cos(5x)$ $f'(x) = e^{2x} \times 5\cos(5x) + \sin(5x) \times 2e^{2x}$ $f'(x) = e^{2x}(2\sin(5x) + 5\cos(5x))$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**U2**
Taking $f'(x) = u'v'$ *when* $f(x) = uv$ $f(x) = e^{2x}\sin(5x)$ $u(x) = e^{2x}$ and $v(x) = \sin(5x)$ $u'(x) = 2e^{2x}$ and $v'(x) = 5\cos(5x)$ $\tilde{f}'(x) = 2e^{2x} \times 5\cos(5x)$ $\tilde{f}'(x) = 10e^{2x}\cos(5x)$		

No. of CSEs /No. incorrect answers **(CSE %)**	13/73**(18%)**	**Date collected**	**2018-19**

3.2.2 Implicit Differentiation

Incorrectly assuming the derivative of the product of two functions is equal to the product of the individual derivatives

Question

Given

$x^4 + 2x^2y^3 = 5y,$

find the derivative $\frac{dy}{dx}$ as a function of x and y.

$\frac{dy}{dx} =$ [ ]

Correct Solution

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**U2**

Taking $\frac{d}{dx}(f(x)g(y)) = \frac{d}{dx}(f(x)) \times \frac{d}{dx}(g(y))$

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 4x \times 3y^2\frac{dy}{dx} + y^4 = 5\frac{dy}{dx}$$

$$\frac{\widetilde{dy}}{dx} = \frac{4x^3}{5 - 12xy^2}$$

No. of CSEs /No. incorrect answers **(CSE %)**	18/158 **(11%)** 15/180 **(8%)**	**Date collected**	**2017-18** **2018-19**

3.2.3 Integration ($\int xe^{ax^n} dx$)

Taking the integration of the product of two functions as the product of individual integrals

Question		
Evaluate the following: $\int xe^{4x^2} dx$ as a function of x, to within an additive constant (i.e. do not put a "+c" in your answer). Enter the answer as a function of x:		
Correct Solution		
Use the method of substitution and put $u = 4x^2$, then $\frac{du}{dx} = 8x$ $I = \int xe^{4x^2} dx = \int \frac{1}{8} e^u du$ $= \frac{1}{8} e^u + c$ $= \frac{1}{8} e^{4x^2} + c$		
CSE 3 related to this question	**CSE Taxonomy Code**:	**U2**
Taking $\int u(x)v(x)dx = \int u(x)dx \times \int v(x)dx$ *and* $\int e^{ax^2} dx = \frac{e^{ax^2}}{a}$ $\tilde{I} = \int xe^{4x^2} dx = \int x\, dx \times \int e^{4x^2} dx$ $= \frac{x^2}{2} \times \frac{e^{4x^2}}{4}$ $= \frac{x^2 e^{4x^2}}{8}$		

No. of CSEs /No. incorrect answers **(CSE %)**	10/192**(5%)**	**Date collected**	**2018-19**

3.2.4 Integration ($\int x \cos(ax)\, dx$)

Taking the integration of the product of two functions as the product of individual integrals

Question

Evaluate the following:

$$\int x \cos(3x) dx$$

as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:

Correct Solution

Use integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Take

$$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$

$$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$

$$I = \int x\cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3} dx$$

$$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 4 related to this question	**CSE Taxonomy Code**:	**U2**

Integrating x and the trigonometric function separately, and then multiplying them together to get the answer

$$I = \int x\,cos(3x) = \left(\frac{x^2}{2}\right) \times \left(\frac{sin(3x)}{3}\right) + c$$

$$\tilde{I} = \frac{x^2 sin(3x)}{6} + c$$

No. of CSEs /No. incorrect answers **(CSE %)**	8/143 **(6%)**	**Date collected**	**2017-18**

3.3 Lack of Consideration of Potential Indeterminate Forms

3.3.1 Complex Numbers (Modulus of $\boldsymbol{z = -a + bj}$)

Taking the square of a negative number to be negative

Question			
Find the modulus $\lvert z \rvert$ of the complex number $z = -2 + 5j$, correct to two decimal places. Enter $\lvert z \rvert$ correct to 2 decimal places: [ ]			
Correct Solution			
$z = -2 + 5j$ $\lvert z \rvert = \sqrt{(-2)^2 + 5^2}$ $= \sqrt{4 + 25}$ $= \sqrt{29}$ $\lvert z \rvert = 5.39$			
CSE 1 related to this question		**CSE Taxonomy Code**:	**U3**
Taking $(-n)^2 = -n^2$ $z = -2 + 5j$ $\lvert z \rvert = \sqrt{(-2)^2 + 5^2}$ $\lvert \tilde{z} \rvert = \sqrt{-4 + 25}$ $= \sqrt{21}$ $\lvert \tilde{z} \rvert = 4.58$			
No. of CSEs /No. incorrect answers **(CSE %)**	40/57**(70%)**	**Date collected**	**2017-18**

3.4 Proposed Solution is not Viable

3.4.1 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Correct working but unfinished solutions and angles is not in the given range

Question
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \leq \pi$. Enter your answers correct to three decimal places. Enter $r = \|z\|$: Enter $\theta = arg(z)$:
Correct Solution
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times0.8-2\times-0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$ $\|z\| = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U4, U6**
Neglecting the required range of $arg(z)$ $z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$ $\|z\| = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\widetilde{arg(z)} = 3.2$		

No. of CSEs /No. incorrect answers **(CSE %)**	66/ 197**(34%)**	**Date collected**	**2017-18**

3.5 Definition/Method/Theorem not Recalled Correctly

3.5.1 Algebra (Completing the Square

Method of completing the square is not recalled correctly

Question

The expression

$t^2 - 12t + 40$

can be expressed in the form:

$a(t-b)^2 + c$

where a, b and c are constants.

Calculate the values of these constants - note that all these solutions are integers:

Enter the value of a [ ]

Enter the value of b [ ]

Enter the value of c [ ]

Correct Solution

$$t^2 - 12t + 40 = (t-6)^2 - 36 + 40$$

$$= (t-6)^2 + 4$$

$$a = 1, b = 6 \text{ and } c = 4$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**

Incorrectly add b^2 instead of subtracting b^2 when completing the square.

$$t^2 - 12t + 40 = (t-6)^2 + 36 + 40$$

$$= (t-6)^2 + 76$$

$$\tilde{c} = 76$$

No. of CSEs /No. incorrect answers **(CSE %)**	6/45 (**13%**) 14/51(**27%**)	**Date collected**	**2017-18** **2018-19**

3.5.2 Wave Forms $\boldsymbol{y = A\sin(\alpha t)}$

Definition of waveform properties not recalled correctly

<table>
<tr><td colspan="4"><u>Question</u></td></tr>
<tr><td colspan="4">Consider the waveform $y = 14\sin(8t)$.

State the amplitude of this waveform and the number of cycles in a interval of length 2π of the waveform.

Enter the amplitude: [ ]
Enter the number of cycles in a 2π interval: [ ]</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$y = 14\sin(8t)$

The amplitude $A = 14$
The number of cycles $\omega = 8$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Swapped the correct answers for the amplitude and the number of cycles.

$y = 14\sin(8t)$

The amplitude $\tilde{A} = 8$

The number of cycles $\tilde{\omega} = 14$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>5/13(38%)
9/19(47%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

3.5.3 Wave Forms $y = A\sin(\alpha t)$

Definition of waveform properties not recalled correctly

Question			
Consider the waveform $y = 14\sin(8t)$. State the amplitude of this waveform and the number of cycles in a interval of length 2π of the waveform. Enter the amplitude: [ ] Enter the number of cycles in a 2π interval: [ ]			
Correct Solution			
$y = 14\sin(8t)$ The amplitude $A = 14$ The number of cycles $\omega = 8$			
CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**	
Taking the number of cycles in a π interval to be ω The number of cycles in a π interval = 8 Therefore, the number of cycles in 2π interval $\tilde{\omega}$ = 16			
No. of CSEs /No. incorrect answers **(CSE %)**	7/29(**24%**)	**Date collected**	**2018-19**

3.5.4 Trigonometric Function $\sin(\theta) = A$

Method of solving a trigonometric equation not recalled correctly

Question
Find the two values of θ in the interval $-180^\circ < \theta \leq 180^\circ$ such that $\sin(\theta) = -0.55$.   Enter your answers to θ_1 and θ_2 **in degrees**, correct to **two decimal places**.   Enter θ_1 : [ ] to two decimal places   Enter θ_2 : [ ] to two decimal places   *(Please note that the order in which you enter your answers does not matter.)*
Correct Solution
$\sin(\theta) = -0.55$ $\theta_1 = \sin^{-1}(-0.55)$ $\theta_1 = -33.37^0$ $\theta_2 = -180^0 + 33.37^0$ $\theta_2 = -146.63^0$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**
Add 180^0 to θ_1 to find θ_2 $\sin(\theta) = -0.55$ $\theta_1 = \sin^{-1}(-0.55)$ $\theta_1 = -33.37^0$ $\tilde{\theta}_2 = -33.37^0 + 180^0$ $\tilde{\theta}_2 = 146.63^0$		
No. of CSEs /No. incorrect answers **(CSE %)**	37/61 **(61 %)**	**Date collected** **2017-18**

3.5.5 Trigonometric Function $\sin(2\theta) = A$

Method of solving a trigonometric equation not recalled correctly

Question
Solve the equation $\sin(2t) = 0.45$ for t, where $0 \leq t \leq \pi$. Enter your two values of t **in radians**, correct to **two decimal places**. Enter t_1 : [ ] to two decimal places Enter t_2 : [ ] to two decimal places
Correct Solution
Let $z = 2t$, then the problem is equivalent to solving the equation: $\sin(z) = 0.45$ for z, where $0 \leq z \leq 2\pi$ $z_1 = \sin^{-1}(0.45)$ and $z_2 = \pi - \sin^{-1}(0.45)$ $z_1 = 0.44477$ and $z_2 = 2.67483$ $t_1 = 0.23$ and $t_2 = 1.34$

CSE 1 related to this question	CSE Taxonomy Code:	U5

Taking $t_2 = \pi - t_1$

$$\sin(2t) = 0.45$$

$$t_1 = \frac{\sin^{-1}(0.45)}{2} = 0.23$$

$$\tilde{t_2} = \pi - t_1 = \pi - 0.23 = 2.91$$

No. of CSEs /No. incorrect answers **(CSE %)**	47/81 **(58 %)**	**Date collected**	**2018-19**

3.5.6 Differentiation ($\boldsymbol{ln(ax)}$)

Method of differentiating a standard function is not recalled correctly

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Obtain the derivative of the function
$f(x) = 3\ln(5x)$
Enter the answer as a function of x:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$f(x) = 3\ln(5x)$
$f'(x) = 3 \times \frac{1}{5x} \times 5$
$f'(x) = \frac{3}{x}$</td></tr>
<tr><td colspan="2">CSE 1 related to this question</td><td>CSE Taxonomy Code:</td><td>U5</td></tr>
<tr><td colspan="4">Taking the differential of $\ln(ax)$ to be $\frac{1}{ax}$
$f(x) = 3\ln(5x)$
$\tilde{f}'(x) = 3 \times \frac{1}{5x}$
$\tilde{f}'(x) = \frac{3}{5x}$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>30/59 (51%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

3.5.7 Differentiation ($\boldsymbol{ln(ax)}$)

Method of differentiating a standard function is not recalled correctly

Question			
Obtain the derivative of the function $f(x) = 8\ \ln(4x) - 3x^2$ and find the value of this derivative when $x = 0.6$. Enter $f'(0.6)$, correct to two decimal places:			
Correct Solution			
$f(x) = 8\ln(4x) - 3x^2$ $f'(x) = 8 \times \frac{1}{4x} \times 4 - 6x$ $f'(x) = \frac{8}{x} - 6x$ $f'(0.6) = \frac{8}{0.6} - 6(0.6)$ $f'(0.6) = 9.73$			
CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**	
Taking differentiation of $ln(ax)$ as $\frac{1}{ax}$ $f(x) = 8\ln(4x) - 3x^2$ $\tilde{f}'(x) = 8 \times \frac{1}{4x} - 6x$ $\tilde{f}'(x) = \frac{2}{x} - 6x$ $\tilde{f}'(x) = \frac{2}{0.6} - 6(0.6)$ $\tilde{f}'(x) = -0.27$			
No. of CSEs /No. incorrect answers **(CSE %)**	30/59 **(51%)** 7/33 **(21%)**	**Date collected**	**2017-18**[1] **2018-19**

[1] In 2017/18 the question asked for an algebraic entry (as opposed) to numerical but CSE was the same.

3.5.8 Differentiation ($\boldsymbol{ln(ax)}$)

Method of differentiating a standard function not recalled correctly

Question			
Obtain the derivative of the function $f(x) = 8\ \ln(4x) - 3x^2$ and find the value of this derivative when $x = 0.6$. Enter $f'(0.6)$, correct to two decimal places: ☐			
Correct Solution			
$f(x) = 8\ln(4x) - 3x^2$ $f'(x) = 8 \times \frac{1}{4x} \times 4 - 6x$ $f'(x) = \frac{8}{x} - 6x$ $f'(0.6) = \frac{8}{0.6} - 6(0.6)$ $f'(0.6) = 9.73$			
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**	
Taking the differential of $ln(ax)$ to be $\frac{a}{x}$ $f(x) = 8\ln(4x) - 3x^2$ $\tilde{f}'(x) = 8 \times \frac{4}{x} - 6x$ $\tilde{f}'(x) = \frac{32}{x} - 6x$ $\tilde{f}'(x) = \frac{32}{0.6} - 6(0.6)$ $\tilde{f}'(x) = 49.73$			
No. of CSEs /No. incorrect answers **(CSE %)**	5/33 **(15%)**	**Date collected**	**2018-19**

3.5.9 Differentiation ($cos(ax^n)$)

Chain rule is not recalled correctly

Question

Select the most appropriate method to use in order to find the derivative of $f(x) = \cos(2x^7)$.

Select ▾

Hence find $\dfrac{df}{dx}$ as a function of x.

Enter the answer as a function of x:

Correct Solution

$$f(x) = \cos(2x^7)$$

$$f'(x) = -\sin(2x^7) \times 14x^6$$

$$f'(x) = -14x^6 \sin(2x^7)$$

CSE 1 related to this question	**CSE Taxonomy Code:**	**U5**

Taking $\frac{d}{dx}(cos(ax^n)) = -a\ sin(ax^n)$;

overgeneralising $\frac{d}{dx}(cos(ax)) = -a\ sin(ax)$

$$f(x) = \cos(2x^7)$$

$$\tilde{f}'(x) = -2\sin(2x^7)$$

No. of CSEs /No. incorrect answers **(CSE %)**	7/56 **(13%)**	**Date collected**	**2017-18**

3.5.10 Differentiation ($\boldsymbol{cos^n(ax)}$)

Chain rule is not recalled correctly

Question			
Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$. Select ▾ Hence find $\frac{df}{dx}$ as a function of x. Enter the answer as a function of x:			
Correct Solution			
$f(x) = cos^4(3x)$ $f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$ $f'(x) = -12\sin(3x)\cos^3(3x)$			
CSE 1 related to this question		**CSE Taxonomy Code**:	**U5**
Taking $\frac{d}{dx}(cos^n(ax)) = -a\ sin^n(ax)$ *;* *overgeneralising* $\frac{d}{dx}(cos(ax)) = -a\ sin(ax)$ $f(x) = cos^4(3x)$ $\tilde{f}'(x) = -\sin^4(3x) \times 3$ $\tilde{f}'(x) = -3\sin^4(3x)$			
No. of CSEs /No. incorrect answers **(CSE %)**	6/73**(8%)**	**Date collected**	**2017-18**

3.5.11 Implicit Differentiation

Method of differentiating implicit functions is not recalled correctly

Question

Given

$x^4 + 2x^2y^3 = 5y,$

find the derivative $\frac{dy}{dx}$ as a function of x and y.

$\frac{dy}{dx} =$ [ ]

Correct Solution

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**

Differentiating functions of x with respect to x and functions of y with respect to y, separately. Equating the answer to $\frac{dy}{dx}$

$$x^4 + 2x^2y^3 = 5y$$

$$4x^3 + 4x \times 3y^2 = 5$$

$$\widetilde{\frac{dy}{dx}} = 4x^3 + 4x \times 3y^2 - 5$$

$$\widetilde{\frac{dy}{dx}} = 4x^3 + 12xy^2 - 5$$

No. of CSEs /No. incorrect answers **(CSE %)**	12/158 **(8%)** 5/180**(3%)**	**Date collected**	**2017-18**

3.5.12 Implicit Differentiation

Method of differentiating implicit functions is not recalled correctly

Question		
Given $x^4 + 2x^2y^3 = 5y,$ find the derivative $\frac{dy}{dx}$ as a function of x and y. $\frac{dy}{dx} =$ [ ]		
Correct Solution		
$x^4 + 2x^2y^3 = 5y$ $4x^3 + 2x^2 \times 3y^2\frac{dy}{dx} + 4x \times y^3 = 5\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{4x^3 + 4xy^3}{5 - 6x^2y^2}$		
CSE 3 related to this question	**CSE Taxonomy Code:**	**U5**
Taking y to be a constant when differenting with respect to x. *Equating the answer to $\frac{dy}{dx}$* $x^4 + 2x^2y^3 = 5y$ $4x^3 + 4x \times y^3$ Therefore, $\widetilde{\frac{dy}{dx}} = 4x^3 + 4xy^3$		

No. of CSEs /No. incorrect answers **(CSE %)**	10/158 **(6%)** 2/180 **(1%)**	**Date collected**	**2017-18** **2018/19**

3.5.13 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

Question
Given $z(x, y) = 3\cos(xy^4)$ find $\frac{\partial z}{\partial x}$. Enter your answer below as a function of x and y :
Correct Solution
$z(x, y) = 3\cos(xy^4)$ $\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$ $\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**

Missing the partial differential of xy^n with respect to x

$$z(x, y) = 3\cos(xy^4)$$

$$\frac{\widetilde{\partial z}}{\partial x} = -3\sin(xy^4)$$

No. of CSEs /No. incorrect answers **(CSE %)**	17/ 103 **(17%)**	**Date collected**	**2017-18**

3.5.14 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x,y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

Question

Given

$z(x,y) = 3\cos(xy^4)$

find $\frac{\partial z}{\partial x}$.

Enter your answer below as a function of x and y :

Correct Solution

$$z(x,y) = 3\cos(xy^4)$$

$$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$$

$$\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U5**

Taking $\frac{\partial}{\partial x}(a\cos(xy^n)) = -a\sin(xy^n) \times ny^{n-1}$

$$z(x,y) = 3\cos(xy^4)$$

$$\frac{\widetilde{\partial z}}{\partial x} = -3\sin(xy^4) \times 4y^3$$

$$\frac{\widetilde{\partial z}}{\partial x} = -12y^3\sin(xy^4)$$

No. of CSEs /No. incorrect answers **(CSE %)**	7/ 103 **(7%)**	**Date collected**	**2017-18**

3.5.15 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

Question

Given

$z(x, y) = 3\cos(xy^4)$

find $\frac{\partial z}{\partial x}$.

Enter your answer below as a function of x and y :

Correct Solution

$$z(x, y) = 3\cos(xy^4)$$

$$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$$

$$\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**U5**

Taking $\frac{\partial}{\partial x}(a\cos(xy^n)) = -a\sin(y^n)$

$$z(x, y) = 3\cos(xy^4)$$

$$\frac{\widetilde{\partial z}}{\partial x} = -3\sin(y^4)$$

No. of CSEs /No. incorrect answers **(CSE %)**	6/ 103 **(6%)**	**Date collected**	**2017-18**

3.5.16 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x,y) = a\cos(x^n y^m)$)

Method of partial differentiation not recalled correctly

Question

Given

$z(x, y) = 3\cos(x^2 y^4)$

find $\dfrac{\partial z}{\partial x}$.

Enter your answer below as a function of x and y :

Correct Solution

$$z(x,y) = 3\cos(x^2y^4)$$

$$\frac{\partial z}{\partial x} = -3\sin(x^2y^4) \times 2xy^4$$

$$= -6xy^4\sin(x^2y^4)$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**

Taking $\dfrac{\partial(a\cos(x^n y^m))}{\partial x} = -a\sin(x^n y^m) \times nx^{n-1} = -anx^{n-1}\sin(x^n y^m)$

$$z(x,y) = 3\,cos(x^2y^4)$$

$$\frac{\partial z}{\partial x} = -3 \times sin(x^2y^4) \times 2x$$

$$= -6x\,sin(x^2y^4)$$

No. of CSEs /No. incorrect answers **(CSE %)**	32/ 145 **(22%)**	**Date collected**	**2018-19**

3.5.17 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x, y) = acos(xy^n)$)

Method of partial differentiation not recalled correctly

Question
Given $z(x, y) = 3\cos(x^2y^4)$ find $\dfrac{\partial z}{\partial x}$. Enter your answer below as a function of x and y :

Correct Solution
$z(x, y) = 3\cos(x^2y^4)$ $\dfrac{\partial z}{\partial x} = -3\sin(x^2y^4) \times 2xy^4$ $= -6xy^4\sin(x^2y^4)$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Missing the partial differential of x^ny^m with respect to x $z(x, y) = 3\cos(x^2y^4)$ $\dfrac{\partial z}{\partial x} = -3\sin(x^2y^4)$		

No. of CSEs /No. incorrect answers **(CSE %)**	11/ 145 **(8%)**	**Date collected**	**2018-19**

3.5.18 Integration Application (Mean Value of $(at^n + b)$)

Mean value theorem is not recalled correctly

Question
Given that $f(t) = 3t^5 + 4$, find the mean value of $f(t)$ in the interval $1 < t < 3$, correct to two decimal places. Enter correct to two decimal places:
Correct Solution
The mean value, m, of $f(t)$ in the interval $a < t < b$ is given by $$\frac{1}{b-a}\int_a^b f(t)dt$$ $$m = \frac{1}{(3-1)}\int_1^3 (3t^5 + 4)dt$$ $$= \frac{1}{2}\left[\frac{t^6}{2} + 4t\right]_1^3$$ $$= 186.00$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**
Taking the mean value as $\int_a^b f(t)dt$ *instead of* $\frac{1}{b-a}\int_a^b f(t)dt$ $$\tilde{m} = \int_1^3 (3t^5 + 4)dt$$ $$= \left[\frac{t^6}{2} + 4t\right]_1^3$$ $$= 372$$		
No. of CSEs /No. incorrect answers **(CSE %)**	10/121 **(8%)**	**Date collected**	**2018-19**

3.5.19 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Method of finding the argument of a complex number is not recalled correctly

Question			
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \leq \pi$. Enter your answers correct to three decimal places. Enter $r = \lvert z \rvert$: [ ] Enter $\theta = arg(z)$: [ ]			
Correct Solution			
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times 0.8 - 2\times -0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$			
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**	
Taking $arg(z) = \theta_1^n - \theta_2^m$ $\widetilde{arg(z)} = (0.8)^3 - (-0.4)^2 = 0.352$			
No. of CSEs /No. incorrect answers **(CSE %)**	9/197**(5%)**	**Date collected**	**2017-18**

3.5.20 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Method of finding the principle value of the argument of a complex number is not recalled correctly

Question			
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \le \pi$. Enter your answers correct to three decimal places. Enter $r = \lvert z \rvert$: [ ] Enter $\theta = arg(z)$: [ ]			
Correct Solution			
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times 0.8 - 2\times -0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050 \text{ and } \arg(z) = 3.2 - 2\pi = -3.083$			
CSE 3 related to this question	**CSE Taxonomy Code**:		**U5**
Taking $arg(z) = \theta - \pi$ $\widetilde{arg(z)} = 3.2 - \pi = 0.058$			
No. of CSEs /No. incorrect answers **(CSE %)**	17/197**(9%)**	**Date collected**	**2017-18**

3.5.21 Complex Numbers (Argument of $z = a - bj$)

Method of finding the principle value of the argument of a complex number is not recalled correctly

Question			
Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \le \pi$. Express your answers correct to three decimal places. Enter r: [ ] Enter θ: [ ]			
Correct Solution			
$z = 2 - 3j$ $r = \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{13}$ $r = 5.39$ z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -0.983$			
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**	
Taking $\theta = \pi + \tan^{-1}\left(\frac{-b}{a}\right)$ *when* $z = a - bj$ $\tilde{\theta} = \pi + \tan^{-1}\left(\frac{-3}{2}\right)$ $= \pi - 0.983$ $= 2.159$			
No. of CSEs /No. incorrect answers **(CSE %)**	13/88**(15%)**	**Date collected**	**2018-19**

3.5.22 Binomial Series

Binomial theorem is incorrectly followed

Question		
Use the binomial theorem to expand $(2x+3y)^5$ and enter the fourth term (that is the full term involving x^2y^3) as a function of x and y :		
Correct Solution		
When n is a positive integrer, the binomial theorem states that $$(ax+by)^n = \binom{n}{0}(ax)^n + \binom{n}{1}(ax)^{n-1}(by)^1 + \binom{n}{2}(ax)^{n-2}(by)^2 + \cdots + \binom{n}{n}(by)^n$$ So in this case, the fourthterm is $$T_4 = \binom{5}{3}(2x)^{5-3}(3y)^{5-2}$$ $$= \frac{5!}{3! \times 2!}(2x)^2(3y)^3$$ $$= 1080x^2y^3$$		
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Taking the r^{th} term as $\binom{n}{r-1}a(x)^{n-r+1}b(y)^{r-1}$ *instead of* $\binom{n}{r-1}(ax)^{n-r+1}(by)^{r-1}$ $$(2x+3y)^5 = \binom{5}{0}2(x)^5 + \binom{5}{1}2(x)^4 3(y)^1 + \binom{5}{2}2(x)^3 3(y)^2 + \binom{5}{3}2(x)^2 3(y)^3 + \binom{5}{4}2(x)^1 3(y)^4 + \binom{5}{5}3(y)^5$$ $$\widetilde{T}_4 = \binom{5}{3}2(x)^2 3(y)^3$$ $$= 60x^2y^3$$		

No. of CSEs /No. incorrect answers **(CSE %)**	8/165**(5%)**	**Date collected**	**2018-19**

3.5.23 Centre of Mass

Definition of Centre of Mass is not recalled correctly

Question		
Masses of 4 kg, 6 kg and 10 kg are located at points with co-ordinates (-2,5) , (1,-4) and (3,1) respectively. Find the co-ordinates of their Centre of Mass,$(\bar{x},\bar{y})$, correct to one decimal place. Enter $\bar{x}$: ____ Enter $\bar{y}$: ____		
Correct Solution		
$$\bar{x} = \frac{\sum_{i=1}^{3} m_i x_i}{\sum_{i=1}^{3} m_i} = \frac{4\times(-2)+6\times 1+10\times 3}{(4+6+10)} = 1.4$$ $$\bar{y} = \frac{\sum_{i=1}^{3} m_i y_i}{\sum_{i=1}^{3} m_i} = \frac{4\times 5+6\times(-4)+10\times 1}{(4+6+10)} = 0.3$$		
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Taking $\bar{x} = \sum_{i=1}^{n} m_i x_i$ *and* $\bar{y} = \sum_{i=1}^{n} m_i y_i$ $\tilde{\bar{x}} = 4\times(-2)+6\times 1+10\times 3 = 28$ $\tilde{\bar{y}} = 4\times 5+6\times(-4)+10\times 1 = 6$		

No. of CSEs /No. incorrect answers **(CSE %)**	11/64**(17%)**	**Date collected**	**2018-19**

3.5.24 Integration ($\int xe^{ax^n}dx$)

Method of integrating not recalled correctly

Question		
Evaluate the following: $$\int xe^{4x^2}dx$$ as a function of x, to within an additive constant (i.e. do not put a "+c" in your answer). Enter the answer as a function of x:		
Correct Solution		
Use the method of substitution and take $u = 4x^2$, then $\frac{du}{dx} = 8x$ $$I = \int xe^{4x^2}dx = \int \frac{1}{8}e^u du$$ $$= \frac{1}{8}e^u + c$$ $$= \frac{1}{8}e^{4x^2} + c$$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**
Use integration by parts and take $\int e^{ax^2}dx = \frac{e^{ax^2}}{4a}$ Let $u = x$, *and* $\frac{dv}{dx} = e^{4x^2}$, *then* $\frac{du}{dx} = 1$ *and* $v = \frac{e^{4x^2}}{4}$ Using integration by parts, $$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$ $$\tilde{I} = \int xe^{4x^2}dx = \frac{xe^{4x^2}}{4} - \int \frac{1}{4}e^{4x^2}dx$$ $$= \frac{xe^{4x^2}}{4} - \frac{e^{4x^2}}{16}$$		
No. of CSEs /No. incorrect answers **(CSE %)**	19/192**(10%)**	**Date collected** — **2018-19**

3.5.25 Integration ($\int xe^{ax^n}dx$)

Method of integrating not recalled correctly

Question		
Evaluate the following: $$\int xe^{4x^2}dx$$ as a function of x, to within an additive constant (i.e. do not put a "+c" in your answer). Enter the answer as a function of x:		
Correct Solution		
Use the method of substitution and put $u = 4x^2$, then $\frac{du}{dx} = 8x$ $$I = \int xe^{4x^2}dx = \int \frac{1}{8}e^u du$$ $$= \frac{1}{8}e^u + c$$ $$= \frac{1}{8}e^{4x^2} + c$$		
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Use integration by parts and take $\int e^{ax^2}dx = \frac{e^{ax^2}}{2ax}$ Let $u = x$, *and* $\frac{dv}{dx} = e^{4x^2}$, *then* $\frac{du}{dx} = 1$ *and* $v = \frac{e^{4x^2}}{8x}$ Using integration by parts, $$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$ $$\tilde{I} = \int xe^{4x^2}dx = \frac{xe^{4x^2}}{8x} - \int \frac{e^{4x^2}}{8x}dx$$ $$= \frac{xe^{4x^2}}{8} - \frac{e^{4x^2}}{8x} \times \frac{1}{8x}$$ $$= \frac{e^{4x^2}}{8} - \frac{e^{4x^2}}{64x^2}$$		

No. of CSEs /No. incorrect answers **(CSE %)**	14/192**(7%)**	**Date collected**	**2018-19**

3.5.26 Integration ($\int x\cos(ax)\,dx$)

Method of integrating not recalled correctly

Question
Evaluate the following: $$\int x\cos(3x)dx$$ as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:
Correct Solution
Use integration by parts $$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$ Take $$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$ Then $$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$ $$I = \int x\cos(3x)\,dx = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$ $$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**
Treating the x in front of the trigonometric function as a constant $$\tilde{I} = \int x\cos(3x)dx = x\left(\frac{\sin(3x)}{3}\right)$$		
No. of CSEs /No. incorrect answers **(CSE %)**	13/143 **(9%)**	**Date collected**	**2017-18**

3.5.27 Integration ($\int \boldsymbol{x}\cos(\boldsymbol{ax})\,\boldsymbol{dx}$)

Method of integrating is not recalled correctly

Question		
Evaluate the following: $$\int x\cos(3x)dx$$ as a function of x, to within an additive constant (do not put a "+c" in your answer). Enter the answer as a function of x:		
Correct Solution		
Use integration by parts $$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$ Take $$u(x) = x, \qquad \frac{dv}{dx} = \cos(3x)$$ $$\frac{du}{dx} = 1, \qquad v(x) = \frac{\sin(3x)}{3}$$ $$I = \int x\cos(3x) = x\left(\frac{\sin(3x)}{3}\right) - \int \frac{\sin(3x)}{3}dx$$ $$I = \frac{x\sin(3x)}{3} + \frac{\cos(3x)}{9} + c$$		
CSE 3 related to this question	**CSE Taxonomy Code**:	**U5**
Missed out x in front of the trigonometric function $$\tilde{I} = \int \mathrm{x}\cos(3\mathrm{x})\mathrm{dx} = \left(\frac{\sin(3\mathrm{x})}{3}\right)$$		

No. of CSEs /No. incorrect answers **(CSE %)**	11/143 **(8%)**	**Date collected**	**2017-18**

3.5.28 Integration Applications (Volume of Revolution)

Definition of volume of revolution is not recalled correctly

Question		
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is: [ ]		
Correct Solution		
The volume of revolution of y over the range $a < x < b$ given by $$V = \pi \int_a^b y^2 dx$$ For given data, $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$= \pi \int_1^4 0.64\, x^3 dx$$ $$= 128.18$$		
CSE 1 related to this question	**CSE Taxonomy Code**:	**U5**
Missing π $$\tilde{V} = \int_a^b y^2 dx$$ $$\tilde{V} = \int_1^4 (0.8x^{1.5})^2 dx = \int_1^4 0.64\, x^3 dx = 40.80$$		

No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 3/135**(2%)**	**Date collected**	**2017-18** **2018-19**

3.5.29 Integration Applications (Volume of Revolution)

Definition of volume of revolution is not recalled correctly

Question		
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is: [ ]		
Correct Solution		
The volume of revolution of y over the range $a < x < b$ given by $$V = \pi \int_a^b y^2 dx$$ For given data, $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$= \pi \int_1^4 0.64\, x^3 dx$$ $$= 128.18$$		
CSE 2 related to this question	**CSE Taxonomy Code**:	**U5**
Taking $V = \int_a^b y\, dx$ $$\tilde{V} = \int_1^4 0.8\, x^{1.5} dx$$ $$= 0.8 \left[\frac{x^{2.5}}{2.5}\right]_1^4$$ $$= 9.92$$		

No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 13/135 **(10%)**	**Date collected**	**2017-18** **2018-19**

3.5.30 Integration Applications (Volume of Revolution)

Definition of volume of revolution is not recalled correctly

Question			
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is:			
Correct Solution			
The volume of revolution of y over the range $a < x < b$ given by $$V = \pi \int_a^b y^2 dx$$ For given data, $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$= \pi \int_1^4 0.64\, x^3 dx$$ $$= 128.18$$			
CSE 3 related to this question	**CSE Taxonomy Code**:	**U5**	
Taking $V = \pi \int_a^b y\, dx$ $$\tilde{V} = \pi \int_1^4 0.8\, x^{1.5} dx$$ $$= 0.8\, \pi \left[\frac{x^{2.5}}{2.5}\right]_1^4$$ $$= 31.16$$			
No. of CSEs /No. incorrect answers **(CSE %)**	9/107 **(8%)** 8/135 **(6%)**	**Date collected**	**2017-18** **2018-19**

3.6 Partial Solution Given

3.6.1 Trigonometric Function $\sin(2\theta) = A$

Correct working but unfinished solutions

Question			
Solve the equation $\sin(2t) = 0.45$ for t, where $0 \leq t \leq \pi$. Enter your two values of t **in radians**, correct to **two decimal places**. Enter t_1 : [ ] to two decimal places Enter t_2 : [ ] to two decimal places *(Please note that the order in which you enter your answers does not matter.)*			
Correct Solution			
Let $z = 2t$, then the problem is equivalent to solving the equation: $\sin(z) = 0.45$ for z, where $0 \leq z \leq 2\pi$ $z_1 = \sin^{-1}(0.45)$ and $z_2 = \pi - \sin^{-1}(0.45)$ $z_1 = 0.44477$ and $z_2 = 2.67483$ $t_1 = 0.23$ and $t_2 = 1.34$			
CSE 2 related to this question	**CSE Taxonomy Code:**	**U6**	
Submitted 2t values for t $\sin(2t) = 0.45$ $\tilde{t_1} = \sin^{-1}(0.45)$ and $\tilde{t_2} = \pi - \sin^{-1}(0.45)$ $\tilde{t_1} = 0.44477$ and $\tilde{t_2} = 2.67483$			
No. of CSEs /No. incorrect answers **(CSE %)**	9/84 **(11 %)**	**Date collected**	**2018-19**

3.6.2 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Correct working but unfinished solutions and angles is not in the given range

Question			
Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of $z = \frac{(w_1)^3}{(w_2)^2}$, where $-\pi < \theta \leq \pi$. Enter your answers correct to three decimal places. Enter $r = \lvert z \rvert$: [ ] Enter $\theta = arg(z)$: [ ]			
Correct Solution			
$z = \frac{(w_1)^3}{(w_2)^2}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times0.8-2\times-0.4)}$ $z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$			
CSE 1 related to this question	**CSE Taxonomy Code**:	**U4, U6**	
Neglecting the required range of $arg(z)$ $z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$ $\lvert z \rvert = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\widetilde{arg(z)} = 3.2$			
No. of CSEs /No. incorrect answers **(CSE %)**	66/ 197**(34%)**	**Date collected**	**2017-18**

3.6.3 Series (Maclaurin Expansion)

Omission of denominator / correct working and unfinished solution is given

Question
Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$, of $f(x) = e^{4x}$ up to and including the cubic term.   Give the values of a_1 and a_2 below.   Enter your answer for a_1 (to three decimal places) here: Enter your answer for a_2 (to three decimal places) here: Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$ Enter your approximate value for $e^{0.8}$ (to three decimal places) here:
Correct Solution
Using MacLaurin expansion, $$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \ldots$$ $$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$ $$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$ $$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**S2, U6**
Missing division by 2! $\widetilde{a_2} = 4^2 = 16.000$		

No. of CSEs /No. incorrect answers **(CSE %)**	11/59**(19%)** 16/80**(20%)**	**Date collected**	**2017-18** **2018-19**

3.7 Incorrect Assumptions

3.7.1 Integration Application (Mean Value of $\boldsymbol{a}\ \mathbf{sin}(\boldsymbol{bt})$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places: [ ]
Correct Solution
$$f(t) = 4\ \sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\ \sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10\text{t})\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$

CSE 2 related to this question	**CSE Taxonomy Code**:	**U7, UM2**

Directly substituting the midpoint of the range of t in degrees into the given function.

$$f(t) = 4\ \sin(10t)$$

Let the mean value of $f(t) = m$. The middle point of the range of $t\ = 4$

$$\tilde{m} = 4\sin(10 \times 4^o) = 4\sin(40^o)$$

$$\tilde{m} = 2.57$$

No. of CSEs /No. incorrect answers **(CSE %)**	44/183 **(24%)**	**Date collected**	**2017-18**

3.7.2 Integration Application (Mean Value of $\boldsymbol{a}\ \mathbf{sin}(\boldsymbol{bt})$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places:
Correct Solution
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10\mathrm{t})\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**U7, UM2**
Substituting the end values of the range t into the given function in degrees and then taking the average. $f(t) = 4\sin(10t)$, let the mean value of $f(t) = m$ $$\tilde{m} = \frac{4\sin(10 \times 2^o) + 4\sin(10 \times 6^o)}{2}$$ $$= 2.42$$		
No. of CSEs /No. incorrect answers **(CSE %)**	8/183 **(4%)**	**Date collected**	**2017-18**

3.7.3 Integration Application (Mean Value of $(at^n + b)$)

Incorrect assumptions on the mean value of a function

Question

Given that $f(t) = 3t^5 + 4$, find the mean value of $f(t)$ in the interval $1 < t < 3$, correct to two decimal places.

Enter correct to two decimal places: [ ]

Correct Solution

The mean value, m, of $f(t)$ in the interval $a < t < b$ is given by

$$\frac{1}{b-a}\int_a^b f(t)dt$$

$$m = \frac{1}{(3-1)}\int_1^3 (3t^5 + 4)dt$$

$$= \frac{1}{2}\left[\frac{t^6}{2} + 4t\right]_1^3$$

$$= 186.00$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**U7**

Directly substitute the midpoint of the range of t into the given function.

$$f(t) = 3t^5 + 4$$

Let the mean value of $f(t) = m$. The midpoint of the range is $t = 2$

$$\tilde{m} = 3 \times 2^5 + 4$$

$$= 100$$

No. of CSEs /No. incorrect answers **(CSE %)**	10/121 **(8%)**	**Date collected**	**2018-19**

3.7.4 Dimensions

Taking dimension of velocity as $[\boldsymbol{v}] = [\boldsymbol{MT^{-1}}]$)

Question			
The quantity $K = \rho^2 r^{-1} v^{1.5}$ where ρ represents density, r represents distance, v represents velocity. The dimensions of K are given by $[M^a L^b T^c]$. Calculate the values of a, b and c. Enter a: [ ] Enter b: [ ] Enter c: [ ]			
Correct Solution			
$K = \rho^2 r^{-1} v^{1.5}$ $[K] = [ML^{-3}]^2[L]^{-1}[LT^{-1}]^{1.5}$ $= [M]^2[L\,]^{-6-1+1.5}[T]^{-1.5}$ $= [M]^2[L]^{-5.5}[T]^{-1.5}$ $a = 2, b = -5.5, c = -1.5$			
CSE 1 related to this question		**CSE Taxonomy Code**:	**U7**
Taking dimensions of v to be $[v] = [MT^{-1}]$ $K = \rho^2 r^{-1} v^{1.5}$ $[K] = [ML^{-3}]^2[L]^{-1}[MT^{-1}]^{1.5}$ $= [M]^{3.5}[L]^{-7}[T]^{-1.5}$ $\tilde{a} = 3.5, \tilde{b} = -7, c = -1.5$			
No. of CSEs /No. incorrect answers **(CSE %)**	6/13 **(46%)**	**Date collected**	**2018-19**

4 Common Student Errors due to Errors in Choice of Method

4.1 Applying an Inappropriate Formula/Method/Theorem

4.1.1 Complex Numbers (Modulus and Argument of $z = \frac{(ae^{bj})^c}{(pe^{qj})^r}$)

Uses the method which is not relevant in the situation

<table>
<tr><td colspan="4"><u>Question</u></td></tr>
<tr><td colspan="4">Given $w_1 = 1.8e^{0.8j}$ and $w_2 = 1.2e^{-0.4j}$ determine the modulus, r, and argument, θ, of
$z = \frac{(w_1)^3}{(w_2)^2}$,
where $-\pi < \theta \leq \pi$.
Enter your answers correct to <u>three</u> decimal places.
Enter $r = |z|$:
Enter $\theta = arg(z)$:</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$z = \frac{(w_1)^3}{(w_2)^2}$
$z = \frac{(1.8)^3}{(1.2)^2} e^{j(3\times0.8-2\times-0.4)}$
$z = \frac{(1.8)^3}{(1.2)^2} e^{-3.2j}$
$|z| = \frac{(1.8)^3}{(1.2)^2} = 4.050$ and $\arg(z) = 3.2 - 2\pi = -3.083$</td></tr>
<tr><td>CSE 4 related to this question</td><td colspan="2">CSE Taxonomy Code:</td><td>CM1</td></tr>
<tr><td colspan="4">Taking $|z| = \sqrt{r_1^2 + r_2^2}$
$\widetilde{|z|} = \sqrt{1.8^2 + 1.2^2}$
$= 2.163$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>11/73(15%)
14/92(15%)</td><td>Date collected</td><td>2017-18
2018-19</td></tr>
</table>

4.1.2 Infinite Geometric Series

Uses a formula which is not relevant in the situation

Question
Consider the following geometic series, S, where: $S = 2 + 2(0.7) + 2(0.7)^2 + 2(0.7)^3 \cdots$. Write down the first term, a and the common ratio, r in the boxes below. Enter a: [ ] Enter r: [ ] Hence calculate the sum, S and enter your result in the box below. Enter S (to three decimal places) here: [ ]
Correct Solution
The first term $a = 2$ The common ratio $r = 0.7$ The sum of an infinite series (S) exists, provided $\lvert r \rvert < 1$ $S = \frac{a}{1-r}$ $= 6.667$

CSE 1 related to this question	**CSE Taxonomy Code**:	**CM1**
Finding the sum of first four terms instead of the sum of the infinite series. $\tilde{S} = \frac{a(1-r^n)}{1-r}$ $\tilde{S} = \frac{2(1-0.7^4)}{1-0.7}$ $\tilde{S} = 5.066$		

No. of CSEs /No. incorrect answers **(CSE %)**	34/67**(51%)**	**Date collected**	**2017-18**

4.1.3 Series (Maclaurin Expansion)

Uses a method which is not valid in the situation

Question

Use the standard MacLaurin expansion to obtain the power series expansion, $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$, of $f(x) = e^{4x}$ up to and including the cubic term.

Give the values of a_1 and a_2 below.

Enter your answer for a_1 (to three decimal places) here: [ ]

Enter your answer for a_2 (to three decimal places) here: [ ]

Use $P_3(x)$ to calculate an approximate value for e^{4x} at $x = 0.2$

Enter your approximate value for $e^{0.8}$ (to three decimal places) here: [ ]

Correct Solution

Using MacLaurin expansion,

$$e^{4x} = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6} \ldots$$

$$P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 = 1 + (4x) + \frac{(4x)^2}{2} + \frac{(4x)^3}{6}$$

$$a_1 = 4.000 \text{ and } a_2 = \frac{4^2}{2!} = 8.000$$

$$P_3(0.2) = 1 + (0.8) + \frac{(0.8)^2}{2} + \frac{(0.8)^3}{6} = 2.205$$

CSE 1 related to this question	**CSE Taxonomy Code**:	**CM1**

Giving the exact value of e^n instead of the approximate value.

$$\widetilde{e^{0.8}} = 2.226$$

No. of CSEs /No. incorrect answers **(CSE %)**	28/116 **(24%)** 29/122**(24%)**	**Date collected**	**2017-18** **2018-19**

5 Common Student Errors due to Errors in Use of Method

5.1 Error in use of an Appropriate Definition/ Method/ Theorem

5.1.1 Differentiation ($cos(ax^n)$)

Error in the use of the chain rule

Question

Select the most appropriate method to use in order to find the derivative of $f(x) = \cos(2x^7)$.

Select ▾

Hence find $\dfrac{df}{dx}$ as a function of x.

Enter the answer as a function of x:

Correct Solution

$$f(x) = \cos(2x^7)$$

$$f'(x) = -\sin(2x^7) \times 14x^6$$

$$f'(x) = -14x^6 \sin(2x^7)$$

CSE 2 related to this question	**CSE Taxonomy Code:**	**UM2**

Taking $\frac{d}{dx}(cos(ax^n)) = -a\,sin(ax^n) \times anx^{n-1} = -a^2nx^{n-1}sin(2x^n)$

$$f(x) = \cos(2x^7)$$

$$\tilde{f}'(x) = -2\sin(2x^7) \times 2 \times 7x^6$$

$$\tilde{f}'(x) = -28x^6 \sin(2x^7)$$

No. of CSEs /No. incorrect answers **(CSE %)**	10/56 **(18%)**	**Date collected**	**2017-18**

5.1.2 Differentiation ($\boldsymbol{cos^n(ax)}$)

Error in the use of the chain rule

<table>
<tr><td colspan="4">Question</td></tr>
<tr><td colspan="4">Select the most appropriate method to use in order to find the derivative of $f(x) = \cos^4(3x)$.
Select ▾
Hence find $\dfrac{df}{dx}$ as a function of x.
Enter the answer as a function of x:
[ ] ?</td></tr>
<tr><td colspan="4">Correct Solution</td></tr>
<tr><td colspan="4">$$f(x) = cos^4(3x)$$ $$f'(x) = -4 \times \cos^3(3x) \times \sin(3x) \times 3$$ $$f'(x) = -12\sin(3x)\cos^3(3x)$$</td></tr>
<tr><td>CSE 2 related to this question</td><td>CSE Taxonomy Code:</td><td colspan="2">UM2</td></tr>
<tr><td colspan="4">Taking $\frac{d}{dx}\left(cos^a(bx)\right) = -a \times sin^{a-1}(bx) \times b = -ab\, sin^{a-1}(bx)$ $$f(x) = cos^4(3x)$$ $$\tilde{f}'(x) = -4 \times \sin^3(3x) \times 3$$ $$\tilde{f}'(x) = -12\sin^3(3x)$$</td></tr>
<tr><td>No. of CSEs /No. incorrect answers (CSE %)</td><td>22/73(30%)</td><td>Date collected</td><td>2017-18</td></tr>
</table>

5.1.3 Partial Differentiation ($\frac{\partial z}{\partial x}$ of $z(x,y) = a\cos(xy^n)$)

Error in use of partial differentiation method

Question

Given

$$z(x,y) = 3\cos(xy^4)$$

find $\frac{\partial z}{\partial x}$.

Enter your answer below as a function of x and y :

Correct Solution

$$z(x,y) = 3\cos(xy^4)$$

$$\frac{\partial z}{\partial x} = -3\sin(xy^4) \times y^4$$

$$\frac{\partial z}{\partial x} = -3y^4\sin(xy^4)$$

CSE 4 related to this question	**CSE Taxonomy Code**:	**UM2**

Taking $\frac{\partial}{\partial x}(a\cos(xy^n)) = -a\sin(xy^n) \times x \times ny^{n-1}$

$$z(x,y) = 3\cos(xy^4)$$

$$\frac{\widetilde{\partial z}}{\partial x} = -3\sin(xy^4) \times x \times 4y^3$$

$$\frac{\widetilde{\partial z}}{\partial x} = -12xy^3\sin(xy^4)$$

No. of CSEs /No. incorrect answers **(CSE %)**	5/103**(5%)**	**Date collected**	**2017-18**

5.1.4 Integration Application (Mean Value of $\boldsymbol{a}\ \mathbf{sin}(\boldsymbol{bt})$)

Incorrect units applied

Question			
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places:			
Correct Solution			
$$f(t) = 4\ \sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\ \sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10\mathrm{t})\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$			
CSE 1 related to this question	**CSE Taxonomy Code**:		**UM2**
Substituting for t in degrees $$f(t) = 4\ \sin(10t)$$ Let the mean value of $f(t) = m,$ $$m = \frac{1}{(6-2)}\int_2^6 4\ \sin(10t)dt$$ $$\tilde{m} = 0.04$$			
No. of CSEs /No. incorrect answers **(CSE %)**	52/183 **(28%)**	**Date collected**	**2017-18**

5.1.5 Integration Application (Mean Value of $\boldsymbol{a}\ \mathbf{sin}(\boldsymbol{bt})$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question		
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places: [ ]		
Correct Solution		
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10t)\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$		
CSE 2 related to this question	**CSE Taxonomy Code**:	**U7, UM2**
Directly substituting the midpoint of the range of t in degrees into the given function. $$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$. The middle point of the range of $t = 4$ $$\tilde{m} = 4\sin(10 \times 4^o) = 4\sin(40^o)$$ $$\tilde{m} = 2.57$$		

No. of CSEs /No. incorrect answers **(CSE %)**	44/183 **(24%)**	**Date collected**	**2017-18**

5.1.6 Integration Application (Mean Value of $\boldsymbol{a}\,\mathbf{sin}(\boldsymbol{bt})$)

Incorrect assumptions on the mean value theorem and incorrect units applied

Question
Given that $f(t) = 4\sin(10t)$, find the mean value of $f(t)$ in the interval $2 < t < 6$, correct to two decimal places. Enter correct to two decimal places:
Correct Solution
$$f(t) = 4\sin(10t)$$ Let the mean value of $f(t) = m$ $$m = \frac{1}{(6-2)}\int_2^6 4\sin(10t)dt$$ $$= \frac{1}{4}\left[-\frac{4}{10}\cos(10\text{t})\right]_2^6$$ $$= -\frac{1}{10}((\cos(60) - \cos(20))$$ $$= 0.14$$

CSE 3 related to this question	**CSE Taxonomy Code**:	**U7, UM2**
Substituting the end values of the range t into the given function in degrees and then taking the average. $f(t) = 4\sin(10t)$, let the mean value of $f(t) = m$ $$\tilde{m} = \frac{4\sin(10\times 2^o) + 4\sin(10\times 6^o)}{2}$$ $$= 2.42$$		

No. of CSEs /No. incorrect answers **(CSE %)**	8/183 **(4%)**	**Date collected**	**2017-18**

5.1.7 Complex Numbers (Argument of $z = a - bj$)

Incorrect units applied

Question			
Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$. Express your answers correct to three decimal places. Enter r: [ ] Enter θ: [ ]			
Correct Solution			
$z = 2 - 3j$ $r = \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{13}$ $r = 5.39$ z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -0.983$			
CSE 3 related to this question	**CSE Taxonomy Code**:	**UM2**	
Calculating $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$ *in degrees (not in radians) when* $z = a - bj$ $\tilde{\theta} = \tan^{-1}\left(\frac{-3}{2}\right)$ $= -56.31^{\circ}$			
No. of CSEs /No. incorrect answers **(CSE %)**	14/88**(16%)**	**Date collected**	**2018-19**

5.1.8 Complex Numbers (Argument of $z = a - bj$)

Overlooking negative sign and incorrect units applied

Question

Express the complex number $z = 2 - 3j$, in the polar form $z = r\angle\theta$, where $-\pi < \theta \leq \pi$.

Express your answers correct to three decimal places.

Enter r: [ ]

Enter θ: [ ]

Correct Solution

$$z = 2 - 3j$$

$$r = \sqrt{(2)^2 + (-3)^2}$$

$$= \sqrt{13}$$

$$r = 5.39$$

z has positive real part, so $\theta = \tan^{-1}\left(\frac{-3}{2}\right)$

$$= -0.983$$

CSE 4 related to this question	**CSE Taxonomy Code**:	**S2, UM2**

Calculating $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ *in degrees (and missing a negative sign) when* $z = a - bj$

$$\tilde{\theta} = \tan^{-1}\left(\frac{3}{2}\right)$$

$$= 56.31^{o}$$

No. of CSEs /No. incorrect answers **(CSE %)**	11/88**(13%)**	**Date collected**	**2018-19**

5.1.9 Integration Applications (Volume of Revolution)

Method finding the volume of revolution is incorrectly followed

Question
Find the volume, V, of the solid formed, when the part of the curve $y = 0.8x^{1.5}$, is rotated about the x-axis between $x = 1$ and $x = 4$. Give your answer correct to two decimal places. The volume of the solid is: [ ]

Correct Solution
The volume of revolution of y over the range $a < x < b$ given by $$V = \pi \int_a^b y^2 dx$$ For given data, $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$= \pi \int_1^4 0.64\, x^3 dx$$ $$= 128.18$$

CSE 5 related to this question	**CSE Taxonomy Code**:	**UM2**
Substitute upper and lower limits without integrating $$V = \pi \int_1^4 (0.8x^{1.5})^2 dx$$ $$\tilde{V} = 0.64\, \pi \left[\frac{x^3}{3}\right]_1^4$$ $$= 126.67$$		
No. of CSEs /No. incorrect answers **(CSE %)**	5/107 **(5%)** 7/135 **(5%)**	**Date collected**	**2017-18** **2018-19**

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