# An Adaptive Fuzzy Control for Human-in-the-loop Operations with Varying Communication Time Delays*

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***Abstract*— Time delay, especially varying time delay, is always an important factor affecting the stability to the human-in-the-loop system. Previous research usually focuses on the performance of the internal signal transmission part, but rarely considers the whole system with human and environmental factors comprehensively. For the problem, we investigate this issue by using an improved proportional-derivative-like plus damping (PD-like + d) control method, the derivative term of which is calculated based on the estimation of time delays. An embedded adaptive fuzzy logic systems (FLS)-based observer is developed to estimate and compensate for the errors caused by time delay estimations and uncertain force/ torque measuring errors. The advantages of the proposed control scheme are discussed by building an environmental input energy function and the effectiveness is also verified by the comparative simulations. The results show that under the same simulation conditions, the follower can track the leader's movements well, and the energy introduced into the environment is the same as that of the leader, which means the extra energy is dissipated to enable the object to be manipulated as desired by the leader side.**

## I. Introduction

Teleoperation is a typical human-in-the-loop control mode widely applied in space manipulation, tele-surgery, deep sea exploitation and nuclear disposition [1]-[7] to connect local robot arms and remote operators to perform operations over long distances or unreachable by humans, such as minimally invasive surgery. Communication between robots and humans is based on the networks, as shown in Fig.1. Human operators operate a joystick or other tools to send operation requests to the follower side via the leader controller and networks. The follower controller receives the signals and forwards them to the robots. The robots and the sensors equipped with them collect environmental and operational information to send the force, position and visual information back through the networks to the humans for future decisions and operations. Since humans and the environment act as transmitters and receivers of input and output signals in a teleoperated system, in simple terms, the system is a two-port system represented by a dashed line in Fig.1.

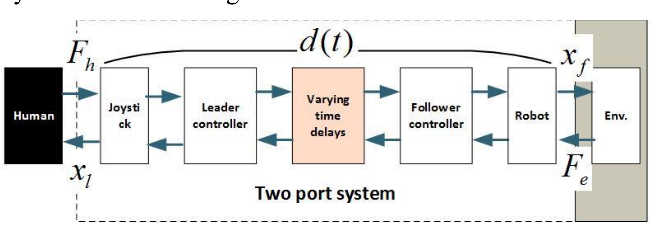


Fig. 1. Illustration of teleoperation/human-in-the-loop framework.

A fair amount of research on teleoperations is based on the structure of two-port systems and discusses system stability and operational transparency by proposing assumptions about the forces exerted and the rates of change of position/velocity of people and the environment.

Time-synchronized stability is a topic undergoing intense study in recent years [8], especially for distributed agent systems [9], and time-delay-induced stability is the key for the research about teleoperation [10]-[16]. The research of Lee and Spong considered constant time delays [10]-[11]. Kebria *et al.* studied teleoperation under the internet environment and proposed a robust adaptive teleoperation control method with time delays and uncertainties [12]. They reviewed the control methods for teleoperation and summarized these methods into several categories: time domain passivity-based control, wave variable-based control, adaptive control, robust control and other control methods [13]. The recent work of Zhai and Xia aimed to solve adaptive control of teleoperation system under asymmetric varying time delays, finite - time limitations and input saturation [14]. A various of methods e.g. robust control [15], neural networks-based control [16] are used for solving the teleoperation system control problems with varying time delays.

As mentioned above, these methods are proposed under the two-port architecture without considering the interaction of robot and environment, especially for human-robot interaction and coordination tasks [17]. Some researchers considered the models of describing interactions between the two-port system and human operator or the environment by a mass-damping-stiffness model, as Passenberg *et al.* claimed in a survey about the environment-, operator- and task-adapted controllers for teleoperation systems [18][17]. Additionally, Yang and Luo *et al.* considered human operational factors to teleoperation process. They used surface electromyograms to extract the human operator's muscle activation and adjust the motion trajectory and stiffness of robot arms to realize perceptual and intelligent teleoperation [19]-[21]. However, these works comprehensively discussed the influence of varying time delays on the teleoperation system and the robot manipulation effect.

In the two-port system shown in Fig. 1, the environment side can be seen as a one-port system that inputs robot position and outputs the contact force. In this paper, we use the human

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as the leader and the robot as the follower, and set time delays between the leader side and the follower side are $d(t)$, $F_h$ and $F_e$ are forces exerted by human hands and the environment. $x_l(t)$ and $x_f(t)$ are the real-time end positions of the joystick handled by humans and the robot. Then, at time $t=T$, the desired manipulation effect is that the operator can feel contact force $F_h(T-d(t))=-F_e(T)$ coming from the follower, while the robot can follow the operator's movements $x_l(T-d(t))=x_f(T)$. However, if time delays are not acquired accurately, the desired delayed information $F_h(T-d(t))$ and $x_l(T-d(t))$ will be accelerated or decelerated to generate force fluctuations and position tracking errors to lead to unsmoothed manipulation effect.

In this paper, we improve the proportional-derivative-like plus damping (PD-like+d) control method for teleoperation, where a factor corresponds to the estimation of the derivatives of the time delays. For the estimation errors caused by this estimations and other influences such as force measuring noises, we will use an adaptive fuzzy logic system (FLS) to comprehensively estimate all the above errors and compensate on both sides.

The rest of this manuscript is organized as follows: Section II presents the basic knowledge of PD+d control and Fuzzy Logic Systems as well as the leader and follower controllers are introduced based on several properties and a lemma. The system stability conditions are concluded in a theorem with two remarks. Section III makes a comparative simulation of the proposed method with other PD +d methods and an experiment in practice. In Section IV, we draw a final conclusion.

## II. Adaptive Fuzzy PD+D Controller Design

### A. System modelling

First, for consistent expressions throughout this paper, we set $l$ and $f$ as the variables of the leader and follower robots. A teleoperation system consisted of two n-link rigid robots is expressed as:

$$\begin{cases} M_l(q_l)\ddot{q}_l + C_l(q_l,\dot{q}_l)\dot{q}_l + G_l = \tau_l - \tau_h \\ M_f(q_f)\ddot{q}_f + C_f(q_f,\dot{q}_f)\dot{q}_f + G_f = \tau_e - \tau_f \end{cases}, \tag{1}$$

where $q_i \in R^n, i=l,f$ is the simplification of $q_i(t)$ at time $t\in R^+$, $M_i(q_i)\in R^{n\times n}$ is the inertia matrix, $C_i(q_i,\dot{q}_i)\in R^n$ is the Coriolis and centrifugal torque matrix, and $G_i\in R^n$ is the gravitational torque. The control torques in the leader and follower sides are $\tau_l$ and $\tau_f$, and $F_h$ and $F_e$ are forces exerted by human hands and the environment, $\tau_h$ and $\tau_e$ are torques calculated by $\tau_h = J_l^T(q_l)F_h$ and $\tau_e = J_f^T(q_f)F_e$, and $J_i^T(q_i), i=l,f$ is a Jacobian matrix. Define $x_i$ as the positions of the human manipulator and robots end in the Cartesian space, then the forward kinematics of robot is expressed as

$$x_i(t)=\phi(q_i(t)), \quad \dot{x}_i(t)=J_i(q_i(t))\dot{q}_i(t), \tag{2}$$

where $\phi(*)$ is a function for joint and position transformation. Time delays between the leader and follower sides are defined as $d_l(t)$ and $d_f(t)$ in forward and backward channels and the contact force is built by a spring-damping linear model as

$$F_e = \begin{cases} K\Delta x_f + B\Delta\dot{x}_f & \Delta x_f \ge 0 \\ 0 & \Delta x_f < 0 \end{cases}. \tag{3}$$

where $K$ and $B$ are constant stiffness and damping factors, and $\Delta x_f = x_f(t) - x_f(0)$ represents the position error to stable postion $x_f(0)$ with an assumption of $\dot{x}_f(0)=0$ at the start time. System (1) satisfies the following properties:

**Property 1**: The time delays are variables with upper and lower boundaries $0\le d_i \le \bar{d}_i, i=l,f$, and the time derivative of $d_i$ satisfies $0<|\dot{d}_i|<\mu_i<1$, $\mu_i$ are positive scalars.

**Property 2:** The matrices in (1) $\dot{M}_l - 2C_l$ and $\dot{M}_f - 2C_f$ are skew -symmetric.

**Property 3:** The inertia matrix is $M(q)$ symmetric positive-definite, and there are two positive constants $m_1$ and $m_2$ satisfying $m_1 I \le M(q) \le m_2 I$.

**Property 4**: For the term $C_i(q_i,\dot{q}_i)$ in (1) with any values of $q_i,\dot{q}_i$, there exists a positive factor $\eta$ satisfying

$$\|C_i(q_i,x(t))\,y(t)\|_2 \le \eta\|x(t)\|_2\|y(t)\|_2. \tag{4}$$

**Property 5**: [25] Following standard considerations, human operator and the environment are assumed passive maps, that is, there exists $\kappa_i \in \mathbf{R}^+$ for all $t>0$, such that

$$\int_0^t \dot{q}_l(\sigma)\tau_h(\sigma)d\sigma \ge -\kappa_l,\ \int_0^t \dot{q}_f(\sigma)\tau_e(\sigma)d\sigma \ge -\kappa_f. \tag{5}$$

**Lemma 1:** For a positive-definite matrix $\Upsilon$, the following inequality holds:

$$-2a^T(t)\int_{t-d(t)}^t b(\xi)d\xi - \int_{t-d(t)}^t b(\xi)^T\Upsilon b(\xi)dt \le \bar{d}a^T(t)\Upsilon^{-1}a(t),$$

where $a(.)$ and $b(.)$ are vector functions, and $d(t)$ is a time-varying scalar with $0\le d(t)\le \bar{d}$.

### B. PD+d controllers

As presented in [21], PD+d is a damping injection controller that ensures position tracking with time delays and passive output interconnection schemes. The standard PD+d controllers in [24] utilized for the case with constant time delays are designed for leader and follower sides as

$$\begin{cases}\tau_l = k_d^l\left(\dot{q}_f(t-d_f(t)) - \dot{q}_l(t)\right) + \\ \qquad k_b^l\left(q_f(t-d_f(t)) - q_l(t)\right) - B_l\dot{q}_l(t) \\ \tau_f = k_d^f\left(\dot{q}_f(t) - \dot{q}_l(t-d_l(t))\right) + \\ \qquad k_b^f\left(q_f(t) - q_l(t-d_l(t))\right) + B_f\dot{q}_f(t)\end{cases}, \qquad (6)$$

with $k_d^i$ and $k_b^i, i = l, f$ are arbitrary nonnegative constants, and $B_i$ represents the damping coefficient. The authors improved (6) by adding time-varying gains $\gamma_l$ and $\gamma_f$ to dissipate the energy generated by varying time delays in [25] and achieved the improved PD+d controllers as

$$\begin{cases}\tau_l = k_d^l\left(\gamma_l\dot{q}_f(t-d_f(t)) - \dot{q}_l(t)\right) + \\ \qquad k_b^l\left(q_f(t-d_f(t)) - q_l(t)\right) - B_l\dot{q}_l(t) \\ \tau_f = k_d^f\left(\dot{q}_f(t) - \gamma_f\dot{q}_l(t-d_l(t))\right) + \\ \qquad k_b^f\left(q_f(t) - q_l(t-d_l(t))\right) + B_f\dot{q}_f(t)\end{cases}, \qquad (7)$$

with $\left\{k_d^i, k_b^i, B_i\right\} \in R^+$, $\gamma_l = \sqrt{1-\dot{d}_l(t)}$ and $\gamma_f = \sqrt{1-\dot{d}_f(t)}$. The similar expression for calculating the velocity errors has been utilized in [26] and [27]. But, most of the research took the same velocity-related expression as [24], even for constant time delays. In [27], the authors claimed that terms $\gamma_l$ and $\gamma_f$ ensures that both robot position tracking errors and velocity asymptotic convergence in the presence of varying time delays. But, using $\dot{q}(t) - \dot{q}(t-d(t))$ instead of $\gamma_l\dot{q}_f(t-d_f(t)) - \dot{q}_l(t)$ will cause input energy error accumulation to influence manipulation of robot end tips, especially for the cases that $\dot{d}_l(t)$ and $\dot{d}_f(t)$ change sharply. Term $\gamma_l$ and $\gamma_f$ is (7) and system stability conditions depend on the $\dot{d}_l(t)$, which is hard to be accurately calculated during robot manipulation process.

### C. Fuzzy Logic Systems

The Fuzzy Logic Systems (FLS) has been widely used for the design of dynamic systems which leave precise models as being unknown. A FLS is a collection of several IF-THEN rules:

$R^{(j)}$ : IF $x_1$ is $A_1^j$ and …and $x_n$ is $A_n^j$, THEN $y$ is $W^j$, $j = 1, 2, .., m$,

where $R^{(j)}$ expresses the $jth$ rule, $\left(x_1, x_2, ..., x_n\right)^T \in U \subset R$ and $y \subset R$ are the linguistic variables associated with the inputs and output respectively. $A_i^j$ and $W^j$ are the fuzzy sets in $U$ and $R$. In this paper, the output of FLS is

$$y(x) = \frac{\sum_{j=1}^{m} y_j\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)}{\sum_{j=1}^{m}\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)} \qquad (8)$$

where $x = \left[x_1, x_2, ..., x_n\right]^T$ and $\mu_{A_i^j}(x_i) = \exp\left(-\frac{\left(x_i - c_{ij}\right)^2}{\sigma_{ij}^2}\right)$ is the membership function of linguistic variable $x_i$, and $y_j$ is the point in $\mathbf{R}$ at $\mu_{B_l^j}$ achieves the maximum value. Eq (8) can be further expressed in a linear form as :

$$y(x) = W^T S(x, c, \sigma) \qquad (9)$$

where $W = \left[y_1, y_2, ..., y_m\right]^T$ and $S(x, c, \sigma) = \left[S_1(x, c, \sigma), S_2(x, c, \sigma), ..., S_m(x, c, \sigma)\right]^T$, where

$$S_1(x, c, \sigma) = \frac{\prod_{i=1}^{n}\mu_{A_i^j}(x_i)}{\sum_{j=1}^{m}\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)} \qquad (10)$$

According to the universal approximation theorem of FLS, there exists the $W^*$ such that $W^{*T}S(x, c, \sigma)$ can approach to a nonlinear $\Psi(x(t))$ in any accuracy that is achieved by the following minimizing function

$$W^* = \arg\min\left(\sup\left|W^T S(x, c, \sigma) - \Psi(x(t))\right|\right) \qquad (11)$$

Then the minimum approaching error satisfies

$$\Psi(x(t)) = W^* S(x, c, \sigma) + \varepsilon(x(t)) \qquad (12)$$

where $\left\|\varepsilon(x(t))\right\| \le \varepsilon^*$ and $\varepsilon^*$ is a positive scalar.

### D. Adaptive Fuzzy PD+d controllers

The solution of this paper is building an adaptive fuzzy PD + damping control based on the estimations of time delays and its derivative values. The general control scheme is designed as shown in Fig.2.

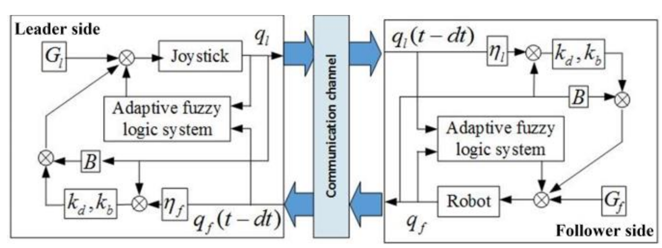


Fig. 2. Teleoperation system with new controllers

In leader and follower sides, the controllers are designed in symmetric form, i.e., an adaptive fuzzy logic system (FLS)-based observer to estimate and compensate the errors caused by estimations of time-delay and force measuring noise. Another characteristics is using the estimated derivative values of time delays to calculate terms $\eta_f$ and $\eta_l$. We set the factors on both leader and follower sides such that $k_d = k_d^l = k_d^f$, $k_b = k_b^l = k_b^f$ and $B_l = B_f = B$ are equal.

Then the controllers are

$$\begin{cases}\tau_l = k_d\left(\eta_f \dot{q}_f(t-d_f(t)) - \dot{q}_l(t)\right) + \\ \quad k_b\left(q_f(t-d_j(t)) - q_l(t)\right) - B\dot{q}_l(t) + G_l + J_l^T(q_l)F_h \\ \quad \Xi_l\left(\dot{q}_f(t-d_f(t)), q_f(t-d_f(t)), \dot{q}_l(t), q_l(t)\right) \\ \tau_f = k_d\left(\dot{q}_f(t) - \eta_l \dot{q}_l(t-d_l(t))\right) + \\ \quad k_b\left(q_f(t) - q_l(t-d_l(t))\right) + B\dot{q}_f(t) + G_f + J_f^T(q_f)F_e \\ \quad \Xi_f\left(\dot{q}_l(t-d_l(t)), q_l(t-d_l(t)), \dot{q}_f(t), q_f(t)\right)\end{cases}, \tag{13}$$

where $\eta_f$ and $\eta_l$ are calculated based on time-delay estimation $\dot{d}_f(t)$ and $\dot{d}_f(t)$ that can be set as constants (such as average values of $\dot{d}_i(t), i = l, f$ ) or time-related terms. $F_e$ and $F_h$ are estimated forces with errors, and $\Xi_i\left(\dot{q}_i(t-d_i(t)), q_i(t-d_i(t)), \dot{q}_{\bar{i}}(t), q_{\bar{i}}(t)\right), i = l, f$ represents the calculating term by FLS for estimating the combined errors caused by time delays and force measures, and $\bar{i}$ represents the opposite role of $i$ in the pair of leader and follower. For clarity, we will use $\Xi_j(*)$ to replace $\Xi_i\left(\dot{q}_i(t-d_i(t)), q_i(t-d_i(t)), \dot{q}_i(t), q_i(t)\right)$ and the exact calculation of $\Xi_i(*), i = l, f$ is

$$\begin{cases}\Xi_l\left(\dot{q}_f(t-d_f(t)), q_f(t-d_f(t)), \dot{q}_l(t), q_l(t)\right) = \\ \quad k_d\left(\sqrt{1-\dot{d}_f(t)} - \eta_l\right)\dot{q}_f(t-d_f(t)) + J_l^T(q_l)\left(F_h - F_h\right) \\ \Xi_f\left(\dot{q}_l(t-d_l(t)), q_l(t-d_l(t)), \dot{q}_f(t), q_f(t)\right) = - \\ \quad k_d\left(\sqrt{1-\dot{d}_l(t)} - \eta_f\right)\dot{q}_l(t-d_l(t)) - J_f^T(q_f)\left(F_e - F_e\right)\end{cases}. \tag{14}$$

According to the universal approximation theorem, fuzzy logic system can be approximated to any nonlinear equations by describing $\Xi_i(*)\big|\Theta_i, i = l, f$ using K terms $\Xi_{j,k}(*)\big|\Theta_{j,k} = \Theta_{j,k}^T \xi\left(x_{j,k}\right), j = l, f$ , where $\Theta_{j,k}^T$ is the weight vector and $\xi\left(x_{j,k}\right)$ is a fuzzy logic function that is introduced in (8), and $x_{j,k} = \left[\dot{q}_{j,k}(t-d_j(t)), q_{j,k}(t-d_j(t)), \dot{q}_{\bar{j},k}(t), q_{\bar{j},k}(t)\right]$ is a variable vector.

Seen from (14), the term $k_d\left(\sqrt{1-\dot{d}_f(t)} - \eta_l\right)\dot{q}_f(t-d_f(t))$ concerns $\dot{q}_f(t-d_f(t))$ and latter term $J_l^T(q_l)\left(F_h - F_h\right)$ relates to $q_l$ . Thus, $\Xi_i(*), i = l, f$ is divided into two parts:

$$\begin{aligned}&\Xi_i\left(\dot{q}_j(t-d_j(t)), q_j(t-d_j(t)), \dot{q}_{\bar{j}}(t), q_{\bar{j}}(t)\right) = \\ &\Xi_i^1\left(\dot{q}_j(t-d_j(t)), q_j(t-d_j(t))\right) + \\ &\Xi_i^2\left(\dot{q}_{\bar{j}}(t), q_{\bar{j}}(t)\right)\end{aligned}. \tag{15}$$

where $\bar{j}$ represents the opposite role of $j$ in the set $[l, f]$ . Then the values of $\Xi_i(*)$ are approximated by multi-input multi-output (MIMO) fuzzy logic system (FLS) as

$$\begin{aligned}&\Xi_i\left(\dot{q}_i(t-d_i(t)), q_i(t-d_i(t)), \dot{q}_{\bar{i}}(t), q_{\bar{i}}(t)\right)\big|\Theta_{i,k} = \\ &{}^1\Xi_i\left(\dot{q}_i(t-d_i(t)), q_i(t-d_i(t))\right)\big|{}^1\Theta_{i,k} + \\ &{}^2\Xi_i\left(\dot{q}_{\bar{i}}(t), q_{\bar{i}}(t)\right)\big|{}^2\Theta_{i,k}\end{aligned}, \tag{16}$$

where $\Theta_{i,k} = [{}^1\Theta_{i,k}, {}^2\Theta_{i,k}]$ , ${}^1\Xi_i\left(\dot{q}_i(t-d_i(t)), q_i(t-d_i(t))\right)\big|{}^1\Theta_{i,k} = {}^1\Theta_{i,k}\xi_{i,k}^1\left(\dot{q}_i(t-d_i(t)), q_i(t-d_i(t))\right)$ , $\Xi_j^2\left(\dot{q}_{\bar{j}}(t), q_{\bar{j}}(t)\right)\big|{}^2\Theta_{j,k} = {}^2\Theta_{j,k}\xi_{j,k}^2\left(\dot{q}_j(t), q_j(t)\right)$.

The vectors ${}^1\Theta_{j,k}$ and ${}^2\Theta_{j,k}$ are adaptively updated by

$$\begin{cases}{}^1\dot{\Theta}_{j,k} = \left(\Gamma_i^1\right)^{-1}\xi_{j,k}^1\left(\dot{q}_j(t-d_j(t)), q_j(t-d_j(t))\right)\dot{q}_j(t) \\ {}^2\Theta_{j,k} = \left(\Gamma_i^2\right)^{-1}\xi_{j,k}^2\left(\dot{q}_j(t), q_j(t)\right)\dot{q}_j(t)\end{cases}, \tag{17}$$

where $\Gamma_i^1$ and $\Gamma_i^2, i = L, K$ are positive-definite scalars.

**Theorem 1**. For a teleoperation system expressed by (1), satisfying Property 1 to Property 5, by using controller (13) and Lemma 1, the sufficient conditions for system stability are

$$\begin{cases}\bar{d}_l < \dfrac{4}{5}\left(\dfrac{k_d + B - k_b}{k_b} - \dfrac{k_d^2\eta_f^2}{4{k_b}^2\gamma_l^2}\right) \\ \bar{d}_f < \dfrac{4}{5}\left(\dfrac{k_d + B - k_b}{k_b} - \dfrac{k_d^2\eta_l^2}{4k_b\gamma_f^2}\right)\end{cases}, \tag{18}$$

where $\bar{d}_l$ and $\bar{d}_f$ are time delay upper boundaries described in Property 1.

**Remark 1:** Following **Theorem 1**, if $\eta_i = 0$, then (18) are simplified as $\bar{d}_l < \frac{4}{5}\left(\frac{k_d + B}{k_b} - 1\right)$ , $\bar{d}_f < \frac{4}{5}\left(\frac{B + k_d}{k_b} - 1\right)$ and the control mode degraded into a P+d control. As $k_d^2\eta_f^2 / 4{k_b}^2\gamma_l^2 > 0$ , with the increase of $\eta_i$ , the upper boundary of $\bar{d}_l$ and $\bar{d}_f$ will decrease, which means the range of stability conditions in $\eta_s = 0$ is wider than the case using $\eta_s = 1$.

**Remark 2**: If the time delay and its derivative can be acquired accurately, that is $\eta_f^2 = \gamma_l^2$ is achieved, then we can get

$$\bar{d}_l, \bar{d}_f < \frac{4}{5}\left(\frac{k_d + B - k_b}{k_b} - \frac{k_d^2}{4{k_b}^2}\right), \tag{19}$$

which is totally calculated based on exact factors. Otherwise, we calculate stability condition based on the upper boundary $\mu_i$ in Property 1 to achieve a narrow stability condition.

## III. Simulation and Experiment

### A. Simulation

In the simulation, we mainly compare the position tracking performance and the input energy to the environment during robot contact manipulation. The simulation is based on a tele-operation system consisted of 2-DOF robots/manipulators in the follower and leader side, as Fig. 3 shown.

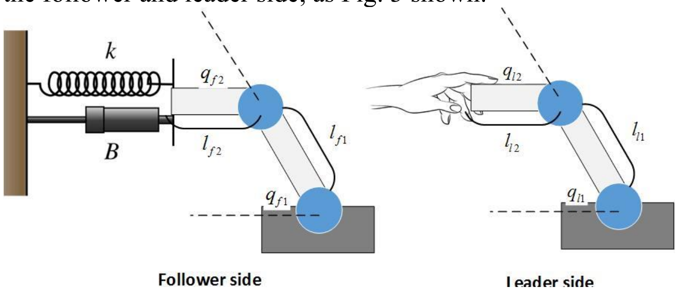


Fig. 3. Simulation settings

The terms in dynamics function of (1) are expressed as

$$M_i(q_i)=\begin{bmatrix} M_i^{11} & M_i^{12} \\ M_i^{21} & M_i^{22} \end{bmatrix}$$

$M_i^{11}=I_{i1}+I_{i2}+1(m_{i1}+m_{i2})l_{i1}^2+m_{i2}l_{i2}^2+2m_{i2}l_{i1}l_{i2}\cos(q_{i2})$,

$M_i^{12}=I_{i2}+m_{i2}l_2^2-m_{i2}l_1 l_2\cos(q_{i2})$,

$M_i^{12}=I_{i2}+m_{i2}l_2^2+m_{i2}l_1 l_2\cos(q_{i2})$ , $M_i^{22}=m_{i2}l_2$ ,

$$C_i(q_i,\dot{q}_i)\dot{q}_i=\begin{bmatrix} -m_{i2}l_{i1}l_{i2}\sin(q_{i2})(2\dot{q}_{i1}\dot{q}_{i2}+\dot{q}_{i2}) \\ -m_{i2}l_{i1}l_{i2}\sin(q_{i2})\dot{q}_{i1}\dot{q}_{i2} \end{bmatrix},$$

$$G_i=\begin{bmatrix} -(m_{i1}+m_{i2})l_{i1}g\sin(q_{i1})-m_{i2}l_{i2}g\sin(q_{i1}+q_{i2}) \\ -m_{i2}l_{i2}g\sin(q_{i1}+q_{i2}) \end{bmatrix}, i=l,f$$

.

The parameters are : $m_{l1}=0.12kg$ , $m_{l2}=0.14kg$ , $m_{f1}=0.23kg$ , $m_{f2}=0.46kg$ , $l_{f1}=l_{l1}=0.3m$ , $l_{f2}=l_{l2}=0.3m$ , $I_{l1}=0.01kg.m^2$ , $I_{l2}=0.02kg.m^2$ , $I_{f1}=0.03kg.m^2$ , $I_{f2}=0.03kg.m^2$ . Time delays are set as $d_l(t)=d_f(t)=0.2\sin 2t+0.3\cos t+1$ in forward and backward channels. Set $\mu_f=\mu_l=0.5$ , $\eta_f=\eta_l=0.7$ for time delay estimation, controller parameters as $k_d=5I^{2\times 2}$, $k_b=10I^{2\times 2}$ , $B=25I^{2\times 2}$ and contact force is modelized as

$$F=20(x_f(t)-x_f(0))+5\dot{x}_f(t), x_f(t)-x_f(0)>0 \quad (20)$$

We choose three controllers: P+damping controller in (6) with $k_d^l=k_d^f=0$, PD+damping controller in (6), and PD+d-controller with time estimations (PD+d-E controller without FLS-based compensations) in (13) and the proposed method in (13) to compare performance of position/joint and input energy tracking. The simulation results are shown in Figs. 4 to 6. Fig. 4 shows the joint tracking performance of four controllers. Choosing the same factors, the position tracking accuracy in P+d controller is much better than the others based on PD control. Fig. 5 shows joint tracking errors. The P+d controller achieves the fastest position tracking effect and smaller tracking errors compared with other PD+d controllers. While for the same PD+d control mode, the joint tracking effect, including error range and convergence time are similar to each other.

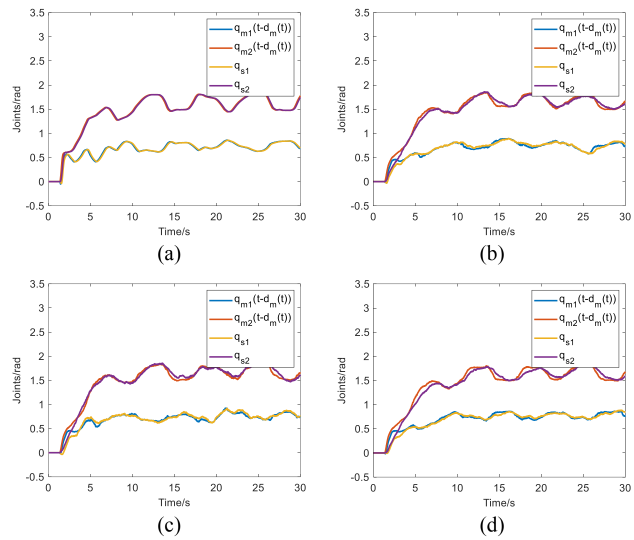


Fig. 4. Joint changes using different controllers (a) Results of P+d control (b) Results of PD+d-like control with $\eta_i=1$ (c) Results of PD+d-like control with $\eta_i=0.7$ (d) Results of PD+d-like control with estimations of time delays and dynamics compensations

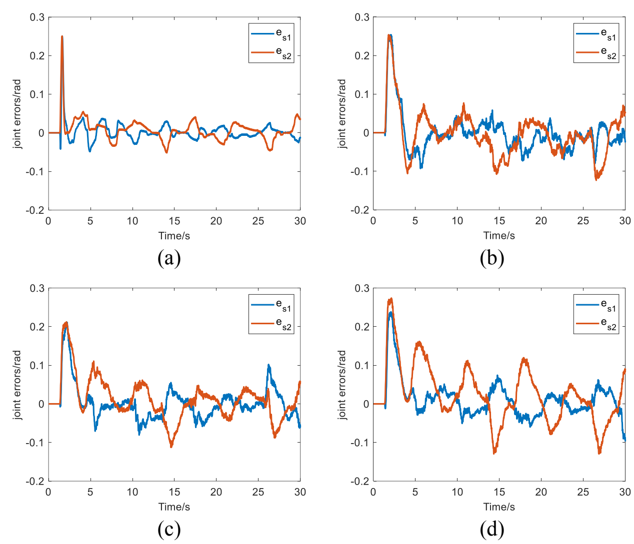


Fig. 5. Joint tracking errors using different controllers (a) Results of P+d control (b) Results of PD+d-like control with $\eta_i=1$ (c) Results of PD+d-like control with $\eta_i=0.7$ (d) Results of PD+d-like control with estimations of time delays and dynamics compensations

The advantages of the proposed control method are shown through minimum input energy tracking errors with varying time delays. By using the contact model in (20), similar to **property 5**, an input energy function of the environment is defined as

$$G(\sigma)=\int_0^t \dot{x}_f(\sigma)F_e(\sigma)d\sigma \tag{21}$$

The result comparison of four controllers is shown in Fig. 6. It can be seen that the energy error of the P+d controller is the largest to the leader delayed orders, due to the lack of velocity tracking. Choosing PD+d controller or PD+d controller with an estimated time-delayed factor helps to reduce the energy difference, especially in the period from start to about 5s (in Fig.6(b)) and 10s (in Fig.6(c)).

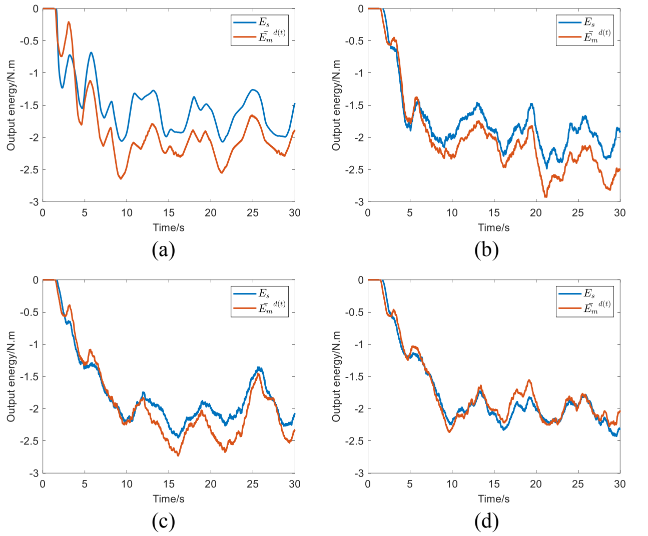


Fig. 6. Environmental input energies by using different controllers (a) Results of P+d control (b) Results of PD+d-like control with $\eta_i = 1$ (c) Results of PD+d-like control with $\eta_i = 0.7$ (d) Results of PD+d-like control with estimations of time delays and dynamics compensations

However, the difference in Fig.6(c) from 21s to about 23s can reach about 0.5N.m. The proposed method can ensure the consistency of the energy transmitted from the leading side with the energy received on the following side (see Fig.6(d)). This ensures that the energy input from the robots or manipulation of the object is the same as that of the operator on the guidance side, which can be affected by time delays.

### B. Experiment

In the experiment, we build the following platform (Fig. 7) to allow the Franka robot equipped with a cutting head to cut a paper tape on a plastic plate. The robot is controlled by the operator via teleoperation using an Omni Touch joystick and a PC. Since the manipulation effects are different in different human cases and the Franka robot is controlled online at a communication rate of 1000 Hz, we first record the teleoperation trajectory and use manually adjustable time delays to generate and send the delayed commands to the robot side for execution to compare the manipulation effects with different controllers and the same input conditions. The cutting head is moved by the robot in a line from the right side to the left side. The cutting task is a surface-touching manipulation that depends on the cutting speed at each trajectory point. The experimental results are shown in Fig. 8.

We choose two controllers: a PD+d controller with constant $\eta_i = 1$ in (13) and the proposed controller with time estimation and dynamics compensation. Using the former controller, there are several discontinuous parts along the cutting trajectory that are shown magnified in subfigure 8 (a) and the cutting edges are rougher than the proposed method under the same conditions in subfigure 8 (b).

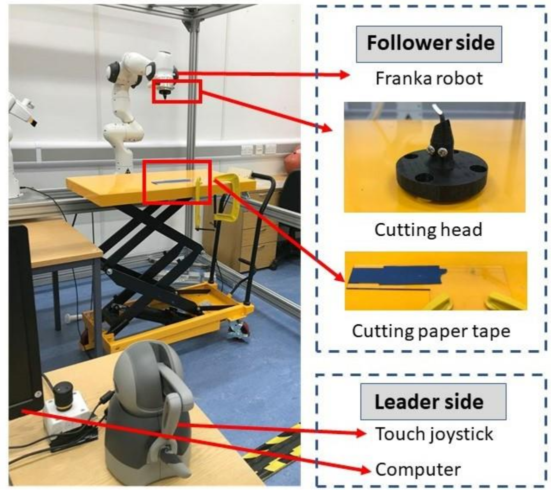


Fig. 7. Experiment setups

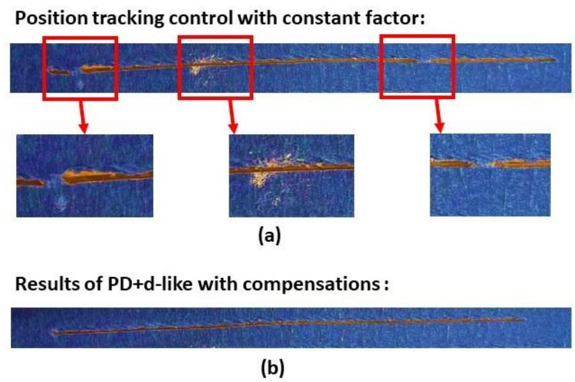


Fig. 8. Object cutting experiment by using different controllers

## IV. Conclusions

In this paper, for the complaint contact of a typical human-in-the-loop system with varying time delays, we improve the traditional PD +d-like controllers based on estimations of the time delays and the dynamics compensation computed by FLS. The system stability is proved by constructing the Lyapnove-Krasovski function to obtain a theorem that gives the upper bound on the time delays for the proposed controller. The comparative simulations confirm the idea proposed in the introduction that the difference of the input energy to the command operations decreases significantly and the position tracking performance does not change significantly. The experiment verifies the effectiveness of the proposed method using a teleoperated cutting task. The experimental results show that the proposed method can improve the smoothness and continuity of the curves, while the comparison group has several discontinuous sections. Future work is expected to combine human skill learning methods such as Dynamic Movement Primitive (DMP) [28] through teleoperation with the proposed method to realize stable human-robot interaction remotely and online modification of robot trajectory and action by human in the loop.

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## APPENDIX

### A. *Proof of Theorem 1*

To simplify the expressions, we set $q_f^{d_f} = q_f(t-d_f(t))$ and $q_l^{d_l} = q_l(t-d_l(t))$ , $1-\dot{d}_f(t)=\gamma_f^2$ and $1-\dot{d}_l(t)=\gamma_l^2$ , then $\frac{d\left(q_f(t-d_f(t))\right)}{dt} = \left(1-\dot{d}_f(t)\right)\dot{q}_f^{d_f} = \gamma_f^2\dot{q}_f^{d_f}$ and $\frac{d\left(q_l(t-d_l(t))\right)}{dt} = \left(1-\dot{d}_l(t)\right)\dot{q}_l^{d_l} = \gamma_l^2\dot{q}_l^{d_l}$.

We build the Lyapnove-Krasovski function as

$$V = V_1(q_i,\dot{q}_i) + V_2(\dot{q}_i) + V_3(q_i,\dot{q}_i) \tag{22}$$

where $V_1(q_i,\dot{q}_i) = \frac{1}{2}\dot{q}_l^T M_l \dot{q}_l + \frac{1}{2}\dot{q}_f^T M_f \dot{q}_f$ ,

$$V_2(\dot{q}_i) = \frac{1}{2}\left(\tilde{\Theta}_{j,l}^T \Gamma_L^{-1}\tilde{\Theta}_{j,l} + \tilde{\Theta}_{j,f}^T \Gamma_F^{-1}\tilde{\Theta}_{j,f}\right),$$

$$\begin{aligned} V_3(q_i,\dot{q}_i) = {} & \frac{k_b}{2}\left(q_l - q_f\right)^2 + k_b\int_{t-d_l}^{t}\dot{q}_l^T\dot{q}_l\,dt + \\ & k_b\int_{t-d_l}^{t}\dot{q}_f^T\dot{q}_f\,dt + k_b\int_{\bar{d}_l}^{0}\int_{t-\theta}^{t}\dot{q}_l^T(s)\dot{q}_l(s)\,ds\,d\theta + . \\ & k_b\int_{\bar{d}_f}^{0}\int_{t-\theta}^{t}\dot{q}_f^T(s)\dot{q}_f(s)\,ds\,d\theta \end{aligned}$$

The time-derivatives of $V_1$, $V_2$ and $V_3$ satisfied

$$\dot{V}_1(q_i,\dot{q}_i) = \dot{q}_l^T M_l \ddot{q}_l + \dot{q}_f^T M_f \ddot{q}_f + \frac{1}{2}\dot{q}_l^T \dot{M}_l \dot{q}_l + \frac{1}{2}\dot{q}_f^T \dot{M}_f \dot{q}_f \, , \tag{23}$$

$$\dot{V}_2(\dot{q}_i) = \tilde{\Theta}_{j,l}^T \Gamma_L^{-1}\dot{\tilde{\Theta}}_{j,l} + \tilde{\Theta}_{j,f}^T \Gamma_F^{-1}\dot{\tilde{\Theta}}_{j,f} \tag{24}$$

$$\begin{aligned}\dot{V}_3(q_i,\dot{q}_i) \le\ & k_b(q_l-q_f)(\dot{q}_l-\dot{q}_f)+\\ & k_b\left(\dot{q}_l^T\dot{q}_l-(1-\dot{d}_l(t))\left(\dot{q}_l^{d_l}\right)^T\dot{q}_l^{d_l}\right)+\\ & k_b\left(\dot{q}_f^T\dot{q}_f-(1-\dot{d}_f(t))\left(\dot{q}_f^{d_f}\right)^T\dot{q}_f^{d_f}\right)+.\\ & k_b\left(\bar{d}_l\dot{q}_l^T\dot{q}_l-\int_{t-d_l}^t\dot{q}_l^T(\theta)\dot{q}_l(\theta)d\theta\right)+\\ & k_b\left(\bar{d}_f\dot{q}_f^T\dot{q}_f-\int_{t-d_f}^t\dot{q}_f^T(\theta)\dot{q}_f(\theta)d\theta\right)\end{aligned} \tag{25}$$

Taking (13) into (1), we can get

$$\begin{cases} M_l(q_l)\ddot{q}_l+C_l(q_l,\dot{q}_l)\dot{q}_l=k_d\eta_f\left(\dot{q}_f^{d_f}-\dot{q}_l\right)+\\ k_b\left(q_f^{d_f}-q_l\right)-\left(B+k_d(1-\eta_f)\right)\dot{q}_l-\tau_h+J_f^T(q_f)F_h+\Xi_l\\ M_f(q_f)\ddot{q}_f+C_f(q_f,\dot{q}_f)\dot{q}_f=k_d\eta_l\left(\dot{q}_l^{d_l}-\dot{q}_f\right)+\\ k_b\left(q_l^{d_l}-q_f\right)-\left(B+k_d(1-\eta_l)\right)\dot{q}_f+\tau_e-J_f^T(q_f)F_e-\Xi_f\end{cases}. \tag{26}$$

Taking (26) into (23) and using **Property 2**, we can get

$$\begin{aligned}\dot{V}_1(q_i,\dot{q}_i)=\ & k_d\eta_f\left(\dot{q}_f^{d_f}-\dot{q}_l\right)^T\dot{q}_l+k_b\left(q_f^{d_f}-q_l\right)^T\dot{q}_l-\\ & \left(B+k_d(1-\eta_f)\right)\dot{q}_l^T\dot{q}_l+\left(\tilde{\Xi}_l\right)^T\dot{q}_l+\\ & k_d\eta_l\left(\dot{q}_l^{d_l}-\dot{q}_f\right)^T\dot{q}_f+k_b\left(q_l^{d_l}-q_f\right)^T\dot{q}_f-\\ & \left(B+k_d(1-\eta_l)\right)\dot{q}_f^T\dot{q}_f+\left(\tilde{\Xi}_f\right)^T\dot{q}_f\end{aligned}, \tag{27}$$

where $\tilde{\Xi}_i=\Xi_i-\Xi_i, i=l,f$ and the time derivative of $V$ satisfies

$$\begin{aligned}\dot{V}=\ & \dot{V}_1(q_i,\dot{q}_i)+\dot{V}_2(q_i,\dot{q}_i)+\dot{V}_3(q_i,\dot{q}_i)\\ \le\ & k_d\eta_f\left(\left(\dot{q}_f^{d_f}\right)^T\dot{q}_l-\dot{q}_l^T\dot{q}_l\right)+k_b\left(\left(q_f^{d_f}\right)^T\dot{q}_l-q_l^T\dot{q}_l\right)-\\ & \left(B+k_d(1-\eta_f)\right)\dot{q}_l^T\dot{q}_l+\tilde{\Xi}_l\dot{q}_f+\\ & k_d\eta_l\left(\left(\dot{q}_l^{d_l}\right)^T\dot{q}_f-\dot{q}_f^T\dot{q}_f\right)+k_b\left(\left(q_l^{d_l}\right)^T\dot{q}_f-q_f^Tq_f\right)-\\ & \left(B+k_d(1-\eta_l)\right)\dot{q}_f^T\dot{q}_f+\tilde{\Xi}_f\dot{q}_f+\\ & k_b\left(q_l^T\dot{q}_l-q_l^T\dot{q}_f-q_f^T\dot{q}_l+q_f^T\dot{q}_f\right)+\\ & k_b\left(1+\bar{d}_l\right)\dot{q}_l^T\dot{q}_l+k_b\left(1+\bar{d}_f\right)\dot{q}_f^T\dot{q}_f-\\ & k_b\gamma_l^2\left(\dot{q}_l^{d_l}\right)^T\dot{q}_l^{d_l}-k_b\gamma_f^2\left(\dot{q}_f^{d_f}\right)^T\dot{q}_f^{d_f}-\\ & k_b\int_{t-d_l}^t\dot{q}_l^T(\theta)\dot{q}_l(\theta)d\theta-k_b\int_{t-d_f}^t\dot{q}_f^T(\theta)\dot{q}_f(\theta)d\theta+\\ & \tilde{\Theta}_{j,l}^T\Gamma_L^{-1}\dot{\tilde{\Theta}}_{j,l}+\tilde{\Theta}_{j,f}^T\Gamma_F^{-1}\dot{\tilde{\Theta}}_{j,f}\end{aligned}. \tag{28}$$

Using the adaptive laws (17) and ${}^i\dot{\Theta}_{j,k}=-{}^i\dot{\tilde{\Theta}}_{j,k}, i=1,2$, $k=l,f$ we have

$$\begin{aligned}\dot{V}\le\ & k_d\eta_f\left(\dot{q}_f^{d_f}\right)^T\dot{q}_l+k_b\left(q_f^{d_f}\right)^T\dot{q}_l+k_d\eta_l\left(\dot{q}_l^{d_l}\right)^T\dot{q}_f+\\ & k_b\left(q_l^{d_l}\right)^T\dot{q}_f+\left(-B-k_d+k_b\left(1+\bar{d}_l\right)\right)\dot{q}_l^T\dot{q}_l+\\ & \left(-B-k_d+k_b\left(1+\bar{d}_f\right)\right)\dot{q}_f^T\dot{q}_f+\tilde{\Xi}_l\dot{q}_l+\tilde{\Xi}_f\dot{q}_f+\\ & k_b\left(-q_l^T\dot{q}_f-q_f^T\dot{q}_l\right)-k_b\gamma_l^2\left(\dot{q}_l^{d_l}\right)^T\dot{q}_l^{d_l}-\\ & k_b\gamma_f^2\left(\dot{q}_f^{d_f}\right)^T\dot{q}_f^{d_f}-k_b\int_{t-d_l}^t\dot{q}_l^T(\theta)\dot{q}_l(\theta)d\theta-\\ & k_b\int_{t-d_f}^t\dot{q}_f^T(\theta)\dot{q}_f(\theta)d\theta+\tilde{\Theta}_{j,l}^T\Gamma_L^{-1}\dot{\tilde{\Theta}}_{j,l}+\tilde{\Theta}_{j,f}^T\Gamma_F^{-1}\dot{\tilde{\Theta}}_{j,f}\\ \le\ & k_d\eta_f\left(\dot{q}_f^{d_f}\right)^T\dot{q}_l+k_d\eta_l\dot{q}_l^{d_l}\dot{q}_f-k_b\gamma_l^2\left(\dot{q}_l^{d_l}\right)^T\dot{q}_l^{d_l}-\\ & k_b\gamma_f^2\left(\dot{q}_f^{d_f}\right)^T\dot{q}_f^{d_f}-\left(-B-k_d+k_b\left(1+\bar{d}_l\right)\right)\dot{q}_l^T\dot{q}_l-\\ & k_b\dot{q}_f\int_{t-d_l}^t\dot{q}_l(\theta)d\theta-k_b\dot{q}_l\int_{t-d_f}^t\dot{q}_f(\theta)d\theta-\\ & \left(-B-k_d+k_b\left(1+\bar{d}_f\right)\right)\dot{q}_f^T\dot{q}_f-\\ & k_b\int_{t-d_l}^t\dot{q}_l^T(\theta)\dot{q}_l(\theta)d\theta-k_b\int_{t-d_f}^t\dot{q}_f^T(\theta)\dot{q}_f(\theta)d\theta\end{aligned} \tag{29}$$

Using Lemma 1, we have

$$\begin{cases}-k_b\dot{q}_f\int_{t-d_l}^t\dot{q}_l(\theta)d\theta-k_b\int_{t-d_l}^t\dot{q}_l^T(\theta)\dot{q}_l(\theta)d\theta\le\dfrac{k_b\bar{d}_l}{4}\dot{q}_f^T\dot{q}_f\\ -k_b\dot{q}_l\int_{t-d_f}^t\dot{q}_f(\theta)d\theta-k_b\int_{t-d_f}^t\dot{q}_f^T(\theta)\dot{q}_f(\theta)d\theta\le\dfrac{k_b\bar{d}_f}{4}\dot{q}_l^T\dot{q}_l\end{cases}. \tag{30}$$

Then

$$\begin{aligned}\dot{V}\le\ & k_d\eta_f\dot{q}_f^{d_f}\dot{q}_l+k_d\eta_l\dot{q}_l^{d_l}\dot{q}_f-k_b\gamma_l^2\left(\dot{q}_l^{d_l}\right)^T\dot{q}_l^{d_l}-\\ & k_b\gamma_f^2\left(\dot{q}_f^{d_f}\right)^T\dot{q}_f^{d_f}+\left(k_b\left(1+\bar{d}_l\right)-B-k_d\right)\dot{q}_l^T\dot{q}_l+\\ & \left(k_b\left(1+\bar{d}_f\right)-B-k_d\right)\dot{q}_f^T\dot{q}_f+\frac{k_b}{4}\bar{d}_l\dot{q}_f^T\dot{q}_f+\\ & \frac{k_b}{4}\bar{d}_f\dot{q}_l^T\dot{q}_l\\ \le\ & -\left(k_b\gamma_l^2\left(\dot{q}_l^{d_l}\right)^T\dot{q}_l^{d_l}-k_d\eta_f\dot{q}_f^{d_f}\dot{q}_l+\frac{k_d^2\eta_f^2}{4k_b\gamma_l^2}\dot{q}_l^T\dot{q}_l\right)-.\\ & \left(k_b\gamma_f^2\left(\dot{q}_f^{d_f}\right)^T\dot{q}_f^{d_f}-k_d\eta_l\dot{q}_l^{d_l}\dot{q}_f+\frac{k_d^2\eta_l^2}{4k_b\gamma_f^2}\dot{q}_l^T\dot{q}_l\right)+\\ & \left(k_b\left(1+\frac{5}{4}\bar{d}_l\right)+\frac{k_d^2\eta_f^2}{4k_b\gamma_l^2}-B-k_d\right)\dot{q}_l^T\dot{q}_l+\\ & \left(k_b\left(1+\frac{5}{4}\bar{d}_f\right)+\frac{k_d^2\eta_l^2}{4k_b\gamma_f^2}-B-k_d\right)\dot{q}_f^T\dot{q}_f\end{aligned} \tag{31}$$

The final stability condition is achieved as **Theorem 1** shown.