The Initial Robustness Analysis of Designed U-Pole Placement Control Systems

Yuan Peng, Xin Liu, Quanmin Zhu, Fengxia Xu, Dongya Zhao

Abstract — **In this study, an initial robust stability analysis procedure is proposed to test the performance of the designed U-pole placement control systems. Unlike the classical design procedures for non-linear control systems, the control-oriented U-model based non-linear control systems cancel the non-linearity of the non-linear models. Therefore, the closed loop transfer function of U-pole placement control system can be regarded as a linear block. Once the internal parameter changes, the parameter variation of the closed loop characteristic equation can be detected by the least squares method to fit measured input and output. The stability margin of the closed loop system can be determined by using LMI (Linear Matrix Inequality) based robust stability analysis procedure. In this study a Hammerstein model is selected to test the robust stability of the U-pole placement control systems. The simulation results demonstrate the proposed procedure effective.**

Index Terms—**Nonlinear control systems, U-Model, robust stability, pole placement controller**

I. Introduction

Control problems arising in a wide variety of engineering fields are characterised by essential nonlinearities. In this case, pole placement approach generally cannot be directly applied because the dynamic behaviour of nonlinear plants cannot be easily determined according to the position of zeros and poles. It is obviously that applying pole placement to nonlinear plants is to synthesise a control system in such a way that the nonlinearities of the nonlinear plant should be removed and the resultant closed loop system behaves linearly.

It must be noted that the main difficulty in the design of nonlinear control systems is the lack of a general modelling framework which allows the synthesis of a simple control law [1]. In some instances linearizing structures have been used but these suffer from 'local applicability' [2], [3] and therefore, are not very attractive. In order to simplify the control law synthesis part in nonlinear modelling, a new control-oriented model termed as the U-Model has recently been suggested [1]. The U-Model has a more general appeal as compared to other non-linear models (NARMAX model and Hammerstein model). Additionally, this model is control-oriented in nature which makes the control synthesis part easier. Specifically, the control law based on the U-model exhibits a polynomial structure in the current input term.

Based on the U-Model, pole placement controllers [1] for non-linear dynamic plants with known parameters have recently been proposed. Some previous works [4], [5] discussed how to design a pole placement based controller using the method of U-Model. Other works [6]-[9] focused on the research of different types of controllers enhanced by U-Model. However, the parameters of non-linear plants in these studies are all regarded as given without considering the uncertainties. Furthermore, the robustness of U-Model based controllers is rarely concerned because it is hard to describe the control-oriented prototype as a state space expression. Motivated by some previous theoretical results [1], in this study a designed U-Model based pole placement control system is used for the robustness test. The uncertainty of the non-linear plant is taken into consideration and the parameter changes of the close loop system are selected for robust stability analysis.

The main contents of this paper are divided into four sections. In section 2 the designed U-Model based pole placement controller is introduced to represent the fundamental methodologies. In section 3 the basic robustness analysis is introduced for implementing system robust stability study. In section 4 a step by step procedure is listing of proposed robustness analysis. In section 5 a Hammerstein model is selected to demonstrate the robustness analysis and the corresponding simulation results are presented with graphical illustrations. In section 6 a summary of the paper is presented.

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Yuan Peng is in the department of Engineering Design and Mathematics, University of the West of England, Bristol, BS16 1QY, UK, (e-mail: Yuan2.Peng@ uwe.ac.uk).

Xin Liu is in the department of Engineering Design and Mathematics, University of the West of England, Bristol, BS16 1QY, UK, (e-mail: Xin2.Liu@live. uwe.ac.uk).

Quanmin Zhu is in the department of Engineering Design and Mathematics, University of the West of England, Bristol, BS16 1QY, UK, (e-mail: Quan.Zhu@uwe.ac.uk).

II. Designed U-Pole Placement Controller

The U-pole placement controller design proposed by Zhu and Guo in [1] will be presented in this section as the fundamental methodology.

Consider single input and single output (SISO) non-linear dynamic plants with a NARMAX (nonlinear auto-regressive moving average with exogenous inputs) representation of the form

$$
\begin{aligned}
y(t) = f(&y(t-1), \ldots, y(t-n), \\
&u(t-1), \ldots, u(t-n), \\
&e(t), \ldots, e(t-n))
\end{aligned} \tag{2.1}
$$

where $y(t)$ and $u(t)$ are the output and input signals of the plant respectively at discrete time instant t, n is the plant order, $f(\cdot)$ is a non-linear function, and the modelling error term $e(t)$. The control oriented model can be obtained by expanding the nonlinear function $f(.)$ as a polynomial with respect to $u(t\text{-}1)$ as follows

$$
y(t) = \sum_{j=0}^{M} \alpha_j(t) u^j(t-1) \tag{2.2}
$$

where M is the degree of model input $u(t-1)$, parameter $\alpha_j(t)$ is a function of past inputs and outputs $u(t\text{-}2)$, …, $u(t\text{-}n)$, $y(t\text{-}1)$, …, $y(t\text{-}n)$, and errors $e(t)$, …, $e(t\text{-}n)$. By this arrangement, the control oriented model can be treated as a pure power series of input $u(t\text{-}1)$ with associated time-varying parameters $\alpha_j(t)$.

Fig. 1 shows the block diagram of the U-model based pole placement control system. In the U-pole placement design, the U-model is firstly transferred from the non-linear model. With the polynomial equation of U-model as a root solver, the Newton-Raphson algorithm can be used to find the controller output.

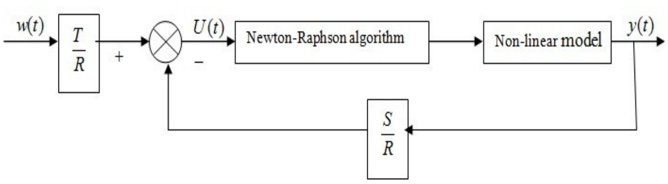

Fig. 1. Block diagram of U-pole placement control system

A standard reference [10] is used to develop following formulations for designing pole placement controller. Consider the U-model of (2.2), a general controller can be described by

$$
RU(t) = Tw(t) - Sy(t) \tag{2.3}
$$

where $w(t)$ is the reference for output target and R, S, and T are the polynomials of the forward shift operator q.

The control law of (2.3) represents a negative feedback with transfer function $-S/R$ and a feedforward with transfer function T/R. It thus has two degree of freedom. The block diagram of the closed loop U-pole placement control system is shown in Fig. 1. The output $y(t)$ can be linked to the reference $w(t)$ as

$$
y(t) = \frac{T}{R+S} w(t) = \frac{T}{A_c} w(t) \tag{2.4}
$$

where polynomial A_c is the closed loop characteristic equation. The polynomials R S and T can be resolved by a Diophantine equation To make control output equals the desired output, which means that the steady state error equal to zero at the control output. The polynomial T is specified with $T = A_c(1)$ from equation (2.4). The key idea of the design is to specify the desired closed loop characteristic polynomial A_c, then resolve. The signal $U(t)$ can be obtained by (2.3) as long as polynomials R, S, and T are determined. With $U(t)$ a root solver, Newton-Raphson algorithm [11], can be used to find the controller output $u(t\text{-}1)$.

The identification error and stability of the controller of the U-pole placement control system have been discussed in [1]. An enhanced Newton-Raphson algorithm is proposed to guarantee the stability of the controller in a minimum phase system [16].

Due to the U-model framework, the non-linearity of the non-linear model is cancelled. The closed loop of U-pole placement control system behaves similarly to that of a linear system. The equivalent block diagram of U-model based pole placement closed loop system is shown in fig. 2

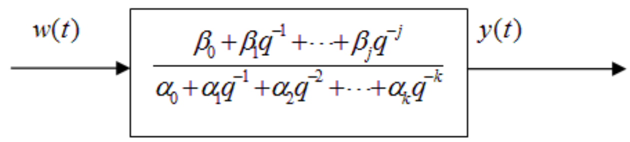

Fig. 2. Equivalent block diagram of U-pole placement control system

III. Basic Robustness Analysis

In the robustness analysis of the control systems, the definition of uncertainty is very significant [13]. To design an effective control system, a complex dynamic plant should be described as a relative simple model. The model uncertainty always exists in the control systems. Besides the uncertain of the simplified model expression, the uncertainties are caused by the environmental change, components aging, parameters drift and unknown errors. This uncertainty is quite different from the external factors such as external disturbance and measurement noise. In this section, the disturbance of internal parameter variation is picked up as the first concerned uncertain factor. The robustness analysis includes two aspects, one is the robust stability, and the other one is robust performance which means make control system not only has stability robustness, but also satisfy some performance constrains.

Consider an uncertain parameter system [12]

$$
\dot{x} = A(\delta)x(t) \tag{3.1}
$$

where $x \in R^n$ is state vector, and $A(\delta)$ is the function of real parameter vector $\delta = [\delta_1, \ldots, \delta_n]^T \in R^k$. Assume that the uncertain parameter δ takes value from the given set Δ, thus the uncertain system (3.1) is always asymptotical stable under this system robust stability condition. According to the uncertain parameter δ is time varying parameter or not, the uncertain system (3.1) is analysed by the different processing methods. A suitable approach of studying the stability of time varying system is the Lyapunov stability theory.

To all the uncertain parameters $\delta \in \Delta$, if and only if the positive definite matrix $P > 0$ satisfies

$$A^T(\delta)P + PA(\delta) < 0 \quad (3.2)$$

The uncertain system (3.1) is quadratic stable. In such a quadratics stable system (3.1), the quadratic form of Lyapunov function is obtained from equation (3.2)

$$V(x) = x^T Px, \ \delta \in \Delta \quad (3.3)$$

The equation (3.3) satisfies

$$V = \frac{dV(x(t))}{dt} < 0 \quad (3.4)$$

According to Lyapunov stability theory, the uncertain system (3.1) is asymptotical stable. Note that the asymptotical stable uncertain system which is so called robustness stability can be obtained from the system quadratics stability.

Generally, Δ is defined by an infinite set. Therefore, the definition of quadratic stability requires testing the feasibility of infinite quantities of matrix inequalities. It is obviously impossible to obtain results in a specific control system.

Consider in an uncertain parameter system (3.1), the matrix $A(\delta)$ is described as

$$A(\delta(t)) = A_0 + \delta_1(t)A_1 + \ldots + \delta_k(t)A_k \quad (3.5)$$

where $A_0, \ldots, A_k$ is known $n \times n$ real constant matrixes, and uncertain parameter $\delta_i(t)$ is the bounded time varying function, where $\delta_i(t) \in [\delta_i^-(t), \delta_i^+], \ i = 1, 2, \ldots, k$.

Define a vertex set as:

$$\Delta_0 = \{\delta = [\delta_1, \ldots, \delta_k]; \delta_i = \delta_i^- or \ \delta_i^+, i = 1, \ldots, k\} \quad (3.6)$$

The allowed value range of uncertain parameter Δ is a convex cell of vertex set, which means that a set is constituted by all convex combinations of the midpoint of vertex set Δ_0.

Theorem the uncertain parameter system (3.1) with the matrix $A(\delta)$ (3.5) has quadratic stability, if and only if a symmetrical positive definite matrix exists, the LMI (3.2) is tenable [12].

It should be noted that this theorem establishes a basis for developing the algorithm for the LMI based robust stability margin analysis used in this study.

The set Δ is infinite, but Δ_0 is a finite set. Following the theorem, the only need is to test if the LMI true or false so that the system quadratic stability can be estimated. The condition of the system quadratic stability is to judge the feasibility of linear matrixes based on a group of LMI. The question of the feasibility of LMI can be solved by the MATLAB LMI tool box.

A simple approach to examine the system quadratic stability is using LMI tool box in MATLAB. It supplies the function to test the quadratic stability of the uncertain parameter system (3.1). This function is determined the maximum range of the uncertain parameter to keep system quadratic stability, which is the maximum quadratic stability area. Denote that $\mu_i = \frac{1}{2}(\delta_i^-, \delta_i^+), \lambda_i = \frac{1}{2}(\delta_i^-, \delta_i^+)$, where $\delta_i(t) \in [\delta_i^-, \delta_i^+]$. The maximum quadratic stability range of closed loop system is to find an estimation θ, which is satisfied the quadratic stability with all $\delta_i \in [\mu_i - \theta\lambda_i, \mu_i + \theta\lambda_i]$.

The least square recurrence method is used to estimate the parameters of the closed loop transfer function via measured input and output [10].

Consider a linear regression model

$$y(k) = h^T(k)\theta \quad (3.7)$$

where $y(t)$ is a matrix of measurable quantity, $h(k) = [y(k-1) \ldots y(k-n) \ u(k) \ldots u(k-m)]^T$, and $\theta = [\alpha_1, \alpha_2, \ \ldots \ \alpha_k, \beta_0, \beta_1, \ \ldots \ \beta_j]^T$ is an n-vector of unknown parameters, called parameter vector.

The algorithm of the least square method is specified as

$$P(k) = [I - K(k)h'(k)]P(k-1)$$

$$K(k) = P(k-1)h(k)[h'(k)P(k-1)h(k) + 1]^{-1}$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - h'(k)\hat{\theta}(k-1)]$$

where $P(k)$ and $K(k)$ are the gain matrixes. The parameter vector $\hat{\theta}$ can be estimated by the previous algorithm via the measured input and output.

In the U-pole placement control system, the closed loop transfer function can be obtained from (2.4)

$$\begin{aligned} y(t) &= \frac{T}{R+S}w(t) = \frac{T}{A_c}w(t) \\ &= \frac{t_0 q^m + t_1 q^{m-1} + \ldots + t_m}{q^n + a_{c1}q^{n-1} + \ldots + a_{cn}} w(t) \end{aligned} \quad (3.8)$$

The least square algorithm can be estimated the $\theta = [a_{c1}, a_{c2}, \ \ldots \ a_{ck}, t_0, t_1, \ \ldots \ t_j]^T$ in $\frac{T}{A_c}$.

Due to the limiting condition of the LMI, the estimated discrete transfer function of closed loop system is necessary to be transformed into continuous transfer function. The following formula is used [15]

$$\begin{aligned} F(s) &= \sum \mathrm{Re}\, s\left[F(z)\frac{z^{-1}}{s - \frac{1}{T}\ln z} \right] \\ &= \sum \left\{ \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z - z_i)^m F(z)\frac{z^{-1}}{s - \frac{1}{T}\ln z} \right] \right\}_{z=z_i} \end{aligned}$$

where *F(s)* is the transfer function of continuous system and *F(z)* is the transfer function of discrete system.

IV. The Proposed Analysis Procedure

A step-by-step procedure for the robust stability study of U-model pole placement control system can be specified as the follows:

Step 1. Design the U-model controller for the selected non-linear model to reach the requirement of desired closed loop characteristic equation [1].

Step 2. Determine the range of the uncertain parameter for the selected non-linear model and test the system response of the control system.

Step 3. According to the measured input and output, use least squares data fitting approach [10] to find the parameter range of the closed loop characteristic equation (2.4).

Step 4. Obtain the continuous system transfer function from the discrete system transfer function (2.4).

Step 5. Use LMI [12] based robust stability analysis equation (3.5) to obtain the stability margin of the closed loop system. Normally MATLAB LMI toolbox can be used to obtain the numerical results.

V. SIMULATED CASE STUDIES

A Hammerstein model is selected for the robust stability test. The closed loop characteristic equation is specified with

$$A_c = q^2 - 1.3205q + 0.4966 \tag{4.1}$$

Therefore the closed loop poles are a complex conjugate pair of $-0.6603 \pm j0.2463$. This design specification corresponds to a natural frequency of 1 rad/sec and a damping ratio of 0.7. To achieve zero steady state error, specify

$$T = A_c(1) = 1 - 1.3205 + 0.4966 = 0.1761 \tag{4.2}$$

For the polynomials *R* and *S*, specify

$$\begin{aligned} R &= q^2 + r_1 q + r_2 \\ S &= s_0 q + s_1 \end{aligned} \tag{4.3}$$

Substitute the specifications of (4.1) and (4.3) into Diophantine equation of (3.5), the coefficients in polynomials *R* and *S* can be expressed with

$$\begin{aligned} r_2 + s_1 &= 0.4966 \\ r_1 + s_0 &= -1.3205 \end{aligned} \tag{4.4}$$

To guarantee the computation convergence of the sequence *U*(*t*), that is to keep the difference equation with stable dynamic, let $r_1 = -0.9 \quad r_2 = 0.009$. This assignment corresponds the characteristic equation of *U*(*t*) as $(q-0.89)(q-0.01) = 0$. Then the coefficients in polynomial *S* can be determined from the Diophantine equation of (4.4)

$$s_0 = -0.4205 \quad s_1 = 0.4876$$

Substitute the coefficients of the polynomials *R* and *S* into controller of (3.1), gives rise to

$$\begin{aligned} U(t+1) &= 0.9U(t) - 0.009U(t-1) \\ &+ 0.1761w(t-1) + 0.4205y(t) - 0.4876y(t-1) \end{aligned}$$

Therefore the controller output *u*(*t*) can be determined.

Consider the following Hammerstein model [1]

$$\begin{aligned} y(t) &= 0.5y(t-1) + x(t-1) + 0.1x(t-2) \\ x(t) &= 1 + u(t) - u^2(t) + 0.2u^3(t) \end{aligned} \tag{4.5}$$

The corresponding control oriented model is obtained from formulation (2.1)

$$y(t) = \alpha_0(t) + \alpha_1(t)u(t-1) + \alpha_2(t)u^2(t-1) + \alpha_3(t)u^3(t-1)$$

where

$$\alpha_0(t) = 0.5y(t-1) + 1 + 0.1x(t-2) \qquad \alpha_1(t) = 1$$

$$\alpha_2(t) = -1 \qquad \alpha_3(t) = 0.2$$

The system response under the proposed pole placement control approach has been analysed in [1]. It can be seen from simulation result that the resultant closed loop system behaves similarly to that of a linear system, which is due to cancellation of the nonlinearity by the proposed control-oriented model and controller design approach.

However, if the internal parameter of the non-linear model is changed, the controller performance will not be same standard and that's the proposed robust stability analysis procedure which is going to be studied in the simulations.

Case I: For the selected Hammerstein model, the time varying parameter $\alpha_0(t)$ was determined as the unknown parameter. The variation of the parameter $\alpha_0(t)$ was selected as

$$\alpha_0(t) \in \left[0.1y(t-1) + 1 + 0.1x(t-2),\ \ 0.8y(t-1) + 1 + 0.1x(t-2)\right]$$

After the least squares data fitting, the closed loop characteristic equation is obtained as

$$A_c' = q^2 + \alpha_1 q + \alpha_2$$

where are respectively

$$A_c'(1) = q^2 + 0.6883q + 0.4336$$

$$A_c'(2) = q^2 + 1.7580q + 1.4700$$

where the variation range of the parameters are respectively $\alpha_1 \in [0.6883,\ \ 1.7580]$ and $\alpha_2 \in [0.4436,\ \ 1.4700]$. The reference input and the plant output of the system ($\alpha_0(t) = 0.8y(t-1) + 1 + 0.1x(t-2)$) is shown in fig. 3.

The result of the robust stability margin is 1.1239 which indicates that the U-model controller can guarantee the system stability within the selected range of $\alpha_0(t)$ and even if the range is increased to 12.39%.

Case II: For the same Hammerstein model, the time varying parameter $\alpha_0(t)$ was still selected as the unknown parameter. The different variation of the parameter $\alpha_0(t)$ was

$$\alpha_0(t) \in \left[0.5y(t-1) + 1 + 0.1x(t-2),\ \ 1.3y(t-1) + 1 + 0.1x(t-2)\right]$$

The closed loop characteristic equation can be obtained by the least squares data fitting method, the characteristic equation expression is

$$A_c' = q^2 + \alpha_1 q + \alpha_2$$

The estimation results are respectively

$$A_c'(1) = q^2 + 0.0752q + 1.9010$$

$$A_c'(2) = q^2 + 0.4229q + 0.8926$$

where the variation range of the parameters are respectively $\alpha_1 \in [0.0752,\ \ 0.4229]$ and $\alpha_2 \in [0.8926,\ \ 1.9010]$. The reference input and the plant output of the system is shown in figures 4 and 5. Fig. 4 shows the U-pole placement control system is in the bound of the robust stability area. And fig. 5 shows the U-pole placement control system is unstable in the range of the internal parameter variation.

The result of the robust stability margin is 0.4139 which indicates that the U-model controller can only guarantee the system stability within 41.39% of the selected parameter range and in the other 58.61% the closed loop system is not stable with the designed U-model controller.

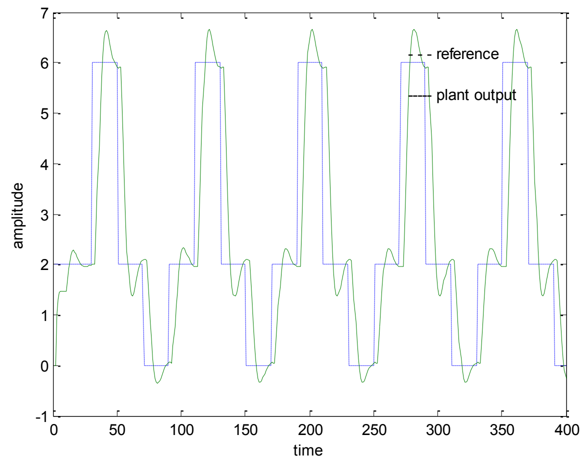

Fig. 3. Performance of Hammerstein model in case I

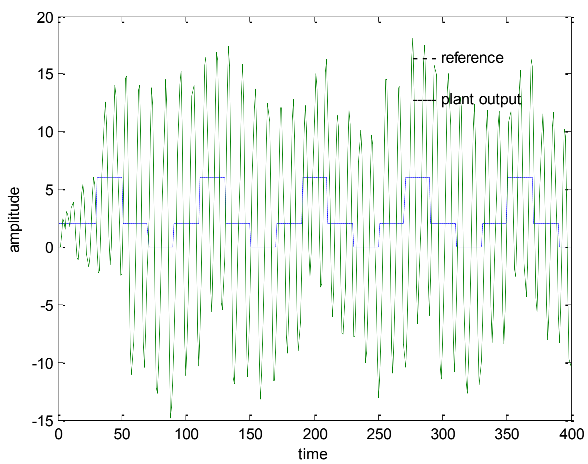

Fig. 4. Performance of Hammerstein model in case II

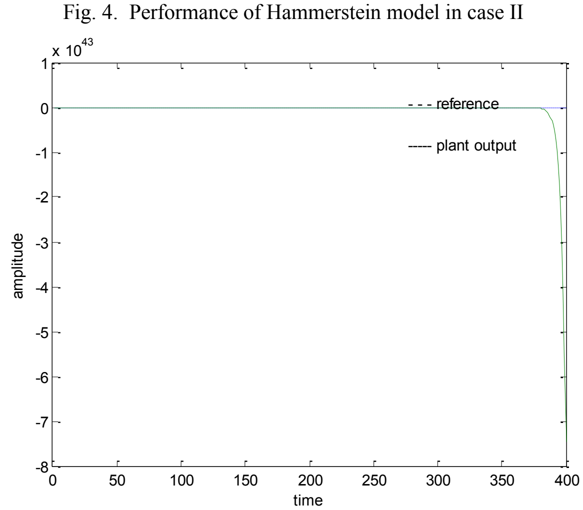

Fig. 5. Performance of Hammerstein model in case II

The simulation results show that the robustness of the U-model based pole placement control system depends on the uncertainty of the non-linear model. The U-model controller can keep the system to be stable within a certain range of the parameter uncertainty. However, if the parameter of the non-linear model is change far away from the original one, the performance of the controller cannot be guaranteed.

VI. Conclusions

A general control oriented U-model and the corresponding pole placement controller design for the dynamic non-linear plants have been introduced to be the fundamental methodologies. With the modularization the procedure of non-linear control system design can be conducted as linear control system design. The robust stability study can be implemented to non-linear models by the help of U-model pole placement method. The least squares data fitting method is necessary in the proposed essential approach and the parameter variation range of the closed loop system can be estimated via measured data. The LMI based robust analysis obtained the stability margin of the closed loop system. The simulation results show that robust stability method can be successfully implemented for U-model based pole placement control systems.

Further studies on the developed methodology, such as H-norm based state feedback controller design for U-model based control systems, expansion to other format controllers, and so on, will be conducted to provide a comprehensive prospectus in designing nonlinear control systems by using linear control system design techniques.

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