Revisiting Voyager 2 Trajectory

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Abstract

This paper presents reconstruction of Voyager 2 trajectory to examine how the mission was optimised for fuel and Time of Flight (TOF), as well as to find out if there is a more efficient trajectory for the full Earth-Jupiter-Saturn-Uranus-Neptune (EJSUN) sequence. Variable TOFs between each interplanetary maneuver for launch dates ± 1 year from the original were considered. Brute force search algorithms were developed to find optimized trajectories, the solution to Lambert's problem for fast computation, and a patched conic integrator to plot the optimal trajectories. Multiple 'unpowered' gravity assist trajectories were found: the minimum total change in velocity (ΔV) at perigee found was less than 0.0035km/s for a total TOF of 4978 days (13.64 years), the shortest TOF trajectory found spanned 3299 days (9.04 years), with a total ΔV at perigee requirement less than 0.11km/s. With consideration to the trajectories found, it was determined that the Voyager 2 trajectory was likely to have been optimized for TOF.

Keywords: Trajectory, Voyager 2, Lambert's Problem, Gravity Assist

Nomenclature:

Symbol	*Description*	*Unit*
a_{in}	Incoming orbit semi-major axis	km
a_{out}	Outgoing orbit semi-major axis	km
δ	Turn angle	°
DSM	Deep space manoeuvre	-
e_{in}	Incoming orbit eccentricity	-
e_{out}	Outgoing orbit eccentricity	-
EJ	Earth-Jupiter	-
EJS	Earth-Jupiter-Saturn	-
EJSUN	Earth-Jupiter-Saturn-Uranus-Neptune	-
GA	Gravity assist	-
JPL	Jet Propulsion Laboratory	-
JS	Jupiter-Saturn	-
LV	Launch velocity	-
MGA	Multiple gravity assist	-
r_p	Perigee radius	km
SOI	Sphere of influence	km
SU	Saturn-Uranus	-
TOF	Time of flight	Days
μ	Standard gravitational parameter	km^3/s^2

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UN	Uranus-Neptune	-
ΔV	Velocity increment	km/s
$\Delta V_{Perigee}$	Required velocity increment at perigee	km/s
$\Delta V_{\infty-in}$	Incoming planetocentric velocity increment	km/s
$\Delta V_{\infty-out}$	Outgoing planetocentric velocity increment	km/s
ΔV_{Planet}	Planet velocity increment	km/s
VIM	Voyager Interstellar Mission	-

1. Introduction

Space exploration has revealed insights into the nature of our solar system and the wider universe, the Voyager missions revealed an extraordinary amount of new information, from Io's constant volcanic activity to Neptune's surprisingly active weather systems [1]. Since the first successful interplanetary mission, the NASA Mariner 2 Venus probe which flew 34,773km over the surface on 14th December 1962, a host of other spacecraft have been launched to continue exploring the universe. Notable missions include Pioneers 10 and 11, the first to visit Jupiter and Saturn respectively; Mariner 10; Galileo; and, of course, Voyagers 1 and 2.

The Voyager missions launched in 1977 from Cape Canaveral, Florida, taking advantage of a rare planetary alignment, in which all four outer planets were aligned on one side of the sun, and the four-planet 'Grand Tour' path proposed by Flandro [2]. After 44 years of operation, both spacecraft are continuing their journey away from our solar system on the Voyager Interstellar Mission (VIM), described in detail by [3].

Trajectory design is an integral part of mission design, a non-optimal trajectory can result in higher than necessary TOF, fuel consumption, and cost [4]. Fuel requirements can be reduced by taking advantage of the 'free' energy gain obtained by gravity assists, instead of traditional chemical propulsion systems [5].

The aim is to investigate if more efficient trajectories than the one flown by the Voyager 2 mission exist, following the same EJSUN sequence, using a similar launch date. A preliminary design approach is adopted, using well known methods for orbit determination and brute force searching to find optimised trajectories. This paper identifies several unpowered GA trajectories using a single impulse ΔV from Earth. Various launch dates ± 1 year to the original and Earth-Jupiter TOFs were considered.

2. Models

Lambert targeting was employed to find the trajectories of each interplanetary leg (planet centre to planet centre); a patched conic integration was used to propagate the departure velocity found from Lambert targeting to plot the trajectory; and the method outlined by Wagner and Wie [6] was utilised to find $\Delta V_{perigee}$ during gravity assist flyby. It was assumed that the spacecraft was affected by the gravitational forces of one body at a time, and flyby was assumed to be less than a day. Planetary SOIs are only considered during calculation of GA parameters.

2.1 Lambert's Problem

Since the advent of spaceflight, many methods have been developed for the solution to Lambert's problem, perhaps most notably Lancaster and Blanchard [7] developed a 'Unified form of Lambert's Theorem'. Izzo [8] proposed an algorithm based on this work, using a Householder iteration scheme and initial guesses derived with two new variables, ξ and τ. Izzo's solution to Lambert's problem is used to find departure and arrival velocities for each interplanetary leg. For simplicity, planet centres are used as trajectory start and end points in Lambert targeting, and the arrival date at the planet for one leg is assumed to be the departure date for the next leg.

2.2 Gravity-Assist (GA) Manoeuvres

The first spacecraft to use GA was the Mariner 10 mission, since then many other missions, including Voyagers 1 and 2, have taken advantage of GA [9]. The ΔV gained from these manoeuvres allowed the Voyager and Pioneer missions gain enough velocity to escape the solar system. The method used is the one proposed by Wagner and Wie [6], described below.

To find the required ΔV at perigee, the planetocentric reference frame is used, heliocentric velocity vectors found by Lambert targeting are converted to planetocentric velocities using Eqn. 1.

$$v_{\infty-in} = V_{s/c-in} - V_{planet}, \qquad v_{\infty-out} = V_{s/c-out} - V_{planet} \qquad \textit{Eqn. 1}$$

The perigee radius, r_p, Eqn. 2 can then be found using the semi-major axes of the incoming, a_{in}, and outgoing hyperbolic orbits, a_{out}, or in terms of the standard gravitational parameter and planetocentric velocities, Eqn. 3. In all phases r_p must be greater than the radius of the planet for impact avoidance. Atmospheric effects, such as drag, were not considered during calculations.

$$r_p = a_{in}(1 - e_{in}) = a_{out}(1 - e_{out}) \qquad \textit{Eqn. 2}$$

$$a_{in} = -\mu_{planet}/v_{\infty-in}^2, \qquad a_{out} = -\mu_{planet}/v_{\infty-out}^2 \qquad \textit{Eqn. 3}$$

The turning angle can be defined as a function of the velocities, Eqn. 4, or in terms of orbit eccentricity, Eqn. 5. These can be combined to form Eqn. 6, and iteratively solved for outbound eccentricity, e_{out}, using the Newton iterative method, which requires the first derivative of the function, Eqn. 7.

$$\delta = \cos^{-1}\left(\frac{\boldsymbol{v}_{\infty-in} \cdot \boldsymbol{v}_{\infty-out}}{v_{\infty-in} \cdot v_{\infty-out}}\right) \qquad \textit{Eqn. 4}$$

$$\delta = \sin^{-1}\left(\frac{1}{e_{in}}\right) + \sin^{-1}\left(\frac{1}{e_{out}}\right) \qquad \textit{Eqn. 5}$$

$$f = \left(\frac{a_{out}}{a_{in}}(e_{out} - 1) + 1\right)\sin\left(\delta - \sin^{-1}\left(\frac{1}{e_{out}}\right)\right) - 1 = 0 \qquad \textit{Eqn. 6}$$

$$\frac{df}{de_{out}} = \left(\frac{a_{out}}{a_{in}} e_{out} - \frac{a_{out}}{a_{in}} + 1\right) \frac{\cos\left(\delta - \sin^{-1}\left(1/e_{out}\right)\right)}{e_{out}^2 \sqrt{1 - 1/e_{out}^2}} + \frac{a_{out}}{a_{in}} \sin\left(\delta - \sin^{-1}\left(\frac{1}{e_{out}}\right)\right) - 1 \qquad \textit{Eqn. 7}$$

Once e_{out} has been found, r_p can be found using Eqn. 2 and used to find ΔV at perigee using Eqn. 8.

$$\Delta V_{perigee} = \left| \sqrt{v_{\infty-in}^2 + \frac{2\mu_{planet}}{r_p}} - \sqrt{v_{\infty-out}^2 + \frac{2\mu_{planet}}{r_p}} \right| \qquad \textit{Eqn. 8}$$

3. Methodology

The investigation consists of two phases. In Phase 1, the original Voyager 2 mission trajectory was replicated as validation for the results found for unpowered MGA trajectories for the same EJSUN sequence (Phase 2). Algorithms developed for these purposes are described in this section. For Phase 2, launch dates one year either side of the original Voyager 2 launch date were considered with a broad range of TOFs between interplanetary legs. The trajectories are solved sequentially starting with Earth-Jupiter (EJ), then Jupiter-Saturn (JS), Saturn-Uranus (SU), and finally Uranus-Neptune (UN).

Trajectory mapping was achieved using a patched conic integrator by propagating the departure velocity. The start date, end date, and spacecraft initial velocity were taken from Lambert targeting, spacecraft initial position was taken as the departure planet centre, and the departure planet SOI was ignored. During propagation, the spacecraft was considered to have reached the destination planet when it reached the edge of that planet's SOI, allowing for a much larger integration time step and therefore lower computation times.

3.1 Phase 1 – Replicating Voyager 2 Trajectory

The Phase 1 approach is outlined below and was implemented in MATLAB with the logic shown in Fig. 1.

1) Find and plot mission trajectory using known launch date and interplanetary TOFs.
2) Find the required $\Delta V_{perigee}$ for GAs.

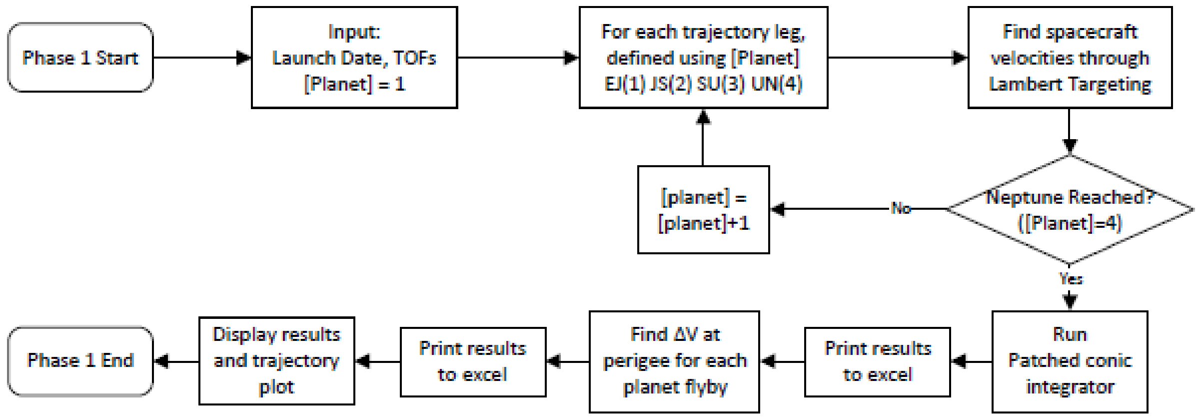

Figure 1 Phase 1 Code Logic Flowchart

Table 1 Phase 1 Inputs and Outputs

No.	Inputs	Outputs (Format)
1	Excel file names for results storage	Calculated velocities from Lambert targeting and Patched conic (excel and MATLAB)
2	Voyager 2 mission launch date	Comparison between calculated velocities against Voyager 2 velocities (excel and MATLAB)
3	Interplanetary TOFs (in days)	r_p, e_{out} and $\Delta V_{perigee}$ results (excel and MATLAB)
4	Integration step size for patched conic integrator	Plot points from patched conic integrator (excel) Trajectory plot (MATLAB figure)

The inputs and outputs for the Phase 1 code are shown in Table 1, these inputs are entered directly into the MATLAB script.

3.2 Phase 2 – Unpowered MGA Trajectory

The second Phase algorithm finds unpowered MGA trajectories with brute force searching. The process is outlined below:

1) Launch dates ±1 year of the original Voyager 2 mission are considered. Launch Velocity (LV) is manually selected within a reasonable range.
2) Using the selected variables, all feasible trajectories to Jupiter are found.
3) For each feasible EJ trajectory, all GA trajectories for subsequent legs are found by lambert targeting, varying TOF between a set range.

A basic layout of the code is given in Fig 2. Each process choice may be executed independently if the results from previous processes, from left to right, are available.

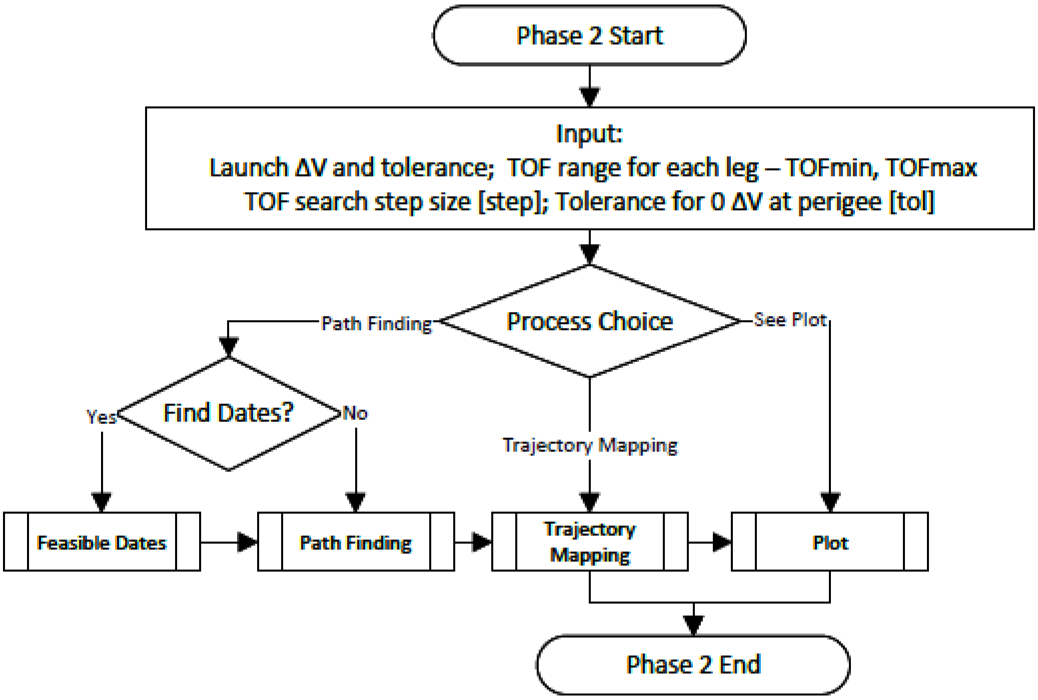

Figure 2 Phase 2 Code Logic Flowchart

The inputs and outputs of the code are shown in Table 2, as with Phase 1, inputs are entered directly into the script. In this report the same input values were used for a range of LVs. Different values of input 6 are used for each interplanetary leg to reduce computation times.

Table 2 Phase 2 Inputs and Outputs

No.	Inputs	Outputs (Format)
1	File path for excel file storage	List of feasible launch dates (excel sheet)
2	Launch ΔV and tolerance	Parameters for unpowered GA trajectories (excel file)
3	Tolerance for $\Delta V_{perigee}$	Plot points from patched conic integrator (excel trajectory file)
4	Patched conic integration step size	Trajectory plots (MATLAB figure)
5	TOF range for Lambert targeting	-
6	Trajectory search step size (*)	-

(*) The step size in this case is not related to TOF, but feasible trajectory number.

The 'Feasible Dates' process from Fig. 2 is shown in more detail in Fig. 3, this finds all the Earth-Jupiter trajectories using the required LV, within a tolerance. Trajectories meeting the requirements are saved to an excel file and send to the 'Path Finding' process, Fig. 4, which finds all possible trajectories from Jupiter to Neptune using the results from the 'Feasible dates' process. The algorithm considers one trajectory leg at a time, upon completion the code finds the two optimum trajectories: shortest TOF and minimum ΔV.

Earth-Jupiter Feasible Dates

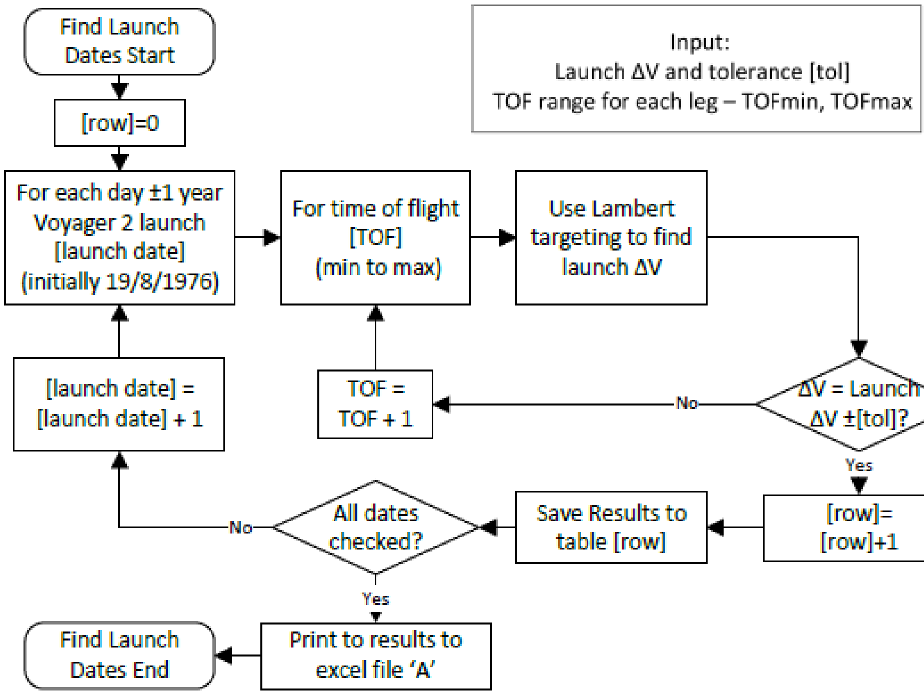

Figure 3 Phase 2 Earth-Jupiter Feasible Dates Flowchart

Path Finding

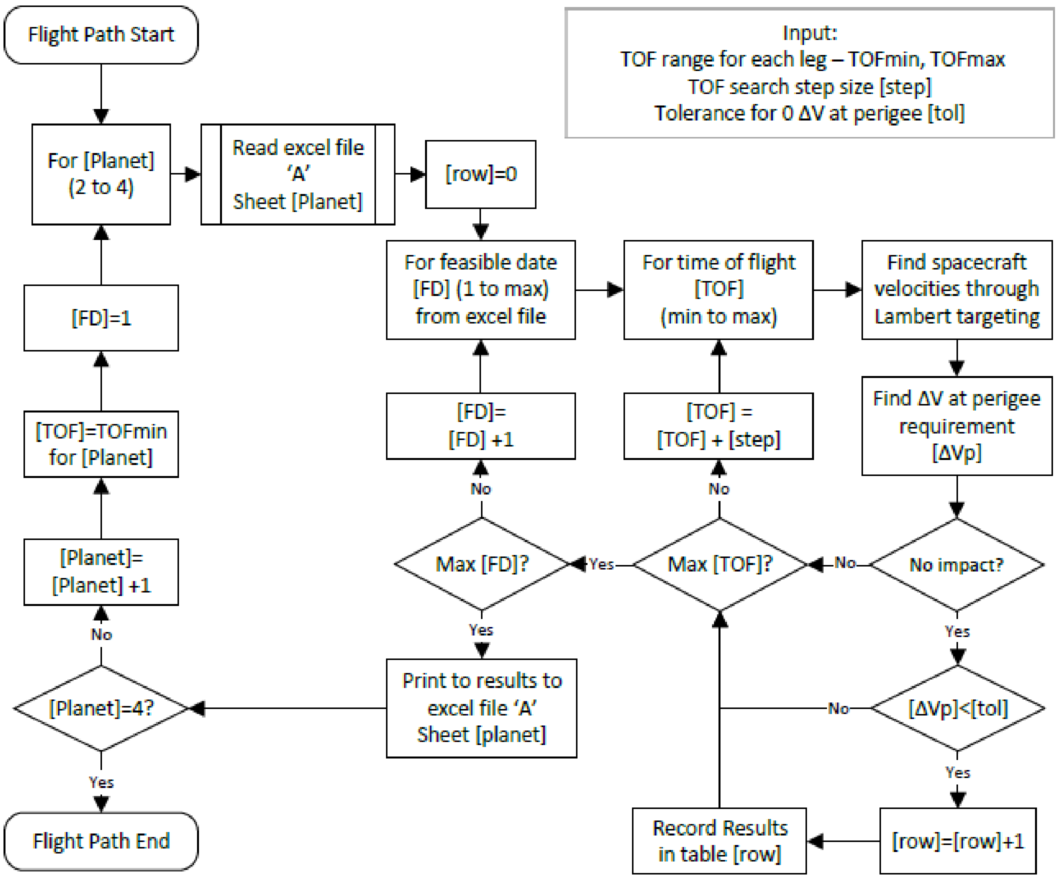

Figure 4 Phase 2 Path Finding Flowchart

Trajectory parameters are then read into the 'Trajectory Mapping' process, which uses a patched conic integrator to solve for the trajectory path. These data points can then be shown graphically via the 'Plot' process, an optimum trajectory for the input LV may be directly chosen, or other trajectories may be selected through a drop-down menu from a list of available files in the Phase 2 folder. Only one plot may be displayed at a time, trajectory variables are displayed in the MATLAB command window.

4. Results and Analysis

4.1 Phase 1 – Replicating Voyager 2 Trajectory

Using the Voyager 2 launch date, 20 August 1977, and GA dates taken from the Voyager 2 in-depth webpage, NASA (2021) to calculate interplanetary TOFs, the Phase 1 code successfully replicates the Voyager 2 trajectory. The shape of the plot produced agrees well with the trajectory map available from the mission website [10], Fig. 5.

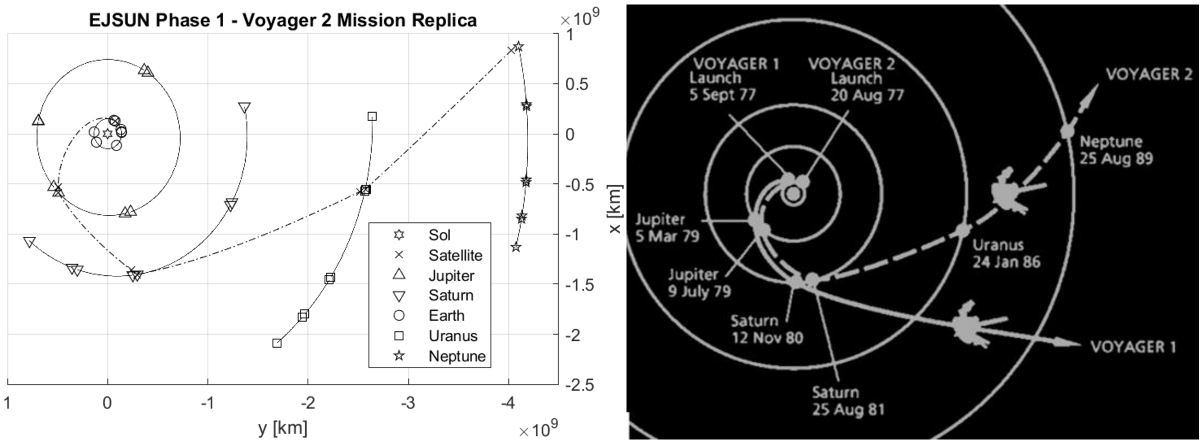

Figure 5 Trajectories – Left: MATLAB. Right: Voyager 1 and 2 Trajectories [10]

From Table 3 Table 3, the arrival dates found by the patched conic integrator are a few months before the dates from NASA, this was expected as the patched conic integrator stops when the satellite reaches the edge of the destination planet SOI. These 'missing' trajectory portions are negligible compared to the heliocentric trajectory.

Table 3 Phase 1 Calculated Velocities

Leg	TOF (days)	GA/Arrival Date		Incoming Velocity (km/s)		
		NASA [1]	Patched Conic	Lambert	Patched Conic	Actual
EJ	688	09/07/1979	30/04/1979	9.64	10.53	10.15
JS	779	26/08/1981	29/06/1981	15.45	15.70	16.34
SU	1612	24/01/1986	15/12/1985	17.80	17.85	20.83
UN	1309	25/08/1989	27/06/1989	18.90	18.93	20.27

The 'actual' velocities in Table 3 were found using the WebPlotDigitizer [11], using Fig. 6 taken from a reference [12]. These values must be taken as approximate.

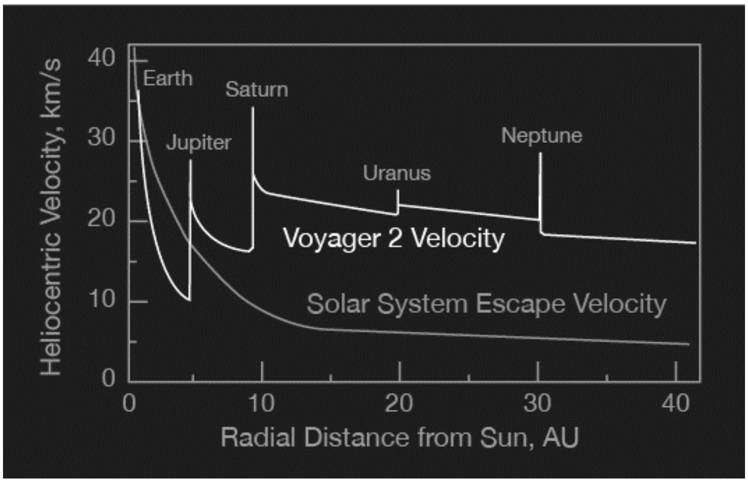

Figure 6 Voyager 2 Heliocentric Velocity [12]

As the methods used are commonly for preliminary design, some discrepancy is expected, part of the found error can be credited to the fact that the 'actual' values may not be wholly accurate, this is difficult to gauge or account for so has not been evaluated. Errors may also be attributed to the combination of simplifying assumptions and models used.

Upon inspection of the Lambert targeting arrival velocities, error generally increases with increasing time and distance, with an exception at SU, where the calculated velocities are well below the 'actual' values. The values obtained from the algorithm agree with the graphical data to within 4.75%, on average, excluding SU for which the error is 14.31%. A mid-course correction was executed by Voyager 2 after its encounter with Uranus [1], which may explain the inconsistency in velocity for the SU leg.

Table 4 Voyager 2 GA Values

Planet	r_p (km)	Closest Approach (km) [1]	$\Delta V_{Perigee}$ (km/s)	δ (°)	e_{out}
Jupiter	686,200	645,000	0.0450	97.29	1.33
Saturn	160,028	101,000	0.0750	84.66	1.48
Uranus	106,571	81,500	0.0982	23.08	5.03

From Table 4, the calculated r_p is larger than the values from the NASA website. This is due, in part, to the fact that the calculated incoming velocities are underestimated, which affected the calculations for r_p as the value for a_{out} increased.

The low $\Delta V_{perigee}$ calculated is indicative of lightly powered GAs. The turn angle, δ and the outgoing eccentricity, e_{out} are sensible values by qualitative analysis. The highly hyperbolic escape orbit at Uranus is expected as Voyager 2 only had a few hours of close observation of the planet, indicative of a more linear orbit [1].

4.2 Phase 2 – Unpowered MGA Trajectory

The unpowered GA algorithm numerical inputs are given in Table 5. LVs ranging from 9km/s to 15km/s were investigated in steps of 1km/s. Trajectories were not found for LV<9.4km/s.

A more restrictive tolerance was used for classification of an unpowered GA for LV=10.5km/s to investigate the effect on execution time.

Table 5 Algorithm Input Values, Phase 2

No.	Inputs	Value		Unit
1	Launch ΔV	9.4, 9.5, 10, 10.5, 11, 12, 13, 14, 15		km/s
2	Launch ΔV tolerance	0.05		km/s
3	Tolerance for $\Delta V_{perigee}$	0.05 (0.01 for LV=10.5)		km/s
4	Patched conic integration step size	0.1		Day
5	TOF range for Lambert targeting	EJ: 1-1000	SU: 1000-3000	Days
		JS: 100-3000	UN: 500-3000	
6	Lambert targeting TOF step size	1 for the first leg (EJ) (*), then 20 (JS SU UN)		Days

(*) The step size in this case is not related to TOF, but feasible trajectory number.

The algorithm found an average of 970 EJ trajectories for each LV. The unpowered GA search time for the subsequent leg was 5.4hrs, using an intel i7-3770 processor. Excluding LV=10.5km/s, total execution time for the full EJSUN sequence ranged from 14hrs to 53hrs. The number of trajectories, per leg, meeting the $\Delta V_{perigee}$<0.05km/s requirement increased by an average of 40% over previous legs, and the number of EJSUN compared to EJ trajectories increased by 200-300% for LV=9.4-10km/s, and 80-140% for LV=11-15km/s. The number of trajectories found decreased with increasing LV. The restrictions on TOF for the SU and UN legs may have led to the omission of more optimal solutions, in terms of required impulse, but were chosen to restrict the maximum total TOF to 10,000 days (approximately 27 years).

While the stricter $\Delta V_{perigee}$ tolerance allows for faster search time for the minimum ΔV requirement, the trade-off with mission length may not be desirable, as the fuel cost savings may not negate costs associated with increased mission length. As the Voyager 2 mission did not terminate at Neptune, and is currently still operating within interstellar space, the increase in mission time to reach Neptune may impact the scientific worth of VIM, as physical equipment may start to degrade earlier and disallow data collection. So, despite the much slower computation times, the larger $\Delta V_{perigee}$ tolerance was used for subsequent searches.

The launch dates for the shortest mission time ranged between the end of July and mid-September 1977 and, for smallest total ΔV, between the start of September and early October 1977. Launch dates for the shortest total TOF coincide with the Voyager 2 launch date of 20th August 1977, and a few weeks after the Voyager 2 date for the lowest mission ΔV.

Table 6 Phase 2 – Shortest TOF

Leg	TOF (days / years)	Arrival Date	ΔV (km/s)
EJ	499 / 1.37	25/01/1979	10.99
JS	580 / 1.59	27/08/1980	0.05
SU	1220 / 3.34	30/12/1983	0.05
UN	1000 / 2.74	25/09/1986	0.01
Total	**3299 / 9.04**	-	**11.09**

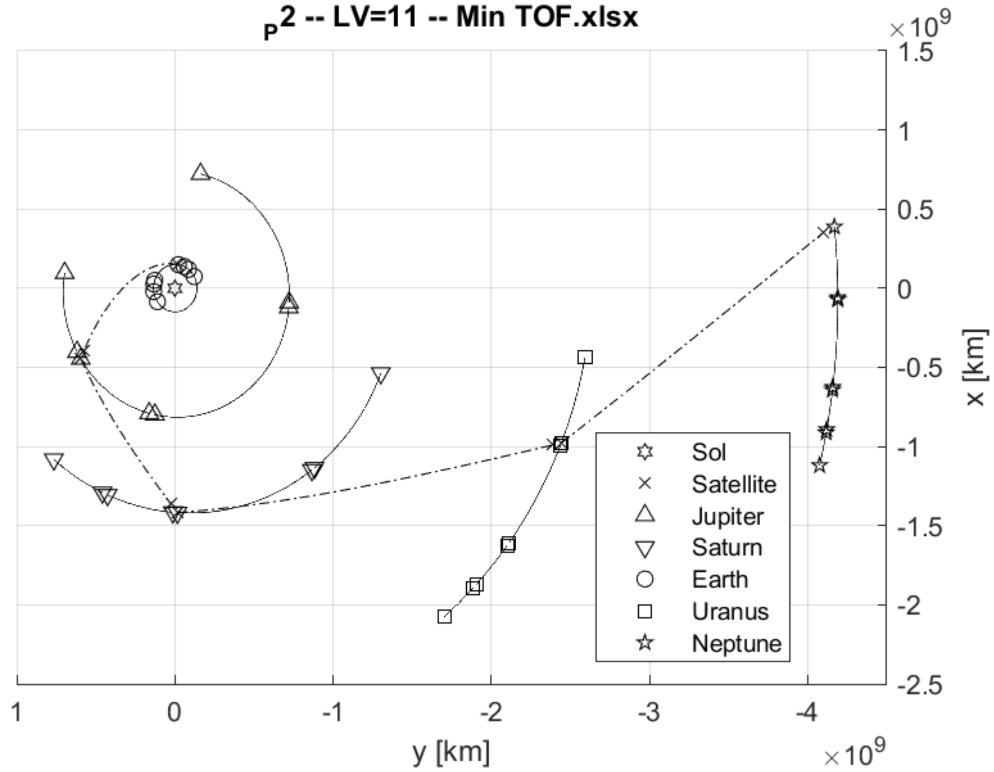

Figure 7 Phase 2 – Shortest TOF Trajectory Plot

Shortest mission times for each launch velocity ranged from 9 to 12.5 years, with mission length generally decreasing with increasing launch velocity, as expected. The shortest overall TOF, 3299 days or approximately 9 years, was found for LV=11km/s, details of this trajectory are shown in Table 6 and Fig. 7.

Without knowing the exact costs associated with increasing LV compared to subsequent engine firing at perigee, it is difficult to understand which would have a greater impact on mission cost. In terms of lowest energy requirements, two trajectories are presented: the first is the lowest overall ΔV requirement including the LV; the second considers only the $\Delta V_{perigee}$ requirements, the lowest of which were found for LV=14km/s. Information on both trajectories is presented in Table 7, and trajectory plots are shown in Fig. 8 and Fig. 9.

Table 7 Phase 2 - Lowest ΔV Trajectories

	Lowest Overall Energy Requirement			**Lowest $\Delta V_{perigee}$ Requirement**		
Leg	TOF (days / years)	Arrival Date	ΔV (km/s)	TOF (days / years)	Arrival Date	ΔV (km/s)
EJ	781 / 2.14	26/10/1979	9.42	738 / 2.02	20/10/1979	13.95
JS	940 / 2.58	23/05/1982	1.22E-03	900 / 2.47	07/04/1982	6.56E-04
SU	1880 / 5.15	16/07/1987	3.71E-03	1820 / 4.99	01/04/1987	2.26E-03
UN	1580 / 4.33	12/11/1991	7.47E-03	1520 / 4.16	30/05/1991	5.64E-04
Total	**5181 / 14.19**	**-**	**9.43**	**4978 / 13.64**	**-**	**13.96**

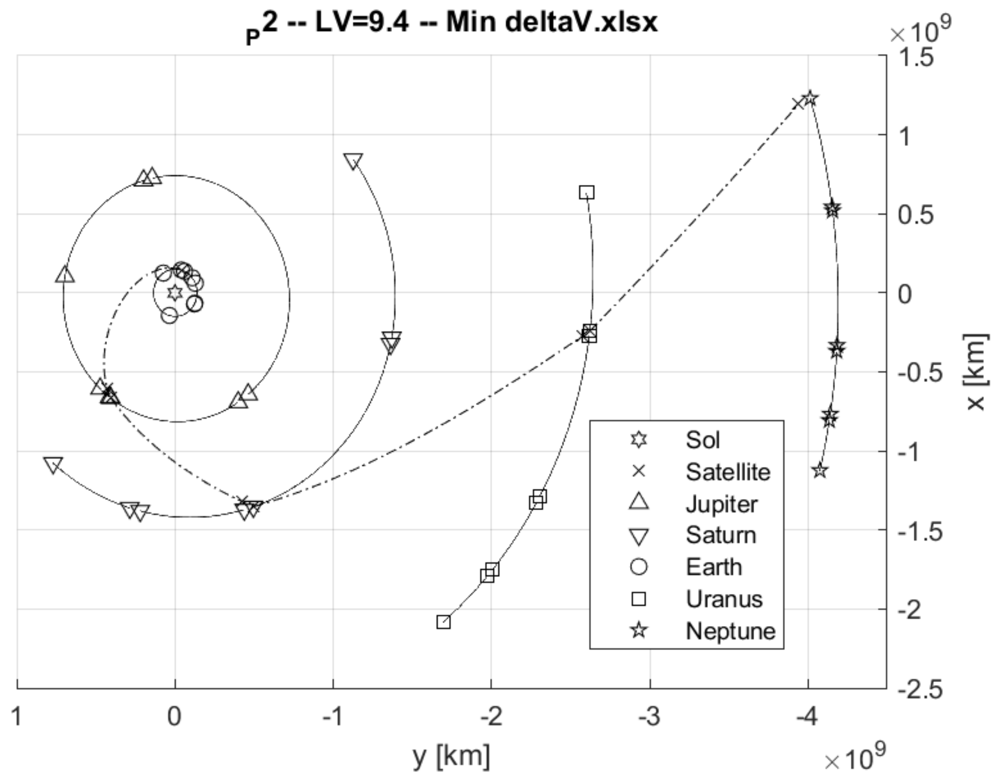

Figure 8 Phase 2 - Lowest overall ΔV

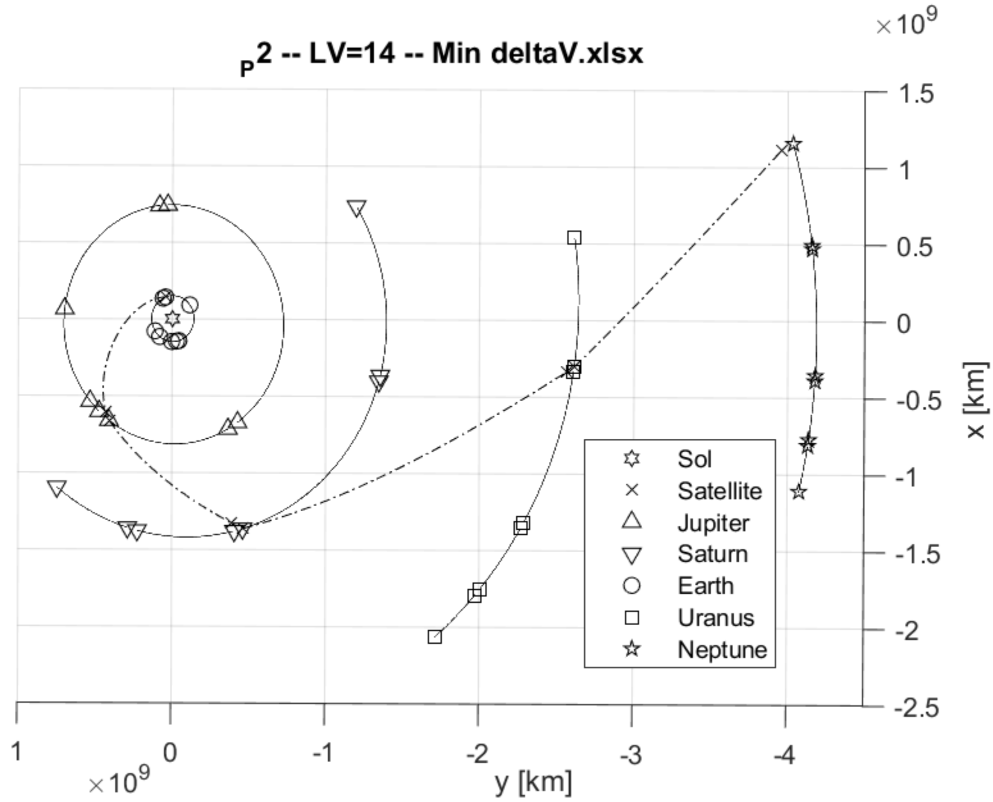

Figure 9 Phase 2 – Lowest $\Delta V_{perigee}$ Requirements

The shortest mission time trajectories are remarkably similar to those found in Phase 1, demonstrated by the similarities in GA orbital properties, where eccentricities match to 1.8%, 0.026%, and 18%, and turn angles matched to 2.6%, 0.37%, and 22%, for JS, SU and UN, respectively. The same cannot be said for the optimal ΔV trajectories, in which the TOFs are generally much larger but the same trends in GA orbital properties and interplanetary TOFs are observed.

From these results it is highly likely that the Voyager 2 trajectory was optimised for TOF, rather than energy requirement. The closest optimal trajectory match found to the Voyager 2 trajectory is the shortest TOF trajectory found with LV=9.5km/s, with a total mission time 194 days longer, with a maximum increase of 5.15% for the TOF of each interplanetary leg, δ and r_p match to a maximum of 3° and 24%, respectively, and the eccentricities of the outbound GA orbit differ by a maximum of 5%.

5. Conclusions

Results from Phase 1 indicate that Lambert targeting correctly found the trajectory for a given TOF between two known points and that the patched conic integrator successfully mapped the trajectory path. The validated methods from Phase 1 were used to find multiple 'unpowered' MGA trajectories for the full EJSUN sequence, the minimum total $\Delta V_{perigee}$ trajectory found was less than 0.0035km/s for a total TOF of 4978 days (13.64 years), the shortest TOF trajectory found spanned 3299 days (9.04 years), with a total $\Delta V_{perigee}$ requirement less than 0.11km/s. It was determined that the Voyager 2 trajectory was likely to be optimised for TOF.

6. References

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