Trimming Simulation of Forming Metal Sheets Isogeometric Models by Using NURBS Surfaces

D. Herrero-Adan[1], Rui P.R. Cardoso[2] and O.B. Adetoro[1]

[1]Department of Engineering Design and Mathematics, University of the West of England, BS16 1QY Bristol, UK

[2]Department of Mechanical, Aerospace and Civil Engineering, Brunel University London, UB8 3PH Uxbridge, London, UK

E-mail: Daniel.Herreroadan@uwe.ac.uk

Abstract. Some metal sheets forming processes need trimming in a final stage for achieving the net-shape specification and for removing micro-cracks and irregularities. In numerical simulation, since the exact final edge location is a priori unknown in the original metal blanket, the trimming needs to be done once the forming is finished. During the forming internal stresses are generated inside the sheet. When trimming those stresses configuration is changed to achieve equilibrium as a consequence of the material removal. In this paper a novel method for simulating the trimming is presented. The part to trim is modelled using isogeometric analysis (IGA). The new surface generated is modelled with non-uniform rational B-splines (NURBS). Due to the IGA characteristics a total geometrical accuracy and an efficient residual stresses recalculation are accomplished.

Keywords. Trimming, IGA, NURBS, Metal sheet forming.

1.- Introduction

Ever since isogeometric analysis (IGA) emerged in 2005 [1] it has been rapidly developed to many applications such as linear elasticity, wave propagation or fluids (see [2] for examples). That fast progression is due to IGA advantages in numerical modelling: geometrical accuracy, efficiency during the modelling process, easy achievable high order continuity between elements and accurate results.

In this work post-forming metal sheet trimming simulation is implemented under IGA. When forming is applied to a metal blanket plastic strains generate residual stresses. These stresses, which are well known (see for example [3]), are the cause of displacements if the sheet is trimmed after the whole process, that means a redistribution of stresses happens due to material removal to achieve a new equilibrium.

The starting stage of this work is the post-formed sheet, which possesses known internal residual stresses but remains in equilibrium. That post-formed part is modelled by a surface called "S". The subsequent stages are: 2) define the trimming 3D surface called "T"; 3) compute the trimming line (called "C"), which is the intersection between both surfaces S and T. Previous calculation of a row of intersection points between S and T is necessary; 4) remove from the S surface the trimmed area confined by the trimming line C; 5) recalculate the stiffness matrix and the forces at control points of S surface; 6) compute the displacements and stresses redistribution due to the removal. Step 2 is the core of this work as provides a novel method for finding the intersection between two NURBS surfaces. Steps 4 and 5 are according to Kim *et al.* [4] and standard finite element analysis procedures. Fig.01 shows the process.

In order to carry out the displacements and stresses analysis of the S surface after the trimming operation an equivalent 3D solid model has been used. This is possible because the thickness is several orders of magnitude lower than the other two dimensions. Initial residual stresses at Gauss points are assumed known. Note that S and T surfaces are mapped to physical space from two different parameter spaces S and T, defined by coordinates (u,v) and (c,d).
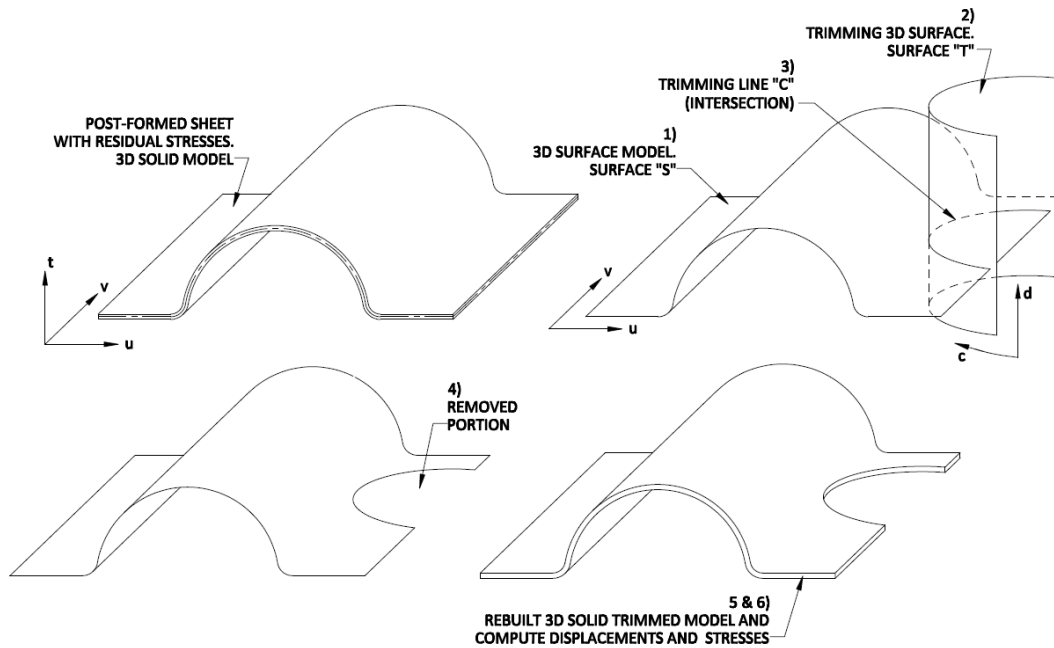
Fig.01.- Post-formed metal sheet trimming process.

2.- Defining 3D trimming T surface

The trimming T surface is defined separately from S, in a different parameter space. Meanwhile S surface parameter space is S (u,v), T surface parameter space is T (c,d) as shown in Fig.01. The requirements for the trimming surface are that it needs to be smooth, it has to trim the metal sheet in the physical space, the resulting trimming curve has to enclose a portion of S surface and it has to extend slightly further than the intersection between the two surfaces. Due to the reduced thickness of the sheet (S surface) compared against the other two dimensions, any trimming is to be assumed perpendicular to S surface, independently of any different values for the angle between S and T.

3.- Trimming curve definition, C curve

Defining the trimming curve on surface S is the core of this work. The idea is to calculate a row of surfaces S-T intersection points obtaining their physical coordinates (x,y,z) and their surface S parameters space coordinates (u,v) in S parameter space. Those points are arranged going from one end to the other of the targer trimming curve C. Then, that row of intersection points can be used to interpolate a curve directly in the parameter space S. Therefore, the output of this stage is a curve definition in S parameter space. By defining C directly in that parameter space instead in the physical space the subsequent computational effort is reduced. This is because both, S surface and C curve, share the same parameter space instead having a different parameter space from S for the C curve. Therefore it is required only one mapping from S to physical space.

3.1.- Computing the S-T intersection points row

Since the two NURBS surfaces S and T are defined in two different parameter spaces, the intersection points calculation needs iterations. Fig.02 shows the pseudocode of the function rowIntersectionPoints implemented for obtaining the row of intersection points. In every step inversion point routine according to [5] is used and it is crucial to select initial searching points in both parameter spaces and physical space reasonably close to the target intersection point. To achieve that, the initial searching points in the parameter spaces are coincident with the latest computed intersection point, and the initial searching point in the physical space is calculated by adding to the latest intersection point and step magnitude (defined by the user) times the derivative of the

trimming curve. The derivative at the trimming curve can be obtained by the cross product of the S and T surfaces normal vectors at that location.
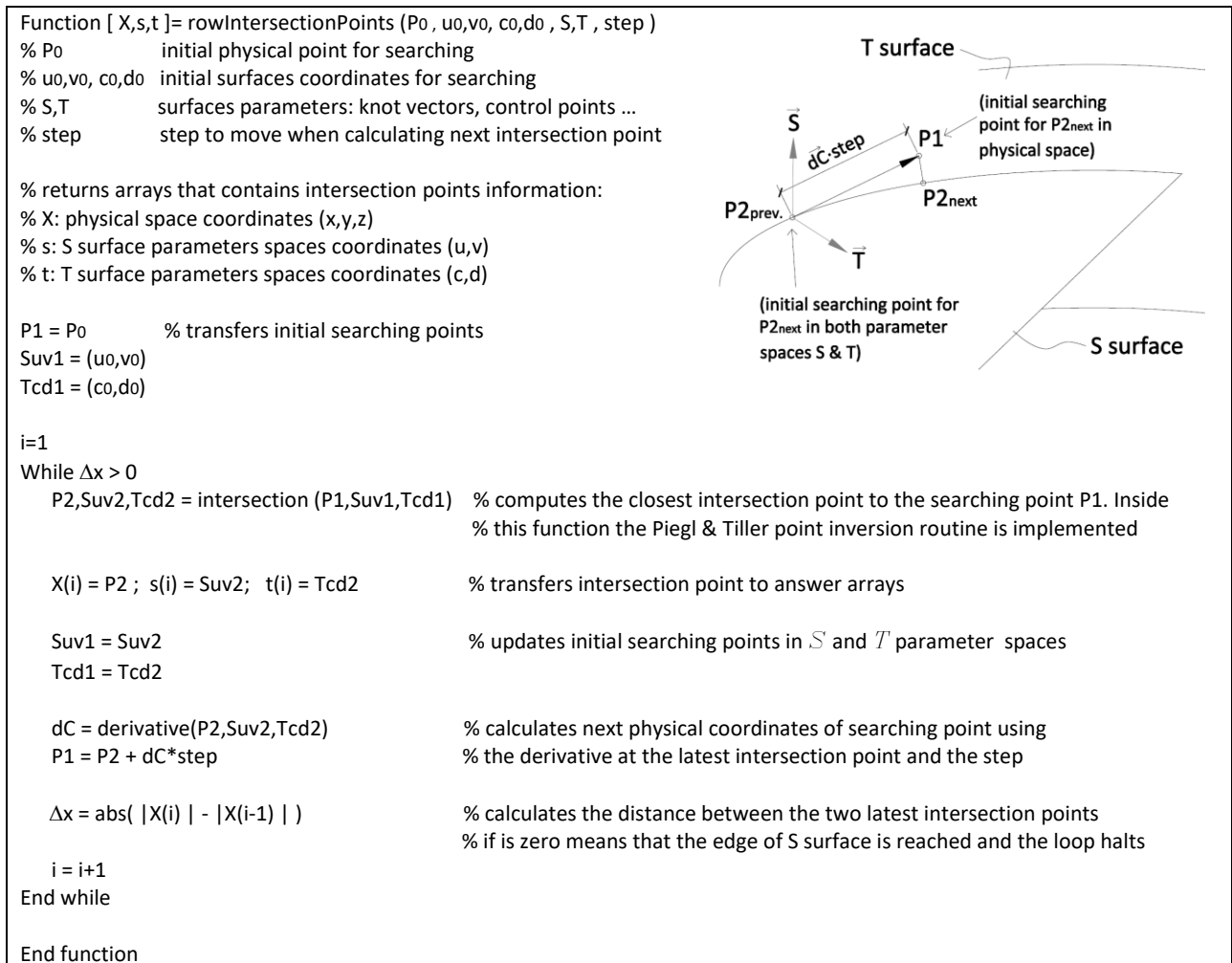
```
Function [ X,s,t ]= rowIntersectionPoints (P0 , u0,v0, c0,d0 , S,T , step )
% P0              initial physical point for searching
% u0,v0, c0,d0   initial surfaces coordinates for searching
% S,T            surfaces parameters: knot vectors, control points …
% step           step to move when calculating next intersection point

% returns arrays that contains intersection points information:
% X: physical space coordinates (x,y,z)
% s: S surface parameters spaces coordinates (u,v)
% t: T surface parameters spaces coordinates (c,d)

P1 = P0          % transfers initial searching points
Suv1 = (u0,v0)
Tcd1 = (c0,d0)

i=1
While Δx > 0
    P2,Suv2,Tcd2 = intersection (P1,Suv1,Tcd1)   % computes the closest intersection point to the searching point P1. Inside
                                                 % this function the Piegl & Tiller point inversion routine is implemented

    X(i) = P2 ;  s(i) = Suv2;   t(i) = Tcd2        % transfers intersection point to answer arrays

    Suv1 = Suv2                                     % updates initial searching points in S and T parameter spaces
    Tcd1 = Tcd2

    dC = derivative(P2,Suv2,Tcd2)                  % calculates next physical coordinates of searching point using
    P1 = P2 + dC*step                              % the derivative at the latest intersection point and the step

    Δx = abs( |X(i) | - |X(i-1) | )                % calculates the distance between the two latest intersection points
                                                   % if is zero means that the edge of S surface is reached and the loop halts

    i = i+1
End while

End function
```


Fig.02.- Intersection points calculation pseudocode.

3.2.- Curve C equation

Once the intersection points are calculated, their S parameter coordinates are used to define the trimming curve equation into that space. In this work a B-spline has been used and the interpolation method is referred in [5] as global curve interpolation to point data. Other equation types are suitable as long as the step used in the intersection points row is fine enough. When the curve equation is obtained is easy to depict in physical space by going through (u,v) coordinates along curve C in S space.

4.- Removing from surface S the trimmed portion and computing displacements and stresses

Elements of S surface on the left of the curve are to remain and elements on the right are to be removed. Direction of C curve is given by its derivative. Elements trimmed by C curve need a more detailed analysis. They are divided into triangles as technique shown in Kim *et al.* [4]. All elements and parts of trimmed element to be removed are deactivated in the subsequent calculations. That means, those deactivated areas (volumes in 3D solid model) are not accounted for in the integrals for stiffness matrix and control points forces calculations. Control points forces due to residual stresses are computed by integration of deformation matrices at each element as shown in (1). If a control point results non-influential due to the deactivated elements it is not considered in the calculations.

The trimmed S surface parameter domain, knot vector and control points structure remains the same, what is varying is that part of the domain (the trimmed domain) turns to a non-active state and is not computed in the calculations as aforementioned.

$$\{f\} = - \int_V [B]^T \{\sigma_0\}\, dV \tag{1}$$

5.- Numerical example

A pipe resulting from deep drawing cup with characteristics from [3] table 4, reference G1, is modelled. Initially residual stresses are assumed as 610 MPa tension in outer surface and compression in inner surface, circumferential direction, resulting a circumferential bending moment. One end is clamped and then a slot is trimmed at the other end. Therefore the slot opens as residual stresses are released to achieve the new equilibrium. Fig.03 and Fig.04 show the process.
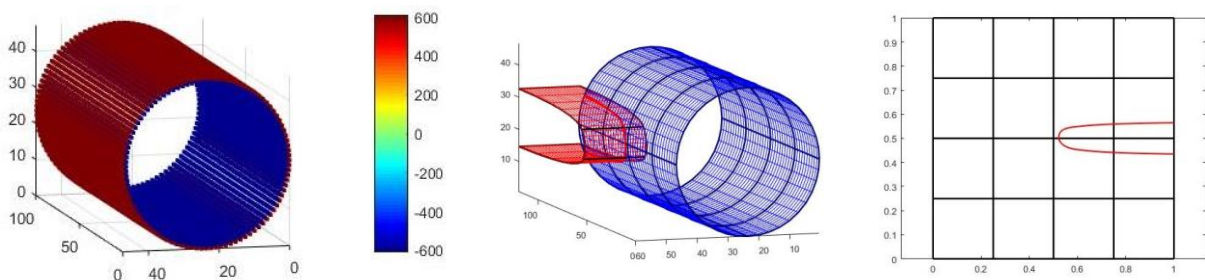

Fig.03.- From left to right: Initial situation with circumferential residual stresses depicted at outer and inner sides. S surface (blue) and T surface (red), polygonal red thick line joints the row of computed intersection points. Parameter space with B-spline adjusted to the intersection points.
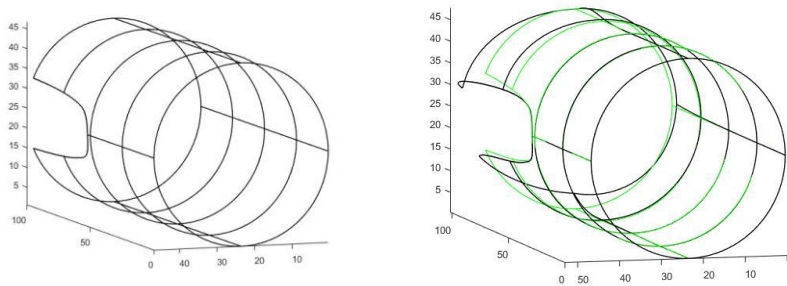

Fig.04.- From left to right: Trimmed pipe before calculation of displacements. Deformed trimmed pipe (un-deformed in green colour). Residual stresses after trimming.

References

[1] Hughes TJR, Cottrell JA, Bazilevs Y. *Isogeometric analysis: CAD finite elements NURBS exact geometry and mesh refinement*. Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135–4195.

[2] Cottrell JA, Hughes TJR, Bazilevs Y. *Isogeometric Analysis, Toward Integration of CAD and FEA*. Wiley, (2009).

[3] Xiao LH, Yuan DH, Xiang JZ, Liu JG, Zhou YC. *Residual stress in the cylindrical drawing cup of SUS304 stainless steel evaluated by split-ring test*. Acta Mech. Sin. (2016) 32(1):125–134

[4] Kim HJ, Seo YD, Youn SK. *Isogeometric analysis for trimmed CAD surfaces*. Comput. Methods Appl. Mech. Engrg. 198 (2009) 2982–2995

[5] Piegl L, Tiller W. *The NURBS Book. Berlin*. Springer-Verlag, (1996).