**Robust and stepwise optimization design for CO2 pipeline transportation**

Qunhong Tian1, Dongya Zhao1\*, Zhaomin Li2, Quanmin Zhu1, 3

1. College of Chemical Engineering, China University of Petroleum, Qingdao, China, 266580

2. School of Petroleum Engineering, China University of Petroleum, Qingdao, China, 266580

3. Department of Engineering Design and Mathematics, University of the West of England, Frenchay Campus, Coldharbour Lane, Bristol, BS16 1QY, UK

\*Corresponding authors’ email: dyzhao@upc.edu.cn; dongyazhao@139.com

**Abstract:** Carbon capture, utilization, storage (CCUS) technology is an effective means to reduce the CO2 emissions. It has been noted that engineering and economic design of the pipeline transportation are important components for CCUS. However, the uncertainties of the pipeline transportation model may make an infeasible design in practice and cause unnecessary cost. In this paper, a novel robust optimization method is proposed for CO2 pipeline transportation design, which can deal with the multiple uncertainties. A stepwise method is presented to further improve the optimization performance. The proposed optimal algorithm is validated by using numerical studies, which show the proposed approach can deal with the multiple uncertainties and improve the design performance in comparison with the existing methods.

**Keywords:** CO2 emissions; pipeline transportation; uncertainties; robust optimization; stepwise optimization

**Nomenclature**

 capital cost of pipelines

 capital cost of compressors

 capital cost of booster pumps

 annual operation and maintenance costs of pipelines, compressors and booster pumps

 annual energy costs of compressors and booster pumps

 capital recovery factor of pipelines

 capital recovery factor of compressors

 capital recovery factor of booster pumps

 price of steel pipeline

 price of electricity

 inner diameter of the pipeline

 longitudinal joint factor

 design factor.

 function of density that depends on the  and 

 function of viscosity that depends on the  and 

 material cost factor

 percentage of the capital cost of booster pumps

 percentage of pipelines capital cost

 Darcy-Weisbach friction

 operation time of the transportation

 base costs for calculating the compressor capital cost

  optimal ideal inner diameter

 inner diameter of the NPS

 length of the pipeline

 maximum distance between the boosting pump stations

 levelized cost of CO2 pipeline transportation

 molar mass

 total number of compression stages

 number of boosting pump stations

 largest number of boosting pump stations

 multiplication exponent

**  operation and maintenance costs

 inlet pressure for each pipe segment

 outlet pressure for each pipe segment

 maximum pressure for CO2 transportation

 injection pressure

 average pressure along the pipeline

 maximum allowable operation pressure

 suction pressure

 discharge pressure

 CO2 mass flow rate

 universal gas constant

 discount rate

 specified minimum yield stress for the pipe material

 average temperature

 soil temperature around the pipeline

 largest soil temperature around the pipeline

 operation time of compressor

 operation time of booster pump

 suction temperature

 wall thickness

 optimal ideal wall thickness

 wall thickness of the NPS

 actual velocity

 minimum velocity

 maximum velocity

 base scale of the compressor

 scaling factor

 lifetime of pipelines

 lifetime of compressors

 lifetime of booster pumps

 CO2 density at average temperature along the pipeline

 actual density along the pipeline

 average CO2 viscosity

 specific heat ratio

 isentropic efficiency of the compressor

 mechanical efficiency of compressor

 booster pump efficiency

 actual pressure drop

1 Introduction

The process of CO2 capture, transportation, enhanced oil recovery (EOR) and storage is one of the best ways to reduce the CO2 emissions, which not only can effectively prevent the increase of CO2 concentrationin the atmosphere, but also bring economic benefit with EOR ([Marston 2013](#_ENREF_11); [Imtiaz et al. 2015](#_ENREF_4)). As a link between the capture source and the storage site, pipelines are attractive approach to transport large amount of CO2 for long distance ([Svensson et al. 2004](#_ENREF_19); [Luo et al. 2014](#_ENREF_10); [Martynov et al. 2015](#_ENREF_12)). It is obvious that engineering and economic pipeline design are important for the CCUS technology ([Knoope et al. 2013](#_ENREF_5)). Most of the existing optimal approaches are model based, whose performance is affected by the uncertainties seriously ([Zhang et al. 2012](#_ENREF_21)). Ignoring the uncertainties may lead to an infeasible design. Consequently, it is central to consider the uncertainties in optimization design for the pipeline transportation.

There are mainly two types of uncertainty of the CO2 pipeline transportation: (1) Engineering model uncertainties. The CO2 temperature along the pipeline changes with the seasons, even the pipeline has insulation and is buried in the soil ([Zhang et al. 2012](#_ENREF_21)). Along with time, booster pumps, compressors and other equipment will be aging and their performance parameters will be varying. The impurities in CO2 such as H2S, SOX and O2 impact the density and viscosity. (2) Economic model uncertainties. There are many changes in labor, material, land prices, and regulations in pipeline lifetime ([Middleton 2013](#_ENREF_14)). For easy calculation, the electricity cost is usually assumed to be constant over time. However, it is an significant uncertainty in the costs of CCUS power plants and in the electricity cost ([Knoope et al. 2014](#_ENREF_7)). If these uncertainties are not considered in the pipeline transportation design, it may degrade the design performance.

To design the pipeline transportation easily, the existing researches always assume the temperature is constant along the pipeline ([Chandel et al. 2010](#_ENREF_1); [Gao et al. 2011](#_ENREF_3); [Knoope et al. 2014](#_ENREF_7)), however, the temperature variation can significantly affect the transport cost ([Chandel et al. 2010](#_ENREF_1)) and/or make the design not well ([Zhang et al. 2012](#_ENREF_21)). To solve this problem, the highest temperature of the soil is used in the pipeline design. In order to simplify the calculation of the pipeline design, the linear optimization is introduced ([Morbee et al. 2012](#_ENREF_17); [Middleton 2013](#_ENREF_14)), in which the modelling uncertainty is not considered. Effects of geologic reservoir uncertainties are analyzed on CO2 transportation ([Middleton et al. 2012](#_ENREF_15)), Monte Carlo trials are used to assess the sensitivity of transport cost to the uncertain model parameters ([McCoy et al. 2008](#_ENREF_13)), but these approaches ([McCoy et al. 2008](#_ENREF_13); [Middleton et al. 2012](#_ENREF_15)) have not discussed the design issues. Considering some uncertainties, an iteration method is proposed for the pipeline design ([Knoope et al. 2015](#_ENREF_6)). However, the method is based on the designer’s experience, which unavoidably exists one-sidedness in the design. In summary, the effects of uncertainties should not be ignored, the existing methods focus on the effects of single uncertainty ([Chandel et al. 2010](#_ENREF_1)) or partial uncertainties ([McCoy et al. 2008](#_ENREF_13); [Middleton et al. 2012](#_ENREF_15); [Knoope et al. 2015](#_ENREF_6)). But these approaches usually lack theoretical analysis. There have not been effective methods to deal with the multiple uncertainties. Therefore, it is necessary to present an approach to cope with multiple uncertainties and improve the design performance. The final selected inner diameter and wall thickness are the nominal pipe size (NPS) in the engineering practice which are larger than the ideal ones in general ([McCoy et al. 2008](#_ENREF_13); [Zhang et al. 2012](#_ENREF_21)), therefore, in order to further improve the design performance, a new algorithm is desired to be explored.

In this paper, multiple uncertainties are transformed into bounded set, a new robust optimization model is initially developed to minimize the levelized cost of the CO2 pipeline design, which is solved by using the linear matrix inequalities (LMI). The proposed robust optimization approach can deal with the effects of multiple uncertainties, which not only include the variable temperature, declined parameter performance, changeable density and viscosity, but also the change in labor, material, land prices etc. A stepwise optimization following the robust optimization is provided to further improve the optimization performance. The proposed approach is validated by using numerical studies. It should be mentioned that this paper further improves the results of ([Zhao et al. 2016](#_ENREF_23)), which has not considered the effects of multiple uncertainties.

The rest of this paper is organized as follows. Uncertain optimization problem is formulated in Section 2. Solutions for robust optimization issue are given in Section 3. The stepwise optimization is presented in Section 4. The computation results and analysis are presented in Section 5. Finally, the conclusions are drawn in Section 6.

2 Uncertain optimization problem description

The optimal design for CO2 pipeline transportation includes the inlet pressure, inner diameter, wall thickness and the number of boosting pump stations. Levelized cost and inlet pressure are selected as the objective function and design variable in the pipeline design, respectively.

The optimization model of CO2 transportation is formulated as follows ([Knoope et al. 2014](#_ENREF_7)):

 (1)

 (2)

 (3)

where  is the levelized cost of CO2 pipeline transportation ();  is the inlet pressure for each pipe segment, which is selected as a decision variable ();  is the outlet pressure for each pipe segment ();  is the maximum pressure for CO2 transportation (); , ,  are the actual, minimum and maximum velocities, respectively ();  is the actual pressure drop ();  is the length of the pipeline ();  is the number of boosting pump stations; , ,  are the capital costs of pipelines, compressors and booster pumps, respectively ();  are the annual operation and maintenance (O&M) costs of pipelines, compressors and booster pumps ();  are the annual energy costs of compressors and booster pumps ().  is the CO2 mass flow rate ();  is the operation time of the transportation (); , ,  are the capital recovery factors of pipelines, compressors and booster pumps, respectively ();  is the discount rate (%); , ,  are the lifetime of pipelines, compressors and booster pumps, respectively (years). In order to make the paper clearly and easily to follow, Table 1 shows the detail models and the related literatures of , , , ,.

Table 1 Detail models and the related literatures

|  |  |
| --- | --- |
| Model | Literature |
|  | ([Gao et al. 2011](#_ENREF_3); [Knoope et al. 2013](#_ENREF_5)) |
|  | ([Knoope et al. 2014](#_ENREF_7)) |
|  | ([Chandel et al. 2010](#_ENREF_1); [Knoope et al. 2013](#_ENREF_5)) |
|  | ([Knoope et al. 2013](#_ENREF_5)) |
|  | ([Knoope et al. 2014](#_ENREF_7)) |

Based on the first order Taylor series, the objective function can be linearized as:

 (4)

where  is a coefficient,  is a constant.

Considering the uncertainties, (4) can be written as:

 (5)

where  and  are nominal, and  are uncertainties, respectively.

Squaring equation (5), the optimization model can be re-written as:

 (6)

where , , .  are nominal parameters, , . , , .  are the uncertainty directions;  are the uncertainties, .

In order to realize the minimization of  under the uncertainties, (6) can be rewritten as:

 (7)

where . (7) is named as robust optimal model of the original issue. The detailed formations of the robust models are given in Section 3 for the stepwise optimization.

**Remark 1:** denotes the uncertainties which is bounded.

**Additional engineering models**

The diameter is an intermediate variable, it can be calculated with the flow rate, variable pressure and temperature along the pipeline, it is implied in  and  of the objective function ([Gao et al. 2011](#_ENREF_3); [Knoope et al. 2013](#_ENREF_5); [Knoope et al. 2014](#_ENREF_7)):

Pipeline inner diameter can be calculated as ([Zhang et al. 2006](#_ENREF_22)):

 (8)

where  is the inner diameter of the pipeline ();  is the CO2 mass flow rate in the pipeline ();  is the CO2 density at average temperature along the pipeline ();  is the average CO2 viscosity (). For CO2 pipeline, the average temperature, , is assumed to be the soil temperature, that is,  ([McCoy et al. 2008](#_ENREF_13)). Pressure and temperature affect the density and viscosity. The current research shows that the change of average temperature (soil temperature) is bounded along the buried pipeline ([Zhang et al. 2012](#_ENREF_21)). This change is small, in this case, the density and viscosity are almost linear with pressure variation ([NIST](#_ENREF_18)). The changes of density and viscosity caused by the variable temperature can be dealt with as the system uncertainties.

Based on the data from National Institute of Standards and Technology ([NIST](#_ENREF_18)), (8) can be converted into:

 (9)

where  is the average pressure along the pipeline ();  is the soil temperature around the pipeline ().  is the function of density that depends on  and  ();  is the function of viscosity that depends on  and  ().

 can be calculated as ([Mohitpour et al. 2003](#_ENREF_16)):

 (10)

The density is given as a function of average pressure and temperature along the pipeline:

 (11)

The viscosity is given as a function of average pressure and temperature along the pipeline:

 (12)

where  and  are known constant matrixes;  is the matrix of ;  is the matrix of .

,,,

By using (9-12), (8) can be re-written as:

 (13)

The pipe wall thickness is given as ([Chandel et al. 2010](#_ENREF_1)):

 (14)

where  is the wall thickness ();  is the maximum allowable operation pressure ();  is the specified minimum yield stress for the pipe material ();  is the longitudinal joint factor;  is the design factor.

Liquid pipeline transportation is researched in this study. Compared with supercritical fluid transportation, liquid transportation is better energy efficiency and lower transportation cost over long distance ([Zhang et al. 2006](#_ENREF_22); [Zhang et al. 2012](#_ENREF_21); [Knoope et al. 2014](#_ENREF_7)). The pressure drop is calculated for all liquid cases as follows ([Knoope et al. 2014](#_ENREF_7)):

 (15)

where  is the Darcy-Weisbach friction;  is the actual density along the pipeline (), it changes with the pressure drop and soil temperature, the change is dealt with as one of the multiple uncertainties.

In this paper, based on the levelized transport cost, the installation of boosting pump stations is an optimization design resulting from tradeoffs between increasing the inlet pressure, enlarging the pipeline diameter, or adding a boosting pump station. The number of boosting pump stations is calculated by ([Knoope et al. 2014](#_ENREF_7)):

 (16)

 (17)

where  is the maximum distance between the boosting pump stations;  is the number of boosting pump stations,  means the largest integer not greater than the enclosed ratio.

CO2 velocity can be calculated as:

 (18)

where  is the actual velocity (),  is the actual density along the pipeline.

3 Robust and stepwise optimization methods

In this Section, the robust optimization issue is solved by using LMI. Combined with robust and stepwise methods, the pipeline transportation optimization design algorithms are presented.

**Theorem 1**: Considering a robust optimization problem (7) with the uncertainties , if there exist the decision variable , auxiliary variables  and  such that

 (19)

(7) can be transformed into an optimization problem that will be an objective function  with constraint (19). Some variables are defined as follows: , , , , . According to (19), the pipeline robust optimization problem can be solved by using LMI. The readers can find the proof for Theorem 1 in Appendix A.

Combined with the proposed robust optimization approach, a stepwise optimization method is presented for designing the pipeline transportation, which can be divided into two steps: (1) The robust optimization of inner diameter and wall thickness; (2) The re-robust optimization of inlet pressure and number of boosting bump stations.

***Algorithm 1: The first step optimization***

*Step 1:* Selecting the minimum operational temperature and inlet pressure as the initial values of  and , respectively.

*Step 2:*Substituting  and  into (13) to compute ; Substituting  into (14) to compute .

*Step 3:* Substituting ,  and  into (18) to compute . If , then go to next step, else if, letting  and go to Step 2.

*Step 4:*Substituting  into (17) to compute . If , then go to next step, else if , letting  and go to Step 2.

*Step 5:* Substituting all known parameters into (2) to get . If , letting , and go to Step 2, else if go to the next step.

*Step 6:* By using the enumeration method, comparing all the computed  and selecting the minimum one as . Then computing ,  and  according to .

*Step 7:* If , then letting ,  and go to Step 2, else if go to next step.

*Step 8*: Dividing the optimal  function with different ‘pieces’ and representing each ‘piece’ with linear function; Obtaining , , , , , , establishing the robust optimization model (7).

*Step 9*: Substituting the parameters of each ‘piece’ into (19), obtaining the robust optimization results by using LMI toolbox, selecting the smallest one.

*Step 10*: Calculating the wall thickness based on (14); Obtaining inner diameter of the NPS () and wall thickness of the NPS () by selecting from the NPS.

End the program.

For the second step optimization, Algorithm 2 calculates the final  and .

***Algorithm 2: The second step optimization***

*Step 1:*Selecting the minimum operational temperature and inlet pressure as the initial values of  and , respectively.

*Step 2:* Substituting , ,  and  into (18) to compute . If , then go to next step, else if, letting  and go to Step 2.

*Step 3:*Substituting ,  into (17) to calculate . If , then go to next step, else if , letting  and go to Step 2.

*Step 4:* Substituting all known parameters into (2) to get . If , letting , and go to Step 2, else if go to the next step.

*Step 5:* By using the enumeration method, comparing all the computed  and selecting the minimum one as .

*Step 6:* If , then letting , and go to Step 2, else if go to next step.

*Step 7*: Dividing the optimal  function with different ‘pieces’ and representing each ‘piece’ with linear function; Obtaining , , , , , , establishing the robust optimization model (7).

*Step 8*: Substituting the parameters of each ‘piece’ into (19), obtaining the robust optimization results by using LMI toolbox, getting the optimal  and related , .

End the program.

After the executing the above algorithms, the optimal  and  can be obtained from the Step 10 of Algorithm 1, the optimal ,  can be obtained from the Step 8 of Algorithm 2.

**Remark 2**: Algorithm 1 and 2 are used to deal with the variable temperature, if considering the other multiple uncertainties, these uncertainties can be considered as perturbations of the nominal parameters.

4 Computation results and analysis

The basic parameters of the transportation are given in Table 2. The other detailed parameters are given in Table 3-5.

Table 2. Basic parameters of the transportation ([Chandel et al. 2010](#_ENREF_1); [Gao et al. 2011](#_ENREF_3); [Zhang et al. 2012](#_ENREF_21))

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Soil temperature () |  | 2~17 |
| CO2 inlet pressure () |  | 8.6~15.3 |
| Pipeline length () |  | 500 |
| Injection pressure () |  | 10 |
| Operation time () |  | 8760 |

Table 3. Detailed parameter values of pipeline ([McCoy et al. 2008](#_ENREF_13); [Vandeginste et al. 2008](#_ENREF_20))

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Specified minimum yield stress for X70 steel () |  | 483  |
| Longitudinal joint factor |  | 1.0 |
| Design factor |  | 0.72 |
| Price of steel pipeline () |  | 1.11 |
| Material cost factor |  | 22.4% |
| Percentage of pipelines capital cost |  | 0.04 |

Table 4. Detailed parameter values of compressor and boosting pump stations ([Zhang et al. 2006](#_ENREF_22); [Kuramochi et al. 2012](#_ENREF_8); [Knoope et al. 2014](#_ENREF_7))

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Universal gas constant() |  | 8.3145 |
| Suction temperature () |  | 313.15 |
| Specific heat ratio () |  | 1.294 |
| Molar mass () |  | 44.01 |
| Total number of compression stages |  | 4 |
| Isentropic efficiency |  | 80% |
| Mechanical efficiency |  | 99% |
| Suction pressure () |  | 0.101 |
| Discharge pressure () |  | 8.6 |
| Base costs for calculating the compressor capital cost () |  | 21.9 |
| Base scale of the compressor () |  | 13 |
| Scaling factor |  | 0.67 |
| Multiplication exponent |  | 0.9 |
| Percentage of the capital cost of booster pumps |  | 0.04 |
| Booster pump efficiency |  | 0.5 |
| Operation time of compressor () |  | 8760 |
| Operation time of booster pump () |  | 8760 |
| Price of electricity () |  | 0.0437 |
| Number of boosting pump stations |  |  |
| Actual velocity () |  |  |

Table 5. Parameter values of the levelized cost model ([Knoope et al. 2013](#_ENREF_5); [Knoope et al. 2014](#_ENREF_7))

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Discount rate (%) |  | 15 |
| Design lifetime of the pipeline () |  | 50 |
| Design lifetime of compressors () |  | 25 |
| Design lifetime of the booster pumps () |  | 25 |

Table 6 Robust optimization results for different design mass flow rates

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mass flow rateMethod  |  () | 120 | 150 | 200 | 250 |
| The first step | () | 13.7717 | 13.6839 | 13.2998 | 13.0346 |
|  () | 0.31115 | 0.33975 | 0.39055 | 0.44135 |
|  () | 0.00635 | 0.007925 | 0.007925 | 0.007925 |
| Total cost () | 1,290,803,170~1,293,016,997 | 1,659,755,448~1,662,475,428 | 2,024,327,376~2,027,654,424 | 2,383,628,850~2,387,551,140 |
| The second step | () | 12.3079 | 12.2640 | 11.9307 | 11.5827 |
| Total cost () | 1,282,572,274~1,284,114,384 | 1,649,762,478~1,651,666,464 | 2,011,444,920~2,013,715,512 | 2,366,520,570~2,369,004,030 |
| ------ | Full Saving cost () | 8,230,896~8,902,613 | 9,992,970~10,808,964 | 12,882,456~13,938,912 | 17,108,280~18,547,110 |



Figure 1. Full saving costs with different temperatures

Table 6 shows the robust optimization results for different design mass flow rates with variable temperature, compared with the first step optimization results, it can be seen that the second step saves the cost. To further illustrate this advantage, the full saving costs are given in Figure 1 for lifetime 25 years, note that full saving cost is the first step total cost minus the second step total cost. For instance, the design mass flow rate is 120 , the full saving costs are 8,230,896~8,902,613  for 25 years. The reasons are given as: In the first step, the optimal ideal inner diameter () and wall thickness () are computed by using the given design conditions. However, the final selected inner diameter and wall thickness are the NPS in the engineering practice which are larger than the ideal ones. Based on the first step optimization, the second stepwise can re-optimize the inlet pressure and the number of boosting pump stations, which can improve the optimal performance.

Table 7 shows the robust optimization results with multiple uncertainties. The flow rate is assumed to be 130 , the CO2 temperature is variable with the seasons. The other multiple uncertainties are considered as perturbations, which are denoted as the random percentages () of the nominal parameters. It can be seen that the levelized cost increases with the uncertainty increases.

Table 7 Robust optimization results with multiple uncertainties

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter |  |  |  |  |  |
|  () | 0.31115 | 0.31115 | 0.31115 | 0.31115 | 0.31115 |
|  () | 0.00635 | 0.00635 | 0.00635 | 0.00635 | 0.00635 |
|  () | 12.6971 | 12.6969 | 12.6966 | 12.6951 | 12.6890 |
|  () | 13.1123 | 13.1438 | 13.1764 | 13.2078 | 13.2688 |

To further illustrate the proposed approach, it will be compared with the existing methods. Two situations are presented as follows: (1) Considering the temperature uncertainty only (2) Not only considering the variable temperature but also the other multiple uncertainties.

Table 8 Comparison results of the existing and proposed methods with variable temperature

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Method | Parameter |    |    |    |
| The existing method ([Chandel et al. 2010](#_ENREF_1)) |   | 13 | 13 | 13 |
|   | 0.31115 | 0.33975 | 0.39055 |
|   | 0.00635 | 0.007925 | 0.007925 |
|   | 10.15900~9.90296 | 10.15905~9.90294 | 10.15901~9.90296 |
| Total cost  | 1,301,231,319~1,303,784,612 | 1,561,273,948~1,563,843,186 | 1,964,157,458~1,968,193,020 |
| The proposed method |   | 12.4857 | 12.1624 | 11.7914 |
|   | 0.31115 | 0.3429 | 0.3937 |
|   | 0.00635 | 0.00635 | 0.00635 |
|   | 10.20830~10 | 10.18282~10 | 10.15295~10 |
| Total cost  | 1,298,899,850~1,300,568,495 | 1,436,135,187~1,438,223,691 | 1785271765~1787428123 |
|  | Total saving cost  | 2,331,469~3,216,117 | 125,138,761~ 125,619,495 | 178,885,693~180764897 |

Assuming the temperature is variable with seasons. The design should satisfy the following constraint:. It is important to note that   is the minimum injection pressure ([Zhang et al. 2012](#_ENREF_21)). The CO2 temperature is assumed to be 12  by using the existing method ([Chandel et al. 2010](#_ENREF_1)). Table 8 shows the comparison results of the existing and proposed methods with variable temperature. It can be seen that  may not satisfy the constraint based on the existing method, compared with the existing method, the proposed method saves the total cost. For example, assuming   and  , based on the existing method, the inlet pressure is 13 , the optimized nominal inner diameter and wall thickness are 0.31115  and 0.00635  respectively,  decreases from 10.15900 to 9.90296  as the temperature increases from 2~17 . Therefore, if the optimization design is applied based on the existing method,  is smaller than 10  at higher temperatures, this lead to an infeasible design. Based on the proposed approach,  decreases from 10.20830 to 10  as the temperature increases. The proposed method satisfy the constraint. That’s because the existing optimization design based on a constant temperature between the variable soil temperature, which ignore the effects of variable temperature. Over the life time of 25 years, the optimal total costs are 1,301,231,319~1,303,784,612 and 1,298,899,850~1,300,568,495  by using the existing and proposed methods, respectively. The proposed method saves 2,331,469~3,216,117 . The proposed method not only satisfies the constraint but also saves total cost. Therefore, the optimal results are more reasonable by using the proposed approach.

Table 9 (a) Comparison results of the existing and proposed methods with multiple uncertainties

|  |  |  |
| --- | --- | --- |
| Method | Parameter |  () |
| 100 | 145 | 180 | 245 |
| The existing research ([Knoope et al. 2014](#_ENREF_7)) |  () | 0.26035 | 0.31115 | 0.33975 | 0.39055 |
|  () | 0.00635 | 0.00635 | 0.007925 | 0.007925 |
|  () | 14.1287 | 13.3920 | 13.2860 | 12.9213 |
| The proposed method |  () | 0.31115 | 0.33975 | 0.39055 | 0.44135 |
|  () | 0.00635 | 0.007925 | 0.007925 | 0.007925 |
|  () | 11.6051 | 12.1170 | 11.56925 | 11.52536 |
| Base optimal results with known uncertainties |  () | 0.26306 | 0.31547 | 0.351186 | 0.403641 |
|  () | 0.00563 | 0.00647 | 0.007042 | 0.007857 |
|  () | 14.2718 | 13.7047 | 13.4079 | 13.0316 |

Assuming the temperature is variable with seasons, other multiple uncertainties are bounded (). In order to further illustrate the effects for dealing with multiple uncertainties by using the proposed method, known perturbations are given as basic reference: the variable temperature is 17 , other multiple uncertainties are 2% of nominal parameters,  and  can be obtained. Table 9 (a) shows the optimization results of the existing and proposed methods with multiple uncertainties. Compared with the results from the basic reference, diameter and wall thickness not satisfy the design by using the existing method ([Knoope et al. 2014](#_ENREF_7)). For example, assuming  ,   is obtained by using the existing method, compared with  ,  cannot satisfy the diameter design requirement. Assuming ,   is obtained by using the existing method, compared with ,  cannot satisfy the wall thickness design requirement, there is no safety guarantee for the pipeline transportation.

The existing method may make an infeasible design, because it cannot deal with the effects caused by the variable temperature and other multiple uncertainties effectively. Note that the determination of diameter and wall thickness depends on temperature indeed (as shown in (13) and (14)). Compared with the results from the basic reference, the proposed method can deal with the multiple uncertainties well and get feasible results.

Table 9 (b) Comparison results of the existing and proposed methods with multiple uncertainties

|  |  |  |
| --- | --- | --- |
| Method | Parameter |  () |
| 120 | 165 | 220 | 290 |
| The existing research ([Zhang et al. 2012](#_ENREF_21)) |  () | 0.31115 | 0.33975 | 0.39055 | 0.44135 |
|  () | 0.00635 | 0.00635 | 0.007925 | 0.007925 |
|  () | 13.9803 | 13.5245 | 13.1588 | 12.8461 |
| Total cost () | 1,335,281,502 | 1,792,087,217 | 2,192,466,945 | 2,660,720,632 |
| The proposed method |  (..) | 0.31115 | 0.33975 | 0.39055 | 0.44135 |
|  () | 0.00635 | 0.007925 | 0.007925 | 0.007925 |
|  () | 12.3098 | 12.6594 | 12.2705 | 12.0464 |
| Total cost () | 1,325,140,505 | 1,784,887,923 | 2,183,147,847 | 2,648,879,152 |
|  | Total saving () | 10,140,997 | 7,199,294 | 9,319,098 | 11,841,480 |

Table 9 (b) also shows the comparison results of the existing and proposed methods with multiple uncertainties. Compared with the existing method, the proposed method saves the total cost. For example, assuming  , the optimal  and   are obtained by using the two methods. The optimal total costs are 1,335,281,502 and 1,325,140,505  over the lifetime of 25 years for the existing and proposed methods, respectively. The proposed method saves 10,140,997 , which improves the optimization performance.

5 Conclusion

In order to minimize LC for pipeline design, a novel robust optimization model is developed by considering multiple uncertainties. The solution for robust optimization problem is obtained by LMI. A stepwise optimization is given to improve the optimization performance. In the numerical studies, comparing with the existing optimization methods, it is verified that the proposed approach can improve the design performance and provides more securities for the pipeline transportation. In the future, the authors will focus on the applications of the proposed approach in the CO2 pipeline design.

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**Appendix A: Proof for Theorem 1**

**Lemma 1**([El Ghaoui 1997](#_ENREF_2)): (*S*-procedure). Letting  be quadratic functions of the variable :



where . If  satisfies the following condition:

 for all  such that 

there are  such that .

**Lemma 2** ([Lin C 2007](#_ENREF_9)) (Schur complement)Letting ,  and  be appropriately dimensional matrices with  and  symmetric. Then,



if and only if any of the following conditions holds:

(i)  and ;

(ii)  and .

**Proof:** Introducing an auxiliary variable , (7) can be reformed as:

 (20)

Defining the following norm:

 (21)

where , . Setting  be an appropriately dimensional matrix, the element of the *j*th row and *j*th column of  is 1, while the rest elements of  are 0. Then .  can be denoted as . Therefore,  can be written as . (20) can be transformed into:

 (22)

 is equivalent to:

 (23)

 can be written as:

 (24)

Defining variables transformation, , , , , . (24) will be:

 (25)

After some straight forward manipulations, (25) becomes:

 (26)

Using the*S*-procedure(Lemma 1), for all , (26) holds if there exist a scalar  such that

 (27)

After some straight forward manipulations, (27) is equivalent to:

 (28)

Using Schur complement (Lemma 2), (28) is transformed into:

 (29)

According to (19), the pipeline robust optimization problem can be solved by using LMI.

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