Inequality and aggregate demand in the IS-LM and IS-MP models*

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Abstract

This paper presents an extension to the textbook IS-LM and IS-MP models that allows the short run effects of an increase in household income inequality to be studied in a simple manner. The income distribution is assumed to be log-normal, and the coefficient of variation of income is assumed to be exogenous. The latter is used as the measure of income inequality, and enters otherwise standard IS and LM curves in a straightforward manner. While the models are highly stylised, they can easily be extended to more complicated variants.

Keywords: Inequality, IS-LM, IS-MP.

JEL Codes: E10, D31, A20.

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1 Introduction

This paper presents an extension to the textbook IS-LM and IS-MP models that allows the short run effects of an increase in household income inequality to be studied in a simple manner. Despite widespread increases in inequality over the last 40 years, and despite the proliferation of models studying the determinants and effects of inequality, there appears to be no simple, tractable model of the short run effects of household income inequality on aggregate demand.

The income distribution has, of course, been an object of study for as long as economics has existed. Understanding the distribution of national income between the various classes of society was the fundamental goal of Ricardo's political economy, and this was also a major goal for Karl Marx. After a brief lull during the heyday of Neoclassical economics, the household income distribution re-emerged as a popular topic in the 1940s and 1950s, with Michał Kalecki (1945) and D. G. Champernowne (1953) publishing notable early studies of stochastic processes in income. Early statistical investigations of the effect of inequality on household consumption include Staehle (1937) and Polak (1939); the history of this literature is discussed in Thomas (1992). Following these empirical studies, the income distribution was regularly discussed in early macroeconomics textbooks, particularly in the context of income determination. This is the case in Tarshis (1947) and Samuelson (1948). Later, Weintraub (1958) discussed distributional effects on liquidity preference, Patinkin (1965) included a general discussion of the issue, and Rowan (1968) included a lengthy discussion of distributional effects on the marginal propensity to consume. In fact, Rowan (1968) proposed a consumption function of the form,

$$C = f(Y, A, r, \alpha),$$

where Y denotes total income, A denotes total assets, r denotes the government bond rate, and α denotes "the distribution of real income", where "the *greater* is α the *more equally* income is distributed" (Rowan 1968: 159-161). Rowan in turn referred to a number of macroeconomics texts which discussed the effects of income distribution on aggregate consumption in the context of Duesenberry's relative income hypothesis (Duesenberry 1949). The savings function in (1) below is very similar to Rowan's formulation.

Although a formal consumption function was proposed in Rowan (1968), and empirical studies of the effects of inequality on consumption behaviour did exist, the early textbook discussions tended to remain at the verbal level. Alongside this literature, a relatively large literature in the Post Keynesian and Marxian traditions examined the effects of changes in the functional distribution of income on aggregate demand. Notable early papers include Kaldor (1956) and Pasinetti (1962). Bhaduri and Marglin (1990) is an important later example of this approach, and has led to an extensive empirical literature surveyed in Stockhammer (2017). Later, more Neoclassical work, tended to discuss the determination of the income distribution and its relation to the supply side, rather than the short run relation between inequality and aggregate demand. Notable early examples are Stiglitz (1969) and Atkinson (1975). The graduate level text of Bertola et al (2006) covers this approach in great detail, and also looks at demand effects in endogenous growth models. Finally, computational models of the income and wealth distributions in general equilibrium have become popular in recent years, following Aiyagari (1994). Heathcote et al (2009) provides a good overview of this literature, and Atkinson (1997) provides a good overview of the literature as a whole.

While the later work on inequality is very sophisticated, it is usually quite complicated, and does not focus on short run demand effects. The present paper aims to fill the gap, providing a formal model of the effects of household income inequality on aggregate demand which is simple and tractable. The income distribution is assumed to be log-normal, and the coefficient of variation of income is assumed to be exogenous. The latter is used as the measure of income inequality, and enters otherwise standard IS and LM curves in a straightforward manner. To the best of the author's knowledge, such a model has not been published before. A recent model in the same spirit is presented in Carvalho and Rezai (2016), which adds Pareto distributed earnings to a Post Keynesian model.

The rest of the paper is organised as follows. Section 2 derives the IS curve, section 3 derives the LM curve, and section 4 studies the IS-LM equilibrium. Section 5 replaces the LM curve with an MP curve, and studies the resulting IS-MP equilibrium. Section 6 concludes.

2 The IS curve

Three assumptions are made in order to derive the IS curve:

A1 Individual income is distributed log-normally: $Y \sim \ln \mathcal{N}(\mu, \sigma^2)$.

A2 Individual saving is a log-linear function of individual income: $S = aY^{\alpha}$.

A3 Per capita investment is a log-linear function of the interest rate: $I = br^{-\beta}$.

Assumptions A1 - A3 are not uncommon in the literature, and ensure both tractability and a degree of realism. Although empirical distributions of consumption expenditure appear to be closer to log-normality than empirical income distributions (Battistin et al 2009), the log-normal distribution is still widely used as a baseline model for empirical income distributions (see Pinkovskiy and Sala-i-Martin (2009) and the references therein). Importantly for the purposes of the present paper, assumption A1 combined with assumptions A2 and A3 ensures a log-linear IS curve without the need for approximation. Importantly, although the focus here is on parametric equations, the argument will hold for a general formulation of the curves.

In assumption A2, α is the income elasticity of saving, which will be a key parameter in determining the behaviour of the model in response to a change in income inequality. To fix intuition at the outset, one would expect α to be greater than one, i.e. one would expect richer households to save more. Evidence in support of this hypothesis is provided in Carroll (1998), Dynan et al (2004), and Carvalho and Rezai (2016).

From assumptions A1 and A2 the following can be derived: expected income $E[Y] = e^{\mu+0.5\sigma^2}$, the coefficient of variation of income $CV[Y] = (e^{\sigma^2} - 1)^{0.5}$, individual saving is distributed log-normally, $S \sim \ln \mathcal{N}(\alpha\mu + \ln a, \alpha^2\sigma^2)$, and expected saving $E[S] = e^{\alpha\mu+\ln a+0.5\alpha^2\sigma^2}$. The IS curve is derived by setting expected saving equal to per capita investment:

$$E[S] = e^{\alpha\mu+\ln a+0.5\alpha^2\sigma^2} = br^{-\beta} = I. \tag{1}$$

Recall the expressions for expected income and the coefficient of variation. The latter is taken as exogenous in order to solve the model. Noting that $\sigma^2 = \ln(CV[Y]^2 + 1)$, and therefore $\mu = \ln(E[Y]) - 0.5\ln(CV[Y]^2 + 1)$, (1) can be rearranged to yield,

$$\ln(E[Y]) - 0.5\ln(CV[Y]^2 + 1) + 0.5\alpha\ln(CV[Y]^2 + 1) = \frac{\ln b - \ln a - \beta\ln r}{\alpha},$$

which in turn can be rearranged to yield the IS curve,

$$\ln(E[Y]) = \frac{\ln b - \ln a}{\alpha} - \frac{\beta\ln r}{\alpha} + (1-\alpha)0.5\ln(CV[Y]^2 + 1). \tag{2}$$

The IS curve shifts out in $(E[Y], r)$ space following an increase in income inequality if $\alpha < 1$, shifts in following an increase in income inequality if $\alpha > 1$, and is unaffected by changes in income inequality if $\alpha = 1$. The intuition is straightforward: if $\alpha > 1$ then the marginal propensity to save is increasing in income; an increase in the coefficient of variation of income shifts income to those with higher marginal propensities to save, and as such the IS curve shifts in. Rothschild and Stiglitz (1971) provides a general discussion of this type of effect.

3 The LM curve

Two further assumptions are made in order to derive the LM curve:

A4 Individual money demand is a log-linear function of individual income and the interest rate: $M^d = cY^\gamma r^{-\delta}$.

A5 Per capita money supply is constant: $M^s = M$.

In assumption A4, γ is the income elasticity of money demand, which will be a key parameter (along with α) in determining the behaviour of the model in response to a change in income inequality. To fix intuition, one would expect γ to be less than one, i.e. one would expect richer households to maintain lower ratios of cash to income than poorer households. Carroll (2000) and Calvet and Sodini (2014) present evidence that wealthier households tend to hold riskier assets, which may be interpreted as supporting evidence for $\gamma < 1$. More direct support for this hypothesis is provided in Fujiki and Hsiao (2008) and Tin (2008), which provide evidence for an income elasticity of money demand between zero and one by estimating money demand functions on cross-sectional data.

From assumptions A1 and A4 the following can be derived: individual money demand is distributed log-normally, $M^d \sim \ln\mathcal{N}(\gamma\mu + \ln c - \delta\ln r, \gamma^2\sigma^2)$, and expected money demand $E[M^d] = e^{\gamma\mu + \ln c - \delta\ln r + 0.5\gamma^2\sigma^2}$. The LM curve is derived by setting expected money demand equal to per capita money supply:

$$E[M^d] = e^{\gamma\mu + \ln c - \delta\ln r + 0.5\gamma^2\sigma^2} = M = M^s. \tag{3}$$

The money market equilibrium condition in (3) is similar to the goods market equilibrium condition in (1), and the LM curve can be derived in the same manner. By using the expressions for σ^2 and μ in expected income and the coefficient of variation of income, (3) can be rearranged to yield,

$$\ln(E[Y]) - 0.5\ln(CV[Y]^2+1) + 0.5\gamma\ln(CV[Y]^2+1) = \frac{\ln M - \ln c + \delta\ln r}{\gamma},$$

which in turn can be rearranged to yield the LM curve,

$$\ln(E[Y]) = \frac{\ln M - \ln c}{\gamma} + \frac{\delta\ln r}{\gamma} + (1-\gamma)0.5\ln(CV[Y]^2+1). \tag{4}$$

The LM curve shifts out in $(E[Y],\, r)$ space following an increase in income inequality if $\gamma < 1$, shifts in following an increase in income inequality if $\gamma > 1$, and is unaffected by changes in income inequality if $\gamma = 1$. The intuition is straightforward: if $\gamma < 1$ then the ratio of desired money holdings to income is decreasing in income; an increase in the coefficient of variation of income shifts income to those with lower liquidity preference, and as such the LM curve shifts out.

4 IS-LM equilibrium

Tidying up the notation, (2) and (4) combine to yield the IS-LM model:

$$y = \Phi_1 - \Phi_2 i + (1-\alpha)f(C), \tag{IS}$$

$$y = \Phi_3 + \Phi_4 i + (1-\gamma)f(C), \tag{LM}$$

where Φ_1, Φ_2, Φ_3, and Φ_4 are reduced form parameters (with $\Phi_2 > 0$ and $\Phi_4 > 0$), y is log per capita income, i is the log interest rate, and C is the coefficient of variation of income (with $f'(C) > 0$). The parameters α and γ are the income elasticity of saving and the income elasticity of money demand, respectively.

Without further information on the magnitude of α and γ, the responses of the IS and LM curves to a change in income inequality is unknown. However, as noted above, Carroll (1998), Dynan et al (2004), and Carvalho and Rezai (2016) present evidence that $\alpha > 1$, and Carroll (2000), Calvet and Sodini (2014), Fujiki and Hsiao (2008), and Tin (2008) present evidence that $\gamma < 1$. As such, one would expect an increase in income inequality to shift the IS curve in and the LM curve out, as the average marginal propensity to save increases on the one hand, and average liquidity preference decreases on the other. The corollary of this is that consumption will decrease, and investment increase, in response to an increase in income inequality in the IS-LM model. This case is illustrated in figure 1.

The net effect of these shifts on per capita income is ambiguous, and depends on the relative magnitude of α and γ and the reduced form elasticities of income with respect to the interest rate. However, Atems and Jones (2015) provides evidence that increases in the

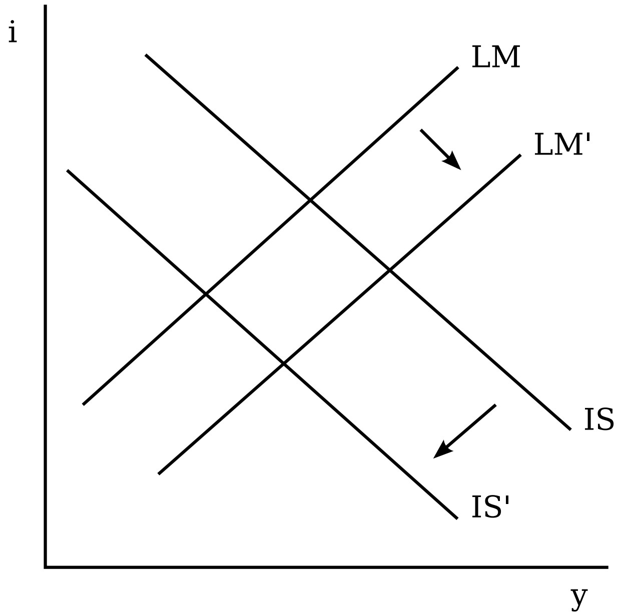

Figure 1: Comparative statics of an increase in income inequality in the IS-LM model, with $\alpha > 1$ and $\gamma < 1$.

Gini coefficient have a small but significantly negative effect on GDP in the short run, using a panel vector autoregression model estimated on US data. This suggests that the scenario illustrated in figure 1 is empirically accurate, at least for the USA.

The net effect of a change in income inequality on the interest rate is predicted to be unambiguously negative in the model. There appears to be no equivalent study to Atems and Jones (2015) exploring the short run effects of changes in inequality on interest rates, but Goda and Lysandrou (2014) argue that the concentration of income and wealth among high net worth individuals depressed bond yields in the run up to the 2008 financial crisis. A similar argument has been made in recent accounts of "secular stagnation" following the 2008 crisis (Summers 2014: 69).

5 IS-MP equilibrium

The only difference between the IS-LM model and the IS-MP model is the replacement of the money market equilibrium condition in the former with a monetary policy rule in the latter. One final assumption is made in order to construct the IS-MP model:

A6 The interest rate rule is a log-linear function of per capita income: $r = kE[Y]^{\rho}$.

Assumption A6 can be used to replace the LM curve with a log-linear interest rate rule, while the IS curve remains unchanged. Using the same notation as in section 4, (2) and the interest rate rule combine to yield the IS-MP model:

$$y = \Phi_1 - \Phi_2 i + (1 - \alpha) f(C), \tag{IS}$$

$$y = \Phi_5 + \Phi_6 i. \tag{MP}$$

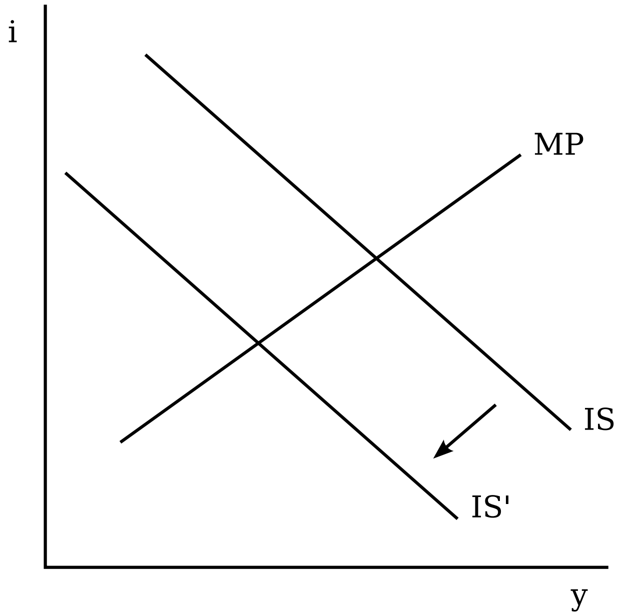

Figure 2: Comparative statics of an increase in income inequality in the IS-MP model, with $\alpha > 1$.

Now, and in distinction to the LM model, the MP curve is unaffected by changes in income inequality. As a result, and again assuming $\alpha > 1$, an increase in income inequality will shift the IS curve in without affecting the MP curve. Per capita income and the interest rate will both unambiguously reduce, although the central bank can limit the effect on output if the interest rate is sufficiently responsive to changes in demand. This case is illustrated in figure 2. Note that this result follows from the central bank ignoring inequality, and because there is only one interest rate in the model. With multiple interest rates, the yield curve would likely change in response to inequality in the IS-MP model, and the results would become qualitatively similar to the IS-LM model discussed above.

6 Concluding Remarks

Over the last 40 years, substantial increases in inequality have occurred within a number of countries. Despite this, and despite the proliferation of models studying the determinants and effects of inequality, there appears to be no simple, tractable model of the short run effects of household income inequality on aggregate demand. This paper aims to fill that gap, by presenting simple extensions of the textbook IS-LM and IS-MP models that take the coefficient of variation of income as an exogenous variable. A key result of the model is the ambiguous effect on output of an increase in income inequality, which results from opposing effects on the IS and LM curves.

There is an interesting parallel between the ambiguous effects of a change in the income distribution in the IS-LM model and the effects of a change in the labour share in the Bhaduri-Marglin model; in both models an increase in inequality should reduce consumption and increase investment, and the total effect depends on the parameterisation. This parallel could be explored further by incorporating wage setting and the functional distribution of income into the models presented here, which would bring the model closer to that of

Carvalho and Rezai (2016). There is a similar parallel between the aggregate demand effect in the IS-LM model and the relation between inequality and growth in endogenous growth models (Bertola et al 2006: 260). This could certainly be explored further, although it would be technically challenging. Finally, it would be useful to reduce the reliance of the model on log-normality, perhaps by the use of Taylor expansions to approximate the moments of the saving and money demand distributions. This is left to future work.

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