Can Indeterminacy and Self-Fulfilling Expectations Solve the International Macro Puzzles?

Stephen McKnight[*] Laura Povoledo[†]
El Colegio de México University of the West of England

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Abstract

We introduce equilibrium indeterminacy into a two-country incomplete asset model with imperfect competition and analyze whether self-fulfilling expectations or beliefs can help resolve the major puzzles of international business cycles. We show that when self-fulfilling beliefs are correlated with technology shocks, the model can account for the counter-cyclical behavior observed for the terms of trade and real net exports, while simultaneously generating higher volatilities relative to output, as in the data. However, the model cannot solve the Backus-Smith puzzle without a strongly negative cross-country correlation for technology shocks.

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[*] Centro de Estudios Económicos, El Colegio de México, Carretera Picacho Ajusco No. 20, Colonia Ampliación Fuentes del Pedregal, Delegación Tlalpan, Mexico City 14110, Mexico. E-mail: mcknight@colmex.mx.

[†] School of Economics, Bristol Business School, University of the West of England, Coldharbour Lane, Bristol, BS16 1QY, UK. E-mail: laura.povoledo@uwe.ac.uk.

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1 Introduction

Business cycle statistics suggest that the terms of trade and net exports are both counter-cyclical and volatile over the cycle, and the real exchange rate is negatively correlated with relative consumption.[1] Standard International Real Business Cycle (IRBC) models struggle to replicate these key properties of the data.[2] In this paper we investigate whether equilibrium indeterminacy, which allows for self-fulfilling expectations or beliefs, can help account for the observed fluctuations in international business cycles.[3] This approach has been successful in quantitatively explaining closed-economy business cycles.[4] We show that a combination of technology shocks and self-fulfilling beliefs can help resolve most of the puzzles relating to international relative prices and cross-country trade flows. However, the model cannot generate a negative correlation between relative consumption and international relative prices without allowing for a negative cross-country correlation for technology shocks.

The model economy we consider is a two-country incomplete asset economy with imperfect competition. In each country, final consumption and investment goods are produced using domestic and foreign intermediate goods. Prices are assumed to be flexible and the real exchange rate deviates from purchasing power parity due to home bias towards domestically-produced intermediate goods. As is common in the IRBC literature, we assume GHH preferences.[5] Indeterminacy is introduced via an increasing returns to scale technology, and thus the marginal cost schedule of intermediate firms is decreasing in output, so the aggregate labor demand schedule of each country slopes upwards. Under indeterminacy, the forecast error to the terms of trade can be the only source of business cycle fluctuations (autonomous beliefs) or correlated with fundamental shocks (correlated beliefs).[6]

Our main findings are summarized as follows. We first show that international business cycle fluctuations driven solely by autonomous beliefs cannot replicate any of the major features of the data. This finding is in stark contrast to Guo and Sturzenegger (1998) and Xiao (2004), who find instead that self-fulfilling expectations can help explain the positive cross-country correlations ob-

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[1] See, e.g., Backus and Smith (1993), Chari et al. (2002), Corsetti et al. (2008), Benigno and Thoenissen (2008), Raffo (2008, 2010), and Engel and Wang (2011).

[2] See Raffo (2010) for an excellent summary of the international business cycle literature.

[3] By indeterminacy we mean that there exists multiple equilibrium paths which converge to the steady state.

[4] See, e.g., Benhabib and Farmer (1994), Farmer and Guo (1994), Jaimovich (2007), and Dufourt et al. (2015).

[5] Due to the absence of an income effect of labor supply, GHH preferences help increase the volatility of consumption in line with the data. See Raffo (2008) for further discussion.

[6] In what follows we use the terms forecast error, expectational error, self-fulfilling expectations or beliefs interchangeably.

served for consumption and output. However, both Guo and Sturzenegger (1998) and Xiao (2004) introduce indeterminacy into a two-country, one-good model, while we generate indeterminacy in a two-good framework, in order to look at a wider set of puzzles related to international relative prices and quantities.

In one-good models self-fulfilling expectations stimulate world demand and generate positive cross-country correlations for consumption and output, as in the data. However, in our two-good model revisions to the terms of trade forecasts are the source of endogenous fluctuations.[7] We show that a belief-induced depreciation of the terms of trade shifts the upward-sloping labor demand schedules in each country in opposite directions, raising domestic output and consumption at the expense of foreign output and consumption. Consequently, in two goods models autonomous beliefs cannot on their own explain the data, since by causing a reallocation of output, they generate counterfactually negative cross-country correlations.

We next show that a number of the empirical irregularities of the data can be resolved by allowing the forecast or expectational errors to be correlated with technology shocks. Now, the indeterminacy model can generate counter-cyclical behavior for the terms of trade and real net exports, while at the same time, increasing significantly the volatility of international relative prices and cross-country trade flows. This improvement in volatility over the business cycle is not at the cost of reduced volatility of the other aggregate variables, whose standard deviations relative to output are also increased.

The improved performance of the model is due to the transmission mechanism of technology shocks which is fundamentally altered under indeterminacy. Standard IRBC models struggle to replicate the international business cycle facts because positive technology shocks generate increases in output and a rise in international relative prices, whereas in the data positive output changes are associated with a fall in international relative prices. In our model, technology shocks induce a change in beliefs by causing agents in both countries to revise their expectations. We find that a very specific transmission of technology shocks, in which there is a negative response of employment to a positive technology shock and a delayed effect on output, best explains the data.[8] In particular, a positive domestic technology shock causes a belief-induced depreciation (increase) of the terms of trade and the delayed expansion generates the desired negative correlation between the terms

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[7] There is sizeable evidence to suggest that terms of trade shocks are an important source of business cycle fluctuations (see, e.g., Mendoza, 1995).

[8] This temporary contractionary transmission mechanism is not without empirical support. See, e.g., Basu et al. (2006) and Giuli and Tancioni (in press).

of trade and output. This transmission mechanism enables the model to also resolve the output-correlation puzzles. Since exports are relatively higher than imports, real net exports are weakly counter-cyclical as in the data. Finally, the depreciation in the terms of trade is sufficiently large relative to output that the model is able to resolve the volatility puzzles.

Our approach is similar to Schmitt-Grohé (2000) and Benhabib and Wang (2013), in that we select the properties of the fundamental and forecast error shocks which best match the key moments of the data, but we specifically target the international macro puzzles. However, one main discrepancy with the data remains, namely the Backus-Smith puzzle. The model predicts a positive correlation between the real exchange rate and relative consumption, whereas in the data this correlation is negative. In our model, a belief-induced depreciation of the terms of trade generates a relatively stronger reduction in employment abroad than in the domestic economy. This increases the ratio of consumption across the two countries, thereby counterfactually implying a positive correlation between international relative prices and relative consumption. We show that to resolve this puzzle the model requires a strong negative cross-country correlation for technology shocks, which is not supported by other studies.[9]

The current paper is also related to several contributions within the indeterminacy literature. Similar to Farmer and Guo (1994), Schmitt-Grohé (1997), Benhabib and Wen (2004), and Jaimovich (2007), among others, we generate indeterminacy under monopolistic competition by assuming increasing returns to scale.[10] Following Wen (1998), we introduce variable capacity utilization so that the model can generate indeterminacy for empirically plausible values for the steady-state markup. Finally, we assume GHH preferences, like Guo and Harrison (2010) and Dufourt et al. (2015). Such modelling features have been successful in quantitatively explaining closed-economy business cycles. For example, Guo and Harrison (2010) and Dufourt et al. (2015) show that under indeterminacy a two-sector RBC model can broadly reproduce several key features of U.S. business cycles. This paper contributes to the indeterminacy literature by examining whether an open-economy model with similar features can also successfully explain the long-standing price and quantity puzzles of the international macro literature.

Similar to Pintus et al. (2016) and Pavlov and Weder (2017), we solve the model under indeterminacy using the Farmer-Khramov-Nicolò (2015) method.[11] However, they estimate their

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[9] See, e.g., Backus et al. (1992) and Heathcote and Perri (2004).

[10] Jaimovich (2007) generates indeterminacy in a imperfect competition model with firm entry rather than increasing returns to scale.

[11] A popular alternative to the Farmer-Khramov-Nicolò solution technique is the method of Lubik and Schorfheide (2003, 2004). As shown by Farmer et al. (2015), these two solution methods are equivalent. See also Bianchi and

models with U.S. data using Bayesian techniques, whereas we use the method of moments approach to try and resolve the international macro puzzles.

Finally, this paper is also related to the recent IRBC studies by Raffo (2010) and Karabarbounis (2014) who also attempt to explain the international macro puzzles in directions different from ours. Karabarbounis (2014) introduces a labor wedge into an otherwise standard IRBC model with complete asset markets, whereas Raffo (2010) considers an additional source of technological variation by including investment-specific technology shocks. In both Raffo (2010) and Karabarbounis (2014) the sources of business cycle fluctuations are due to exogenous shocks. This paper complements these two studies by examining how far endogenous fluctuations can go in explaining international business cycles. Similar to them, we cannot solve all the puzzles. However, while these studies show that additional sources of exogenous fluctuations can be successful in resolving the Backus-Smith puzzle, we show that endogenous fluctuations can solve the volatility and output-correlation puzzles.

The remainder of the paper is organized as follows. Section 2 outlines the model economy and Section 3 discusses the calibration of the model and the solution method employed. Section 4 discusses the results obtained under autonomous beliefs, whereas Section 5 discusses the findings when beliefs are also assumed to be correlated with productivity shocks. Finally, Section 6 briefly concludes.

2 Model

We develop a two-country extension of the imperfect competition model studied by Farmer and Guo (1994), Schmitt-Grohé (1997), and Benhabib and Wen (2004) for the closed economy. Following Wen (1998), we assume variable capacity utilization which significantly reduces the size of the steady-state markup needed to generate indeterminacy. Within each country there exists a representative agent, two final-good producers, and a continuum of intermediate-good producing firms. Intermediate firms operate under monopolistic competition and use domestic labor and capital as inputs to produce tradeable goods. The competitive final good producers use domestic and imported intermediate goods to produce non-tradeable consumption or investment goods, which are subsequently purchased by the domestic agent. However, final good producers are assumed to have a bias for domestically produced intermediate goods. While the law of one price is assumed

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Nicolò (2017).

to hold for all intermediate goods, with home bias, the real exchange rate deviates from purchasing power parity. The following presents the features of the model for the Home country on the understanding that the Foreign case can be analogously derived. All Foreign country variables are denoted by an asterisk.

2.1 Final good producers

In each country, there are two final goods, consumption and investment, which are produced with homogenous of degree one production functions using intermediate goods as the only inputs. The Home consumption final good C_t is produced by a competitive firm that uses $C_{H,t}$ and $C_{F,t}$ as inputs according to the following CES aggregation technology index:

$$C_t = \left[a^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the constant elasticity of substitution between aggregate Home and Foreign intermediate goods is $\theta > 0$ and the relative share of domestic and imported intermediate inputs used in the production process is $0 < a < 1$. The Home investment final good I_t is produced according to the following CES aggregation technology index:

$$I_t = \left[b^{\frac{1}{\rho}} I_{H,t}^{\frac{\rho-1}{\rho}} + (1-b)^{\frac{1}{\rho}} I_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{2}$$

where $\rho > 0$ and $0 < b < 1$. The inputs $C_{H,t}$, $C_{F,t}$, $I_{H,t}$, and $I_{F,t}$ are defined as the quantity indices of domestic and imported intermediate goods respectively:

$$\begin{aligned} C_{H,t} &= \left[\int_0^1 c_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad & C_{F,t} &= \left[\int_0^1 c_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}}, \\ I_{H,t} &= \left[\int_0^1 i_t(j)^{\frac{\kappa-1}{\kappa}} dj \right]^{\frac{\kappa}{\kappa-1}}, \qquad & I_{F,t} &= \left[\int_0^1 i_t(j^*)^{\frac{\kappa-1}{\kappa}} dj^* \right]^{\frac{\kappa}{\kappa-1}}, \end{aligned}$$

where the elasticity of substitution across domestic (imported) intermediate goods is $\kappa > 1$, and $c_t(j)$, $i_t(j)$, $c_t(j^*)$, $i_t(j^*)$ are the respective quantities of the domestic and imported type j and j^* intermediate goods. Intermediate firms sell their products to both consumption and investment final-good producers, where it is assumed that the law of one price holds. Cost minimization in final good production yields the demand conditions for Home and Foreign goods:

$$C_{H,t} = a\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, \qquad C_{F,t} = (1-a)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t, \tag{3}$$

$$I_{H,t} = b\left(\frac{P^I_{H,t}}{P^I_t}\right)^{-\rho} I_t, \qquad I_{F,t} = (1-b)\left(\frac{P^I_{F,t}}{P^I_t}\right)^{-\rho} I_t, \tag{4}$$

and the corresponding aggregate price indices are given by:

$$P_t = \left[aP_{H,t}^{1-\theta} + (1-a)P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \quad P^I_t = \left[b(P^I_{H,t})^{1-\rho} + (1-b)(P^I_{F,t})^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{5}$$

where P_t is the consumer price index, P^I_t is the price of investment goods, and $P_{H,t}$, $P^I_{H,t}$, $P_{F,t}$, $P^I_{F,t}$ are the respective price indices of Home and Foreign intermediate goods.

2.2 Intermediate goods producers

All intermediate firms have access to the same technology. A Home firm of type j has a production technology given by

$$Y_t(j) = Z_t\left(u_t(j)K_t(j)\right)^{\alpha} L_t(j)^{\gamma} - \phi, \quad j \in [0,1] \tag{6}$$

where K_t and L_t represent capital and labor usage, respectively, Z_t is the exogenous level of technology or productivity, and the input share is $\alpha + \gamma \geq 1$. The rate of capacity utilization $u_t \in (0,1)$ is endogenously determined. Following Greenwood et al. (1988), it is assumed that the depreciation rate of capital δ_t is higher if it is used more intensively:

$$\delta_t = \frac{1}{\eta} u_t^{\eta}, \tag{7}$$

where $\eta > 1$. A fixed cost of production $\phi > 0$ is also included in the production technology (6). Therefore, regardless of how much output Y_t is produced, a proportion ϕ of the intermediate good is used up in each period. As in Schmitt-Grohé (1997), allowing for a fixed production cost enables the model to generate zero profits without imposing any restrictions on the size of the steady-state markup.[12] Given competitive prices of labor and capital, cost-minimization yields:

$$w_t = \gamma mc_t(j) Z_t\left(u_t(j)K_t(j)\right)^{\alpha} L_t(j)^{\gamma-1}, \tag{8}$$

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[12] As discussed by Rotemberg and Woodford (1996), Schmitt-Grohé (1997), and Jaimovich (2007), positive profits are not observed in the U.S. economy despite the presence of market power.

$$rr_t + \delta_t = \alpha mc_t(j) Z_t u_t^{\alpha}(j) K_t(j)^{\alpha-1} L_t(j)^{\gamma}, \tag{9}$$

$$u_t^{\eta} = \alpha mc_t(j) Z_t u_t^{\alpha}(j) K_t(j)^{\alpha-1} L_t(j)^{\gamma}, \tag{10}$$

where mc_t is real marginal cost, w_t is the real wage, and $rr_t+\delta_t$ is the user cost of capital.

Given that the total demand for firm j's output can be expressed as:

$$Y_t(j) = \left(\frac{p_t(j)}{P_{H,t}}\right)^{-\kappa} \left[C_{H,t} + C^*_{H,t}\right] + \left(\frac{p_t(j)}{P^I_{H,t}}\right)^{-\kappa} \left[I_{H,t} + I^*_{H,t}\right],$$

it follows from the firm's profit maximization problem that the optimal price-setting rule is:

$$p_t(j) = \chi mc_t(j) P_t, \tag{11}$$

where $\chi \equiv \frac{\kappa}{\kappa-1}$ is the markup.

2.3 Representative agent

The representative agent has an expected utility function of the form:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right),$$

where C_t and L_t are consumption and work effort, respectively, and the discount factor is $0 < \beta < 1$. Following Greenwood et al. (1988), we assume that the period utility function is given by:

$$U(C_t, L_t) = \frac{1}{1-\sigma}\left[\left(C_t - \frac{\psi}{1+\nu} L_t^{1+\nu}\right)^{1-\sigma} - 1\right],$$

where $\sigma > 0$ is the relative risk aversion in consumption, $\nu \geq 0$ is the inverse of the Frisch elasticity of labor supply, and $\psi > 0$.

The representative agent during period t supplies labor and capital to intermediate-good producing firms, receiving real income from wages w_t, a rental return on capital rr_t, and nominal profits from the ownership of domestic intermediate firms $\mathit{\Pi}_t$. The agent then uses these resources to purchase the two final goods, dividing purchases between consumption C_t and investment I_t. The purchase of an investment good forms next period's capital according to the law of motion

$$K_{t+1} = (1-\delta_t) K_t + I_t. \tag{12}$$

The asset market structure is assumed to be incomplete. The Foreign agent is able to trade two non-state contingent bonds $B^*_{H,t}$ and $B^*_{F,t}$, whereas the Home agent can only purchase domestic bonds $B_{H,t}$. All bonds are denominated in units of the domestic aggregate consumption index. For the Foreign agent, there is a transaction cost $\varPsi$ of adjusting the internationally traded bond $B^*_{H,t}$, where it is assumed that $\varPsi$ is a positive and differentiable function.[13] This transaction cost, which is paid to financial firms, captures the costs of adjusting bond holdings and is sufficient to ensure that bond holdings are stationary.[14] Consequently, the period budget constraints of the Home and Foreign agent can be expressed in real terms as:

$$\frac{B_{H,t}}{r_t} + C_t + \frac{P^I_t}{P_t} I_t \leq B_{H,t-1} + \int_0^1 w_t L_t(j) dj + \int_0^1 (rr_t + \delta_t(j)) K_t(j) dj + \int_0^1 \varPi_t(j) dj + R_t, \quad (13)$$

$$\begin{aligned} \frac{B^*_{H,t}}{Q_t r_t} \frac{1}{\varPsi(B^*_{H,t})} + \frac{B^*_{F,t}}{r^*_t} + C^*_t + \frac{P^{*I}_t}{P^*_t} I^*_t \leq \frac{B^*_{H,t-1}}{Q_t} + B^*_{F,t-1} + \int_0^1 w^*_t L^*_t(j^*) dj^* \\ + \int_0^1 (rr^*_t + \delta^*_t(j^*)) K^*_t(j^*) dj^* + \int_0^1 \varPi^*_t(j^*) dj^* + R^*_t, \end{aligned} \quad (14)$$

where R_t and R^*_t denote rebates from financial firms, r_t and r^*_t are the Home and Foreign (gross) real interest rates, and Q_t is the CPI-based real exchange rate.

The Home agent's maximization problem yields:

$$U_c(C_t, L_t) = \left(C_t - \frac{\psi L_t^{1+\nu}}{1+\nu} \right)^{-\sigma} = \lambda_t, \quad (15)$$

$$-\frac{U_L(C_t, L_t)}{U_c(C_t, L_t)} = \psi L_t^{\nu} = w_t, \quad (16)$$

$$\lambda_t \frac{P^I_t}{P_t} = \beta E_t \lambda_{t+1} \left[rr_{t+1} + \delta_{t+1} + (1 - \delta_{t+1}) \frac{P^I_{t+1}}{P_{t+1}} \right], \quad (17)$$

$$\beta r_t E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] = 1, \quad (18)$$

where λ_t denotes the shadow price of wealth. Analogous conditions to (15)-(18) apply for the Foreign agent, where the following interest-rate parity condition can be derived:

$$r_t = \frac{r^*_t}{\varPsi(B^*_{H,t})} E_t \left[\frac{Q_{t+1}}{Q_t} \right]. \quad (19)$$

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[13] Following Benigno (2009), we assume that $\varPsi = 1$ when bond holdings are at their steady-state level and $\varPsi$ is positive, differentiable, and strictly decreasing in a neighborhood of the steady state.

[14] For an in-depth discussion of the stationary problem of incomplete market, open-economy models, see Schmitt-Grohé and Uribe (2003) and Ghironi (2006).

Optimizing behavior implies that the budget constraints (13) and (14) hold with equality in each period and the appropriate transversality conditions are satisfied.

2.4 Market clearing and equilibrium

We now focus on a symmetric equilibrium where all firms in Home and Foreign set the same price in each period t, rent the same amount of capital, and employ the same amount of labor. Consequently, $p_t(j) = P_{H,t} = P^I_{H,t}$ and the index j can be dropped from all variables. Market clearing in the Home goods market requires:

$$Y_t = C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t}, \tag{20}$$

and assuming that the Foreign non-state contingent bond is in zero net supply, bond market clearing requires:

$$B_{H,t} + B^*_{H,t} = 0, \qquad B^*_{F,t} = 0. \tag{21}$$

The aggregate resource constraint is given by:[15]

$$C_t + \frac{P^I_t}{P_t} I_t + \frac{B_{H,t}}{r_t} = B_{H,t-1} + \frac{P_{H,t}}{P_t} Y_t, \tag{22}$$

where

$$\frac{P^I_t}{P_t} = \frac{\left[b + (1-b)T_t^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[a + (1-a)T_t^{1-\theta}\right]^{\frac{1}{1-\theta}}}, \qquad \frac{P_{H,t}}{P_t} = \left[a + (1-a)T_t^{1-\theta}\right]^{\frac{1}{\theta-1}} \tag{23}$$

follow from the aggregate price indices (5). The terms of trade T_t can be expressed as:

$$T_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{\left[a + (1-a)T_t^{\theta-1}\right]^{\frac{1}{\theta-1}}}{\left[a + (1-a)T_t^{1-\theta}\right]^{\frac{1}{\theta-1}}} Q_t. \tag{24}$$

In what follows, we call an increase (decrease) in the terms of trade, or the real exchange rate, a depreciation (appreciation). Finally, we measure net exports as the difference between exports and imports, divided by total output (all evaluated at steady state prices):[16]

$$NX_t = \frac{C^*_{H,t} + I^*_{H,t} - \overline{T}\left(C_{F,t} + I_{F,t}\right)}{C_{H,t} + C^*_{H,t} + I_{H,t} + I^*_{H,t}}. \tag{25}$$

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[15] By Walras' Law, the aggregate resource constraint of the Foreign country is redundant.

[16] Thus, our measure of net exports is unaffected by fluctuations in relative prices.

Equilibrium. An equilibrium for the world economy consists of a set of real prices r_t, r_t^*, w_t, w_t^*, rr_t, rr_t^*, δ_t, δ_t^*, mc_t, mc_t^*, λ_t, λ_t^*; a set of relative prices $\frac{P_{H,t}}{P_t}$, $\frac{P_{F,t}^*}{P_t^*}$, $\frac{P_t^I}{P_t}$, $\frac{P_t^{*I}}{P_t^*}$, Q_t, T_t; a collection of allocations for the Home and Foreign agent C_t, C_t^*, I_t, I_t^*, L_t, L_t^*, K_t, K_t^*, u_t, u_t^*, $B_{H,t}$, $B_{H,t}^*$, $B_{F,t}^*$; and a collection of allocations for Home and Foreign final and intermediate good producers Y_t, Y_t^*, $C_{H,t}$, $C_{F,t}$, $C_{H,t}^*$, $C_{F,t}^*$, $I_{H,t}$, $I_{F,t}$, $I_{H,t}^*$, $I_{F,t}^*$, NX_t satisfying (i) the optimality conditions of each agent; (ii) the optimality conditions of final and intermediate good producing firms; (iii) all markets clear; and (iv) the aggregate resource constraints of both countries.

2.5 The determinacy model and the international macro puzzles

To help motivate the indeterminacy analysis that follows, we briefly summarize the international macro puzzles that arise under determinacy. Marginal costs are assumed to be constant (i.e., $\alpha + \gamma = 1$), sunspot shocks do not exist and the dynamics of our imperfect competition model become very similar to standard IRBC models.[17] In terms of the steady state, the output-capital ratio and consumption-output ratio are the same. The only difference relates to levels where steady-state output and capital are lower because of the presence of monopoly power. In terms of the log-linearized model, the major difference relates to the aggregate production technology condition:

$$\widehat{Y}_t = \chi\alpha\widehat{K}_t + \chi\gamma\widehat{L}_t + \chi\widehat{Z}_t, \tag{26}$$

where output fluctuations generated by technology shocks are amplified under imperfect competition $\chi > 1$.

The simulation results for the determinacy version of the model are summarized in Table 1.[18] As is standard in the IRBC literature, technology shocks are assumed to follow an AR(1) process with zero mean. The estimated moments for the data, given in column 2, are for the period 1973(1) − 2007(4) and are taken from Gao et al. (2014), except for the moments for real net exports and first-order autocorrelations, which we compute using data from the Quarterly National Accounts of the OECD.[19] Column 3 reports the unconditional second moments generated under a fixed capacity utilization rate with unitary values for the trade elasticities (*determinacy baseline*).

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[17] In standard IRBC models, $\chi = \alpha + \gamma = 1$, given the absence of monopolistic competition.

[18] The parameter values used in the simulations are the same as in Table 2 of Section 3.2 below with the exception that $\alpha + \gamma = 1$.

[19] All series are logged, except real net exports, and Hodrick-Prescott (HP) filtered with a smoothing parameter of 1600. While there are a number of drawbacks to using the HP filter (see, e.g., Hamilton, 2017), we adopt it to ensure the comparability of our results with the existing IRBC literature. The statistics in Gao et al. (2014) are computed where the U.S. is taken as the Home country and the Foreign country is the aggregate of Canada, Japan, and 19 European countries.

Table 1: Second moments of the determinacy model

			Variations on the determinacy baseline			
			Trade elasticity§		Variable capacity utilization	
	Data†	Determinacy baseline$^\diamondsuit$	$\theta = 0.5$	$\theta = 1.5$	Constant MC	Declining MC*
Standard deviations‡						
Consumption	0.62	0.91	0.86	0.93	0.86	0.81
Investment	2.92	1.38	1.48	1.44	1.83	1.97
Employment	0.68	0.80	0.75	0.82	0.83	0.91
Terms of Trade	1.77	0.59	2.11	0.27	0.36	0.21
Real Exchange Rate	2.38	0.45	1.60	0.21	0.27	0.16
Real Net Exports	0.38*	0.05	0.22	0.09	0.09	0.15
First-order autocorrelations						
Output	0.87*	0.72	0.72	0.72	0.71	0.46
Real Exchange Rate	0.82*	0.75	0.73	0.81	0.74	0.77
Real Net Exports	0.85*	0.95	0.75	0.78	0.74	0.70
Correlations with output						
Consumption	0.82	0.99	0.94	1.00	1.00	1.00
Investment	0.94	0.96	0.94	0.95	0.96	0.97
Employment	0.85	1.00	0.95	1.00	1.00	1.00
Terms of Trade	-0.16	0.45	0.45	0.42	0.44	0.41
Real Net Exports	-0.47*	0.20	0.44	-0.31	-0.41	-0.44
Cross-country correlations						
Output	0.58	0.58	0.58	0.58	0.58	0.58
Consumption	0.43	0.77	0.97	0.70	0.71	0.60
Investment	0.41	0.07	-0.06	-0.01	0.05	0.15
Employment	0.45	0.70	0.93	0.63	0.64	0.61
Correlation with the real exchange rate						
Relative Consumption	-0.17	0.97	0.90	0.91	0.96	0.89

Notes:

† The estimated sample moments for the data are taken from Gao et al. (2014), except for values denoted by $*$ which are from the authors' own calculations.

$\diamondsuit$ In the determinacy baseline, the standard deviations of Home and Foreign technology shocks are estimated to be 0.317 with a cross-country correlation of 0.315. The autocorrelation parameters are set equal to $\upsilon = \upsilon^* = 0.96$. In every alternative parameterization, we keep υ and υ^* unchanged and recalibrate the standard deviations and cross-country correlation of the shocks. The parameter values used in the simulations are given in Table 2 after setting $\alpha + \gamma = 1$.

§ For all variations in the trade price elasticities we set $\theta = \rho$.

$\star$ In the presence of declining marginal costs, we set $\alpha + \gamma = 1.099$.

‡ The standard deviations of all variables are divided by the standard deviation of output, except for the standard deviation of real net exports which is expressed in absolute terms.

The remaining columns of Table 1 evaluate the robustness of the results to variations in the trade elasticity parameters θ and ρ and the introduction of variable capacity utilization under bothconstant and declining marginal costs.[20]

Comparison of columns 2 and 3 of Table 1 shows that the determinacy model suffers from the same well-established discrepancies with the data for international relative prices and quantities as standard IRBC models. First, while the data suggests that both the terms of trade and real net exports are counter-cyclical, the model counterfactually predicts that real net exports and the terms of trade are pro-cyclical (*output-correlation puzzles*). Second, a *volatility puzzle* arises where the predicted volatilities generated by the model are significantly lower than the data. In the data, both the terms of trade and the real exchange rate are more volatile than output, whereas the model predicts the opposite. Furthermore, the model can only generate 13% of the observed standard deviation of real net exports. Third, the data suggests the cross-country correlation of output is greater than the cross-country correlation of consumption, whereas the model predicts the opposite (*cross-country correlation puzzle*). Finally, the model suffers from the so-called *Backus-Smith puzzle*, where the model predicts a high positive correlation between relative consumption and the real exchange rate (0.97), whereas in the data this correlation is negative (-0.17).

The international macro puzzles are robust to alternative assumptions for the trade price elasticities and capacity utilization. Similar to conventional IRBC models, the determinacy model faces an unpleasant trade-off: relatively high trade elasticities can be selected to help generate counter-cyclical real net exports, or relatively low trade elasticities can be chosen to help improve the volatilities of international relative prices, but the choice of trade elasticities cannot solve both puzzles simultaneously (columns 4 and 5). Variable capacity utilization increases the ability of the model to generate a negative correlation between net exports and output, but it cannot remedy any other discrepancy with the data (columns 6 and 7).[21]

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[20] Under declining marginal costs, the values of α and γ are chosen to be sufficiently small to ensure determinacy.

[21] A concise explanation of why these empirical failures arise in the determinacy model is given in a separate Appendix, available from the authors on request.

3 Numerical solution and calibration

3.1 The solution method under indeterminacy

To solve the indeterminacy model, we log-linearize the equilibrium conditions around a symmetric, deterministic steady state, where bond holdings are zero and the steady-state terms of trade is equal to 1.[22] Let $\boldsymbol{s}_t = \left[\widehat{K}_{t+1}, \widehat{K}^*_{t+1}, \tilde{B}_{H,t}, \widehat{T}_t, \widehat{C}_t, \widehat{C}^*_t, E_t\widehat{T}_{t+1}, E_t\widehat{C}_{t+1}, E_t\widehat{C}^*_{t+1}, \widehat{Z}_t, \widehat{Z}^*_t\right]'$ denote the vector of endogenous variables expressed in terms of percentage deviations from their steady state values.[23] The linearized system can be written as:

$$\boldsymbol{\Gamma}_0 \boldsymbol{s}_t = \boldsymbol{\Gamma}_1 \boldsymbol{s}_{t-1} + \boldsymbol{\Psi}\boldsymbol{\varepsilon}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t, \tag{27}$$

where $\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}$, and $\boldsymbol{\Pi}$ are matrices of structural parameters, $\boldsymbol{\varepsilon}_t = [\varepsilon_t, \varepsilon^*_t]'$ is the vector of fundamental or exogenous technology shocks, and $\boldsymbol{\eta}_t = \left[\eta^T_t, \eta^C_t, \eta^{C^*}_t\right]'$ is the vector of non-fundamental or endogenous shocks, which collects the one-step ahead forecast errors for the expectational variables of the system.[24] We assume that $E_{t-1}(\boldsymbol{\varepsilon}_t) = 0$ and $E_{t-1}(\boldsymbol{\eta}_t) = 0$.

If the marginal cost is assumed to be decreasing in output (i.e., $\alpha + \gamma > 1$), then the system (27) may not have a unique solution. With our chosen value of increasing returns to scale (see Section 3.2 below), the number of non-predetermined variables exceeds the number of unstable roots by one, and thus we have one degree of indeterminacy.

The model is solved using the Farmer-Khramov-Nicolò (2015) solution method, whereby we redefine one expectational error $\eta_{f,t}$ of vector $\boldsymbol{\eta}_t$ as a new fundamental disturbance.[25] In this way the number of non-predetermined variables is decreased by one. This transformation enables us to treat the indeterminacy model as determinate and we use the popular algorithm of Uhlig (1999) to solve the model. Importantly, Farmer et al. (2015) show that the choice of which expectational error to redefine as a new fundamental shock is irrelevant. They demonstrate that the same solution can be obtained under alternative specifications of $\eta_{f,t}$. We choose the the forecast error of the terms of trade as the new fundamental: $\eta_{f,t} = \eta^T_t \equiv \widehat{T}_t - E_{t-1}\widehat{T}_t$.[26] We refer to the forecast

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[22] In the steady state, the degree of increasing returns to scale can be expressed as the ratio between average and marginal costs, which is equal to the markup: i.e., $\frac{(\alpha+\gamma)(\overline{Y}+\phi)}{\overline{Y}} = \chi$. Consequently, for a steady state to exist, the steady-state markup cannot be lower than the degree of diminishing marginal cost i.e., $\chi \equiv \frac{\kappa}{\kappa-1} \geq \alpha + \gamma$.

[23] For bond holdings $\tilde{B}_{H,t}$, we take the linear deviation relative to steady-state Home consumption.

[24] The log of technology in both countries is assumed to follow an AR(1) process with zero mean.

[25] Pintus et al. (2016) and Pavlov and Weder (2017) adopt a similar solution method.

[26] In an earlier version of this paper (available from the authors on request), we selected the forecast error of Home consumption as the new fundamental $\left(\eta_{f,t} = \widehat{C}_t - E_{t-1}\widehat{C}_t\right)$, and obtained similar results.

error η_t^T as a self-fulfilling expectation or belief.

An equilibrium is characterized by $\theta^* \in \boldsymbol{\Theta}$, where $\boldsymbol{\Theta}$ is a parameter space which includes the parameters of the structural equations, the variance covariance matrix of the original fundamental shocks, and the variance and covariances of the new fundamental shock with the original set of fundamentals:

$$\boldsymbol{\Theta} \equiv \left\{\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Omega}_{\varepsilon\varepsilon}, \boldsymbol{\omega}_{\eta\varepsilon}, \sigma_\eta^2\right\}, \tag{28}$$

where $\boldsymbol{\Omega}_{\varepsilon\varepsilon} \equiv E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)$, $\boldsymbol{\omega}_{\eta\varepsilon} \equiv \left[E\left(\varepsilon_t \eta_t^T\right), E\left(\varepsilon_t^* \eta_t^T\right)\right] = E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right)$, and $\sigma_\eta^2 \equiv E\left[\left(\eta_t^T\right)^2\right]$. By specifying a new fundamental shock together with $\boldsymbol{\omega}_{\eta\varepsilon}$ and σ_η^2 we select a unique rational expectations equilibrium. The covariance of η_t^T with $\boldsymbol{\varepsilon}_t$ represents the response of beliefs to the original set of fundamentals, which amplify or attenuate the effects of technological shocks in the economy (Dufourt et al., 2015).

Farmer et al. (2015) demonstrate that this representation of equilibrium under indeterminacy can be alternatively characterized in terms of a linear forecasting rule that expresses the forecast errors as a function of fundamentals and sunspot shocks. This alternative solution methodology has been proposed in the seminal contributions of Lubik and Schorfheide (2003, 2004). As shown by Farmer et al. (2015), the two representations of equilibrium indeterminacy are entirely equivalent, because for each indeterminate equilibrium $\theta^* \in \boldsymbol{\Theta}$ there exists a unique linear forecasting rule that implements equilibrium θ^*, and vice versa.

The equivalence between the two solution methods enables us to compute the parameters of a linear forecasting rule à la Lubik and Schorfheide, in order to illustrate the relationship between fundamental and sunspot disturbances. For our purposes, we specify the linear forecasting rule as follows:

$$\eta_t^T = [\beta_1, \quad \beta_2] \cdot \boldsymbol{\varepsilon}_t + \zeta_t = \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t + \zeta_t, \tag{29}$$

where the residual ζ_t can be interpreted as a "pure"sunspot shock, uncorrelated with fundamentals: $E\left(\zeta_t\right) = 0$, $E\left(\zeta_t^2\right) \equiv \sigma_\zeta^2 > 0$, and $E\left(\zeta_t \boldsymbol{\varepsilon}_t\right) = \mathbf{0}$.

To aid our understanding of the indeterminacy model we consider two alternative assumptions. (i) *Autonomous beliefs*: shocks to the forecast error of the terms of trade η_t^T are the only source of business cycle fluctuations ($\boldsymbol{\Omega}_{\varepsilon\varepsilon} = \mathbf{0}$ and $\boldsymbol{\omega}_{\eta\varepsilon} = \mathbf{0}$); (ii) *Correlated beliefs*: the forecast error η_t^T is correlated with fundamentals, thus both $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$ and $\boldsymbol{\omega}_{\eta\varepsilon}$ are not restricted to be zero.[27] In this case,

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[27] In the indeterminacy literature (e.g., Dufourt et al., 2015), the forecast error is assumed to be perfectly correlated

we can use the equivalence between the Farmer-Khramov-Nicolò and the Lubik and Schorfheide solution methods to recover $\boldsymbol{\beta}$ and σ_ζ^2 pertaining to equation (29).[28] Multiplying equation (29) by $\boldsymbol{\varepsilon}_t'$ and taking expectations yields:

$$\boldsymbol{\beta} = E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right) E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)^{-1} = \boldsymbol{\omega}_{\eta\varepsilon} \boldsymbol{\Omega}_{\varepsilon\varepsilon}^{-1}. \tag{30}$$

To compute the variance of the pure sunspot shock, first note:

$$\sigma_\zeta^2 = E\left(\zeta_t \zeta_t'\right) = E\left(\left[\eta_t^T - \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t\right]\left[\eta_t^T - \boldsymbol{\beta} \cdot \boldsymbol{\varepsilon}_t\right]'\right),$$

and since $E\left(\eta_t^T \boldsymbol{\varepsilon}_t'\right) = \boldsymbol{\beta} E\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right)$ it follows that:

$$\sigma_\zeta^2 = E\left[\left(\eta_t^T\right)^2\right] - \boldsymbol{\beta} E\left(\boldsymbol{\varepsilon}_t \eta_t^T\right) = \sigma_\eta^2 - \boldsymbol{\beta}\boldsymbol{\omega}_{\eta\varepsilon}'. \tag{31}$$

Next, we describe how we calibrate the structural parameters of matrices $\boldsymbol{\Gamma}_0$, $\boldsymbol{\Gamma}_1$, and $\boldsymbol{\Psi}$ in Section 3.2 below. Since the alternative assumptions of autonomous and correlated beliefs imply different strategies for the calibration of the stochastic processes, we discuss the calibration of σ_η^2, $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$, and $\boldsymbol{\omega}_{\eta\varepsilon}$ separately in Sections 4 and 5.

3.2 Parameterization

The baseline parameter values used to compute the indeterminate equilibrium are summarized in Table 2. The U.S. is assumed to be the Home country and the rest of the world represents the Foreign country. As is standard in the literature, we set the time interval to be a quarter, the discount factor $\beta = 0.99$, and the steady-state depreciation rate $\overline{\delta} = 0.025$ (which implies $\eta \simeq 1.4$). The labor share in production is set equal to 0.7 and we set the inverse elasticity of labor supply $\nu = 0$ (i.e., indivisible labor) to help generate indeterminacy for a small degree of returns to scale, a standard assumption of the indeterminacy literature. The preference parameter ψ is set so that

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with fundamentals, i.e. $\sigma_\zeta = 0$ in (29). While this assumption can be imposed by placing appropriate restrictions on the covariance vector $\boldsymbol{\omega}_{\eta\varepsilon}$, we choose to leave it unrestricted so as not to lose any degree of freedom in our calibration strategy.

[28] Notice that under a linear forecasting rule the equilibrium is characterized by an alternative parameter space $\tilde{\boldsymbol{\Theta}}$ whereby $\boldsymbol{\beta}$ and σ_ζ^2 replace $\boldsymbol{\omega}_{\eta\varepsilon}$ and σ_η^2 in (28):

$$\tilde{\boldsymbol{\Theta}} \equiv \left\{\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Psi}, \boldsymbol{\Omega}_{\varepsilon\varepsilon}, \boldsymbol{\beta}, \sigma_\zeta^2\right\}.$$

Alternatively, a researcher may want to consider a linear transformation of equation (29) and adjust the parameter space accordingly.

Table 2: Baseline parameter values

β	0.99	Discount factor
$\overline{\delta}$	0.025	Steady state depreciation rate of capital
ν	0	Inverse elasticity of labor supply
σ	2	Inverse of the intertemporal substitution elasticity of consumption
θ	1	Elasticity of substitution between home & foreign consumption goods
ρ	1	Elasticity of substitution between home & foreign investment goods
a	0.88	Home bias in consumption goods
b	0.88	Home bias in investment goods
ω	0.001	Bond adjustment cost
$\overline{L}$	1/3	Steady state hours worked
S_L	0.7	Labor share in production
χ	1.2	Steady state markup
α	0.36	Elasticity of output with respect to capital
γ	0.84	Elasticity of output with respect to labor

in the steady state the agent in each country allocates one-third of their time to market activities. In the existing literature, the risk aversion parameter typically chosen lies between $1 \leq \sigma \leq 2$. Following Stockman and Tesar (1995), we set $\sigma = 2$. In line with Benigno and Thoenissen (2008), we set the bond adjustment cost $\omega = 0.001$ and the steady-state terms of trade equal to 1. We set $a = b = 0.88$ to ensure that the ratio of imports to GDP is equal to 0.12, consistent with the U.S. economy.

Empirical studies offer no clear conclusion on the magnitude of the trade price elasticities, θ and ρ. We initially set $\theta = \rho = 1$ broadly consistent with the empirical estimates of Heathcote and Perri (2002).[29] However, the robustness of the numerical results are examined for variations in these parameters. In particular, we consider a low trade elasticity parameterization $\theta = \rho = 0.5$ roughly consistent with the estimates of Anderton et al. (2004) and Corsetti et al. (2008).

A key issue is to generate equilibrium indeterminacy with empirically plausible values for the steady-state markup χ. Since intermediate firms use only capital and labor in the production process (6), this implies that the markup is value added. As discussed by Jaimovich (2007), value-added markups are estimated for the U.S. economy to lie between 1.2 to 1.4. We set the steady-state markup $\chi = 1.2$, consistent with the lower range of these empirical estimates.[30] The numerical analysis suggests that under the baseline parameterization there are many values of α

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[29] Heathcote and Perri (2002) estimate the trade elasticity for the U.S. to be approximately 0.9.

[30] A sensitivity analysis was conducted using a higher value for the steady-state markup $\chi = 1.3$ with little significant change in the results found.

and γ that generate indeterminacy for empirically plausible values of the steady-state markup. For simplicity, we follow Hornstein (1993) and set $\alpha + \gamma = \chi = 1.2$, which implies that profits are zero in every period.

4 Autonomous beliefs

When the forecast error is assumed to be the only source of business cycle fluctuations, the standard deviations and correlations with technology shocks are set equal to zero: $\boldsymbol{\Omega}_{\varepsilon\varepsilon} = \mathbf{0}$ and $\boldsymbol{\omega}_{\eta\varepsilon} = \mathbf{0}$. Since we choose $\eta_{f,t} = \eta_t^T$, under autonomous beliefs equation (29) is simply:

$$\widehat{T}_t - E_{t-1}\widehat{T}_t = \zeta_t.$$

In this scenario we treat the standard deviation σ_ζ as a free parameter and we calibrate it so as to match the standard deviation of U.S. output in all our experiments. For example, in the baseline parametrization we set $\sigma_\zeta = 0.832$ in order to produce a standard deviation of output of 1.49.

Table 3 presents the simulation results under autonomous beliefs. In column 3 (Baseline), the model moments are computed employing unitary values for the trade price elasticities $\theta = \rho = 1$, whereas the remaining two columns either assume $\theta = \rho = 0.5$ or $\theta = \rho = 1.24$.[31] By inspection of Table 3, the model is unable to resolve any major empirical irregularity of the data for international relative prices and quantities. The terms of trade and the real exchange rate are less volatile than output and the model fails to generate sufficient volatility for real net exports. The terms of trade and output are predicted to move in the same direction leading to a counterfactual positive correlation. The model generates cross-country correlations which are equal to -1 and the correlation between the real exchange rate and relative consumption is positive and close to 1 such that the Backus-Smith puzzle arises. While the model can generate counter-cyclical real net exports, the negative correlation generated between net exports and output is very close to -1, which is much stronger than the data (-0.47).

To understand the poor performance of the indeterminacy model under autonomous beliefs, Figure 1 depicts a selection of impulse response functions to a one percent positive shock to the terms of trade forecast. An important element in understanding how self-fulfilling beliefs are transmitted relates to the labor market. The log-linearized Home and Foreign aggregate labor

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[31] We set $\theta = \rho = 1.24$ as this is the highest value for the trade elasticities that generate indeterminacy with $\chi = 1.2$.

Table 3: Second moments under autonomous beliefs

			Trade elasticity[§]	
	Data[†]	Baseline	$\theta = 0.5$	$\theta = 1.24$[⋆]
Standard deviations[‡]				
Consumption	0.62	0.81	0.81	0.81
Investment	2.92	2.24	2.72	2.00
Employment	0.68	0.91	0.91	0.91
Terms of Trade	1.77	0.73	0.73	0.73
Real Exchange Rate	2.38	0.56	0.55	0.56
Real Net Exports	0.38*	0.17	0.32	0.10
First-order autocorrelations				
Output	0.87*	0.73	0.74	0.72
Real Exchange Rate	0.82*	0.74	0.75	0.73
Real Net Exports	0.85*	0.73	0.74	0.75
Correlations with output				
Consumption	0.82	1.00	1.00	1.00
Investment	0.94	1.00	1.00	1.00
Employment	0.85	1.00	1.00	1.00
Terms of Trade	-0.16	0.99	0.98	0.99
Real Net Exports	-0.47*	-0.97	-1.00	-0.92
Cross-country correlations				
Output	0.58	-1.00	-1.00	-1.00
Consumption	0.43	-1.00	-1.00	-1.00
Investment	0.41	-1.00	-1.00	-1.00
Employment	0.45	-1.00	-1.00	-1.00
Correlation with the real exchange rate				
Relative Consumption	-0.17	0.99	0.98	0.99

Notes:

[†] The estimated sample moments for the data are taken from Gao et al. (2014), except for values denoted by $*$ which are from the authors' own calculations.

[§] For all variations in the trade price elasticities we set $\theta = \rho$.

[⋆] We set $\theta = \rho = 1.24$ as this is the highest value for which indeterminacy is possible.

[‡] The standard deviations of all variables are divided by the standard deviation of output, except for the standard deviation of real net exports which is expressed in absolute terms.

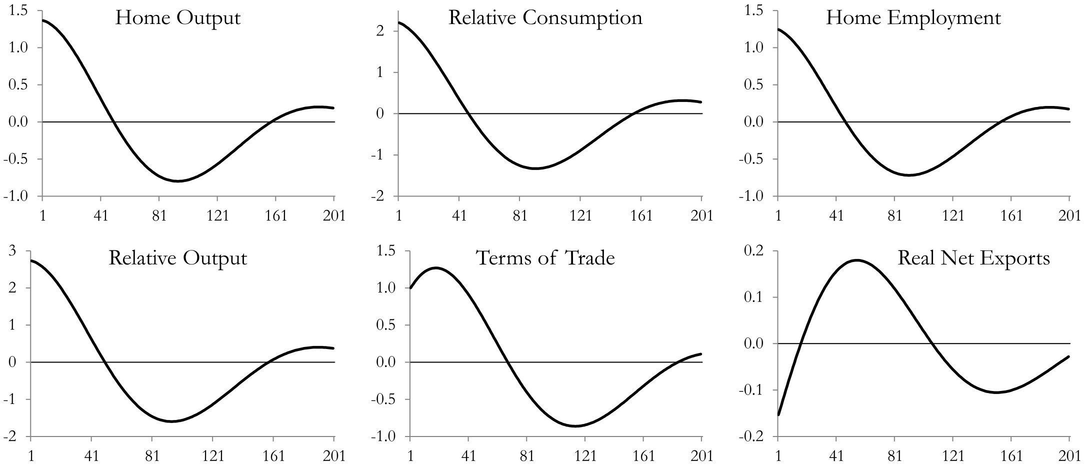

Figure 1: Impulse responses under a positive 1% shock to the terms of trade forecast. Vertical axes: % deviation from the steady state; Horizontal axes: quarters.

demands can be expressed as:

$$\widehat{w}_t = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t + \left[\frac{\eta\gamma}{\eta-\alpha}-1\right]\widehat{L}_t - \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t, \tag{32}$$

$$\widehat{w}_t^* = \left[\frac{\alpha(\eta-1)}{\eta-\alpha}\right]\widehat{K}_t^* + \left[\frac{\eta\gamma}{\eta-\alpha}-1\right]\widehat{L}_t^* + \left[\frac{(1-a)\eta}{\eta-\alpha}\right]\widehat{T}_t + \left[\frac{\eta}{\eta-\alpha}\right]\widehat{Z}_t^*, \tag{33}$$

where in our parameterization $\eta - \alpha > 0$, $\frac{\eta\gamma}{\eta-\alpha} - 1 > 0$, and $\widehat{Z}_t = \widehat{Z}_t^* = 0$ under autonomous beliefs. With decreasing marginal costs, the source of indeterminacy arises from an upward-sloping aggregate labor demand schedule, which is steeper than the horizontal aggregate labor supply schedule (arising from an infinite elasticity of labor supply parameterization). A positive revision to the terms of trade forecast, results in a depreciation (increase) in the terms of trade $\widehat{T}_t$. From equation (32) the upward-sloping Home aggregate labor demand schedule shifts down, increasing Home employment, which raises Home output and consumption. Consequently, belief-induced fluctuations counterfactually generate a positive correlation between the terms of trade and output. As the demand for imports increases in the Home country due to higher consumption, real net exports decrease. For the Foreign country, from equation (33) the Foreign aggregate labor demand schedule shifts up, and as a result, Foreign employment decreases, reducing Foreign output and consumption. This explains the perfect negative cross-country correlations generated under autonomous beliefs. Furthermore, while the data suggests that relative consumption increases in response to an appreciation of international relative prices, self-fulfilling expectations induce a counterfactual positive correlation between relative consumption and the terms of trade. Overall, the indeterminacy model under autonomous beliefs cannot replicate the observed behavior for international relative prices and quantities nor solve the Backus-Smith puzzle.

This finding is in stark contrast to the two-country, one-good models of Guo and Sturzenegger (1998) and Xiao (2004), where self-fulfilling expectations result in positive cross-country correlations for consumption and output. Due to the absence of international relative prices in these models, belief-induced fluctuations stimulate consumption and output in both counties. In our two-good model, self-fulfilling beliefs are global extrinsic shocks that affect the terms of trade, inducing an output reallocation between the two countries. Consequently, cross-country correlations for consumption and output are negative.

5 Correlated beliefs

The quantitative results from the previous section showed that self-fulfilling beliefs alone cannot help in resolving the international macro puzzles. However, when the forecast error of the terms of trade is correlated with productivity shocks the analysis differs significantly. Inspection of the aggregate labor demand equations (32) and (33) suggest that the indeterminacy model should perform better under correlated shocks. In this case, a Home technology shock $\widehat{Z}_t$ causes a revision of expectations (Equation 29) and therefore a belief-induced change in the terms of trade $\widehat{T}_t$. Since both $\widehat{Z}_t$ and $\widehat{T}_t$ affect the aggregate labor demand schedule (32) in opposite directions, the positive response of domestic employment and output will not be as strong as under autonomous beliefs. Indeed, both domestic employment and output could actually fall provided the shocks to Home technology and the expectational error are sufficiently positively correlated to generate an upward shift of the Home labor demand schedule (32), and consequently, the correlation between the terms of trade and output would become negative, as in the data. Furthermore, if a temporary domestic contraction results in low Home imports then the model would also generate countercyclical real net exports solving another output-correlation puzzle. Finally, if the model can induce a large enough adjustment in the terms of trade relative to output then the model could potentially generate sizeable volatility improvements for international relative prices and quantities helping to resolve the volatility puzzles.

Shock processes To test the above conjecture we introduce technology shocks and leave the covariances between the fundamental shocks and the forecast error η_t^T unrestricted, and therefore, the matrix $\boldsymbol{\Omega}_{\varepsilon\varepsilon}$ and the vector $\boldsymbol{\omega}_{\eta\varepsilon}$ are not assumed to be zero. As a result, we have six free parameters: the standard deviations of the technology shocks and forecast error (σ_ε, σ_{ε^*}, and σ_η), and the cross correlations between the shocks ($\rho_{\varepsilon,\varepsilon^*}$, $\rho_{\eta,\varepsilon}$, and $\rho_{\eta,\varepsilon^*}$). The vector $\boldsymbol{\omega}_{\eta\varepsilon}$ of the covariances between η_t^T and the technology shocks can be interpreted as a coordination mechanism for revising expectations, which amplify (or attenuate) the effects of technological shocks in the economy.

In line with the IRBC literature, we assume that the stochastic processes for productivity are quite persistent and we set the Home and Foreign autocorrelation parameters equal to $\upsilon = \upsilon^* = 0.96$. Similar to Schmitt-Grohé (2000) and Benhabib and Wang (2013), the standard deviations and cross-correlations of the stochastic processes are calibrated using a method of moments approach, where we include all the moments that define the international macro puzzles in the objective function. In this way, we explicitly look for the shock properties that maximize the model's ability to explain the puzzles. Specifically, we calibrate the volatility and cross-correlations of the shocks so as to minimize the distance between selected model moments and data moments.[32] Consistent with the empirical evidence of Backus et al. (1992) and Heathcote and Perri (2004), the cross-country correlation of technology shocks is restricted to be non-negative. We check that the covariance matrix of the shocks that minimizes the objective function is positive semi-definite.[33]

The objective function is computed as the sum of the squared differences between HP-filtered model moments and data moments, with the identity matrix as the weighting matrix. The following eight moments are included in the objective function: the standard deviations of output, the terms of trade, and net exports; the correlations with output of the terms of trade and net exports; the cross-country correlations of output and consumption; and the correlation of the real exchange rate with relative consumption. Therefore, the number of moment conditions exceeds the number of parameters to be estimated by two.

The calibrated standard deviations and shock cross-correlations that maximize the model's ability to explain the puzzles are summarized in Table 4 for variations in the trade elasticity parameters θ and ρ. The results of Table 4 confirm our previous conjecture: in order to match the international macro puzzles revisions to the terms of trade forecasts must be positively correlated

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[32] Model moments are computed using frequency domain techniques as described in Uhlig (1999).

[33] In a small number of cases the estimated covariance matrix is not positive semi-definite. In these cases, we replace the estimated covariance matrix with its closest positive semi-definite matrix.

Table 4: The shock processes under correlated beliefs

		Trade elasticity	
	Baseline	$\theta = \rho = 0.5$	$\theta = \rho = 1.24$
Technology shocks (ε_t, ε_t^*)			
s.d. of ε_t (σ_ε)	0.290	0.284	0.284
s.d. of ε_t^* (σ_{ε^*})	0.285	0.283	0.285
AR parameters (υ, υ^*)	0.960	0.960	0.960
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$	0.001	0.001	0.000
Beliefs			
s.d. (σ_η)	1.798	1.744	1.772
cross-correlation $\rho_{\eta,\varepsilon}$	0.960	0.949	0.961
cross-correlation $\rho_{\eta,\varepsilon^*}$	-0.112	-0.092	-0.133

with Home productivity shocks. We also find that the correlation of beliefs with Home productivity shocks must be near one, and the standard deviation of beliefs must be relatively high.[34] The high values of $\rho_{\eta,\varepsilon}$ and σ_η drive the coefficient β_1 in equation (29) above unity, and the relative low value of $\rho_{\eta,\varepsilon^*}$ drives the coefficient β_2 near zero. In fact, in the baseline scenario the implied vector $\boldsymbol{\beta}$ is $[5.95,\ -0.71]$. These results indicate that domestic productivity shocks, amplified by self-fulfilling beliefs (revisions to the terms of trade forecasts), have a stronger effect on the business cycle than foreign productivity shocks.

Results For the parameter values given in Table 2 and the shock processes given in Table 4, Table 5 summarizes the simulation results when self-fulfilling expectations are correlated with technology shocks. As before, the robustness of the results are checked using alternative values for the trade elasticity parameters. Under correlated beliefs, the quantitative performance of the indeterminacy model improves significantly in terms of replicating the data. Now, both the terms of trade and the real exchange rate are more volatile than output generating over 86% of the observed standard deviation of the terms of trade. The volatilities of international relative prices have increased by a factor of 2 in comparison to autonomous beliefs, and by a factor of over 2.5 relative to the determinacy baseline model.[35] Furthermore, the model can also simultaneously

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[34] In our model we find that a one percent shock to the forecast error has a relatively modest impact on the variables compared to technology shocks. Consequently, the estimation procedure selects a relatively higher standard deviation for the forecast error in order to match the selected moments.

[35] Under correlated beliefs, the model still generates only half the volatility for the real exchange rate relative to the data. This is unsurprising since the real exchange rate in our model is a linear transformation of the terms of trade (due to the assumption of the law of one price and the absence of non-traded goods). See Corsetti et al. (2008) for

Table 5: Second moments under correlated beliefs

			Trade elasticity[§]	
	Data[†]	Baseline	$\theta = 0.5$	$\theta = 1.24^\star$
Standard deviations[‡]				
Consumption	0.62	0.93	0.92	0.92
Investment	2.92	2.43	2.47	2.61
Employment	0.68	1.04	1.03	1.03
Terms of Trade	1.77	1.53	1.52	1.50
Real Exchange Rate	2.38	1.17	1.15	1.14
Real Net Exports	0.38*	0.65	0.60	0.72
First-order autocorrelations				
Output	0.87*	0.76	0.74	0.77
Real Exchange Rate	0.82*	0.72	0.73	0.71
Real Net Exports	0.85*	0.71	0.73	0.71
Correlations with output				
Consumption	0.82	0.99	0.99	0.99
Investment	0.94	0.63	0.52	0.64
Employment	0.85	0.99	0.99	0.99
Terms of Trade	-0.16	-0.40	-0.34	-0.39
Real Net Exports	-0.47*	-0.22	-0.09	-0.25
Cross-country correlations				
Output	0.58	0.45	0.37	0.47
Consumption	0.43	0.48	0.42	0.50
Investment	0.41	-0.50	-0.69	-0.43
Employment	0.45	0.48	0.41	0.49
Correlation with the real exchange rate				
Relative Consumption	-0.17	0.42	0.43	0.41

Notes: See Table 3 above.

generate sufficient volatility for real net exports. Not only is the volatility of real net exports larger than the data, but it is robust to the choice of trade elasticity. Remarkably, under correlated beliefs the indeterminacy model results in a standard deviation for real net exports nearly 4 times larger than under autonomous beliefs and 13 times larger than the determinacy baseline model.

In terms of output correlations, the indeterminacy model with correlated beliefs correctly predicts that both real net exports and the terms of trade are counter-cyclical, regardless of the value for the trade elasticities. In stark contrast to autonomous beliefs, the nearly perfect negative corre-

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further discussion.

lation between net exports and output no longer arises. While the indeterminacy model struggles to generate cross-country output correlations higher than cross-country consumption correlations, it does much better compared to the determinacy baseline model. The main discrepancy between the model and the data relates to the correlation between the real exchange rate and relative consumption. Although the model generates a significantly lower positive correlation than under autonomous beliefs (0.42 vs. 0.99) and the determinacy baseline model (0.42 vs. 0.97), this correlation remains counterfactual with the data where a negative correlation is observed (-0.17).

Inspecting the mechanism With the notable exception of the Backus-Smith puzzle, our results show that when self-fulfilling beliefs are correlated with productivity shocks, the indeterminacy model can solve several international relative price and quantity puzzles. To understand these findings, first note that indeterminacy alters the propagation of technology shocks via the expectations formation mechanism described in (29).

Letting $\overline{\Phi}_{\varepsilon}^{X}$ denote the impulse response of variable X to an uncorrelated Home technology shock and $\overline{\Phi}_{\eta_T}^{X}$ denote the impulse response of variable X to a shock to the terms of trade forecast η^T, then the combined impulse response Φ_{ε}^{X} to a positive productivity shock is given by:[36]

$$\Phi_{\varepsilon}^{X} = \overline{\Phi}_{\varepsilon}^{X} + E_t\left(\eta_t^T|\varepsilon_t = 1\right)\overline{\Phi}_{\eta_T}^{X} = \overline{\Phi}_{\varepsilon}^{X} + \beta_1\overline{\Phi}_{\eta_T}^{X}. \tag{34}$$

For simplicity we have abstracted from Foreign technology shocks.[37] Figure 2 depicts selected impulse response functions, which combine the effect of Home technology shocks with the revision of expectations.

When shocks to the terms of trade are positively correlated with Home technology shocks, a positive Home technology shock results in a belief-induced increase (depreciation) in the terms of trade $\widehat{T}_t$. From inspection of (33), the rise in $\widehat{Z}_t$ and $\widehat{T}_t$ shift the Home aggregate labor demand schedule in opposite directions. The top-right panel of Figure 2 suggests that the increase in employment caused by an increase in $\widehat{T}_t$ is more than offset by the rise in $\widehat{Z}_t$. Consequently, the Home aggregate labor demand schedule (33) shifts inwards causing a fall in Home employment and thus Home output. However, as shown in Figure 2, the negative effect on these variables is small and temporary. This finding that technology improvements can have temporary contractionary

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[36] $\overline{\Phi}_{\varepsilon}^{X}$ and $\overline{\Phi}_{\eta_T}^{X}$ are obtained under the Farmer et al. (2015) solution method under the assumption that all shocks are uncorrelated.

[37] We ignore the cross-country correlation of Home and Foreign technology shocks, since in our simulated results $\rho_{\varepsilon,\varepsilon^*} = 0.001$.

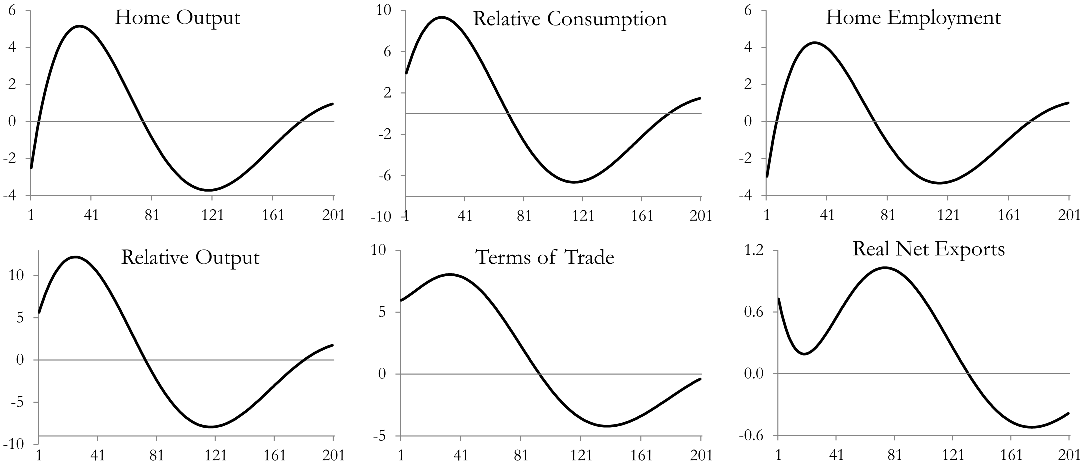

Figure 2: Impulse responses under a positive 1% productivity shock in the Home country. Vertical axes: % deviation from the steady state; Horizontal axes: quarters.

effects is supported by a number of empirical studies (see, for example, Galí, 1999; Francis and Ramey, 2005; Pesavento and Rossi, 2005; Basu et al., 2006, Fernald, 2007; Giuli and Tancioni; in press) and is crucial for the model to solve the output correlation puzzles.[38] Furthermore, since the model generates a large change in the terms of trade relative to domestic output, international relative prices are now more volatile than output as in the data. This is in stark contrast to autonomous beliefs which predicts a near perfect negative correlation between real net exports and output and insufficient volatility. Recall that when the international business cycle is driven only by self-fulfilling beliefs, domestic output and consumption are stimulated and imports rise more than exports such that real net exports fall. Under correlated beliefs, the deterioration in the terms of trade and the temporary fall in output implies that imports fall more than exports, thereby generating a weak negative correlation between real net exports and output. The delayed effect on output is key for generating sufficient volatility for real net exports.

Finally, the perfect negative cross-country correlations generated under autonomous beliefs no longer arises with correlated beliefs. Under autonomous beliefs, a belief-induced increase in $\widehat{T}_t$ stimulates Home output. In the Foreign country, the rise in $\widehat{T}_t$ causes the Foreign aggregate labor demand curve (34) to shift up, and the resulting fall in Foreign employment and output generates counterfactual negative cross-country correlations. However, when self-fulfilling expectations and Home technology shocks are sufficiently positively correlated, the increase in $\widehat{Z}_t$ more than offsets

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[38] For example, Giuli and Tancioni (in press) show that the short-term response of both hours and investment to a positive technology shock is negative and the contraction is significant over approximately four to five quarters.

the belief-induced rise in $\widehat{T}_t$. Now, the aggregate labor demand schedules in both countries shift upwards resulting in positive cross-country correlations for employment and output.

Indeterminacy and the Backus-Smith puzzle The main discrepancy between the indeterminacy model and the data is that the model generates a positive correlation between the real exchange rate and relative consumption, whereas in the data this relationship is negative. To understand why correlated self-fulfilling expectations and technology shocks fail to resolve the Backus-Smith puzzle,we concentrate on the transmission of Home technology shocks which have a more marked effect on the revision to the terms of trade forecasts than Foreign technology shocks.[39] First, recall that a positive Home technology shock causes a belief-induced increase (deterioration) in the terms of trade, and therefore an increase (depreciation) in the real exchange rate. Consequently, in order to solve the Backus-Smith puzzle the response of Foreign consumption must be above the response of Home consumption for relative consumption to fall, thereby generating a negative correlation with the real exchange rate. However, this cannot happen in our calibrated model despite the delayed effect on output of technology shocks. The upward shift of the Foreign aggregate labor demand schedule (34) caused by the belief-induced increase in $\widehat{T}_t$ is always greater than that of the Home country, since the the rise in $\widehat{T}_t$ partially offsets the upward shift of (33) due to $\widehat{Z}_t$. With Foreign employment now relatively lower than Home employment, this implies that the response of Foreign consumption must be below Home consumption.

The above analysis suggests that by allowing for a negative correlation between the Home and Foreign technology shocks, the indeterminacy model could generate a response for Foreign consumption greater than Home consumption. In this case, the upward shift of the Foreign aggregate labor demand schedule (34) caused by the belief-induced increase in $\widehat{T}_t$ is now offset by a fall in $\widehat{Z}_t^*$. To verify this conjecture, we re-estimate the shock properties of the indeterminacy model without restricting the cross-country correlations for the productivity shocks to be non-negative. Table 6 summarizes the second moments and shock processes estimated for this exercise. Indeed, we find that our method of moments approach selects a negative correlation between Home-Foreign technology shocks, as we have hypothesized. By inspection, the indeterminacy baseline can indeed generate a negative correlation between the real exchange rate and relative consumption (-0.12) almost matching the data (-0.17). This finding is robust to alternative calibrations for the trade

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[39] With correlated beliefs the vector β (Equation 29) controls how expectations are affected by technology shocks. In our calibration β_2 is close to zero.

Table 6: Second moments under correlated beliefs: unrestricted cross-country correlations for the productivity shocks, $\rho(\varepsilon_t,\, \varepsilon_t^*)$

			Trade elasticity	
	Data	Baseline	$\theta = 0.5$	$\theta = 1.24$
Standard deviations				
Consumption	0.62	0.91	0.91	0.90
Investment	2.92	2.43	1.87	2.74
Employment	0.68	1.02	1.01	1.02
Terms of Trade	1.77	1.65	1.77	1.56
Real Exchange Rate	2.38	1.26	1.34	1.19
Real Net Exports	0.38	0.66	0.38	0.77
First-order autocorrelations				
Output	0.87	0.78	0.75	0.78
Real Exchange Rate	0.82	0.71	0.71	0.70
Real Net Exports	0.85	0.71	0.72	0.71
Correlations with output				
Consumption	0.82	0.99	0.99	0.99
Investment	0.94	0.88	0.94	0.79
Employment	0.85	0.99	0.99	0.99
Terms of Trade	-0.16	-0.19	-0.17	-0.23
Real Net Exports	-0.47	-0.44	-0.46	-0.39
Cross-country correlations				
Output	0.58	0.40	0.50	0.49
Consumption	0.43	0.42	0.50	0.50
Investment	0.41	-0.56	-0.16	-0.57
Employment	0.45	0.42	0.50	0.49
Correlation with the real exchange rate				
Relative Consumption	-0.17	-0.12	-0.19	-0.08
Shock processes				
s.d. of ε_t (σ_ε)		0.29	0.30	0.28
s.d. of ε_t^* (σ_{ε^*})		0.22	0.25	0.21
s.d. (σ_η)		1.97	2.07	1.87
cross-correlation $\rho_{\varepsilon,\varepsilon^*}$		-0.84	-0.79	-0.76
cross-correlation $\rho_{\eta,\varepsilon}$		0.95	0.95	0.95
cross-correlation $\rho_{\eta,\varepsilon^*}$		-0.97	-0.93	-0.92

Notes: See Table 3 above.

elasticities and is not at the expense of any of the other international puzzles.

Since the influential paper of Corsetti et al. (2008), it is now well established in the IRBC literature that models driven by technology shocks can only be reconciled with most of the features of the data under an unconventional negative international transmission mechanism. If the values of the trade elasticity are restricted to be sufficiently low, IRBC models under incomplete asset markets can generate large uninsurable wealth effects such that the terms of trade appreciates when domestic production expands. Recently, Raffo (2010) and Karabarbounis (2014) have attempted to explain the international macro puzzles without resorting to this negative transmission mechanism. Raffo (2010) investigates the role of investment-specific technology shocks, whereas Karabarbounis (2014) modifies the standard IRBC model by including home production to generate a labor wedge. Both these papers have been successful in resolving the Backus-Smith puzzle but have struggled to resolve the volatility and output correlation puzzles. The model of Karabarbounis (2014) counterfactually predicts procyclical terms of trade and generates no additional volatility for international relative prices than the standard workhorse model. To resolve the Backus-Smith puzzle, Raffo (2010) requires standard deviations of the investment-specific technology shocks three times higher than found in the data by Mandelman et al. (2011). Even then, his model fails to generate enough volatility for the terms of trade. Our findings show that indeterminacy and self-fulfilling expectations offer one possible transmission mechanism to generate counter-cyclical and volatile terms of trade and real net exports in line with the data. However, similar to the findings of Raffo (2010) and Karabarbounis (2014), neither determinate nor indeterminate models to date can adequately explain all the major anomalies with the data.

6 Conclusion

We have analyzed whether equilibrium indeterminacy and self-fulfilling beliefs can help resolve the international macro puzzles. We have found that the indeterminacy model can solve the volatility and output-correlation puzzles and generate significantly improved statistics for the cross-correlation anomaly than standard IRBC models. However, the model cannot solve the Backus-Smith puzzle without a negative cross-country correlation for technology shocks.

While significant progress has been made in the determinacy literature in reconciling IRBC models with the data, at present no single modelling strategy has been able to explain all the puzzles. By comparison, there are very few studies that have explored the role of indeterminacy and self-fulfilling expectations in the open economy.

The positive performance of our model in generating counter-cyclical and volatile terms of trade and real net exports suggests that indeterminacy and endogenous fluctuations could be successful in explaining international business cycles. The success of this line of research will crucially depend on the ability of indeterminacy models to also solve the Backus-Smith puzzle. Our analysis shows that in order to solve this puzzle the transmission mechanism of at least one shock must induce a negative co-movement between the real exchange rate and relative consumption. For example, this may be possible in models that permit two self-fulfilling beliefs. In this case, the cross-country correlations for consumption and output will now depend on how these endogenous shocks are related. Consequently, depending on the properties of the two self-fulfilling beliefs, it may be possible to generate a negative correlation between international relative prices and relative consumption. We leave this topic for future research.

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Appendix to “Can Indeterminacy and Self-Fulfilling Expectations Solve the International Macro Puzzles?”

October 2017

1 The Determinacy Model and the International Macro Puzzles

This appendix gives a more detailed explanation on why the empirical failures arise in the determinacy model. The determinacy model cannot replicate the negative correlation between the terms of trade and output found in the data. As in the standard IRBC model, a positive Home technology shock is followed by a depreciation of the terms of trade. This happens because Foreign goods become relatively scarce, and a terms of trade depreciation is needed to clear the market. Foreign households raise their consumption of Home goods taking advantage of better import prices, thus the international transmission is positive.

As shown in Table 1 of the main text, in terms of volatility, the performance of the determinacy model can be improved by choosing a lower value for the trade elasticity parameters. For example, by setting $\theta = \rho = 0.5$ (column 4 of Table 1) this more than triples the volatilities of the terms of trade and the real exchange rate, and more than quadruples the volatility of real net exports relative to the determinacy baseline. Yet, despite these improvements, the model still generates less than 60 percent of the volatility observed for real net exports and less than 70 percent of the volatility observed for the real exchange rate. By setting $\theta = \rho = 1.5$ (column 5 of Table 1), the model can generate counter-cyclical real net exports (-0.31) almost matching the data. However, this is at the cost of further reducing the volatility of relative prices relative to the baseline.[1] Therefore, similar to conventional IRBC models, the determinacy model faces an unpleasant trade-off. Relatively high trade elasticities can be selected to help generate counter-cyclical real net exports, or relatively low trade elasticities can be chosen to help improve the volatilities of real net exports and relative prices.

To understand this trade-off, Figure A1 of this appendix reports selected impulse response functions for the Home country after a 1% positive technology shock. In each panel of Figure A1, the impulse responses are plotted under three alternative values for the trade price elasticity $\theta = \rho = 0.5, 1.0, 1.5$. By inspection, the trade elasticity parameter crucially affects the response of the terms of trade after a productivity shock. If this parameter is relatively low, Home and Foreign goods are less substitutable for one another. Consequently, a positive technology shock results in a large deterioration in the terms of trade (i.e., a fall in the relative price of Home-produced goods) and a lower increase in domestic output. Hence, the lower the trade elasticities, the higher the volatility of international relative prices and the lower the volatility of output in response to productivity changes. Exports rise more than imports, and real net exports, in contrast to the data, are consequently

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[1] As shown by the final two columns of Table 1, allowing for variable capacity utilization has a similar effect.

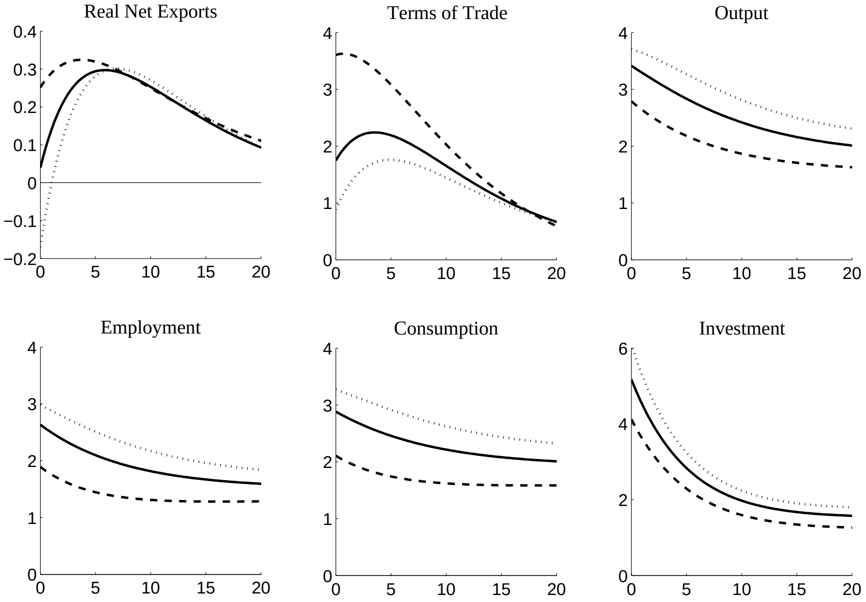

Figure A1: Dynamic responses for the determinacy model (Home country) to a positive 1% Home productivity shock: baseline (—); high trade elasticity (· · ·); low trade elasticity (- - -). Vertical axes: % deviation from the steady state; Horizontal axes: years.

pro-cyclical. With higher trade elasticities, productivity shocks will have a lower impact on relative prices and a higher impact on output, thereby generating counter-cyclical real net exports. Therefore, in order to match the volatility of relative prices *and* the correlation between real net exports and output, standard IRBC models require a negative international transmission mechanism whereby the terms of trade appreciate when domestic production expands. However, as shown by Thoenissen (2010), this negative transmission mechanism only arises under low values of the trade elasticities $\theta = \rho$ and for a very narrow range $0.4113 \leq \theta \leq 0.4678$.