Research on Intelligent Controller Design for MIMO Spatially-Distributed Systems with Applications

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Abstract

Spatially dynamic distributed systems have been attracting increasing attention from researchers in the field of system modelling and control since their introduction as an alternative to simple systems to meet the ever-greater requirements to make industrial systems more precise and energy-efficient and to overcome process complexities. An approach whereby complex systems with multi-dimensional parameters, inputs or outputs are simply disregarded or simplified with the help of convenient mathematical models is no longer feasible. Therefore, the purpose of the present study is to contribute to the advancement of both theoretical and empirical knowledge in this field through the means of theoretical analysis, application simulations and case studies.

From a theoretical perspective, this study focuses primarily on the design methodology of control systems. To this end, the first step is identification of requirements from the applications, followed by the implementation of an original approach underpinned by data prediction for type-2 T-S fuzzy control with the purpose of making the control system design more convenient. With this aim in mind, the study creates an interface/platform to link or anticipate spatially dynamic distributed system output from lumped system data by taking advantage of the three-dimensional character of type-2 fuzzy sets. Moreover, on the basis of a decoupled spatially dynamic distributed system, this study applies Mamdani-type and interval type-2 T-S type fuzzy control, and extends a discussion about the results of simulation and analysis.

With regard to application examination, the study contributes to primarily with system analysis and modelling. Along with the progress of physical analysis, a MIMO model is customized for the plant by expanding from the lumped physical character to a distributed system. Furthermore, the coupling feature of the object is addressed based on the decoupling approach and the pole placement approach, while the SISO approach is expanded to a universally acknowledged MIMO approach and Matlab is used to produce the simulation results.

As a conclusion, in this research, firstly a state space model was established to expand the SISO system into a MIMO system and the interacted inputs and outputs have been decoupled using decoupling method; and then a Mamdani-type fuzzy control was designed for temperature control and an Interval Type-2 fuzzy control was designed for pressure control, using a simple state-space model instead of a fuzzy model, accordance with the practical plant in use, and very satisfied, very robust control performances were obtained.

Key words: Spatially dynamic distributed systems; State-space approach; Decoupling; Fuzzy Logic Systems; Interval Type-2 T-S Fuzzy Control, Biochemical Process.

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I would like to take this opportunity to express my sincere gratitude to my supervisors, Professors Quanmin Zhu and Mokhtar Nibouche, for their encouragement and guidance during the past years of my studies not only to my research but also to my English, and I also give my thanks to staff from the Faculty of Environment and Technology of UWE with whom I share the excellent and comfortable research environment.

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Table of Content

Nomenclature

Variables:

c	Specific Heat Capacity	$J/(Kg*^{\circ}C)$
Q	Quantity of heat	J
M	Mass of object	Kg
T	Temperature	$^{\circ}C$
t	Time	s
S	Area	m^2
d	Diameter	m
ρ	Density	Kg/m^3
h	Height	m
q	Volume rate	m^3
η	Constant	J/m^3

Subscript:

o	Object (here refers to vials)
a	Air
p	Pump

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1 Introduction

This chapter introduces the motivation of the present research based on a general review. For clarity purposes, this chapter also provides the layout of the thesis and highlights its contributions. The chapter concludes with the research outcomes.

1.1 Overview

Advances and developments in medicine and medical treatment have led to a longer human lifespan and a lower death rate. For instance, drugs of higher purity have a better healing effect, such as more accurate efficacy, or relieve symptoms more effectively, or even contain fewer side-effects. However, the effectiveness of each section of manufacture can influence the desired results either positively or negatively, and sometimes such an influence is linked to multiple disciplines rather than a single cause and effect. Partially, such increasing multi-disciplinary requirements have directly promoted the development of pharmaceutical engineering, as one of the important factors. In addition, different from the chemical pharmaceutical process, the biochemical pharmaceutical process presents a more complicated nature, due to its complex biochemical reactions, such as more than one reactor involved in a single reaction, as well as the time-delay, unobservable and hardly controllable process during manufacture. Along with such nature, the biochemical pharmaceutical industry has transformed itself throughout the years, with its technologies and processes expanding and changing constantly to suit today's needs. The development process for high performance equipment for use in the industry has now become subject to tighter and stricter regulations, especially for safety concerns, and this in turn has made the control process of such equipment more demanding, complex and challenging. The control process must ensure the highest form of purity of the products going through the machine, all while reducing the overall consumption of energy and raw materials (Koveos et al., 2013).

In the biochemical pharmaceutical industry, sterilization is among the most critical processes not only for drug products, but also for the containers. Drugs administered orally or through injection, which are known as pharmaceutical preparations, are made from Active Pharmaceutical Ingredients (APIs), and the containers, including vials, syringes, capsules, and other packaging materials, are the last protection from contamination before the drugs enter the human body. Therefore, sterilization must ensure the elimination of any bio-contamination from the containers. A device widely accepted in the biochemical pharmaceutical industry and exclusively intended for sterilisation and drying of vials and syringes conveyed through it is the depyrogenation tunnel. This device can consume a large amount of power resources when in operation, and thus, energy consumption has become an important factor when designing a depyrogenation tunnel along with its control system design. To minimize power consumption and to maintain its reliability of the sterilisation process, modelling and control strategies are required. The unstable pressure field within the depyrogenation tunnel is an additional key issue, alongside the temperature issue. As the vials are conveyed through, the open zone within is connected with the room, which will be affected significantly by the combination of room pressure and wind pressure. In other words, if the pressure fluctuates beyond a certain value, the small vials made of glass are likely to fall off, causing the device to stop and resulting in significant loss. In addition, more critical than the issue of energy consumption is the issue of physical structure deficiency caused by the complex nature of the device and known as a spatially dynamic distributed system.

The spatially dynamic distributed system has attracted considerable research attention in recent years. Using the depyrogenation tunnel as an instance, for safety concerns, the equipment shall be validated to ensure that every item conveyed through it will satisfy the requirements of temperature and time. However, in order to reduce costs, the structure of this equipment is designed as a lumped parameter system. Meanwhile, the design is also deficient in the validation method. So far, there is no such efficient and theoretically feasible test method to validate the results. Instead, operators are using a practical method (e.g. periodically conducting comparison test). Thus, in order to pass the comparison test (and also meet the manufacture requirements), operators have to practically degrade the priority of energy consumption. For instance, they have to use a much higher temperature at the check point to ensure the test results.

Since control system identification and design were introduced in the last century, many scientists successfully established a variety of methods for constructing efficient, robust, flexible and general purpose control system approaches, such as PID control, optimal control, robust control, self-adapted control, fuzzy control, neural network-based control, expert control, and various combinations of them. In recent years, among these methods, the application of fuzzy system has become one of the most popular research interests in this area. Fuzzy control theory is a derivative from Professor Zadeh's important research result in 1965, where he proposed the concept of fuzzy set known from traditional sets. More specifically, Zadeh (1965) assigned a degree to the traditional set that could decide the degree of the element belonging to the conventional set. This made it possible for computers to deal with human uncertainty and ambiguity. In this case, one of the significant applications evolved into fuzzy control theory. After years of development, the fuzzy control approach has been made suitable for the complex plants for non-model-based control system design method and many researchers have pushed this domain forward to many successes in both theory and practice. Henceforth, following the introduction of type-2 fuzzy sets by Zadeh in 1975, related applications and advancements were prompted to address a range of issues of high complexity, such as a spatially dynamic distributed system that cannot be simplified as a lumped parameter system.

1.2 Motivation

Based on the problems and issues identified above, the research question is how to minimize the consumption of energy (not only power) without affecting the system performance and to resolve the issues using some kind of framework that can be applied to many practices rather than a "customized" method suitable only for a single case. Therefore, this research is derived from the practical industrial application identified or required as spatially dynamic distributed system, while was initially constructed based on a SISO structure. This is partly due to the complicated and uncertain nature of the objects, which are usually concerned with data distribution and time variation within a certain area. As usual, there are several ways to deal with such issues: some complicated objects are easy to model in software and in laboratory application where researchers can assume infinite number of sensors to obtain many data. Some employ mathematical methods for simplification, whilst others sacrifice energy consumption in order to achieve the desired results. However, as mentioned above, along with the gradually complex requirements of the manufacture processes, it is not so feasible to install many sensors in such an object for the reason of manufacture cost; or so accurate after simplification when the object contains significant variables that cannot be idealized as known formulas; or so earthy to sacrifice energy consumption especially when it becomes the main kernel in view of the plant running cost. In conclusion, sometimes it is not feasible to obtain the effect from the cause via the cause-and-effect relationship. However, a different way of dealing with the effect can be conjured up. On some occasions, difficulties in obtaining precise models drive the other ways to meet the system design requirements. Under such prerequisites, it is significant to open up a snap course for such purposes. For the above-mentioned systems, and especially when it is not possible to accurately model them mathematically, control system design can focus on the results using servo method, such as fuzzy control.

Apart from the physical analysis and application design to deal with the identified issue, the theoretical challenges as associated with the spatial distributed system with MIMO nature and its control approaches are also identified. One of the challenges of this thesis is to make sure that there is a balance of accuracy and response within the system, so the total number of sensors used in the system needs to be limited to reduce the cost, but must still be high enough to retrieve the amount of data with acceptable levels of accuracy. In accordance with the complicated and even coupling nature of the variables associated with spatially dynamic distributed systems, the control system is designed to deal with the issues instead of considering the objects directly.

At the practical level, another important motivation driving that justifies the present research is that there is a need to address the current concerns about designing principles and limitations. This project also aims to create a good balance between accuracy and the response the machine produces and to reduce the amount of energy the machine consumes to operate, which will ultimately reduce the total cost of machine operation. Having more sensors will allow more data with higher rate of accuracy. However, the downside of this is the lack of quick response from the machine. On the other hand, reducing the number of sensors used in a system will result in excellent system response, but it can adversely affect accuracy. This is usually known as a lumped parameter system.

1.3 Research Questions

The logical framework of the research, throughout the modelling and assumption and to the facilitation of control systems, is outlined in the following research questions.

1.3.1 Modelling: how can a SISO system be expanded into a MIMO system?

For purposes of analysis, only one input and output (temperature) is considered and established for the original physical plant. However, as recommended by technicians and engineers, it is worth considering an additional input and output to prevent accidents that could disrupt the entire production line. Furthermore, it is well-known that the best way to enhance precision is to optimize the physical structure design of the plant, such as adding more sensors, when the issue of cost is of no concern. Therefore, the issue this research is faced with is how to expand the mathematical design of a SISO system into a MIMO system without changing the original physical structure of the plant. Additionally, in case the introduced parameter, input or output, is somehow coupled in nature with the existing parameter, input or output, the research must also figure out how to deal with such a situation.

1.3.2 How can the coupled nature be solved?

After the expansion work is done, two inputs, two outputs and the two states, which is not a commonly seen, and standard mathematical state space model, are coupled together. That is to say, how can the coupled nature of this plant can be solved in case to facilitate the design of control systems?

1.3.3 How can the energy consumption be reduced?

As for the practical-in-use plant, it is mounted with only two sensors, one for temperature control and one for pressure monitor. In other word, this plant is designed as a lumped parameter system, for cost-down purpose, as the more sensors, the more cost. in order to meet FDA's requirements (to heat the vials to 300ºC and keep them for 5 minutes under this temperature), the most secure way is to set the desired temperature to 340ºC. And it is well known that to generate the high temperature environment even in a small space will consume large amount of energy, and even reference temperature is lower down by 1ºC, energy consumption will be accumulatively reduced. Therefore, the research question that how can the energy consumption be reduced is also a very practical issue to settle down.

1.3.4 How can type-2 fuzzy control be implemented?

Although the three-dimensional nature of type-2 fuzzy sets is well-known, the research question for each design here is how to determine the primary and secondary fuzzy sets and membership functions. This is greatly important because it is common knowledge that one of the drawbacks of fuzzy control is one-on-one design, with no two plants having the same fuzzy control. Therefore, the research must determine how to choose the appropriate fuzzy sets (parameters) and how to fuzzify the parameter in order to facilitate the design and make it popular. On the other hand, in view of process requirements, the research questions are not only considering popularising the design, but also how to reduce the calculation time and how to simplify the type-2 fuzzy control design process, such as the type-reduction process.

1.4 Contributions

The main contributions of this thesis are summarised as follows:

- Based on a comprehensive literature review of both theory and practice, the concepts and the definitions of the related subjects are revisited and clarified with reasonable justifications/revisions and improvements demonstrated through descriptions and examples.
- Expansion of the SISO spatially dynamic distributed system to MIMO spatially dynamic distributed system, to introduce another variable based on the existing physical structure of the machine.
- The creation of a platform for a MIMO spatially dynamic distributed system control scheme.
- Development of a solution to the system's coupling nature
- Application of matrix transformation to expand the SISO pole placement method to MIMO pole placement
- Controlling the decoupling system based on the design of a Mamdani-type fuzzy control scheme
- Application of data-based design to create interval type-2 T-S fuzzy control for the system and development of an interface for 2D control conversion to 3D control

1.5 Structure of Thesis

This thesis is divided into seven chapters. Apart from Chapter 1, which is the overview and introduction to this research, and Chapter 7, which provides the conclusions drawn from the research and recommendations for further study, the rest of the chapters are separated into two parts, namely, theoretical study and application area design and development. Chapters 2 and 3 provide the background and methodology for the research thesis, respectively; Chapter 4 is concerned with the application area design and development, including model expansion design, pole placement control and decoupling control design; Chapter 5 addresses the theory-based study of fuzzy control and type-2 fuzzy control system design, which will be facilitated for MIMO spatially dynamic distributed system, and the designed fuzzy control system will be validated with the model established in Chapter 4; Chapter 6 presents the simulation results and analysis, finalising the research content. The outline of the thesis is as follows:

Chapter 1 Introduction and outline of the thesis.

Chapter 2 The literature review covers the practical and theoretical background, including research related to the modelling and simulation of MIMO spatially dynamic distributed systems based on the conventional control theory, as well as conventional and type-2 fuzzy control to tackle the problems of control of spatially dynamic distributed systems.

Chapter 3 The fundamental knowledge and preliminary methodologies related to the research work in this thesis are set out, and some concepts and definitions are clarified.

Chapter 4 MIMO spatially dynamic distributed system analysis and modelling are generated using the state space approach, and relevant pole placement and decoupling control method based on state feedback analysis are developed for this application.

Chapter 5 The mathematical analysis and control system design for MIMO spatially dynamic distributed systems is achieved using the data-based predictive control approach. Furthermore, this chapter is also concerned with the development of an interval type-2 fuzzy control framework without reliance on objects.

Chapter 6 Comparison and analysis of the simulation results.

Chapter 7 Conclusions and recommendations for further research.

1.6 Published Papers

1) Y. Z. Wang, Q. M. Zhu and M. Nibouche, "State-Space Modelling and Control of a MIMO Depyrogenation Tunnel", accepted by 34th Chinese Control Conference and SICE Annual Conference 2015 (CCC&SICE2015),27-31, July, Hangzhou.
2) Y. Z. Wang, Q. M. Zhu and M. Nibouche, "Mamdani Type Controller Design for MIMO Systems with Case Study", accepted by 7th International Conference of Modelling, Identification and Control 2015 (ICMIC 2015), 18-20, December, Tunisia.
3) Y. Z. Wang, Q. M. Zhu and M. Nibouche, Intelligent Control of MIMO Spatially-Distributed Systems with Applications, Book, (under preparation)

2 Background and Literature Review

This chapter introduces the general background to the practice and applications of the research. It also briefly reviews the most commonly used control algorithms provided by conventional theory, fuzzy control theory, and spatially dynamic distributed systems and control. Further, the research aims, objectives, contributions and motivation of this research are presented. The chapter concludes with an outline of the thesis.

2.1 Spatially Dynamic Distributed Systems

2.1.1 Introduction

The system identification is an art and the science for constructing a mathematical model of a system (Ljung, 2010). From one perspective, a suitable mathematical model will afford comprehensive insight into system structure and core elements, thus ensuring ample data on consequential sections, signal transmission, controller design and other aspects. The system identification and modelling should be the foundation of all work, because a well-designed system model will deliver the most elaborate precision in validation output for a proposed controller.

As the software industry is becoming increasingly more developed, various reputable applications have begun to be exploited for the purposes of system modelling. Generally, AutoCAD is proper to graphic design and ANSYS is tailored for finite element design, which is extensively used in industrial design. Matlab, as one of the powerful simulation tools in system design, provides capacious space and freedom for engineers to construct their own options. In conclusion, the problem is how to identify the system and what software to use to model it.

The control theory of spatially dynamic distributed systems was first introduced in the mid-twentieth century. Initially, the research in this field focused on linear and half-linear systems, without considering the state boundary of control, due to the complexity of spatial distributed systems. Wang (1964) discussed the properties of spatially dynamic distributed systems, including stability, controllability, observability and the issue of optimal control. Consequently, Butkovsky (1969) generalized the extremum principle to specific spatially dynamic distributed systems, which had originally been used in the theory of lumped parameter system control theory. Meanwhile, he utilized the moment method to achieve optimal control over spatially dynamic distributed systems. Subsequently, Lions (1971) developed the optimal control and identification theory of spatially dynamic distributed systems and he conducted further study on the definite theory of the partial differential equations, which was used to describe the distributed parameter systems, including elliptic type, parabola type and hyperbolic type of partial differential equations.

As the control of spatially dynamic distributed systems is becoming an increasingly prominent research field, an abundance of research results is being produced. The relevant analysis of optimal control, tuning, random control, self-adaptive control, robust control is gradually supplemented in the control of linear spatially dynamic distributed systems and the important theories of stability, controllability and observability have been perfected in this field as well (Curtain, 1978). The main research methods of linear spatially dynamic distributed systems consist of abstract space theory, functional analysis, spectral method, frequency domain analysis methods, finite difference method and finite element method, among others.

Consequently, in keeping with the developmental trajectory of contemporary science and technology as well as practical engineering control systems, the control of non-linear spatially dynamic distributed systems has been the focus of ample research and analyses, fostering the development and implementation of effective control methods. These methods include the stability control based on Lyapunov (Christofides, 2001), PID control (Alvarez, 2001), model control (Chen and Chang, 1992), geometric control (Kravaris, 1991), control based on finite dimension system theory (Hoo, 2001), model predictive control (Zheng, 2004), self-adaptive control (King, 2003), sliding model control (Sira, 1989) and optimal control (Park, 1995). Meanwhile, some methods that were originally intended for linear spatially dynamic distributed systems are now used in the controller design of non-linear spatially dynamic distributed systems.

Spatially dynamic distributed system is a term that may not be familiar to the general public, but it permeates every aspect of daily life. One example is a room which is heated with a central heater or is cooled with an air conditioner, and another example is a reactor which is pressured with compressed air. The temperature field and pressure field identified from these two examples are two typical cases of spatially dynamic distributed systems. In general, spatially dynamic distributed system is considered as a system with parameters that span over space and time, which means that, within a certain space and within a certain period of time, the parameters vary with time, or space, or both. As a result, spatially dynamic distributed system is also known as parameter distributed system. Hence, when considering the Energy Conservation Law, most of the spatially dynamic distributed systems can be represented as follows:

$$F(\mathrm{z}, \mathrm{t}, \mathrm{x}, \frac{\partial x}{\partial z}, \frac{\partial x}{\partial t}, \frac{\partial^2 x}{\partial z^2}, \frac{\partial^2 x}{\partial t^2}, ..., \frac{\partial^n x}{\partial z^n}, \frac{\partial^n x}{\partial t^n}) = 0 \tag{2.1.1}$$

Where z and t are independent variables, $\mathrm{z} \in [l_a, l_b]$ denotes the variable of space, and l_a, l_b are constants, $t \geq 0$ denotes the time constant, x is the dependent variable, therefore $F(\cdot)$ is a nonlinear function with regard to the independent variables z and t, dependant variable x, and partial derivative formula of x with regard to independent variables from the first order to n-order.

2.1.2 Development

The control theory of spatial distributed systems was first introduced during the mid-twentieth century. Initially, the research in this field focused on linear and half-linear systems, without considering the state boundary of control, due to the complexity of spatial distributed systems. In 1954, Xuesen Qian launched the discussion about heat conduction process in distributed parameter systems, finally leading to the application of the concept of infinite transfer function. Wang (1964) discussed the properties of spatially dynamic distributed systems, including stability, controllability, observability and the issue of optimal control. Consequently, Butkovsky (1969) generalized the extremum principle to specific spatially dynamic distributed systems, which had been initially used in the control theory associated with lumped parameter systems. Meanwhile, he utilized the moment method to optimally control spatially dynamic distributed systems. Subsequently, Lions (1971) developed the optimal control and identification theory of spatially dynamic distributed systems, and he conducted further study on the definite theory of the partial differential equations, which was used to describe the distributed parameter systems, including elliptic type, parabola type and hyperbolic type of partial differential equations.

As the control of spatially dynamic distributed systems is becoming an increasingly prominent research field, an abundance of research results is being produced. The relevant analysis of optimal control, tuning, random control, self-adaptive control, robust control was gradually supplemented in the control of linear spatially dynamic distributed systems and the important theories of stability, controllability and observability have been perfected in this field as well (Curtain, 1978). The main research methods of linear spatially dynamic distributed systems consist of abstract space theory, functional analysis, spectral method, frequency domain analysis methods, finite difference method and finite element method. Glowinski et al. (2008) provided an overview of quantitative research conducted on the controllability of distributed parameter systems as well as applications.

The research and development on partial differential equations and functional analysis support the theoretical research of distributed parameter systems as well as providing the research with powerful analysis tools. Until now, research on the tuning, optimal control, controllability, observability, identity of distributed parameters and filtering of partial distributed systems has achieved similar results to lumped parameter systems, which can be considered as an expansion of relevant research results.

Consequently, in keeping with the developmental trajectory of contemporary science and technology as well as practical engineering control systems, the control of non-linear spatially dynamic distributed systems has been the focus of ample research and analyses, fostering the development and implementation of effective control methods. These methods include the stability control based on Lyapunov (Christofides, 2001), PID control (Alvarez, 2001), model control (Chen and Chang, 1992), geometric control (Kravaris, 1991), control based on finite dimension system theory (Hoo, 2001), model predictive control (Zheng, 2004), self-adaptive control (King, 2003), sliding model control (Sira, 1989) and optimal control (Park, 1995). Meanwhile, some methods initially intended for linear spatially dynamic distributed systems are now used in the controller design of nonlinear spatially dynamic distributed systems.

2.1.3 Applications and Problems

In engineering, it occasionally happens that control objects may need to be altered as distributed parameter systems due to the control system structure or actuators. For instance, if a hydraulic pressure actuator or pneumatic actuator is designed with complex structure or requires over distance, when performing modelling according to the actuator movement principle, the state transition of fluid or other media must be considered as well. Furthermore, such state transition is also described by a distributed parameter, which is not expected. In practice, the parameter-distributed controller is seldom adopted due to the difficulty involved in implementing it. In most cases, the control objects are parameter-distributed systems when a lumped parameter system serves as controller. There are generally three types of control approaches for distributed parameter systems:

1) Point control approach: put the control effort on several independent points of control objects, such as light control panel in a room;

2) Distributed control approach: put the control effort on several sections or sub-area of control objects, such as the central heating system of a building, using distributed panels of heater to increase the building's temperature;

3) Boundary control approach: put the control effort on the boundary of the control objects, such as flooring heating system.

However, existing studies present obvious shortcomings. On the one hand, in the area of distributed parameter systems, research results are still far away from application in practice. On the other hand, a large gap in the research on spatially dynamic distributed systems remains to be filled, namely, systems with spatially dynamic distributed inputs and outputs.

2.2 State-space Approach

The state-space modelling approach is probably one of the most powerful modelling methods associated with the modelling and control of MIMO systems. This approach has undergone numerous developments over the years and has been successfully applied to many MIMO systems (Gueguen et al., 1985; Cassell and Choi, 2012). Due to its convenient transformation and simplification, the state-space approach is the preferred approach for characterising systems with dynamic parameters.

2.2.1 Definition

Once the matrices are defined, the next step is determining their controllability. Wonham (1967) proved that if a linear time-invariant multivariable dynamical equation is controllable, new eigenvalues of a new matrix can be chosen arbitrarily by introducing the state variable feedback. This has been further proven by Chen et al. (1968), who mentioned that the concepts of cyclic and model were deemed unnecessary. The main idea that emerged from their research results was that new and sufficient conditions could be derived from the non-singular transformation of the input vector. The current implementation will draw largely from methods described by Wonham and Chen. According to MathWorks (2015), a state-space model can be defined as a way of describing a system using state variables by a set of first-order differential equations or difference equations, as opposed to having one or more nth-order differential or difference equations. Systems theory is the main foundation of the state-space model. Its early applications include famous programs like the Apollo and Polaris aeronautics and aerospace programs (Hutchinson, 1984). For example, it has seen application in the Kalman filter, which is an algorithm developed based on the work conducted by Kalman (1960, 1963). One version of the Kalman filter, known as the Kalman-Bucy filter (named after Richard Snowden Bucy), which is a continuous time version of the Kalman filter, has its algorithm based on the state-space model. It uses the measured data to decrease or even eliminate stochastic disturbance, and rebuilt the dynamic matrix (Gu and Yung, 2013). The state-space model used in Kalman filter takes measured data as recursive state variables. The state-space model has also been used in fuzzy logic control, especially in the Takagi-Sugeno (TS) inference type (Wang et al., 2015). Apart from the previously-mentioned scientific and engineering applications, the state-space model has also been used in other industries, such as finance (Mergner, 2009).

For the most part, the state-space model is particularly used in systems where there are multiple states to be dealt with. Within a state-space system, the state equation must be defined explicitly as the internal state of the system. Then, an output equation needs to be defined by combining the current state of the system and the current input of the system. The two equations will form a series of equations known collectively as the state-space equations (wikibook, 2015). A vector that consists of all the internal states of a system is known as the state space. In order to model a system using the state-space method, the system must be lumped. A lumped system is a system where a finite-dimensional state-space vector that characterizes all internal states of the system in entirety. In linear state-space systems, the state and output equations obey the superposition principle and the state space is linear. However, the state-space method is also suitable to be implemented in non-linear systems, but requires a somewhat different approach to tackling them.

A general state-space model is shown in Figure 2-1:

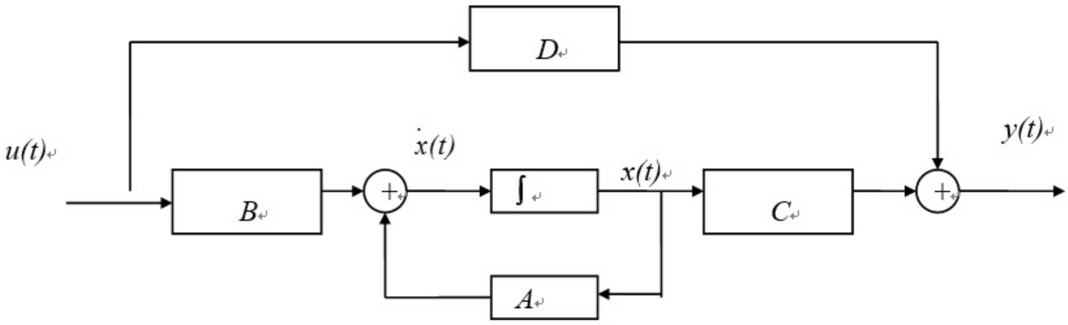

Figure 2-1 General Diagram for State Space Model

In a state-space model, the state of a system represents the very core of the model. A state of a system can be described as the current value of an internal element of a system, which changes its values independently from the system output. When using the state-space equations to model a system, three vectors need to be defined beforehand:

Input Variables: These represent the inputs of the system. The type of system that is being modelled will determine the number of inputs that need to be defined. In general, there are two possible systems: SISO (single input single output) and MIMO (multiple input multiple output). In SISO systems, only one input variable needs to be defined, while in a MIMO system, more than one of inputs can to be defined. Once all the inputs have been defined, they need to be arranged in a vector form.

Output Variables: These represent the outputs that the system produces. In a SISO system, only one output is produced, whereas in a MIMO system, more than one output is produced. The output variables are dependent on a combination of the input vector and the state vector. The outputs should be ideally independent of any of the others. That is, one input variable should only affect one output variable.

State Variables: State variables represent the parameters within the system that change over time. State variables can be either linear or non-linear, depending on the results of system analysis.

The state-space model has been developed over a long period of time and it is one of the most powerful modelling methods in dealing with complex MIMO systems (Wang et al., 2015). MIMO systems that are linear and lumped can be easily represented using the state-space approach.

Let *S* be the state-space model, then the general representation of *S* is as follows:

$$S(A,B,C,D): \quad \dot{x}(t) = Ax(t) + Bu(t) \quad (State \;\; Equation)$$
$$y(t) = Cx(t) + Du(t) \quad (Output \; Equation) \tag{2.2.1}$$

The inputs, outputs, and state variables are defined respectively as follows:

$$u(t) = [u_1(t) \ldots u_m(t)]^T$$
$$y(t) = [y_1(t) \ldots y_l(t)]^T$$
$$\dot{x}(t) = [x_1(t) \ldots x_n(t)]^T \tag{2.2.2}$$

The matrices A, B, C and D are defined as follows:

Dynamic Matrix A_{n*n} : Also known as the state matrix, it is generally used to describe the dynamics of the system as well as to control the trajectory of the state vector $x(t)$.

Input Matrix B_{n*m} : denotes how each control input affects the state variables of the systems.

Output Matrix C_{l*n} : it represents output vector $y(t)$ with state vector $x(t)$.

Transmission Matrix D_{l*m}: It indicates the feedforward effect of control inputs to output vector $y(t)$.

State Feedback Approach

Based on the open loop diagram, the closed loop with state feedback approach, which is different from the output feedback, is also generated, as shown in Figure 2-2 General diagram of State Feedback Approach

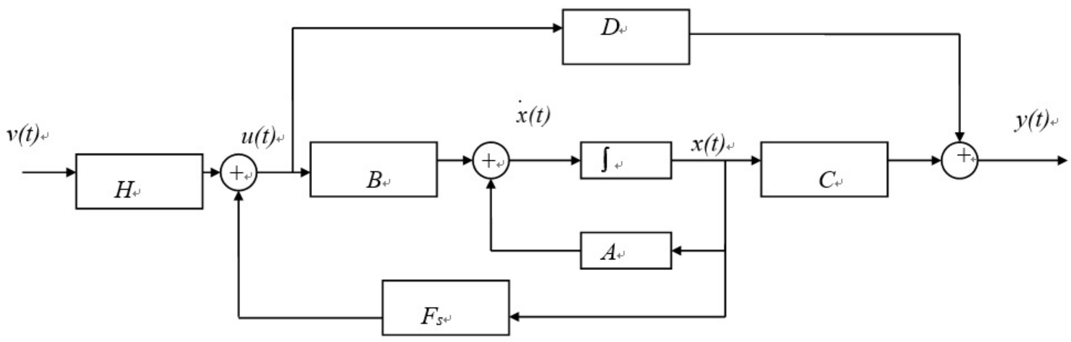

Figure 2-2 General diagram of State Feedback Approach

$$v(t) = \begin{bmatrix} v_1(t) & \ldots & v_m(t) \end{bmatrix}^T \quad (2.2.3)$$

Which is called reference vector and it should be noticed that there are two gain matrices are introduced,

F_s is the $m*n$ **state feedback gain matrix** to specify the poles of the closed loop system.

H: is the $m*m$ **input feedforward gain matrix** to specify the zeros of the closed loop system.

2.2.2 Applications

Start from kalman filter (1964), state space approach is firstly introduced to research area. Any change in any input within a multivariable system will result in changes in all outputs most of the time and this means that the system is known as a "coupled system". According to Zheng et. al (2009), there are times where such interaction is not desired in an organization and this has led to decoupled systems, where multivariable systems being in the limelight in the last decade or so. To design a decoupled system, one has to ensure that each input maps to one output, effectively creating single input/output (IO) channel.

In state observation, a system's state space model is not a true feedback system and this means that a feedback mechanism that x' relates to x represents a single internal mechanism to the plant. To put it simply, the matrices A, B, C and D are part of a single device and not separate "components" per se. These matrices are immutable, meaning that they cannot be altered during the operation of the machine since they are intrinsic parts of the plant (Siekmann et. al, 2015). However, if the entire plant has been modified, the matrices will change. Since the matrices are immutable, a method that modifies the system externally is needed, and that is the feedback loop. In a state feedback, the state vector's value is returned back to the input channel of the system. If an external feedback element exists in the system, the system is a closed-loop system. If otherwise, the system is known as an open-loop system. (WikiBooks, 2015).

2.2.3 Recent Research Outcomes

According to Marconato et al. (2014), precise models that are capable of capturing details of most systems is needed in the field of measurement, since there is a demand for them. Development of good models allows for better understanding and further analysis of the plant equations. By nature, depyrogenation tunnels have a non-linear behaviour, which is the source of their dynamic properties. This dynamism is due to the increase in the total number of physical properties of the machine, resulting in greater difficulty to model mathematical equations that can accurately capture system details. Depyrogenation tunnels used to be modelled in a SISO fashion and though the SISO method is easy to model, their limitation arises from the fact that the number of parameters enhances their complexity.

Consequently, MIMO-based models need to be developed to complement the machine's complex non-linear nature.

In this regard, one of the best models is the one proposed by Jiang (2008), which is a new multi-rate fuzzy control technique for a continuous-time non-linear system. Jiang (2008) used the lifted Takagi-Sugeno (T-S) fuzzy logic model for the plant's construction, which involved linear matrix inequalities (LMIs) and a multi-rate input controller for the multiple T-S linear model. It was based on the design of local feedback controllers using optimal disk-pole (D-Pole) placement and showed promising results in the closed-loop control system. Another powerful model for modelling a MIMO system is the state-space model. Over the years, the state-space model approach has been successfully applied in many different MIMO systems (Gueguen et al., 1980; Cassell et al., 2012). In this case, it becomes a very sound solution when trying to implement that state-space model in the control system design of a depyrogenation tunnel.

2.2.4 Discretization

As mentioned before, the discrete value plays a significant role in the system modelling. The discretization of a continuous system not only reduces the amount of data that need to be sampled and sent to the computer, but also enhances the modelling flexibility of the computer. There are several methods used Among the several methods that are used, the most popular to discrete a continuous model of a temperature field is the finite volume method, which represents and evaluates partial differential equations in the form of algebraic equations (LeVeque, 2002; Toro, 1999). The effective methods for discretization are Taylor series expansion method and thermal balance method, which are suitable for application in stable heat transfer process. For the transient conduction process, the effective methods are the explicit difference scheme and implicit difference scheme. Under this situation, as the element temperature varies with both space and time, the discretization must be done from both space and time. Space discretization is similar to steady-state conduction, while time discretization involves dividing the time by $\Delta\tau$.

2.3 Fuzzy Logic and Fuzzy Control

2.3.1 Introduction

The accuracy of the acquired knowledge and the possibility to employ traditional control approaches of confirmed precision are both minimised as control objects are becoming increasingly more complex, non-linear, and presenting a hysteretic quality and coupling nature. As stated in the Exclusive Principle, the more complex a system is, the more difficult it is to obtain crispy results. In other words, complexity and clarity are mutually exclusive. However, the human brain has managed to overcome these problems successfully.

Considering the mechanism underpinning the human brain's decision-making ability, it differs from the computer's mechanism because it does not rely on numbers mainly, but on concepts, patterns, images and thoughts. Furthermore, human language is replete with ambiguities, which the computer cannot understand. The introduction of fuzzy logic made it possible to translate the knowledge and experience presented in natural language into computer language, namely mathematical functions and expressions, so that the computer can now understand and process this information. Fuzzy logic theory is designed to perform inference based on certain rules, while the logic value can be any real number between 0 and 1, which can be easily identified and processed via numerical computational approach. In this case, it facilitates the combination of physical systems represented by mathematics and human intelligence represented by fuzzy theory, embedding the human ideas into control system design.

The theory of fuzzy control considers the control objects as a "black box". After translating the manipulating experience of the "black box" into "fuzzy rules", and following the specific scheme, computers can imitate the actions of experts for the purpose of automatic control. Therefore, it introduces the theory of fuzzy control, which is a computer system control technology based on natural language control rules and fuzzy logic inference. Fuzzy control is theoretically independent from mathematical models of traditional control systems but relies greatly on experience manipulation and knowledge base. It is worth mentioning that, although fuzzy control and expert system are both based on knowledge from experts, the expert system transfers the human language symbols directly to computer language, while fuzzy control rules transfer the language into numbers or mathematical expressions in advance of utilization.

Due to its convenient application, fuzzy control theory has been widely accepted since the 1980s and 1990s. As mentioned before, the development of fuzzy control system minimizes the precise mathematical requirements.

2.3.2 Fuzzy Logic

1. Definition of fuzzy sets

The traditional theory of sets describes the clear, determined objects distinct to each other. However, clarity, determination and distinction are often the exception rather than the rule, as there is not always a clear boundary between objects. Ambiguity is especially pronounced when it comes to those matters that are on a transitory stage to each other. Unfortunately, studies are always performed on the basis of quantity, indicating the impossibility of quality research. Therefore, the theory of fuzzy sets is the mathematical theory for description and investigation of ambiguous matters with clear mathematical methods. This definition was provided by Zadeh (1965), who expanded the traditional sets containing two values $\{0,1\}$ into fuzzy ones with the value discourse of [0,1], whilst also giving the following mathematical definition: in a given domain of discourse U , there exists a mapping:

$$\overline{A}: U \to [0,1], x \mapsto \mu_{\overline{A}}(x) \tag{2.3.1}$$

Where $\overline{A}$ denotes a fuzzy set or a sub-fuzzy set in the domain U ; $\mu_{\overline{A}}(x)$ denotes the degree of each element of x belonging to the fuzzy set $\overline{A}$ and is known as the membership function of element x in fuzzy set $\overline{A}$. When x is a determined element x_0 , $\mu_{\overline{A}}(x_0)$ is denoted as the degree of membership against fuzzy set $\overline{A}$. Such definition has given the degree to fuzzy set $\overline{A}$, whose boundary is not clear from any determined element x_0 and made the degree mathematized. If memberships of any fuzzy set only have two values, 0 and 1, the fuzzy set $\overline{A}$ is sharpened as a traditional set. Obviously, a traditional set is a typical case for fuzzy sets.

2. Membership Functions

Before a fuzzy set can be defined, the membership needs to be defined first. However, there is no single definition of membership, but multiple ones, due to differences in perception and language.

After years of development and trial-and-test efforts, the most widely used membership functions are listed as follows:

1) Triangle

$$f(x,a,b,c) = \begin{cases} 0 & x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & x \ge c \end{cases} \tag{2.3.2}$$

where requires $a \le b \le c$.

2) Bell

$$f(x,a,b,c)=\frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \tag{2.3.3}$$

Where *c* determines the central position of the function, and *a*, *b* determines the shape of function.

3) Gaussian

$$f(x,\sigma,c)=e^{\frac{(x-c)^2}{2\sigma^2}} \tag{2.3.4}$$

Where *c* determines the central position of the function and σ determines the width of the curve.

4) Ladder

$$f(x,a,b,c,d)=\begin{cases}0 & x\le a\\ \frac{x-a}{b-a} & a\le x\le b\\ 1 & b\le x\le c\\ \frac{d-x}{d-c} & c\le x\le d\\ 0 & x\ge d\end{cases} \tag{2.3.5}$$

Where requires $a\le b$ and $c\le d$.

5) Sigmoid

$$f(x,a,c)=\frac{1}{1+e^{-a(x-c)}} \tag{2.3.6}$$

Where a and c determines the shape of function, and the function is central symmetry with respect to the point $(a,0.5)$.

3. Fuzzy Relationship

1) Definition

Within the traditional set theory, if some relationship exists between the elements with regard to the two sets, then it is not difficult to describe that relationship with some functions. However, as stated by the definition of traditional sets, the relationship between two sets either exists or not. However, after expansion to fuzzy set theory, the relationship between elements in two fuzzy sets came to denote the degree to which the elements were correlated to each other, namely, the fuzzy relationship. The fuzzy relationship is defined as follows:

$$R(x,y):A\times B\rightarrow[0,1] \tag{2.3.7}$$

Where R is a fuzzy subset in $A\times B$, and the relationship determines the degree of correlation between elements x in fuzzy set A and elements y in fuzzy set B. Therefore, $R(x,y)$ denotes the binary fuzzy relationship, and abbreviated as *Fuzzy Relationship*.

2) Fuzzy Relationship Composition

Suppose relationship $\boldsymbol{P}$ and $\boldsymbol{Q}$ are respectively two fuzzy relationships, where $P\in\mathbb{F}(X\times Y)$ and $Q\in\mathbb{F}(Y\times Z)$; hence, the composition of the fuzzy relationship $\boldsymbol{P}$

to ***Q*** denotes the fuzzy relationship from fuzzy set X to Z, which can be represented as $P \circ Q$.

4. Defuzzification

Defuzzification is a process whereby a single number is used to represent a fuzzy set. The single number shall be an element in the fuzzy set, and can, in a manner, represent the fuzzy set. The most commonly employed methods of defuzzification are outlined below:

1) Method of Area Centre (Centroid)

This method is designed to determine the centroid of the area encircled by the membership function curve and the horizontal coordinate, using the abscissa value of this point as the representation of the fuzzy set.

Suppose the membership function of fuzzy set A in domain U is $A(u),\ u \in U$. If the abscissa value of the centroid is u_{cen} , then the value is calculated as follows:

$$u_{cen} = \frac{\int_U A(u)u\,du}{\int_U A(u)\,du} \tag{2.3.8}$$

If domain U is discrete as $U = \{u_1, u_2, \cdots, u_n\}$ and the membership of u_j is $A(u_j)$, so that u_{cen} can obtain as follows:

$$u_{cen} = \frac{\sum_{j=1}^{n} u_j A(u_j)}{\sum_{j=1}^{n} A(u_j)} \tag{2.3.9}$$

Despite being considerably accurate and reasonable, the centroid method is time-consuming in terms of calculation.

2) Bisector Method

This method first determines the area encircled by the membership function curve and the horizontal coordinate and then determines the abscissa value of bisector that can divide the area into equal parts. The abscissa value is used to represent the fuzzy set.

Suppose the membership function of fuzzy set A in domain U is $A(u),\ u \in U$. If the abscissa value of the bisector line is u_{bis} and $u \in [a,b]$ then the value is calculated as follows:

$$\int_a^{u_{bis}} A(u)du = \int_{u_{bis}}^b A(u)du = \frac{1}{2}\int_a^b A(u)du \qquad (2.3.10)$$

If domain U is discrete as $U = \{u_1, u_2, \cdots, u_n\}$, the area under the membership function is triangles, ladders or squares; therefore, the matter is reduced to determining the position in relation to half of the area of elements. This method is widely used in fuzzy control design.

3) Method of Maximum

In some cases, the fuzzy sets may not be regular or convex, while their membership function may not be a continuous curve either. In such cases, the most reasonable approach is to use a point with the largest degree of membership to represent the fuzzy sets. There are three commonly used sub-methods based on the Maximum Method.

a. Mean Value of Maximum (MOM)

If there are more than one point with the maximum value of membership, the abscissa value of the average value u_{mom} should be taken as the representation.

Suppose $A(u_j) = \max(A(u))$, where $j = 1, 2, \cdots, n$, and there are n points in the largest membership degree, therefore:

$$u_{\max} = \frac{\sum_{j=1}^{n} u_j}{n} \tag{2.3.11}$$

b. Largest Value of Maximum (LOM)

If there are more than one point with the maximum value of membership in the domain, the point with the largest absolute value among those points should be selected and its abscissa value u_{lom} should be used for the representation.

Suppose $A(u_j) = \max(A(u))$, where $j = 1, 2, \cdots, n$, and there are n points in the largest membership degree; the point with the largest absolute value $\max(|u_j|) = |u_k|$ should be chosen, namely:

$$u_{lom} = u_k \tag{2.3.12}$$

c. Smallest Value of Maximum (SOM)

Similar to LOM, if there are more than one point with the maximum value of membership in the domain, the point with the smallest absolute value among those points should be selected, and its abscissa value of u_{som} should be used for the representation.

Suppose $A(u_j) = \max(A(u))$, where $j = 1, 2, \cdots, n$, and there are n points in the largest membership degree; therefore, the point with the largest absolute value $\min(|u_j|) = |u_k|$ should be selected, namely: $u_{som} = u_k$.

5. Fuzzy Inference and Implication Relationship

A fuzzy proposition is classified as a simple proposition if it cannot be divided into simpler propositions. Therefore, suppose $A(a)$ and $B(b)$ are two simple propositions; if there is a fuzzy dependant relationship between the two propositions, such as "if $A(a)$, then $B(b)$", the compound proposition is called fuzzy condition statement, also known as fuzzy condition proposition. Suppose $A(a)$, $B(b)$ and $U(u)$ are fuzzy propositions, therefore there are two types of widely used statements, namely:

1) If A, then U.

It indicates the proposition: if *a* is *A*, then *u* is *U*, or if *A(a)* then *U(u)*, can be represented as $A \to U$.

In order to obtain the fuzzy implication relationship of $A \to U$, there are several widely used algorithms summarised as follows:

Zadeh algorithm:

$$\begin{aligned} R(a,u) &= (A \to U)(a,u) \\ &= \max((1 - A(a)), \min(A(a), U(u))) \\ &= (1 - A(a)) \vee (A(a) \wedge U(u)) \end{aligned} \tag{2.3.13}$$

Mamdani algorithm:

$$R(a,u) = (A \to U)(a,u)$$
$$= \min(A(a), U(u)) = A(a) \wedge U(u) \tag{2.3.14}$$

Larsen Algorithm:

$$R(a,u) = (A \to U)(a,u) = A(a) * U(u) \tag{2.3.15}$$

Bounded Sum Algorithm:

$$R(a,u) = (A \to U)(a,u)$$
$$= 1 \wedge (A(a) + U(u)) = \min(1, (A(a) + U(u))) \tag{2.3.16}$$

Mizumoto-s Algorithm:

$$R(a,u) = (A \to U)(a,u)$$
$$= \begin{cases} 1 & A(a) \le U(u) \\ 0 & A(a) > U(u) \end{cases} \tag{2.3.17}$$

Mizumoto-g Algorithm:

$$R(a,u) = (A \to U)(a,u)$$
$$= \begin{cases} 1 & A(a) \le U(u) \\ U(u) & A(a) > U(u) \end{cases} \tag{2.3.18}$$

Among the six algorithms, the Mamdani algorithm is the most successful and thus the most acceptable algorithm in industrial practices. However, the specific implication algorithm depends on the continuity of the fuzzy sets $A(a)$ and $U(u)$. This gives rise to three specific situations: both $A(a)$ and $U(u)$ are discrete; $A(a)$ is discrete while $U(u)$ is continuous, and both $A(a)$ and $U(u)$ are continuous. The first one is the most popular in practice and the calculation associated with it is performed as follows:

According to the Mamdani algorithm, the result of the fuzzy implication relationship $R(a,u) = A(a) \wedge U(u)$, when $A(a)$ and $U(u)$ are discrete, will be a fuzzy subset of direct product $A \times U$, namely $R(a,u) \in \mathbb{F}(A \times U)$. In this case, the fuzzy relationship $R(a,u)$ can be represented by a $m*n$ fuzzy relationship matrix. Therefore, the process consists of two steps:

The first step involves performance of transposition of $A(a)$ to obtain $\vec{A}(a)$, which is a column vector. This is to ensure that every element a_i in A matches with every u_i in U .

The second step involves performance of calculation $\vec{A}(a) \circ U(u)$ (choose the smaller one) as follows:

$$
\begin{aligned}
R(a,u) &= A(a) \wedge U(u) = \vec{A}(a) \circ U(u) = A(a)^T \circ U(u) \\
&= \begin{bmatrix} A(a_1) \\ A(a_2) \\ \vdots \\ A(a_m) \end{bmatrix} \circ \begin{bmatrix} U(u_1) & U(u_2) & \cdots & U(u_n) \end{bmatrix} \\
&= \begin{bmatrix} A(a_1) \wedge U(u_1) & A(a_1) \wedge U(u_2) & \cdots & A(a_1) \wedge U(u_n) \\ A(a_2) \wedge U(u_1) & A(a_2) \wedge U(u_2) & \cdots & A(a_2) \wedge U(u_n) \\ \vdots & \vdots & \vdots & \vdots \\ A(a_m) \wedge U(u_1) & A(a_m) \wedge U(u_2) & \cdots & A(a_m) \wedge U(u_n) \end{bmatrix}
\end{aligned}
\tag{2.3.19}
$$

Therefore, determine $R(a_i,u_j) = A(a_i) \wedge U(u_j)$, where $i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$, then:

$$R(a,u) = A(a) \wedge U(u) = \vec{A}(a) \circ U(u)$$
$$= \begin{bmatrix} R(a_1,u_1) & R(a_1,u_2) & \cdots & R(a_1,u_n) \\ R(a_2,u_1) & R(a_2,u_2) & \cdots & R(a_2,u_n) \\ \vdots & \vdots & \vdots & \vdots \\ R(a_m,u_1) & R(a_m,u_2) & \cdots & R(a_m,u_n) \end{bmatrix} \quad (2.3.20)$$

2) *If A and B, then U.*

It indicates the proposition if a is A and b is B, then u is U, or if $A(a)$ and $B(b)$ then $U(u)$, which can be represented as $A \wedge B \rightarrow U$. The calculation is similar to the first type of proposition.

It is worth mentioning that either of the inference results U is a fuzzy set, which cannot be used directly until defuzzification.

The concept of fuzzy logic control (FLC) involves basing the design of a practical controller on qualitative system knowledge. FLC is generally applicable to plants whose mathematical model cannot be found, but the qualitative knowledge of experienced operators provides enough information for control system design. It is particularly suitable for those systems with uncertain and/or complex dynamics.

2.3.3 Mamdani Fuzzy Control

Mamdani (1974) was the first to introduce fuzzy control based on fuzzy condition statement to successfully manage to control a boiler-steam engine. This was the milestone that marked the birth of fuzzy control theory. In fact, fuzzy control inherits the basic structure of the traditional control systems, performing as the extension and supplement to the tradition. Similar to the traditional control systems, the general block diagram of a fuzzy control system is as shown below:

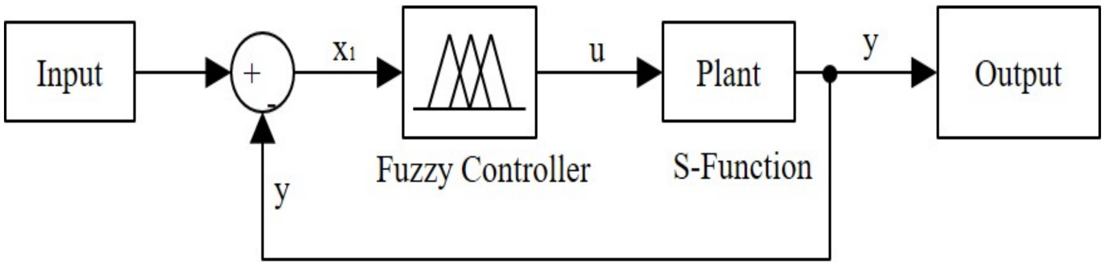

Figure 2-3 Block Diagram of General Fuzzy Control System

As shown in the figure, the fuzzy controller constitutes the core element of the control system. Furthermore, the input $x_1 = r - v$ and output u of the fuzzy controller are crisp numbers.

With the development of fuzzy logic, fuzzy control theory has become a key branch of modern control theory, and the two most widely accepted methods are the Mamdani method (Mamdani and Assilian, 1974) and the T-S method (Takagi-Sugeno, 1985). Mamdani fuzzy control architecture performs control of a physical system based on fuzzy rules, and the rules are provided either by the expert or by the database of the physical system. The T-S method requires a large amount of data to obtain the control discipline and it is better used for system identification and modelling. On the other hand, despite being flexible and imposing fewer requirements on the system nature, the Mamdani method presents greater complexity than the T-S method. Ideally, complex systems should be addressed. The structure of a typical Mamdani-based fuzzy controller is shown below.

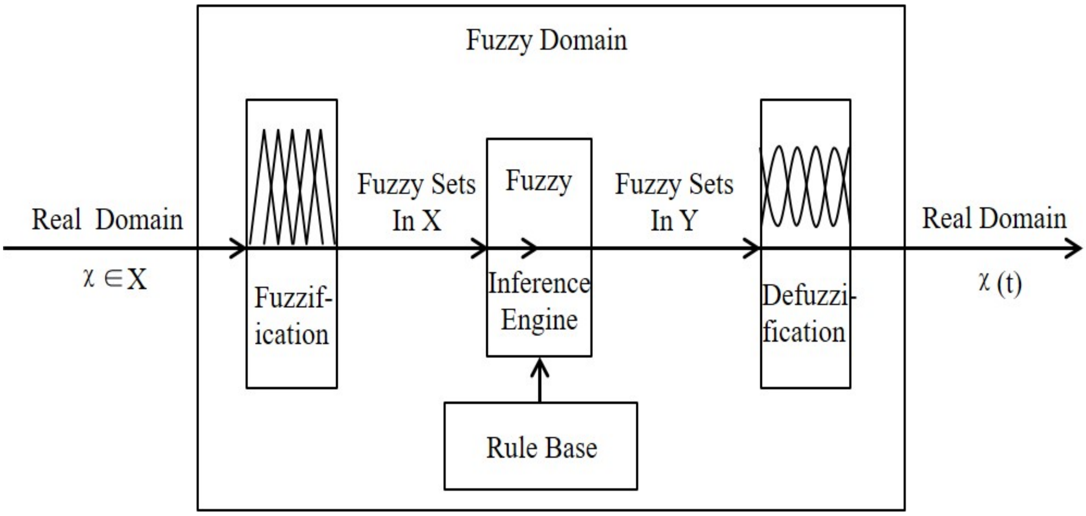

Figure 2-4 General Diagram of Fuzzy Control System

It is assumed that the fuzzy system has inputs $x_i \in X_i$ (where $i = 1, 2, \cdots, n$) and outputs $y_j \in Y_j$ (where $j = 1, 2, \cdots, m$). The inputs x_i and the outputs y_j are real numbers; they are also called "crisp".

3) Mamdani Type Control

A typical Mamdani Type control system always contains the modules as shown in the following figure:

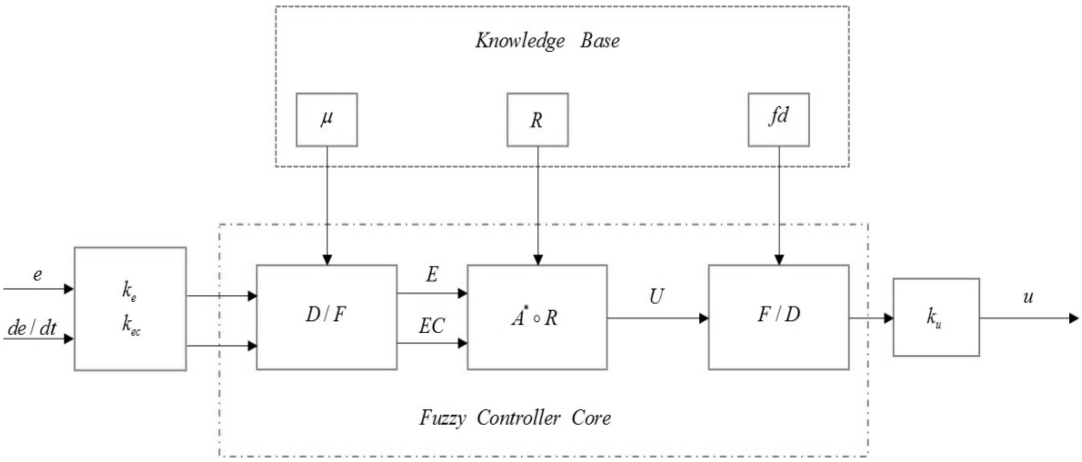

Figure 2-5 2-D Mamdani type Control Block Diagram

A standard Mamdani-type controller includes a fuzzification module (D/F), an inference module ($\circ R$), and a defuzzification module (F/D); two additional modules, namely, the scale factor module and the proportion factor module, are required to convert the data from measurement into data that can be identified by the controller. The Fuzzifier converts the crisp inputs into its membership values for fuzzy sets, the Inference Engine uses the fuzzy rules in the Rule Base to produce fuzzy conclusions, and the Defuzzifier converts these conclusions into crisp outputs. According to Chen (2001), Tshe fuzzification is defining a map from the natural domain of discourse of the inputs and outputs to the fuzzy domain of discourse by giving the scale factors accordingly to represent the experience from experts for the inputs and outputs to 'translate' (fuzzify) the natural domain to fuzzy domain, which is easy to 're-translate' (defuzzify) reversely.

Fuzzification Module

The fuzzifier maps the crisp input into a fuzzy set (Karnik, 1999). For fuzzy logic systems, it is important to choose the appropriate method to crisp the human expertise and knowledge and construct the fuzzy sets, and the output is not unique. With the development of science and technology, various mathematical methods have been developed and applied to the fuzzification procedure. In this context, the main issue is choosing a method that is appropriate for obtaining the optimal control output. Additionally, the fuzzy inference engine determines the judgment and output of the fuzzy controller directly. For the purposes of this research, the fuzzy inference plays a very important role in controller design. The controller would be much less popular without a well-designed rule base and fuzzy inference.

Scale Factor Module:

Suppose the real input is x_0^* and the real input range is $[x_{\max}^*, x_{\min}^*]$, while the desire fuzzy input range is $[x_{\max}, x_{\min}]$, therefore:

$$x_0 = \frac{x_{\min} + x_{\max}}{2} + k(x_0^* - \frac{x_{\min}^* + x_{\max}^*}{2}) \tag{2.3.21}$$

where k is the scale factor and is obtained from:

$$k = \frac{x_{\max} - x_{\min}}{x_{\max}^* - x_{\min}^*} \tag{2.3.22}$$

Suppose the inputs are $x \in X$, $y \in Y$ and the output is $z \in Z$, where X, Y, Z are respectively the fuzzy domain of discourse representing *x*, *y* and *z*. The rules in the rule base have the general expression:

where $i \in [1, m]$; $j \in [1, n]$; $k \in [1, o]$ and $p \in [1, q]$; X_i , Y_j and Z_k are the defined membership functions for *x*, *y* and *z*, respectively, and *p* denotes the number of rules. Here, X_i and Y_j are defined as antecedent membership functions while Z_k is defined as the consequent membership function, so that the results of rule inferences will be fuzzy.

Rules Inference Module

The fuzzy control rule base is generated from language, either in a series of propositions “if… then …”, or in a control rule table. For a 2-D Mamdani-type controller, the if-then rule always takes the following form:

Rule p: if x is X_i and y is Y_j, then z is Z_k

After the construction of the rules, the rules composition principles are triggered accordingly.

Proportion Factor Module

The coefficient used to transfer the fuzzy domain of discourse to physical domain of discourse is called proportion factor, denoted as k_u .

Defuzzification Module

The fuzzy results must be subjected to defuzzification as they cannot be used directly as system inputs. As previously mentioned, defuzzification can be achieved through several methods: the centroid method, maximum of membership method, bisector method, etc.

Therefore, the design procedure of the Mamdani fuzzy control system consists of the following steps:

- Determine the natural domain of discourse for inputs and outputs
- Determine scale factors and obtain the fuzzy domain
- Determine inference engine (language base and fuzzy rules base).
- Determine defuzzification method and obtain the distinct data.
- Connect the controller output to the system.

2.3.4 T-S Type Fuzzy Control

The fuzzy values that are the inference results of the Mamdani-type controller cannot be used directly to drive control objects without defuzzification. Furthermore, it is not convenient to perform mathematical analysis for the system containing fuzzy values. To overcome these limitations, Takagi and Sugeno (1985) introduced a new type of fuzzy control inference, namely, the T-S-type fuzzy inference. This model is especially feasible for control of sectional-type systems, as well as for fuzzy modelling.

Consider a fuzzy proposition as: If x_1 is A_1 and x_2 is A_2 , then u is U . For this proposition, if we further know that x_1 is A_1^* and x_2 is A_2^* , then we can infer the new proposition that u is U^* .

Again, considering a linear system that can be controlled piecewise, the stated inference can be modified as: "If x_1 is A_1 and x_2 is A_2, then $u = f(x_1, x_2)$ ", where output u is a numerical function related to the actuation of the inputs x_1 and x_2 (without defuzzification), while A_1 and A_2 are fuzzy sets.

1) Commonly used T-S type fuzzy control:

There are two applications of T-S fuzzy control:

0-order T-S type fuzzy controls: If x_1 is A_1 and x_2 is A_2, then $u = k$

1-order T-S type fuzzy control: If x_1 is A_1 and x_2 is A_2, then $u = px_1 + qx_2 + r$

Where *k*, *p*, *q* and *r* are constants.

When using *n* T-S fuzzy rules to describe a system, suppose the input is x_i , it is impossible to correlate with only one rule but several rules. Therefore, suppose the i^{th} rule is denoted as R^i , then:

R^i: If x_1 is A_1^i and x_2 is A_2^i, then $u_i = k_i$ ($i = 1, 2, \ldots, n$) (for 0-order)

Or: R^i : If x_1 is A_1^i and x_2 is A_2^i, then $u_i = p_i x_1 + q_i x_2 + r_i$ ($i = 1, 2, \ldots, n$) (for 1-order)

2) Algorithms to obtain output u

Weighted Summation (wtsum):

$$U = \sum_{i=1}^{m} w_i u_i = w_1 u_1 + w_2 u_2 + \cdots + w_n u_m \tag{2.3.23}$$

where w denotes the weight of the rule in the total output.

Weighted Average (wtaver):

$$U = \frac{\sum_{i=1}^{m} w_i u_i}{\sum_{i=1}^{m} w_i} = \frac{w_1 u_1 + w_2 u_2 + \cdots + w_n u_m}{w_1 + w_2 + \cdots + w_n} \tag{2.3.24}$$

The typical T-S fuzzy control diagram is shown as follows:

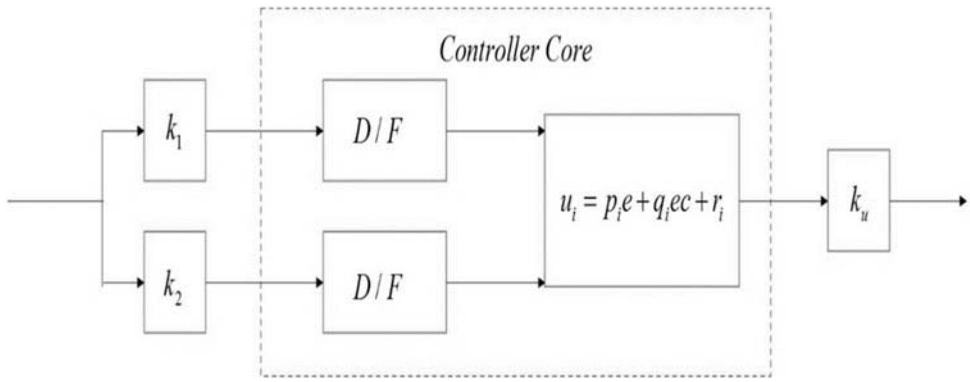

Figure 2-6 T-S Type Control Block Diagram

2.3.5 Type-2 Fuzzy Control

The introduction of the type-2 fuzzy set fostered the development of the algorithm of type-2 fuzzy logic system in recent years. The defining characteristic of a type-2 fuzzy set is that its membership is not clear number or boundaries, but it is derived from another series of membership functions. First introduced by Zadeh in 1975, the type-2 fuzzy set has enabled computers to effectively deal with both linguistic and numerical uncertainty. Since then, there has been a proliferation of studies on type-2 fuzzy sets. Liang and Mendel (1999) were the first to introduce type-2 TSK fuzzy models, outlining the difference between those models and the Mamdani fuzzy model. Consequently, Karnil and Mendel (2001) introduced the operations on type-2 fuzzy sets and Mendel (2002) provided an overview of type-2 fuzzy control. Since the emergence of fuzzy control, it has been used in many applications, such as robots (Lu and Liu, 2016), water tank level control (Galluzzo and Cosenza, 2011), decision-making (Naim and Hagras, 2012) and database design (Niewiadomski, 2010). According to Li et al (2008, 2009), there are also several methods to realize the 3-D type of control. They use three levels of fuzzy sets, which is also called type-3 fuzzy control. However, most of them are firstly using the first (primary) level of set to create the model of the plant. It is very efficiency indeed, to facilitate the consequent fuzzy control systems design process. As it is well known that fuzzy control is very customized type rather than a universal type for design, such fuzzy control system design can be only applied to that plant and cannot be generalized from the very beginning. Secondly, the establishment the three levels of fuzzy sets is usually adopted three different dimensions independently, which cannot efficiently exhibit the link between different levels of dimensions (or, variables). Type-2 fuzzy sets shows clearly the relationship and the influence that the secondary fuzzy sets to the primary fuzzy sets.

Figure 2-6 shows the basic structure of the type-2 fuzzy control. By contrast to type-1 fuzzy control, the fuzzy output sets must undergo the step of type reduction to obtain the explicit output. The type-2 fuzzy control has been the focus of numerous theoretical and experimental studies and it has been employed in various areas. Hargas (2004) put forward a hierarchical type-2 fuzzy logic control architecture and applied it to the control of autonomous mobile robots. Li et al. (2007) proposed a decoupled interval type-2 fuzzy sliding-mode controller for controlling the chaos in systems and obtaining a better control performance. Wang and Li (2008) presented fuzzy modelling of dynamic systems with measurement noise. Abbadi et al. (2013) developed an interval type-2 T-S fuzzy controller for non-linear voltage. The proposed controller has been applied to two-generator infinite bus power system. Apart from engineering applications, the type-2 fuzzy control is also used in other areas, such as in exchange rate modelling and prediction (Medina, 2006), and in ATM networks via type-2 fuzzy logic systems (Liang, 2000).

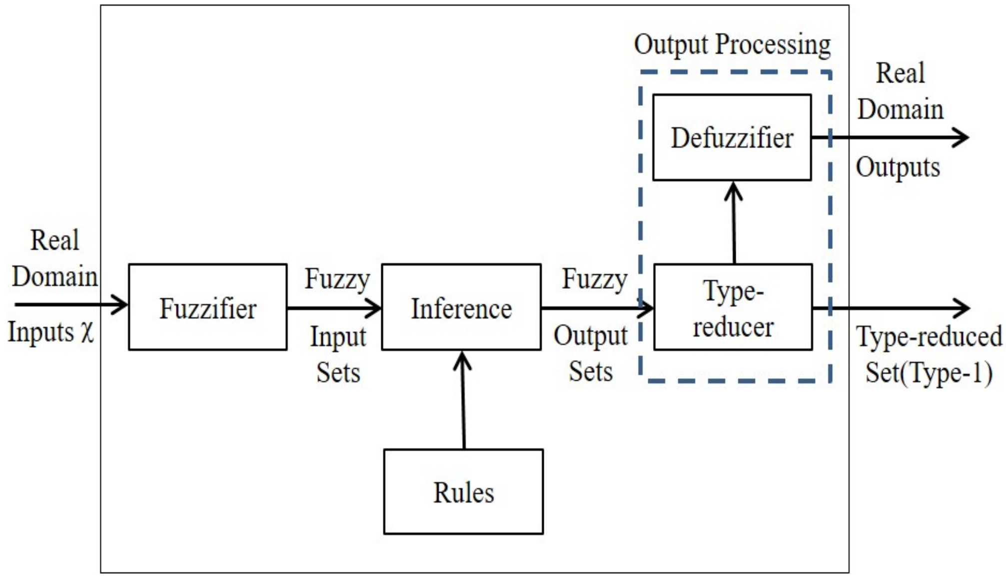

Figure 2-7 The Structure of Type-2 fuzzy control

Footprint of Uncertainty

Figure 2-8(a) shows a curve for type-1 triangle membership function, while Figure 2-8(b) shows a projection for type-2 triangle membership function. In (a), $\mu_A(x)$ is the membership function of the degree that a certain value x' belongs to fuzzy set A, from which it is very clear that the degree of membership can be immediately determined by determining the value of x' and the membership function. However, in (b), $\mu_A(x)$ is the so-called blurred membership function, which means that it is not a certain curve but covers a range of space. The space is determined by secondary membership function while correspondingly $\mu_A(x)$ is called main membership function. Consequently, the shadow shown in (b) can also be considered as the possible position or footprint of the membership. Therefore, a type-2 fuzzy set is defined mathematically as follows:

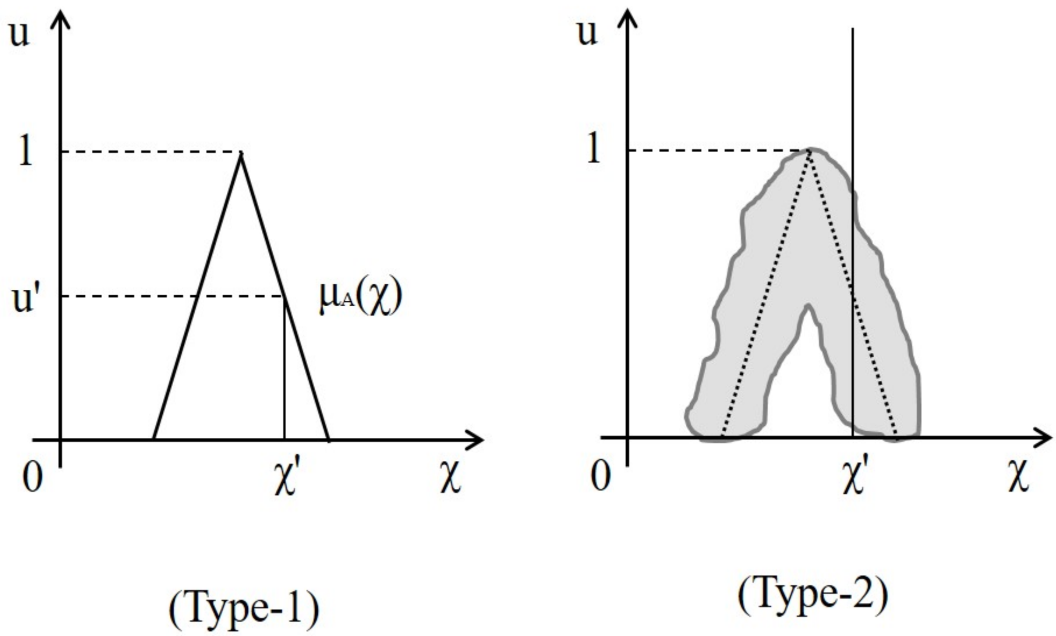

Figure 2-8 Comparison of Type-1 and Type-2 Membership Functions (© 2006 IEEE reprinted from Mendel et. al, 2006, Fig. 1. (a) Type-1 MF. (b) Blurred type-1 MF.)

Therefore, a type-2 fuzzy set is defined mathematically as follows:

$$\tilde{A} = \left\{((x,u), \mu_{\tilde{A}}(x,u)) \,\middle|\, \forall x \in X, \forall u \in J_x \subseteq [0,1]\right\} \quad (2.3.25)$$

Where $\tilde{A}$ denotes a type-2 fuzzy set, the main membership is $0 \le \mu_{\tilde{A}}(x) \le 1$, and the secondary membership is $0 \le \mu_{\tilde{A}}(x,u) \le 1$. If the value of every secondary membership is equal to 1 (namely $u_i = 1$), $\tilde{A}$ is called Interval Type-2 fuzzy set (IT2-FS).

Apart from the fact that they rely on type-2 fuzzy sets, type-2 fuzzy rules are similar to type-1 fuzzy rules in terms of the "if-then" structure.

Interval Type-2 Fuzzy Relationship

Define $U,V \neq \varnothing$ as two domain of discourses, and $\overset{\circ}{F}(U \times V)$ denotes the sum of all interval type-2 fuzzy sets in domain of discourse $U \times V$ (Robert, et al, 2006). Therefore, use $\overset{\circ}{R}$ to denote interval type-2 fuzzy relationship from domain U to V, and the relationship is denoted as:

$$\overset{\circ}{R} \in \overset{\circ}{F}(U \times V) \tag{2.3.26}$$

Therefore, if $\forall (u,v) \in U \times V$, then use $\overset{\circ}{\mu}(u,v)$ to denote the degree of u and v with relationship $\overset{\circ}{R}$ as follows:

$$\overset{\circ}{\mu}(u,v) = [\underline{\mu}(u,v), \overline{\mu}(u,v)] \subseteq [0,1] \tag{2.3.27}$$

Where $\underline{\mu}(u,v)$ denotes the lower boundary and $\overline{\mu}(u,v)$ denotes the upper boundary of membership, and shall meet the following relationship:

$$0 \leq \underline{\mu}(u,v) \leq \overline{\mu}(u,v) \leq 1 \tag{2.3.28}$$

Consider most of the situations in practice, number of membership function is finite, and use discrete domain of discourse as follows:

$$\begin{aligned} U &= \{u_1, u_2, \ldots, u_m\} \\ V &= \{v_1, v_2, \ldots, v_n\} \end{aligned} \tag{2.3.29}$$

Therefore, substitute (2.3.29) into (2.3.26), interval type-2 fuzzy relationship can be represented as a matrix as:

$$\tilde{R} = \begin{pmatrix} \tilde{\mu}(u_1, v_1) & \ldots & \tilde{\mu}(u_1, v_n) \\ \vdots & \ddots & \vdots \\ \tilde{\mu}(u_m, v_1) & \ldots & \tilde{\mu}(u_m, v_n) \end{pmatrix} = \begin{pmatrix} [\underline{\mu}(u_1, v_1), \overline{\mu}(u_1, v_1)] & \cdots & [\underline{\mu}(u_1, v_n), \overline{\mu}(u_1, v_n)] \\ \vdots & \ddots & \vdots \\ [\underline{\mu}(u_m, v_1), \overline{\mu}(u_m, v_1)] & \cdots & [\underline{\mu}(u_m, v_n), \overline{\mu}(u_m, v_n)] \end{pmatrix} \tag{2.3.30}$$

Where $\tilde{\mu}(u_i, v_j) = [\underline{\mu}(u_i, v_j), \overline{\mu}(u_i, v_j)] \subseteq [0,1]$ and $i = 1, 2, \cdots, m; j = 1, 2, \cdots, n$.

Type-2 Fuzzy System Inference

Consider a type-2 fuzzy system with M rules, and for each rule there are p inputs and 1 output. Use R^l to indicate the l^{th} rule as follows:

R^l : If x_1 is $\tilde{F}^l_1$, x_2 is $\tilde{F}^l_2$, … and x_p is $\tilde{F}^l_p$, Then y^l is $\tilde{G}^l$ (Mamdani Type)

R^l: If x_1 is $\tilde{F}^l_1$, x_2 is $\tilde{F}^l_2$, … and x_p is $\tilde{F}^l_p$, Then $y^l = f(x_1, x_2, \cdots, x_n)$ (T-S Type)

If R^l is activated, membership function, $\mu_{\tilde{B}^l}(y)$, of type-2 fuzzy set is:

$$\mu_{\tilde{B}^l}(y) = \sqcup_{x \in X} \left[\mu_{\tilde{A}_x}(x) \sqcap \mu_{\tilde{A}^l \to \tilde{B}^l}(x, y) \right] \tag{2.3.31}$$

Where X is a Cartesian Product Space with p dimensions for $X = X_1 \times \cdots \times X_p$, and X_k is the domain of discourse of input x_k (k=1,…,p).

Type Reduction

For type-1 fuzzy control, the deffuzifier provides the crisp output and enables a type-0 set (traditional set) to be obtained from the type-1 fuzzy set to determine the movements of actuators. On the other hand, for type-2 fuzzy control, the direct output of the deffuzifier produces a type-1 fuzzy set, which cannot be applied to actuators directly. Hence, Zadeh proposed an extension principle (Dubois and Prade, 1980) to provide type-1 fuzzy sets. This subsequently fostered research on type reduction of type-2 fuzzy sets, considered to be an important method. Liu (2008) proposed an efficient centroid type-reduction strategy and generalized the strategy to generic type-2 fuzzy logic systems. Sepulveda et al. (2007) obtained an interval set of type reducer and placed the experimental study of type-1 and type-2 fuzzy controllers. The authors concluded that type-2 fuzzy sets were powerfully robust, owing to their flexibility to achieve modelling even when data were ambiguous (Sepulveda et al., 2007).

Although type-2 fuzzy sets have been developed for several decades, they are yet to achieve the level of development of type-1 fuzzy sets. Referring to the reasons behind the phenomenon, the biggest obstacle is the excessive calculation and possible time delay during simulation. This is because of the three-dimensional nature of a type-2 fuzzy set which is difficult to display via image, and also introduces difficulty into rule inference. However, type-2 fuzzy logic system is definitely advantageous in describing systems beyond mathematical language.

2.4 Deprygenation Tunnel

In this section, the depyrogenation tunnel will be introduced in brief detail, in order to understand how the machine operates in general. The machine’s specifications will also be introduced and will serve as a reference or measuring point for the calculations that will be used throughout the course of this project. The methods for modelling the plant and the design of the control system will also be touched on alongside with some of the general ideas and formulas.

2.4.1 Application

Over the years, the biochemical pharmaceutical industries have experienced a variety of changes, ranging from new development processes and high performance equipment to stricter and tight regulations. As a consequence, the control processes are becoming more various, more demanding, as well as more challenging. The aims of such control processes are to ensure a higher purity of finished products while minimising the consumption of raw materials and consequently the consumption of energy. For example, in the case of a depyrogenation tunnel, which is widely used in pharmaceutical filling lines for sterilization and drying purposes, the energy consumption is one of the most important factors behind choosing a model or another. One way of dealing with such a problem is to develop modelling and control strategies that will help in minimising the energy consumption burden.

In the field of measurement and instrumentation, there is always a need for precise models capable of capturing most details of a system - if not all of them. In fact, as observed by Marconato et al. (2014), "there is an ever-increasing demand for good models". The development of good models is driven by either the need to increase understanding of the plant or the need to support further analysis and design objectives.

High-performance depyrogenation tunnels are non-linear by nature. Closely related to the machines' dynamic properties, this non-linear behaviour is engendered by an increase in the number of the machines' physical properties. As a result of such changes, basic mathematical models have grown in complexity, making it increasingly more difficult to capture the exact details of these systems using existing physical structures (Zhu et al., 2015). Traditionally, depyrogenation tunnels have been modelled as single input single output systems (SISO), making them relatively very easy to model and control. Unfortunately, with the increase in the number of parameters and thus in complexity, MIMO-based models are now required. This will require the development a different strategy for modelling and controlling them, which will be significantly challenging because of their non-linear nature.

2.4.2 Description

A depyrogenation tunnel is a universally accepted and widely used device for sterilisation purposes. More specifically, it is used to remove the pyrogen, also known as bacterial endotoxin, from the physical components, such as stoppers, tubing or vials, which then come directly into contact with the injection drug products (FDA, 2015). Although all the physical components are pre-washed and washed with water for injection, the pyrogen adhering to the inner surface of these physical components is rather difficult to eliminate completely by dilution. Therefore, according to some regulations, such as Chinese Pharmacopoeia (2010 Ed.), a FH value is specified to represent the relationship between temperature and duration of depyrogenation rate: destruction of the pyrogen can be achieved by exposing the components to a temperature of 320ºC for at least five minutes.

The general structure and functions of a depyrogenation tunnel are divided into three sections, pre-heating, heating and cooling. The schematic diagram of the tunnel operation is shown below.

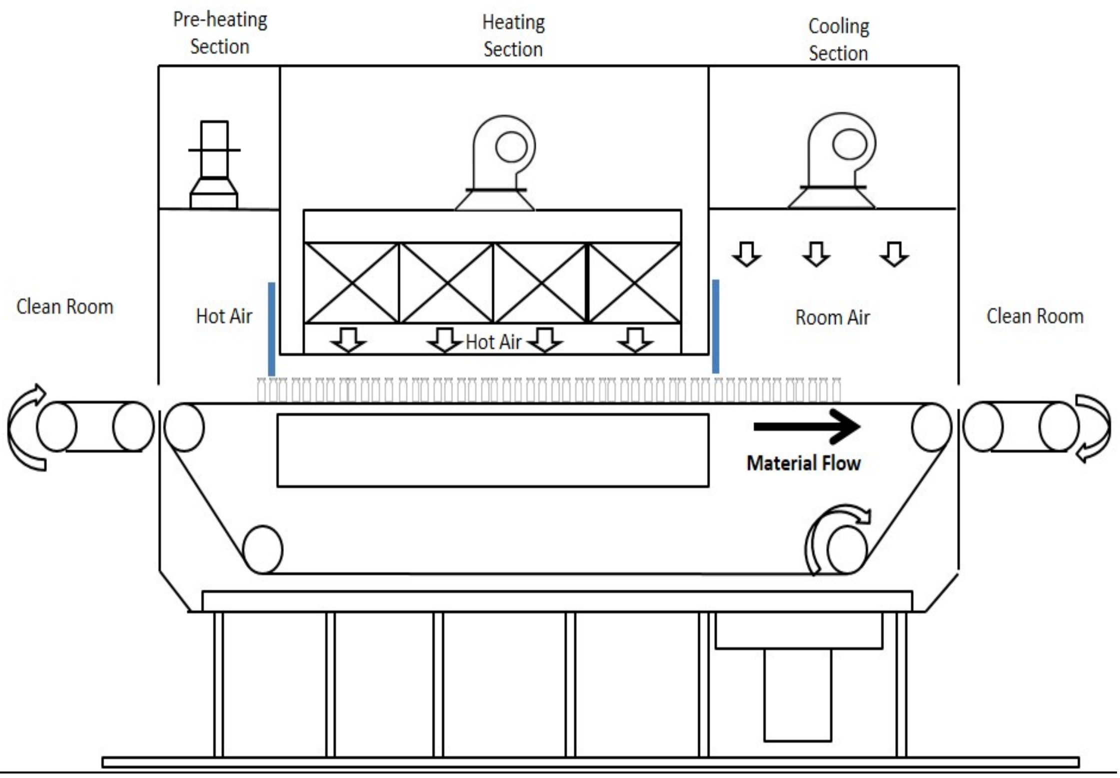

Figure 2-9 Systematic Diagram of Depyrogenation Tunnel

Requirements and problems specification

The components to be sterilised are transferred by conveyor belt through the whole equipment, beginning with the preheating section, then the heating section and finally the cooling section. The components are required to be kept in the heating section for at least five minutes at a temperature of 320ºC. In view of the accepted industrial process, the depyrogenation tunnel is designed with one hot air inlet for each section, with two sensors located at the pipe exit, one for temperature control and the other one for air pressure observation, which is obviously a typical lumped parameter system structure. However, it has been observed that the components of stoppers, vials and tubing of minimal weight are very sensitive to air pressure variation. Therefore, the pressure should be considered as a critical parameter instead of just a parameter for observation.

2.5 Conclusions

This chapter mainly consists of background description and literature review of this research, and the content is discussed from four dimensions:

- Spatially dynamic distributed systems, which is also the object of this research.
- State-space approach, which is used for system analysis and modelling.
- Fuzzy logic and fuzzy control, which is adopted for control system design.
- The Dyprygenation tunnel, which is the application case study.

In the part of spatially dynamic distributed systems, a general definition or description is given: a system with parameters, or outputs, or inputs spatially distributing, is categorised as a spatially dynamic distributed system. According to this description, in fact most of our systems, especially those commonly used in industry, are spatially dynamic distributed systems. However, for many reasons, in practical almost all the systems treated as lumped parameter ones, which will undoubtedly lose some precision. Sometimes such simplification is a feasible shortcut to deal with specific issues, but in some other situations it will expose the whole system to unknown risks, as the change of conditions. This section answers the question that why there's a need to study spatially dynamic distributed systems when there are already mature simplification approaches.

The consequent section is state space approach. In this chapter, state space approach is given for modelling, not only for its feasibility but also for its feasible superposition and the nature of involving many parameters / states. This is because the state space approach treat the parameters as state, thus the state is easily overlaid onto the state equations. As to this research, State-space approach suits the purpose of expanding SISO to MIMO, and taking another parameter into account. Therefore in this chapter the brief introduction is firstly made to reveal its characteristics and then the recent outcomes and development are reviewed; the results revealed that the work for 2 by 2 non-standard form of pole placement approach has not been studied very much. In this case, in this research the relevant work has been developed as well.

The third part is fuzzy logic and fuzzy control. It is employed for the design of the control systems for the sake that: on one hand, it can be established without a mathematical model, which would possibly introduce other facets affecting the precision of control system performance. In this chapter, two approaches are referred, Mamdani type and interval T-S type-2 fuzzy control, and also in this part the difference between a type-2 fuzzy control and a 3-D fuzzy design (though both of them are of three-dimensional nature) is also addressed.

The last section is the introduction and application of a Deprygenation Tunnel. As this term rare shows up in common use, it is well explained in this research including its description and application. It is a specific plant used mainly in biochemical pharmaceutical industry, and is a live, typical sample of a spatially dynamic distributed system. The consecutive conveying of vials (glass bottles) represents spatial distribution, while the physical structure of this plant is a design of a lumped parameter system.

3 Methodology

In the following part, the methodology adopted to address the issue is introduced, and the general procedure is outlined in Figure 3.1.

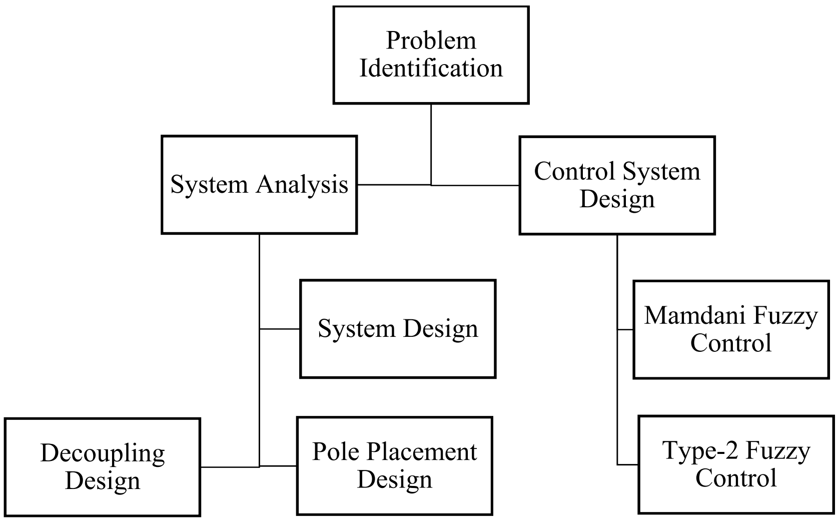

Figure 3-1 Design Methodology

3.1 State Space Modelling Approach

3.1.1 Model assumption

According to Wang et al. (2014), when modelling plant equations for a system, there are several critical requirements that need to be taken into consideration beforehand. Assuming that the vials are to be fed into the machine, the follow requirements and simplifications must be complied with:

1. All batches of vials must be heated to the required temperature for a specified duration of time, as per the specification mentioned by the regulation.

2. All the vials must meet the requirements where the temperature is kept higher than the one specified by the regulation for a set period of time.

3. Vials are extremely sensitive to pressure and therefore careful attention must be paid when they are subjected to sterilisation. If the pressure is increased abruptly or the pressure difference in the plant exceeds the safety limit, the vials may be damaged, resulting in system error. This must be avoided since production may need to be halted.

4. There should only be one inlet for hot and dry air.

5. A pair of sensors to check for temperature and pressure is needed to use as reference for the controller design.

6. The system assumes that it is possible to control the process based on the lumped parameter formulations.

3.1.2 System Identification

In addition to the specifications mentioned above, it is necessary to simplify the plant to ensure that the modelling and design process are facilitated. To this end, several assumptions have to be applied. Although the three segments of the plant are similar in terms of operability, the heating segment is considered to be the most important of the three. Therefore, this segment will be utilized for modelling and simulation during the course of this project.

The target object entering the machine does not have its own internal heat source.

The initial temperature of the object is maintained after passing through the pre-heating segment of the machine.

The plant is assumed to be operating in a stable manner with a fixed supply of air.

Heat convection and heat radiation are ignored and only heat conduction between the hot air and vials is considered.

3.2 Pole Placement Control

Research and development on transfer functions revealed that the stability of the designed system depended on the poles of the denominators as well as that alterations in system performance could be achieved by allocating the pole positions correctly.

By contrast to SISO, linear and time-invariable systems, which have been the focus of the majority of studies on pole placement, dynamic systems have not been studied yet.

3.2.1 Assign closed loop poles

The following expression gives the transfer function matrix between output *Y(s)* and reference *V(s)*:

$$\frac{Y(s)}{V(s)} = G(s) = \left\{ \left[C + DF_s \right] \left[sI - A - BF_s \right]^{-1} B + D \right\} H$$

$$= \left\{ \left[C + DF_s \right] \left[sI - \overline{F}_s \right]^{-1} B + D \right\} H \tag{3.2.1}$$

$$= \left\{ \left[C + DF_s \right] \frac{adj\left[sI - \overline{F}_s \right]}{\det\left[sI - \overline{F}_s \right]} B + D \right\} H$$

With

$$\overline{F}_s = A + BF_s \tag{3.2.2}$$

Thus, based on certain specifications, it is possible to separate the controller design into allocation of poles and zeroes, which is referred to as the pole and zero allocation approach.

The state feedback control law underpins the derivation of (3.2.1).

$$u(t) = F_s x(t) + Hv(t) \tag{3.2.3}$$

A closer look at (3.2.1) reveals that the denominator determines the closed loop poles (closed loop characteristic polynomial).

$$\Delta(s) = \det\left[sI - \overline{F}_s \right] \tag{3.2.4}$$

Let the allocation of the target closed loop poles be undertaken in the following way:

$$\Delta(s) = \prod_{i}^{n} (s + a_i) = \sum_{i=0}^{n} \alpha_i s^{n-i} \tag{3.2.5}$$

There is equivalence between (3.2.4) and (3.2.5) in order to derive the state feedback matrix F_s.

$$\det\left[sI - \overline{F}_s \right] = \sum_{i=0}^{n} \alpha_i s^{n-i} \tag{3.2.6}$$

To obtain a series of n linear equations, the coefficients related to s^i on both sides of (3.2.6) must be equivalent (since $\alpha^0 = 1$, the equation related to s^n can be eliminated):

$$\begin{aligned} \mu_1(F_s) &= \alpha_1 \\ &\vdots \\ \mu_n(F_s) &= \alpha_n \end{aligned} \tag{3.2.7}$$

With the linear function of F_s being denoted by μ_i (.).

Theorem: The state feedback law in (3.2.3) can be used to allocate the closed loop poles at random, provided that the system *S (A, B, C, D)* from (2.2.1) is fully state controllable.

3.2.2 Assign closed loop zeros

To make sure that precise equivalence exists between the actual output *y(t)* and the reference *v(t)* upon satisfaction of stable state requirements, the input feedforward gain *H* has to be selected. In other words, the innate stable state gain of the closed loop system is restrained based on selection of *H*. Hence, regarding (3.2.1) and based on the premise of a step reference *v(t)*, it is logical to choose:

$$H = \left\{ \left[C + DF_s \right] \left[sI - \overline{F}_s \right]^{-1} B + D \right\}_{s=0}^{-1} = -\left\{ \left[C + DF_s \right] \left[\overline{F}_s \right]^{-1} B - D \right\}^{-1} \qquad (3.2.8)$$

The first step in the case of MIMO systems is to verify that they are controllable. If controllability is ascertained, system transformation into two SISO state-space systems can be undertaken to achieve the pole placement design.

3.3 Decoupling Control

In linear multivariable systems, a change in any input will usually result in changes in all outputs. Such systems are characterized by coupling or interaction. It may be useful for certain applications to obtain a system in which such interaction between controls does not occur (Soponariu and Lupu, 2014). In the last ten years, considerable attention has been paid to designing multivariable systems in such a way so as to prevent interaction or coupling (Zheng et al., 2009). The design objective of non-interacting (or decoupled) systems is to obtain a system in which each input affects only one output. The primary advantage of such a design is that once non-interaction is achieved, the system is reduced to a number of single input/single output channels (subsystems) to which the well-established design techniques may be applied.

In the time domain analysis, the implication of non-interaction is not apparent and can only be expressed through a complicated mathematical relation. Therefore, it is essential to apply state feedback to convert the open loop state-space model into a closed loop model (Siekmann et al., 2003).

To ensure simplicity without losing generality, the plant model *S(A, B, C, D)* (matrix *D* = 0) is considered. The first step is to obtain the state feedback controller in the following way:

$$u(t) = F_s x(t) + Hv(t) \tag{3.3.1}$$

Where *v(t)* is the reference input of the controller. Let the differential equations of a linear multivariable system that is decoupled into m first order subsystems be expressed as

$$\begin{aligned} \overset{g}{y}(t) &= M_o y(t) + \Gamma v(t) = M_o Cx(t) + \Gamma \\ \Gamma &= diag\{\gamma_i\} \end{aligned} \tag{3.3.2}$$

where $\Gamma = diag\{\gamma_i\}$, such that M_0, that is, any i^{th} output is affected by the i^{th} input, and M_0 is a diagonal matrix. The characteristic polynomial of this decoupled system is

$$\det(sI - M_o) \tag{3.3.3}$$

If the decoupled structure above were to be obtained by using a state feedback control law in (3.3.1), the closed loop differential equations

$$\dot{y}(t) = C\dot{x}(t) = C(A + BF_s)x(t) + CBHv(t) \tag{3.3.4}$$

And comparing (3.3.4) with (3.3.2) to have the equivalent equations

$$\begin{cases} M_o C = C(A + BF_s) \\ CBH = \Gamma \end{cases} \tag{3.3.5}$$

Then the state feedback matrix Fs and the input forward matrix H can be designed.

$$\begin{aligned} F_s &= (CB)^{-1}[M_o C - CA] \\ H &= (CB)^{-1}\Gamma \end{aligned} \tag{3.3.6}$$

Non-singularity of CB is thus the primary requirement for the existence of the pair of matrices F_s and H, called the decoupling pair. The control law of (3.3.1) formed out of these two matrices is known as the state feedback decoupling control law. With regard to (3.3.6), it is important to mention that the elements of M_0 and Γ, specifying the poles and gains of the m decoupled channels, respectively, can be chosen freely while preserving input and output non-interaction.

After application of the decoupling approach, the following system is obtained:

$$G_o(s) = G_c(s)G_p(s) = \begin{bmatrix} G_{c11}(s) & G_{c12}(s) \\ G_{c21}(s) & G_{c22}(s) \end{bmatrix} \begin{bmatrix} G_{o1}(s) & 0 \\ 0 & G_{o2}(s) \end{bmatrix}$$

(3.3.7)

Where $G_o(s)$ denotes the control system, $G_c(s)$ denotes the controller, and $G_p(s)$ denotes the plant or object, and the design methodology diagram for case study is shown in Figure 3-2.

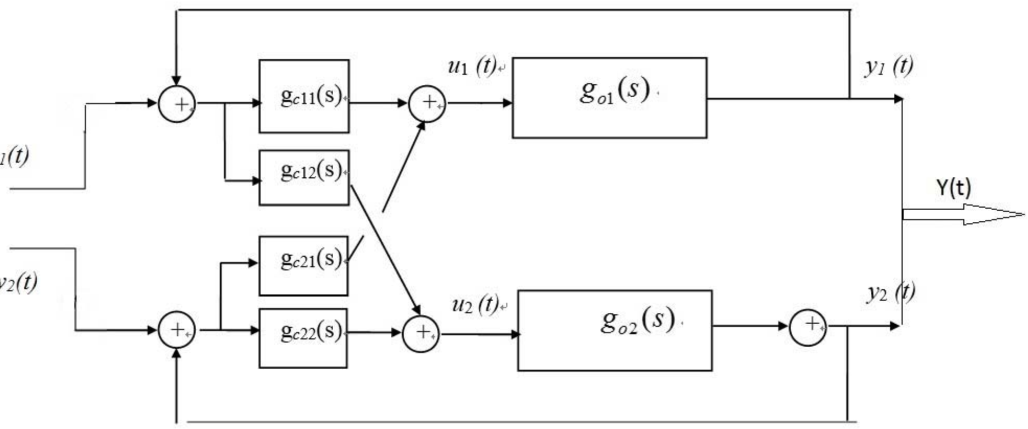

Figure 3-2 General Diagram of Decoupling Approach

3.4 Mamdani Fuzzy Control

Considering the following system after the decoupling approach:

$$G_o(s) = G_c(s)G_p(s) = \begin{bmatrix} G_{c11}(s) & G_{c12}(s) \\ G_{c21}(s) & G_{c22}(s) \end{bmatrix} \begin{bmatrix} G_{o1}(s) & 0 \\ 0 & G_{o2}(s) \end{bmatrix} \tag{3.4.1}$$

Where $G_o(s)$ denotes the control system, $G_c(s)$ denotes the controller, and $G_p(s)$ denotes the plant or object, and the design methodology diagram for the case study is shown in Figure 3-2.

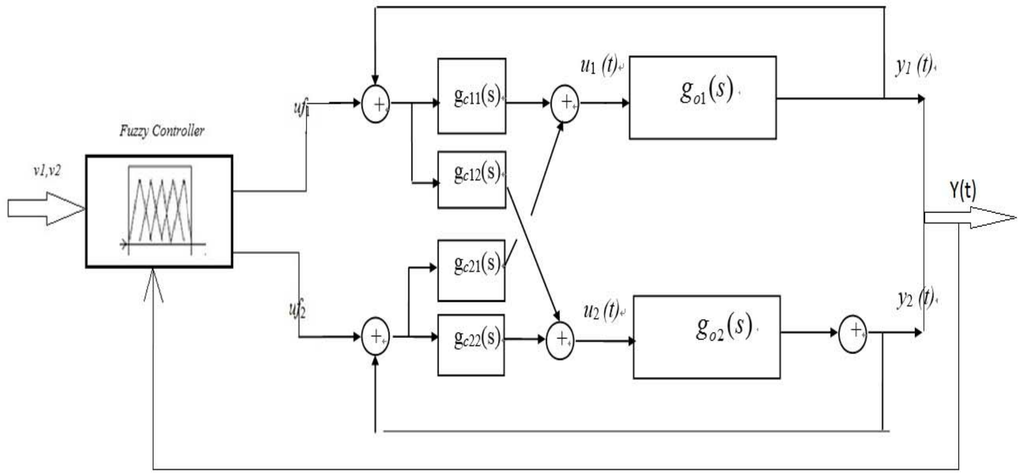

Figure 3-3 General Diagram of Fuzzy Control on Decoupling Approach

As the depyrogenation tunnel model is using the state-space approach and implementing state feedback decoupling method for first phase design, the controller design should contain the following key points:

As the physical model is sensitive to the environment and inputs, this paper uses the maximum of membership (MOM) approach for defuzzification because it provides greater ease of access compared to the other methods of defuzzification. For simplicity, the design includes two inputs dealing with Error and Error speed, and one output as input of the mathematical model. It is supposed that *e* and *ec* are the two inputs of the controller and *uf* is the output, while *r* is the number of rules in the knowledge base. *UF [uf1, uf2]* is defined as an interface connecting to the two inputs of the model, while in this design phase the *uf2* is set as 1, reserved for the following design.

The state feedback approach is applied to determine the natural domain of Error E (t) (e1(t), e2(t)) and Error Change EC(t) (ec1(t), ec2(t)) from the two states X(t) (x1(t), x2(t)) as the given reference input V (v1, v2). Error E and EC are expressed as follows:

$$E(\mathrm{t}):\begin{cases} e_1(\mathrm{t}) = x_1(\mathrm{t}) - v_1 \\ e_2(\mathrm{t}) = x_2(\mathrm{t}) - v_2 \end{cases} \tag{3.4.2}$$

$$EC(\mathrm{t}):\begin{cases} ec_1(\mathrm{t}) = \dfrac{e_1(\mathrm{t}) - \mathrm{e}_1(\mathrm{t}-1)}{Ts} \\ ec_2(\mathrm{t}) = \dfrac{e_2(\mathrm{t}) - \mathrm{e}_2(\mathrm{t}-1)}{Ts} \end{cases} \tag{3.4.3}$$

Where *Ts* is the sampling time interval and denoted as 1 in this design.

To create the control system, the output of controller UF(t) (uf1(t), uf2(t)) must be connected to the decoupling system.

The expression of the designed system is denoted as follows:

$$S(A,B,C,D):\begin{cases} u(t) = Fs * X(t-1) + H * v(t-1) & (input) \\ X(t) = A_d * X(t-1) + B_d * u(t-1) & (state) \\ Y(t) = C * X(t-1) & (output) \end{cases} \tag{3.4.4}$$

Therefore, the fuzzy control system is expressed as follows:

$$\begin{cases} u(t) = Fs * X(t-1) + H * UF(t-1) & (input\ equation) \\ X(t) = Ad * X(t-1) + Bd * u(t-1) & (state\ equation) \\ Y(t) = C * X(t-1) & (output\ equation) \end{cases} \tag{3.4.5}$$

Where $\begin{matrix} u(t) = [u_1(t) \quad \dots \quad u_m(t)]^T \\ y(t) = [y_1(t) \quad \dots \quad y_l(t)]^T \\ \dot{x}(t) = [x_1(t) \quad \dots \quad x_n(t)]^T \end{matrix}$, Ad and Bd indicate the discrete state-space structure. UF(t) is the output of the controller and u(t) is the output of the decoupling controller.

Since this controller consists of two inputs and one output, the fuzzy inference can be established in the following way:

$$\underline{U}F = (\underline{E} \times \underline{E}C) \circ R \tag{3.4.6}$$

Where E and EC are the antecedent membership functions and UF is the consequent membership function, R is the rule base, and o denotes the fuzzy implication operator, which is the relation composition.

3.5 Interval Type-2 Fuzzy Control

3.5.1 Interval Type-2 Fuzzy Control

The results of system analysis revealed air pressure control to be a significant issue. Compared to temperature control, the nature of pressure control is as follows:

1. Sensitive to pressure variation. As the whole process is in an open space connected with the room, the pressure within the tunnel will be influenced by

many factors, such as occasional opening of the door, movements of inspectors or operators, slight change of pressure difference between the room and the tunnel for no apparent reason, and so on.

2. Due to its sensitivity to pressure change, air flow simulation and anticipation of its impact on the vials during pressure variation are challenging.

3. Compared with temperature control, there is no time delay, which is known as awful robustness.

4. Requires fast response of actuators.

Therefore, type-2 fuzzy control is used here to combine the experts' language with such uncertainty to optimise the performance of the control system.

3.5.2 Type-2 Fuzzification

In general, the input of a system is determined and a fuzzifier is designed to map the determined input values into series of fuzzy sets, which is similar to type-1 fuzzy sets. In the case of type-2 fuzzy control, the type-2 fuzzifier is intended to map the determined input values into a series of type-2 fuzzy sets.

3.5.3 Inference

In practice, the application of interval type-2 fuzzy inference can be simplified significantly. According to Karnik and Mendel (2001), in the case of an interval type-2 fuzzy system with N rules, the rules are considered as follows:

$$R^n: \text{If } x_1 \text{ is } \tilde{X}_1^n, \text{ and } \ldots \text{ and } x_I \text{ is } \tilde{X}_I^n, \text{ Then } y \text{ is } Y^n, \text{ where } n = 1, 2, \ldots, N. \quad (3.5.1)$$

Here, $\tilde{X}_i^n$ denotes interval type-2 fuzzy sets where $(i = 1, \ldots, I)$, and $Y^n = \left[\underline{y}^n, \overline{y}^n\right]$ denotes the interval output. Let $\underline{y}^n = \overline{y}^n$ refer to the first simplification, so that the output of each rule inference will become a number.

Give input vector as follows:

$$x' = (x_1', x_2', \cdots, x_I') \tag{3.5.2}$$

The inference is performed as the following steps:

Step 1: Calculation of the membership of x_i' on each X_I':

$$\left[\mu_{\underline{X}_i^n}\left(x_i'\right), \mu_{\overline{X}_i^n}\left(x_i'\right)\right] \qquad i = 1, 2, \cdots, I,\ n = 1, 2, \cdots, N \tag{3.5.3}$$

Step 2: Determination of the firing interval of the n^{th} rule: $F^n(x')$:

$$\begin{aligned} &F^n\left(x'\right) \\ &= \left[\mu_{\underline{X}_1^n}\left(x_1'\right) \times \cdots \times \mu_{\underline{X}_I^n}\left(x_I'\right), \mu_{\overline{X}_1^n}\left(x_1'\right) \times \cdots \times \mu_{\overline{X}_I^n}\left(x_I'\right)\right] \\ &= \left[\underline{f}^n, \overline{f}^n\right] \qquad (n = 1, \cdots, N) \end{aligned} \tag{3.5.4}$$

Step 3: Type Reduction

Normally, type reduction performs the combination of fired $F^n\left(x'\right)$ and the corresponding consequent of rules. This combination is usually achieved with the method of centre-of-sets type-reducer (Mendel, 2001), as shown below:

$$Y_{\cos}\left(x'\right)=\underset{\substack{f^{n}\in F^{n}(x')\\ y^{n}\in Y^{n}}}{\mathrm{U}}\frac{\sum_{n=1}^{N}f^{n}Y^{n}}{\sum_{n=1}^{N}f^{n}}=\left[y_{l},y_{r}\right] \qquad (3.5.5)$$

Here $\left[y_{l},y_{r}\right]$ is considered as minimum and maximum value of output membership function (also called switch point), which is normally calculated with the Karnik-Mendel (KM) algorithm (Karnik and Mendel, 2001).

3.6 Conclusions

This chapter introduces the methodologies that are used in this research, and the content is divided into two main parts, one is the part of modelling and system analysis, and the other is the control system design. As mentioned in Chapter 2, the state-space approach is employed for modelling and system analysis, and fuzzy control is used to design the control system. For modelling, to obtain a simple model that is closed to the mathematical model in practical use, the primary way of modelling is employed. In order to facilitate modelling, the relevant assumptions are made, and inputs, outputs and parameters are identified. And considered there are probably a coupled nature involved in this model, the pole placement control and decoupling control approach are used. Furthermore, in order to mount pole placement onto a non-standard-form state space model, an equivalent matrix transformation is designed.

The second part is the control system design. In this research, two inputs, two outputs and two states are identified, based on decoupling control. Furthermore, two control systems are designed for two inputs and outputs separately. For temperature control, the most commonly used 2-dimentional Mamdani type fuzzy control is proposed and also the inputs of the control system are determined as error (E) and error change (EC), and the approach of Maximum of Membership for defuzzification method is employed, due to the rigid lower limit of temperature requirements.

For pressure control, the designed system must be very sensitive to change, that is why the interval type-2 fuzzy control is chosen. There are two reasons to use interval type-2 fuzzy method. The first one is, as the response speed is highly demanded, large quantities of calculations which will cause the delay of response will be put away. Secondly, as the turbulence is spatially dynamic distributed, a type-2 fuzzy control to deal with the situation, to realize a 3-D control is required. Therefore, the primary membership in this chapter is determined by the pressure difference itself; and the secondary membership is determined by a 3-D random turbulence.

4 Plant Modelling

In this chapter, how the physical model is transformed into a state space model will be described step by step; after the SISO and MIMO model is created, the state feedback approach, discrecization will be applied to the model in order to test the response of this model. Furthermore, pole placement will be designed for the MIMO model to obtain its performance; decoupling approach will be also used to solve the coupled nature between inputs and outputs. It is worthy mention that, the work accomplished in this chapter is somehow independent to the following intelligent control system design, as the intelligent control system design and tune process doesn’t require a precise mathematical model of objects.

4.1 Plant Modelling

Plant modelling has a number of key specifications, which are outlined in the following part, with the vials serving as illustrations:

- Heating of all vial batches at a specified temperature and for a certain amount of time;
- The requirement of temperature exceeding regulation until a given time must be satisfied by each vial in every batch;
- Vials intended for sterilisation display sensitivity to pressure, and therefore the vials may collapse, resulting in system error or even disruption of batch production, if the pressure is unintentionally increased significantly or if the pressure difference in the plant is higher than the safety limit;
- A single inlet for hot and dry air is designed;
- A temperature and a pressure sensors serve as reference for the controller design;
- The working premise is that process control can be achieved based on lumped parameter formulations.

Apart from the above requirements, the plant must be simplified to make the modelling and design process easier, based on the following premises:

- Plant operation is assumed to be stable, with more or less the same amount of supply air;
- Solely heat conduction must be assumed to occur between the hot air and the vials, ignoring heat convection or heat radiation;
- There are close similarities between the three plant sections regarding operating mechanism and the most important is the heating section, which will be employed for modelling and simulation in the present study.

4.1.1 Plant Analysis

According to the systematic requirements, the temperature distribution and pressure difference must be homogeneous:

- Inputs: T_0, denoting heating air temperature, and P_0, denoting heating air pressure;

- Outputs: T_t, denoting the final object temperature, and P_t, denoting the final object pressure.

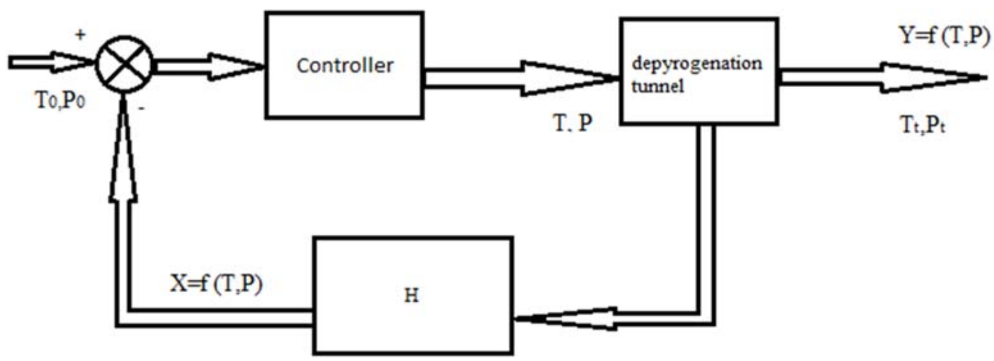

Figure 4-1 System Analysis of Depyrogenation Tunnel

The principle of energy conservation and the principle of pressure changing analysis are the two principles underpinning plant description.

The principle of energy conservation: The energy input from the hot air and the energy possessed by the vials must be the same as the final post-heating energy possessed by the air and by the vials:

$$Q_0(t)+\theta_0(t)=\theta_t(t)+Q_t(t) \quad (4.1.1)$$

$$Q_0(t) - Q_t(t) = \theta_t(t) - \theta_0(t) \tag{4.1.2}$$

The pre-heating amount of hot air heat is denoted by $Q_0(t)$, while the post-heating amount of hot air heat is denoted by $Q_t(t)$. The amount of object heat is denoted by $\theta_t(t)$.

The formula of Specific Heat Capacity is as follows:

$$c = \frac{Q}{M * \Delta T} \tag{4.1.3}$$

The specific heat capacity, vial mass and temperature discrepancy are respectively denoted by c, M, and ΔT.

Therefore, after a simple transform, the quantity of heat can be denoted as follows:

$$Q = c * M * \Delta T \tag{4.1.3}$$

By integrating (4.1.3) into (4.1.2) and modifying the format, (4.1.4) is obtained, illustrating the energy conservation of the plant:

$$c_a * M_a * (T_{0(a)} - (T_{t(a)}) = c_o * M_o * (T_{t(o)} - (T_{0(o)}) \tag{4.1.4}$$

The specific heat capacity and mass of hot air are respectively denoted by c_o and M_a, while the particular heat capacity and mass of the vials are denoted by c_a and M_o, respectively; the original and last temperature of the air are respectively denoted by $T_{0(a)}$ and $T_{t(a)}$, while the original and last temperature of the vials are respectively denoted by Q_a and $T_{t(o)}$.

(4.1.4) is simplified to (4.1.5), following the differential against time to the right-hand side of the equation:

$$c_a * M_a * (T_{0(a)} - (T_{t(a)}) = c_o * M_o * \frac{d\Delta T_o}{dt} \tag{4.1.5}$$

According to a rated quantity from equipment:

$$\dot{Q}_a = c_a * M_a * T_a = c_a * T_a * \rho_a * \dot{q}_a \tag{4.1.5-1}$$

Where $\dot{q}_a$ is denoted the volume speed of hot air. As c_a, T_a, ρ_a comes from the selected equipment, hereby $\eta = c_a * T_a * \rho_a$ where η is a constant directly from the equipment.

Therefore, (4.1.5-1) can be simplified as:

$$\dot{Q}_a = \eta \dot{q}_a \tag{4.1.5-2}$$

Considered the influence brought in by pump:

$$C_o M_o T_o = c_a M_a \rho_{ap} \Delta T_a \tag{4.1.5-3}$$

Where

$$M_a = \rho_a q_a \tag{4.1.5-4}$$

Therefore, taking deferential on both sides of (4.1.5-3), it will become the following:

The volume velocity of air can be used to determine its velocity, since c_a, c_o and M_o are constants.

Pressure State Analysis

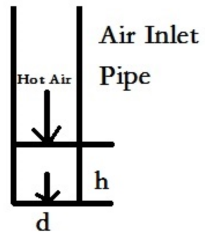

Figure 4-2 Pressure analysis

Newton's second law of movement specifies that:

$$P*S=\rho*g*S*\Delta h \tag{4.1.6}$$

With P denoting the pressure at the pipe inlet; Δh and ρ respectively denoting the distance travelled by the hot air and the air density. The latter can be deemed to be constant, since it is assumed that the operation of the plant takes place under stable state. The cross-section area of the pipe inlet, S, is determined with the following formula:

$$S=\pi*\left(\frac{d}{2}\right)^2 \tag{4.1.7}$$

Equation (4.1.5) can subsequently be integrated in:

$$P=\rho*g*\Delta h \tag{4.1.8}$$

The two sides of (4.1.7) can be modified following the differential against time:

$$\frac{dP_1}{dt}=\rho_a*g*\frac{d\Delta h}{dt} \tag{4.1.9}$$

The diameter of the hot air pipe, the pipe cross-section, and the distance travelled by the hot air are respectively denoted by d, S, and Δh.

If the pump influence is taken into account:

The pump will increase the speed of the hot air, enabling the reformulation of (4.1.9) as:

$$\frac{dP_{pump}}{dt} = \rho_a * g * (at) = \rho_a * g * \frac{d^2 \Delta h}{dt^2} * t \qquad (4.1.9\text{-}1)$$

Where a is denoted the pump's acceleration.

Therefore, after putting them together, the equation 4.1.10 has been concluded from equations 4.1.9 and 4.1.9-1 as:

$$\frac{dP}{dt} = \frac{dP_1}{dt} + \frac{dP_{pump}}{dt} = \rho g_a \left(\frac{d\Delta h}{dt} + \frac{d^2 \Delta h}{dt^2} * t\right) \qquad (4.1.10)$$

Here t is denoted as a time unit, with a value of 1m/s.

Since the total hot air volume rate, q_a, is known from the equipment, the system representation can be derived based on the volume conservation:

$$\dot{q}_{ap} = \frac{d^2 \Delta h}{dt^2} * t * S \qquad (4.1.11)$$

4.1.2 State Variable Determination

Based on the above analysis, the state variables selected for this study are:

$x_1(t) = v(t) = \frac{d\Delta h}{dt}$, denoting the hot air speed indicative of pressure.

$x_2(t) = \Delta T_o$, denoting temperature fluctuation of the objectives.

4.1.3 State Space Equations

While considering the pump influence, by integrating (4.1.11-1) into (4.1.5), the mathematical analysis for the plant can be simplified as:

$$c_a * M_a * \Delta T_a = c_o * M_o * \Delta T_o \tag{4.1.12}$$

Where

$$M_a = \rho_a q_{ap} \tag{4.1.12-1}$$

Put (4.1.12-1) into (4.1.12), equation 4.1.12-2 is obtained as follows:

$$c_a * \rho_a * q_{ap} * \Delta T_a = c_o * M_o * \Delta T_o \tag{4.1.12-2}$$

As q_{ap} and ΔT_o are the variables on both sides, take deferential on both sides against time, equation 4.1.13 is obtained as follows:

$$c_o * M_o * \frac{d\Delta T_o}{dt} = c_a * \rho_a * \frac{d^2\Delta h}{dt^2} * t * S * \Delta T_a \tag{4.1.13}$$

The volume velocity of the hot air, $\frac{d\Delta h}{dt}$, is denoted by x_1, while x_2 denotes the temperature discrepancy between the vials, ΔT_o. Reformulation of (4.1.13) and (4.1.12) based on state equations takes the following form:

$$\begin{cases} \dot{x}_1 = -\dfrac{x_1}{t} + \dfrac{\dot{Q}_a}{\eta St} \\ \dot{x}_2 = -\dfrac{c_a \rho_a \Delta T_a}{c_o M_o} x_1 + \dfrac{c_a \Delta T_a \dot{Q}_a}{gt\eta S^* c_o M_o} \end{cases} \quad \text{(State Equations)} \qquad (4.1.14)$$

Let ΔT_a and $\dot{Q}_a$ to be u_1 and u_2, respectively (the plant input and controller output), the state space equation can be reformulated as in (4.1.15):

$$\begin{cases} \dot{x}_1 = -\dfrac{x_1}{t} + \dfrac{1}{\eta St} u_1 \\ \dot{x}_2 = -\dfrac{c_a \rho_a}{gtc_o M_o} x_1 u_1 + \dfrac{c_a}{gt\eta S^* c_o M_o} u_1 u_2 \end{cases} \qquad (4.1.15)$$

The output equation is as follows:

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (4.1.16)$$

Equation (4.1.17) illustrates the state space model for plant following format simplification:

$$\begin{cases} \dot{X} = \begin{bmatrix} -\dfrac{1}{t} & 0 \\ -\dfrac{c_a \rho_a}{gtc_o M_o} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{\eta St} & 0 \\ 0 & \dfrac{c_a}{gt\eta S^* c_o M_o} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ \dot{Y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \end{cases} \qquad (4.1.17)$$

It seems that this is a non-linear system model with inputs and outputs that are coupled. To decouple the inputs and outputs, the control system with decoupling technique is applied based on the state feedback law in the next part.

4.1.4 Control System Design

Based on (4.1.17), the following matrices have been established for this model:

$$\begin{cases} A = \begin{bmatrix} -\frac{1}{t} & 0 \\ -\frac{\rho_a \Delta T_a}{gtc_o M_o} & 0 \end{bmatrix} \\ B = \begin{bmatrix} \frac{1}{\eta St} & 0 \\ 0 & \frac{c_a}{gt\eta S * c_o M_o} \end{bmatrix} \\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D = 0 \end{cases} \tag{4.1.18}$$

(1) Discretization

Conversion of the state space continuous model to a discretised model is necessary for computational purposes. The expression of the discretised state space equations takes the following form:

$$X[(t+1)T_s] = A_d * X\left(tT_s\right) + B_d * u\left(tT_s\right) \tag{4.1.19}$$

The sampling time interval is denoted by T_s = 0.1. Hence, based on how differential equations are defined,

$$\overset{g}{X}(tT_s) = \lim_{\Delta t \to 0} \frac{X[(t+\Delta t)T_s] - X(tT_s)}{\Delta t} \tag{4.1.20}$$

The value of Δt is set to be 1, allowing equation (4.1.20) to be expressed as:

$$\begin{aligned} &\dot{X}(tT_s) = X[(t+1)T_s] - X(tT_s) = T_s AX(t) + T_s Bu(tT_s) \\ &\dot{X}[(t+1)T_s] = \{[I + A]T_s\}X(tT_s) + BT_s u(tT_s) \end{aligned} \tag{4.1.21}$$

Comparison of (4.1.21) with (4.1.19) gives:

$$\begin{aligned} &A_d = T_s[I + A] \\ &B_d = BT_s \end{aligned} \tag{4.1.22}$$

where I is unit matrix.

4.2 System Performance with Case Study

The next step following the mathematical modelling of the system is development of computational experiments for plant modelling and simulation of control system operations with the use of the MATLAB software. Table 1 lists the simulation parameters and constants based on the assumption of a scenario in which 1500 pieces/batch of petri dishes are introduced into the machine. Furthermore, the data from the table draw on the work conducted by Wang (2015) regarding the production process in a Chinese pharmaceutical plant.

Given the type of gravity model employed in the present settings, gravity may be considered a constant because of its acceleration value, in keeping with data from the World Geodetic System 1984 (WGS-84), where the value of g was established to be 9.8 m/s^2 (Stevens and Lewis 2003). After incorporation of the parameters with the above values in the plant model, the following model representation is obtained:

$$\begin{cases} A=\begin{bmatrix} -1 & 0 \\ -0.027 & 0 \end{bmatrix} \\ \text{B}=\begin{bmatrix} 0.0012 & 0 \\ 0 & 0.000051 \end{bmatrix} \\ C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D=0 \end{cases} \tag{4.1.23}$$

Discretisation of the state space equations is the following step. Discretisation is understood as the division of continuous functions, model and equations into a limited number of discrete elements in order to enable those continuous functions, model and equations to be computed and implemented on digital computers. Furthermore, quantisation is essential for processing on a digital computer.

Under the present circumstances, discretisation of the state space model is undertaken to obtain the equation below:

$$X[(t+1)T_s] = A_d * X(tT_s) + B_d * u(tT_s) \tag{4.1.24}$$

The sampling time interval is denoted by T_s = 0.1. Based on how differential equations are defined,

$$\dot{X}(tT_s) = \lim_{\Delta t \to 0} \frac{X[(t+\Delta t)T_s] - X(tT_s)}{\Delta t} \tag{4.1.25}$$

If Δt = 1, then the previous equation can be expressed as:

$$\begin{aligned} \dot{X}(tT_s) &= X[(t+\Delta t)T_s] - X(tT_s) \\ &= T_s AX(t) + T_s AX(t) + T_s Bu(tT_s) \end{aligned} \tag{4.1.26}$$

$$\overset{g}{X}[(t+1)T_s] = \left\{[I+A]T_s\right\} X(tT_s) + BT_s u(tT_s) \quad (4.1.27)$$

Comparison of the above equation with (4.1.27) gives the following:

$$\begin{cases} A_d = T_s[I+A] \\ B_d = BT_s \end{cases} \quad (4.1.28)$$

In the above, *I* is unit matrix.

4.2 Pole Placement State Feedback System Design

4.2.1 Observability

If reconstruction of any particular state or control vector can be achieved solely based on data derived from the system output, then the system is considered to be fully observable. As it will become clear, observability is directly correlated with controllability. Conversely, if a system is not observable, then it will be difficult to gauge the impact of system stabilisation through a control signal on the plant, leading to the system being categorised as uncontrollable. The criterion for verifying if a system is observable is given in (4.2.1), where the matrix rank and the dimensions of matrix *A* are respectively denoted by rank (*R*) and *n* (Dutton, 1998). Equivalence must exist between the rank of matrix *R* and *n* for system *A* to be completely observable.

$$R = [C \quad CA \quad \ldots \quad CA^{n-1}]^T \quad (4.2.1)$$

Matrices *A* and *C* can thus be integrated in (4.2.1):

$$R=[C \quad CA]^T=\left[\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}-1 & 0\\-0.027 & 0\end{bmatrix}\right]^T$$

$$=\begin{bmatrix}1 & 0\\0 & 1\\-1 & -0.027\end{bmatrix} \tag{4.2.2}$$

$$rank(R)=rank\begin{bmatrix}1 & 0\\0 & 1\\-1 & -0.027\end{bmatrix}=2 \tag{4.2.3}$$

The observability of the system is confirmed as MATLAB indicates that the rank of matrix R is 2, meaning that it is compatible with the dimensions of matrix A.

4.2.2 Controllability

After the system is suitably modelled mathematically, the next step is to determine how controllable the system is by inspecting some of its structural characteristics based on the generated model. State and output are the two existing types of controllability. Only state controllability is addressed in the present study. It can be understood as the controller's capacity to alter any internal state vector with an original value to any final value in a specific time frame. More specifically, for a dynamic system to be fully state controllable, an unconstrained control vector u(t) must be able to be created to enable the transfer of a particular original state x(t0) to a final state x(T) in a limited time frame t0 < t < T (Zhu, 2014).

As shown in (4.2.4), there are close similarities between the criterion for assessment of state controllability and that of observability. The matrix rank and the dimensions of matrix A are respectively denoted by rank (P) and n (Zhu, 2014).

$$P=[B \quad AB \quad \ldots \quad A^{n-1}B]^T \tag{4.2.4}$$

By introducing the values of matrix *A* and matrix *B* into (4.2.4), the following is obtained:

$$
\begin{aligned}
P &= [B \quad AB] \\
&= \left[\begin{bmatrix} 0.0012 & 0 \\ 0 & 0.00051 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ -0.027 & 0 \end{bmatrix}\begin{bmatrix} 0.0012 & 0 \\ 0 & 0.00051 \end{bmatrix}\right]^T \\
&= \begin{bmatrix} 0.0012 & 0 & -0.0012 \\ 0 & 0.00051 & -0.0000324 \end{bmatrix} \\
&= 2
\end{aligned}
\tag{4.2.5}
$$

The controllability of the system is confirmed as MATLAB indicates that the rank of *P* is 2, meaning that it is compatible with the dimensions of matrix *A*. The system was additionally validated to be fully state controllable based on running the code (see Appendix) in MATLAB.

On the basis of the above results, it can be said that the plant is appropriate for adopting the pole placement technique for the allocation of random poles.

4.2.3 Matrix Transformation

The values allocated to the design damping ratio and undammed natural frequency are respectively $\xi = 0.5$ and $\omega_n = 0.2$. The Appendix provides the calculation of the assigned poles.

MATLAB is applied in the present section to determine the system closed loop feedback gain matrix.

A determinant is set based on the state feedback and (4.2.6) is derived from the previous section.

$$\begin{cases} A=\begin{bmatrix} -1 & 0 \\ -0.027 & 0 \end{bmatrix} \\ B=\begin{bmatrix} 0.0012 & 0 \\ 0 & 0.000051 \end{bmatrix} \\ C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D=0 \end{cases} \tag{4.3.6}$$

System controllability is unaffected by matrix modification. The transformation block diagrams are illustrated in the figures below.

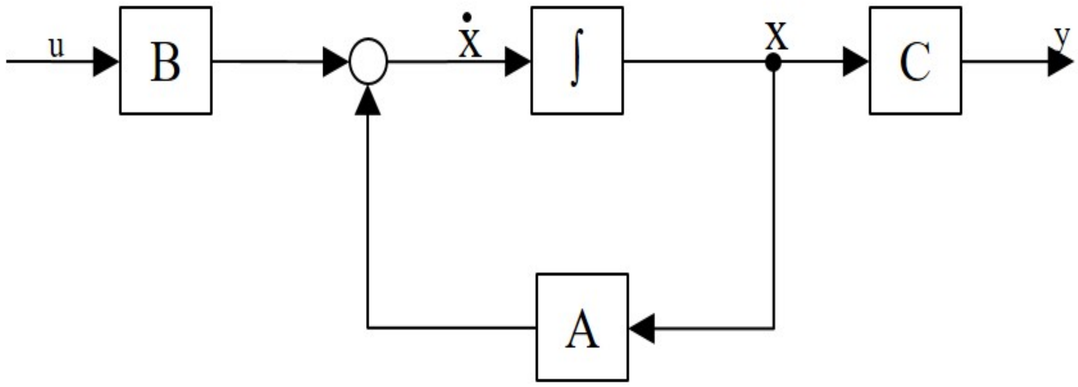

Figure 4-3 Block Diagram of State Space

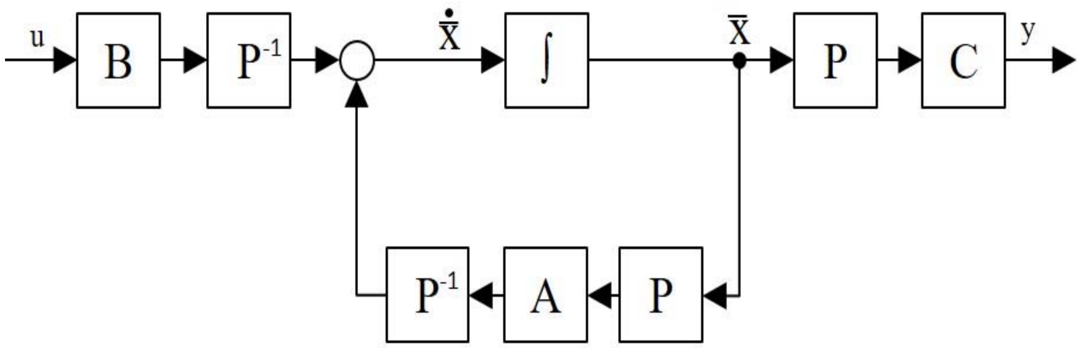

Figure 4-4 Block Diagram of State Space after Transformation

The system controllability matrix is denoted by matrix *P*. The transformation-related equations below have been obtained from the preceding figures.

A determinant is identified based on (4.2.7) before the gain matrix can be identified.

$$sI-(A-BK)$$
$$=\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}-\left\{\begin{pmatrix} -1 & 0 \\ -0.027 & 0 \end{pmatrix}-\begin{pmatrix} 0.0012 & 0 \\ 0 & 0.000051 \end{pmatrix}\begin{pmatrix} k_1 & 0 \\ k_2 & 0 \end{pmatrix}\right\} \quad (4.2.7)$$

Matrix *I* constitute of a 2x2 unit matrix and it is assumed that the gain matrix is matrix $K=\begin{pmatrix} k_1 & 0 \\ k_2 & 0 \end{pmatrix}$.

Hence, expression of the determinant of $sI-(A-BK)$ takes the following form:

$$sI-(A-BK)=\begin{pmatrix} s+1+0.00121k_1 & 0 \\ 0.027+0.000051k_2 & s \end{pmatrix} \quad (4.2.8)$$

The determinant of $sI-(A-BK)$ can be expressed as:

$$\begin{aligned} f^*(s) &= \det[sI-(A-BK)] \\ &= s^2+(0.00121k_1+1)s \end{aligned} \quad (4.2.9)$$

The target poles can be identified with (4.3.10).

$$f^*(s)=\prod_{i=1}^{n}(s-s_i^*)=s^n+a_{n-1}^*s^{n-1}+\cdots+a_0^* \quad (4.3.10)$$

The preceeding two equations lead to:

$$\begin{cases} a_1^* = 0.0012k_1 + 1 \\ a_0^* = 0 \end{cases} \tag{4.2.11}$$

As indicated before, the system controllability has been clearly demonstrated, resulting in a non-singular transformation given in (4.2.12):

$$x = T\,\bar{x} \tag{4.2.12}$$

Where matrix T is represented as:

$$T = [B \quad AB \; \cdots \quad A^{n-1}B]\begin{bmatrix} a_1 & \cdots & a_{n-1} & 1 \\ \cdots & \ddots & & \\ a_{n-1} & \ddots & & \\ 1 & & & \end{bmatrix} = cam * w \tag{4.2.13}$$

The function *ctrb()* was applied in MATLAB to obtain the controllability matrix *cam*, while the function *hankel()* gave the controllability matrix *w*.

$$\begin{cases} cam = \begin{bmatrix} 0.0012 & 0 & 0 & -0.0012 \\ 0 & 0.00051 & 0 & -0.0000324 \end{bmatrix} \\ w = \begin{bmatrix} -6.1500 & -0.0027 & 0 & 0 \\ -0.0027 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00051 \end{bmatrix} \end{cases} \tag{4.2.14}$$

Hence,

$$T = cam * w = 1*10^{-3} * \begin{bmatrix} -1.2 & -0.0324 \\ -0.006 & 0 \end{bmatrix} \tag{4.2.15}$$

The state space equations can be converted to the Standard Controllable Form I, as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\,\bar{x} + \bar{B}\,u \\ y = \bar{C}\,\bar{x} \end{cases} \tag{4.2.16}$$

Where

$$\begin{cases} \bar{A} = T_{cl}^{-1} A T_{cl} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \\ \bar{B} = T_{cl}^{-1} B = \begin{bmatrix} 0 \\ \cdots \\ 0 \\ 1 \end{bmatrix} \\ \bar{C} = C T_{cl} = \begin{bmatrix} b_0 & b_1 & \cdots & b_{n-1} \end{bmatrix} \end{cases} \tag{4.2.17}$$

The expression of the state feedback gain matrix is:

$$\bar{K} = \begin{bmatrix} \bar{k}_0 & \bar{k}_1 & \cdots & \bar{k}_{n-1} \end{bmatrix} \tag{4.2.18}$$

This enables the equation of the closed loop state space to be derived:

$$\begin{cases} \dot{\bar{x}} = (\bar{A} - \bar{B}\bar{K})\bar{x} + \bar{B}v \\ y = \bar{C}\bar{x} \end{cases} \tag{4.2.19}$$

Where

$$(\overline{A}-\overline{B}\overline{K})=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -(a_0+\overline{k}_0) & -(a_1+\overline{k}_1) & \cdots & \cdots & -(a_{n-1}+\overline{k}_{n-1}) \end{bmatrix} \tag{4.2.20}$$

The system Standard Controllable Form I is derived from (4.2.17):

$$\begin{cases} \overline{A}=T_{cl}^{-1}AT_{cl}=\begin{bmatrix} 0 & 1 \\ -0.0245 & -0.34 \end{bmatrix} \\ \overline{B}=T_{cl}^{-1}B=1*10^{-3}\begin{bmatrix} 0 \\ -5.7 \end{bmatrix} \\ \overline{A}-\overline{B}\overline{K}=\begin{bmatrix} 0 & 1 \\ 0.0025\overline{k}_0-0.0245 & 0.0025\overline{k}_1-0.34 \end{bmatrix} \end{cases} \tag{4.2.21}$$

The characteristic polynomial of the system closed loop is expressed as:

$$\begin{aligned} f(s)&=\left|sI=(\overline{A}+\overline{B}\overline{K})\right| \\ &=s^2+(0.0025\overline{k}_1+1)s+(-0.034\overline{k}_0+0.018\overline{k}_1+0.0002) \end{aligned} \tag{4.2.22}$$

To ensure compatibility between the closed loop poles and the target poles, the formulas below must be satisfied:

$$f(s)=\left|sI-(\overline{A}+\overline{B}\overline{K})\right|=f^*(s) \tag{4.2.23}$$

With $f^*(s)$ being derived from equation (4.2.9). The gain matrix can be determined based on the coefficient of the same power factors on both sides of the equation. Thus,

$$\overline{K}=\begin{bmatrix} -3.24 & -24.8 \end{bmatrix} \tag{4.2.24}$$

Therefore, the input will be:

$$u = \left| v - \overline{K}x \right| = v - \overline{K}T^{-1}x \tag{4.2.25}$$

Hence, the closed loop gain matrix associated with the pole placement design will be:

$$K = \overline{K}T^{-1} = 1*10^{3}\begin{bmatrix} 410.4822 & 0.0101 \end{bmatrix}^{T} \tag{4.2.26}$$

4.3 Decoupling Control System Design

Equation (3.3.6) gives the decoupling pair F_s and H. Following replacement, the control system is represented as in (4.2.1):

$$\begin{cases} Y(tT_s) = M_o * y(tT_s) + \Gamma * u(tT_s) = M_o * C * X(t) + R * u(tT_s) \\ \overset{g}{X}(tT_s) = A_d * x(tT_s) + B_d * u(tT_s) \\ u(tT_s) = F_s * X(tT_s) + H * v(t) \end{cases} \tag{4.3.1}$$

where

$$\begin{aligned} F_s &= (CB_d)^{-1}[M_oC - CA_d] \\ H &= (CB_d)^{-1}\Gamma \end{aligned} \tag{4.3.2}$$

M_o and T are calculated and then replaced in F_s and H:

$$\begin{cases} M_o = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \\ \Gamma = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \end{cases} \tag{4.3.3}$$

Discretisation of the system representation against time from continuous equations is required to enabled modelling and simulation in MATLAB. Modification of system representation can be undertaken in the following way:

$$\begin{cases} u[(t+1)T_s] = F_s * X\left(tT_s\right) + H * v(t) \\ X[(t+1)T_s] = A_d * X\left(tT_s\right) + B_d * u\left(tT_s\right) \\ Y[(t+1)T_s] = C * X\left(tT_s\right) \end{cases} \tag{4.3.4}$$

As highlighted previously, to divide the inputs and the outputs, the state space models have to be subjected to decoupling. However, a common triangular decoupling issue may arise during this procedure. To solve this issue, Shen et al. (2015) suggested that the canonical decomposition of right invertible system [C, A, B] should be applied. The formulas provided by the authors define every achievable transfer function matrix for delineating the decoupling issue as well as the pole allocation issue. The premise underpinning the triangular decoupling issue is that the state variable feedback possesses a non-singular lower triangle form up to row permutation. This study adopts the recommendations of Shen et al. (2015) in the approach used to delineate the decoupling procedure.

4.4 Conclusions

In this chapter, the design of plant is given using state-space approach. Firstly is the plant modelling which firstly includes the analysis of this plant. This step includes the analysis of this plant which is to identify the inputs, outputs and states, also the relevant simplification and assumptions are figured out. Then this chapter follows the general process to establish a state-space model, defines two state variables, velocity of hot air, $\frac{d\Delta h}{dt}$, and $\Delta T_{o,}$ the tempeature discrepancy of vials.And then based on ***the principle of energy conservation***, the state equations are established. After the model is established, the coupled nature between inputs and outputs is proved, the relevant system analysis and control methods are implemented, respectively the pole placement and decoupling method. The designed state space model is non-standard form, therefore in the process of pole placement for the non-standard model, a matrix transform is designed to transfer the model to a standard form.

5 Control Systems Design

In this chapter, control systems are designed for two inputs separately. Mamdani-type fuzzy control is adopted for temperature control, while the interval type-2 fuzzy control is adopted for pressure control, based on previous designed decoupling control.

5.1 Mamdani-type fuzzy control system design

5.1.1 Control System Design

Temperature and air pressure are the system control variables. The fuzzy control system in the present design is intended for temperature (*e1*), while the air pressure (*e2*) control is reserved for the following design. Hence, temperature and air pressure are respectively denoted by corner mark 1 and corner mark 2. Based on the design procedure, the control system design is approached in the following way:

5.1.2 Define the domains

E1: The natural domain is [-20, 20], as the required temperature is 320 ºC, with an error range of ±20ºC.

The series of language variables associated with E1 is determined in the following way:

$$E_1 = \{NB, NM, NS, NZ, PZ, PS, PM, PB\} \tag{5.1.1}$$

Figure 5-1 presents the membership function for E1 based on a fuzzy domain of [-6, 6], with the scale factor being:

$$K_e = \frac{12}{20-(-20)} = 0.3 \tag{5.1.2}$$

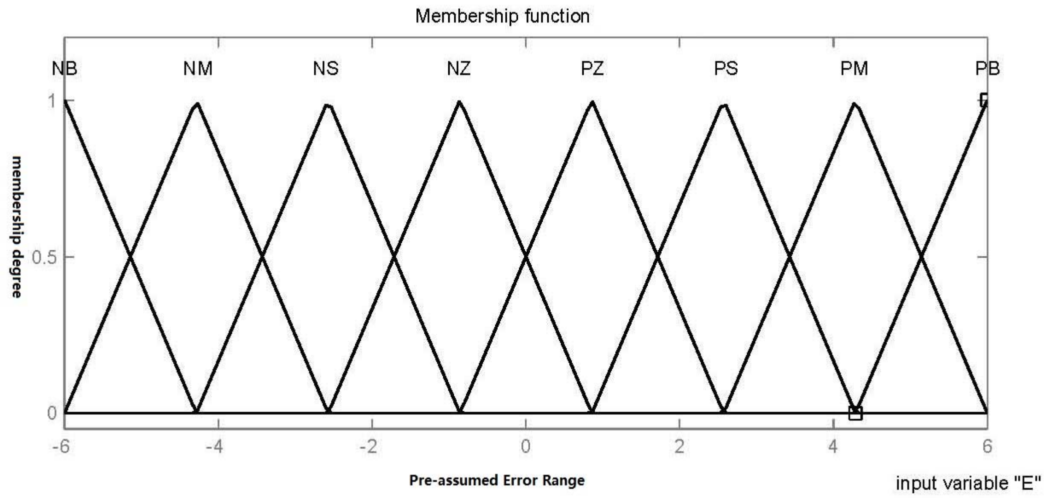

Figure 5-1 Membership Function for Input Variable E1

EC1: As in the case of E1, the natural domain is established to be [-15, 15] and the series of language variables for EC1 can be determined in the following way:

$$EC_1 = \{NB, NM, NS, \mathrm{ZO}, PS, PM, PB\} \tag{5.1.3}$$

Figure 5-2 presents the membership function for EC1 based on a fuzzy domain of [-6, 6], with the scale factor being:

$$K_{ec1} = \frac{12}{15-(-15)} = 0.4 \tag{5.1.4}$$

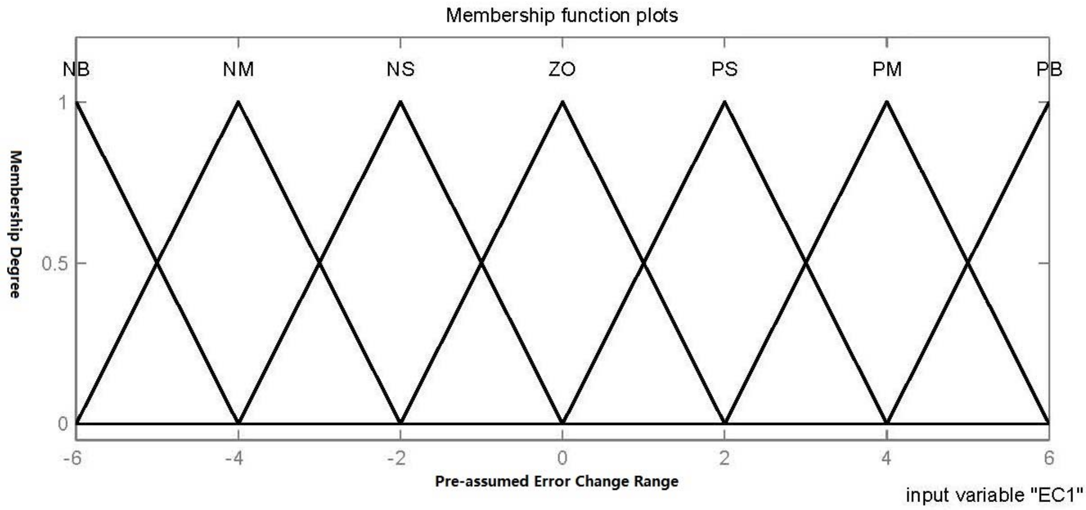

Figure 5-2 Membership Function for Input Variable EC1

UF1: The natural output domain is established to be [-15, 15] and the series of language variables for UF1 can be determined in the following way:

$$UF_1 = \{NB, NM, NS, \mathrm{ZO}, PS, PM, PB\} \tag{5.1.5}$$

Hence, as indicated in Figure 5-3, the membership function and scale factor are identical to those of EC1.

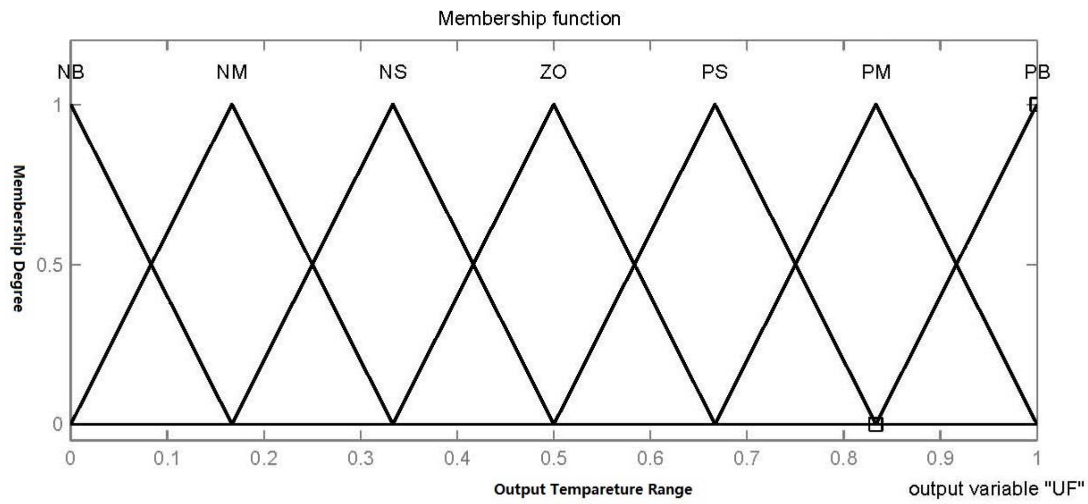

Figure 5-3 Membership Function for output UF

5.1.3 Fuzzy Rule Base

The experts' experience has informed the development of the fuzzy rule base, which is underpinned by the following principles:

E=NB and EC<0

The temperature is not as high as required and exhibits a downward trend when error (x1-v1) is negative big (NB) and the error change rate $\frac{e_1(t)-e_1(\mathrm{t}-1)}{Ts}$ is negative as well. Under such circumstances, the controller must increase the temperature to offset the error and avoid the down trend.

E= NB and EC>0

The temperature is not as high as required but exhibits an upward trend when error (x1-v1) is negative big (NB) while the error change rate $\frac{e_1(t)-e_1(\mathrm{t}-1)}{Ts}$ is positive.

Under such circumstances, the controller must manage the small control output to offset the error and regulate the overshoot.

E=NM

The temperature is not as high as required when error (x1-v1) is negative middle (NM). To address the issue, the same measures should be adopted by the controller as in the case of E=NB.

E=NS

The system state is near the desired state when error (x1-v1) is negative small (NS). To stop the error from becoming positive if the error change rate is negative small, the controller needs to maintain the middle position (PM). To remove the error change rate if it is positive, the controller should adopt the positive small position (PS).

Table 1 provides an overview of the fuzzy rule base developed in keeping with the above-mentioned principles and taking the form of an 8*7 matrix with 56 rules.

Table 1: Fuzzy Rule Base

E / U / EC	NB	NM	NS	NO	PO	PS	PM	PB
NB	PB	PB	PM	PM	PM	PS	ZO	ZO
NM	PB	PB	PM	PM	PM	PS	ZO	ZO
NS	PB	PB	PM	PS	PS	ZO	NS	NM
ZO	PB	PM	PS	ZO	ZO	NS	NM	NB
PS	PM	PM	PS	ZO	NS	NS	NM	NB
PM	PM	PS	ZO	NS	NM	NM	NB	NB
PB	PS	ZO	NS	NM	NM	NB	NB	NB

Hence, the temperature error (E1), temperature change rate (EC1), and the controller action (U) are connected by the following fuzzy relationship (R):

$$
\begin{aligned}
& R = (\underset{-}{E}\times\underset{-}{U})\,\mathrm{o}(\underset{-}{EC}\times\underset{-}{U}) = \underset{-}{E}\times\underset{-}{EC}\times\underset{-}{U} \\
& = (NB_E \times NB_{EC} \times PB_U)\,\mathrm{U} \\
& (NM_E \times NB_{EC} \times PB_U)\,\mathrm{U} \\
& (NS_E \times NB_{EC} \times PM_U)\,\mathrm{UL\ L}
\end{aligned}
\qquad (5.1.6)
$$

The error, error change rate, and controller output are respectively denoted by E, EC, and U.

5.1.4 Maximum of Membership Approach

When just one calculation requires more than one rule, the most prominent one should be established as the controller output.

5.2 Type-2 Fuzzy Control System Design

As discussed in the third chapter, it is challenging to apply a mathematical method to describe pressure modification in the tunnel. This limitation can be offset with the type-2 fuzzy control system, which is constructed in the present part. By taking into account the value of pressure change and the position where pressure fluctuates, the pressure change should be established as the fuzzy variable, while the pressure fluctuation position should serve as the primary membership function. This position can be determined with the secondary membership function.

5.2.1 Fuzzification

The natural domain is [0, 5], because the pressure difference (*PD'*) is positive and its value does not exceed 5.

The series of language variables associated with *PD* can be determined in the following way:

$$PD' = \{S, M, L\} \tag{5.2.1}$$

Figure 5-4 presents the membership function based on a fuzzy domain of [0, 5], with the scale factor being:

$$K_{PD'} = \frac{5}{5} = 1 \tag{5.2.2}$$

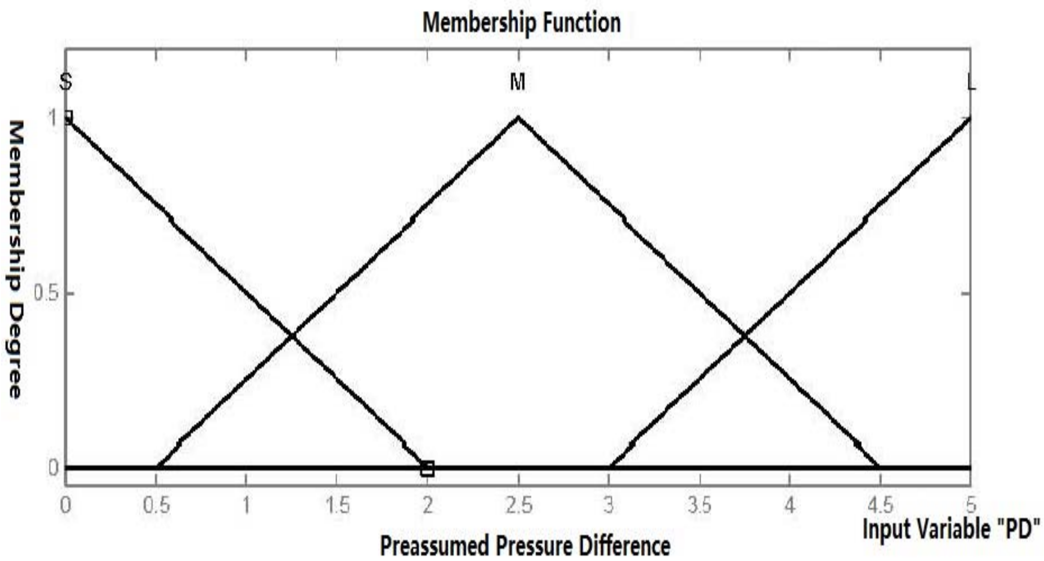

Figure 5-4 Membership Function for PD

The likelihood of vials falling off (p_v) due to modifications in pressure is defined as the primary membership function, which is derived based on technicians' language. This is applicable to the design outer layer as well.

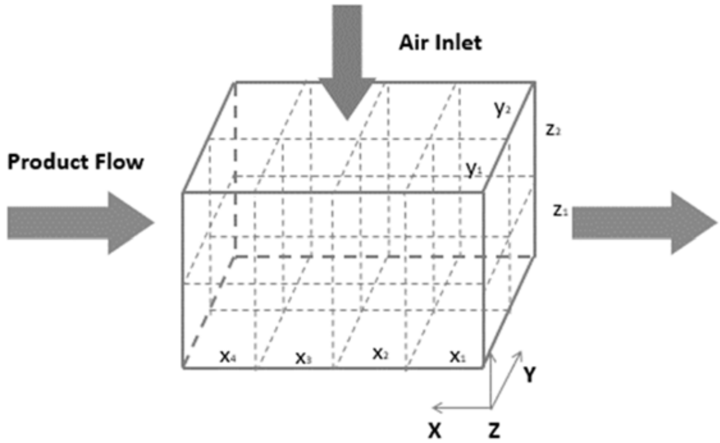

Figure 5-5 Secondary Variables (Coordinates)

The most significant pressure change at observation point $(x, y, z) \in (X, Y, Z)$ is the secondary fuzzy variable. For the design inner layer, sixteen spatially dynamic distributed intervals should be defined in the direction of the product flow, as indicated in Figure 5-5.

Pressure change (*PC*) is established to be the output, constituting a crisp number.

5.2.2 Inference

The technicians' experience-based language is used to formulate the rules for the outer layer:

The pressure difference exceeds 5 if *PD'* > 5, p_v = 1, meaning that vials ***must*** be falling off the conveyor.

If 3< PD' <5, p_v =[0.5,1], which indicates if pressure difference is between 3 to 5, Some vials will ***probably*** fall down from the conveyor.

The pressure difference is in the range 0-5 if 0 < *PD'* < 3, p_v = [0, 0.5], meaning that some vials ***might*** fall off the conveyor, which denotes a very small possibility.

The system is on the point of disruption if *PD'* is less than or equal to 0, which is the ***warning state***.

The position or zone in which the vials might fall off from is indicated by the inner layer. As shown in Figure 5-5, the zone is separated into 16 spaces, which are organised in the following way:

$$N_i = \sum_{i=1}^{16} X_o Y_r Z_q, \ \ \mathrm{o} = 1,2,3,4; r = 1,2; q = 1,2; \qquad (5.2.3)$$

The technicians' experience-based language is used to formulate the rules. There is a direct correlation between the likelihood of the vials falling off and their position. Hence, the location where pressure change occurs dictates the primary membership function.

5.2.3 Rule Base

If the pressure difference is considered to be denoted by the output, the rules included in the rule base are as follows:

R^1 : If pd is $\tilde{P}D1$, then uf_2 is U_1;

R^2 : If pd is $\tilde{P}D2$, then uf_2 is U_2;

R^3 : If pd is $\tilde{P}D3$, then uf_2 is U_3;

The control variable that represents the action of the valve is denoted by uf_2.

5.3 Conclusions

In this chapter, two control systems are respectively designed for temperature control and pressure control. Mamdani type fuzzy model is designed for temperature control. Based on two-dimensional process of error (Error and Error change), the control system is designed with two inputs. Therefore, for each input there is a fuzzification, after determine the fuzzy domain of the two inputs as well as the output, using scale factors respectively 0.3 and 0.4 to scale the real domain data to fuzzy domain inputs. Consequently, given two groups of language variables, the membership functions are determined for each input; establish the rule base according to experts' language and then the rule base is also determined with 8*7 (56) rules inside, accordingly the language variables for output are determined. When a real domain datum entered the control system, it will be firstly transferred a datum that a fuzzy system can identify, then will be inferenced according to the fuzzy rules. Therefore, in order to scale the fuzzy type output datum into a real domain datum that can be identified by actuators, the Maximum of Membership defuzzification method with a fuzzy type output datum.

An Interval type-2 fuzzy control has been designed in this Chapter for pressure control. There are two levels of fuzzy sets. The first level (primary level) is based on the input-output data, with similar "translation process" as Mamdani type, and still use Triangle type of membership functions; however, the secondary fuzzy sets are established based on limited work points spatially dynamic distributed, therefore the noise, or interference are considered as an impulse input added to the original step input.

6 Simulation Results and Analysis

In this chapter, simulation results will be displayed and illustrated from two aspects, systems response and control system performance. System responses are listed from SISO state space model, MIMO state space model with state feedback, to pole placement and decoupling control design; each one is given a step input and input with random interference to illustrate system's resistance to random interference. Control system performances are listed from mamdani type fuzzy control for temperature to interval type-2 fuzzy control for pressure. Each of them is also given step input and random interference to illustrate the robustness to disturbance. Additionally, the cross comparison among state feedback method, decoupling method and fuzzy control method is also made in this chapter.

6.1 System Performance

In this research, Matlab programs were developed for computational experiments of plant modelling and control system operation simulation. In advance of modelling and simulation, the parameters and constants were given as follows, taking the situation of 5ml vials with 1500pcs per batch as simulation example:

Table 2: Initial Value of Parameters

Parameters	Value
ρ_a	0.615 kg/m^3 (at 300 °C)
M_o	2.4kg (5ml Vials*1500pcs for one batch)
c_a	1.0×10^{3}J/ (kg. °C)
c_o	9.66*10^{2}J/ (kg °C)
$T_{0(a)}$	350 °C (specified)
$T_{t(a)}$	320 °C (Ideally)
$T_{0(o)}$	280 °C
$T_{t(o)}$	320 °C (Required)
d	0.3m
g	9.8m/s^2

After Substitution of these values into the plant model, the specific model representation was acquired for this case:

$$\begin{cases} A=\begin{bmatrix} -1 & 0 \\ -0.027 & 0 \end{bmatrix} \\ \mathrm{B}=\begin{bmatrix} -0.0012 & 0 \\ 0 & 0.000051 \end{bmatrix} \\ C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D=0 \end{cases} \tag{6.1.1}$$

Given reference input *v(t)* as follows for SISO (because *u1* is measured heat):

$$v(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6.1.2}$$

Therefore, for this case study, F_s and H is calculated in (6.1.3):

$$\begin{cases} F_s = \begin{bmatrix} 0.4167 & 0 \\ 0.0529 & 7.8431 \end{bmatrix} \\ H = \begin{bmatrix} 0.4167 & 0 \\ 0 & 9.8039 \end{bmatrix} \end{cases} \tag{6.1.3}$$

Based on the primary data, the following work related to system analysis has been done.

6.1.1 SISO (temperature) System Performance

After given three reference inputs respectively with u=0.5, u=0.8 and u=1, the simulation results show in Figure 6-1. When only one input and one output is considered, according to the figure, the system response is monotonous, without oscillation, which is close to the practical occasion, as the object will be heated gradually to a certain temperature rather than oscillated between a range. It is worthy of mention that the shown response and duration is a proportional data (as sampling data, the interval between sampling time can be defined accordingly). Furthermore, Fig 6-2 shows the SISO system with considerable resistance to noises, at the work point of 1 with noise=0.0583 and noise = -0.0348. The performance of SISO system is concluded as follows:

- Fig 6-1 and Fig 6-2 show very stable results. Figures show a monotonous curve no damping and no overshoot, which is very similar to the practical PID control results. It also explains that in order to control the cost of energy, the design of machine will sacrifice the response speed to privilege the accuracy. That is why the results are monotonous curves (large delay).

- However, there exists a large steady state error, around 20%. This is partly due to the introduction of the second state; at this stage it is a SISO model but with two states.

- Given interference at work point u=1, the designed SISO model shows a strong robustness in interference resistance. The obtained performance shows a very stable response, no overshoot or damping response.

In general, such a response matches the practical response using Classic PID. Furthermore, as the model shows a large positive steady error, it means if the pressure is considered into this model, such steady error will introduce a significant false high temperature.

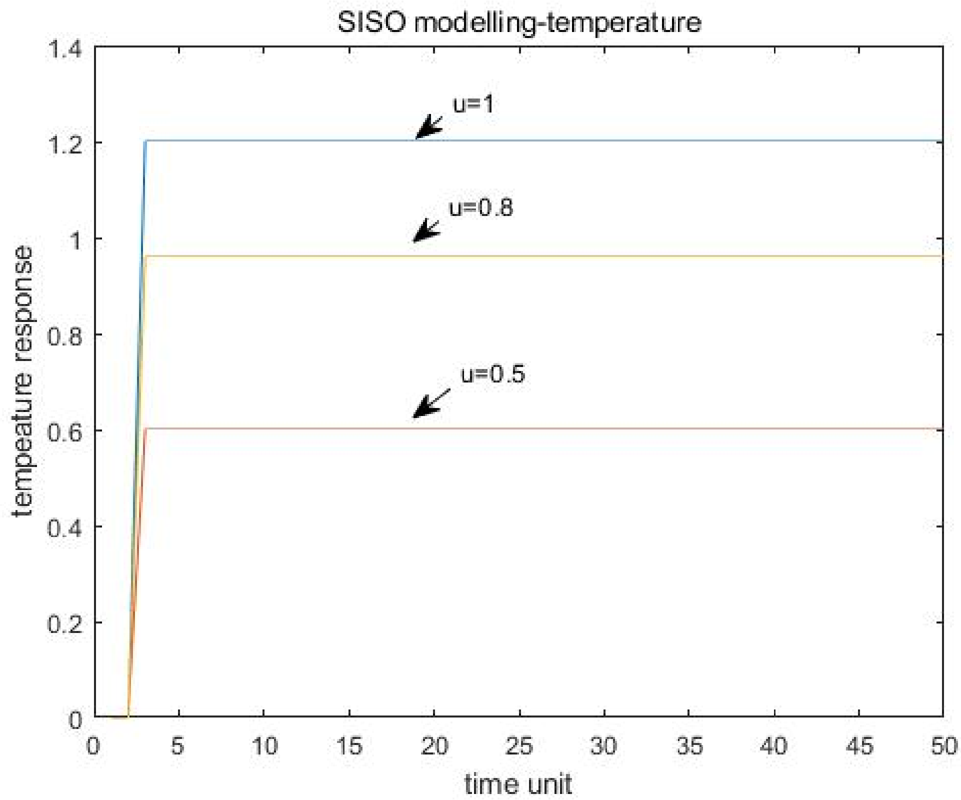

Figure 6-1 Time Response for SISO State Space Model under 0.5,0.8, 1

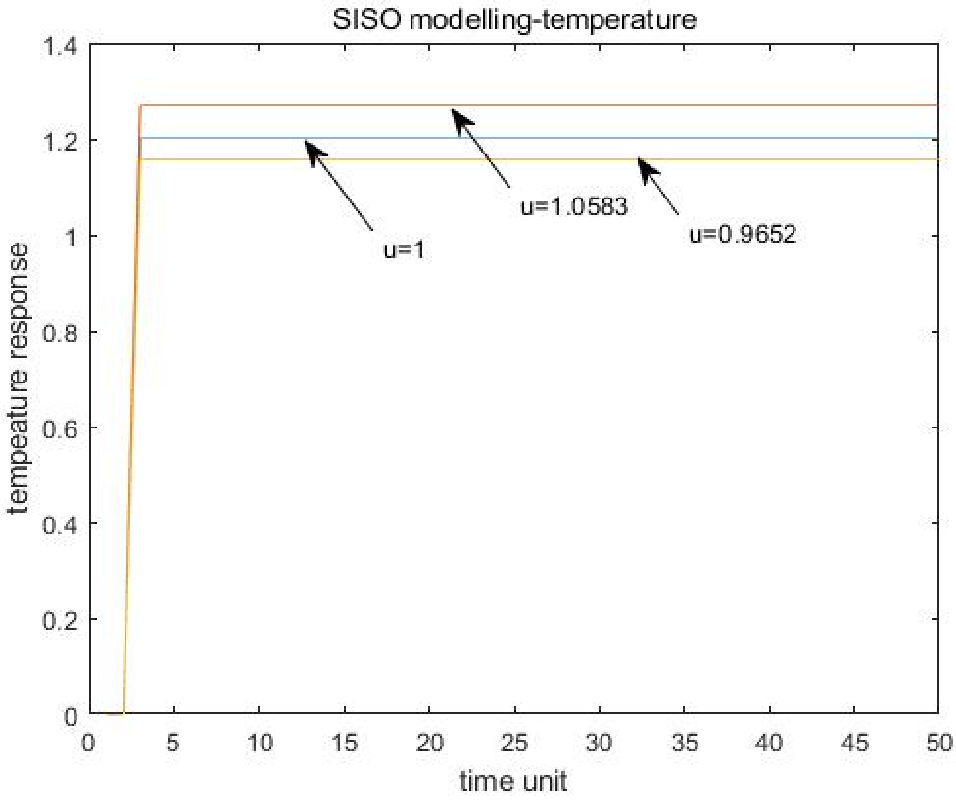

Figure 6-2 Time Response for SISO State Space Model under u=1 and noises

6.1.2 MIMO (Temperature and Pressure) System Performance (state feedback)

After introducing the input and output of pressure, the model is given two groups of reference inputs, which are respectively:

$$v(t) = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix} \tag{6.1.4}$$

And:

$$v(t) = \begin{bmatrix} 3 \\ 0.5 \end{bmatrix} \tag{6.1.5}$$

The results are shown respectively in fig 6-3 and 6-4. Although the curves return a steady response, such a significant steady error on both curves are not acceptable and it should be considered not stable.

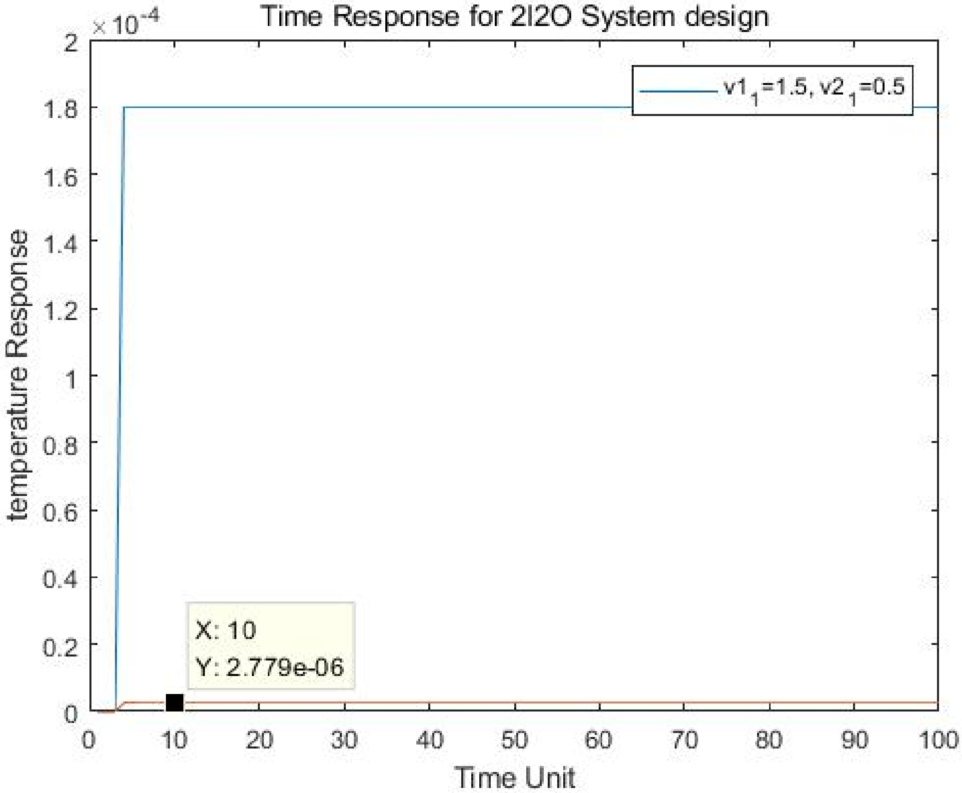

Figure 6-3 System Response for MIMO State-Space Model (v1=1.5,v2=0.5)

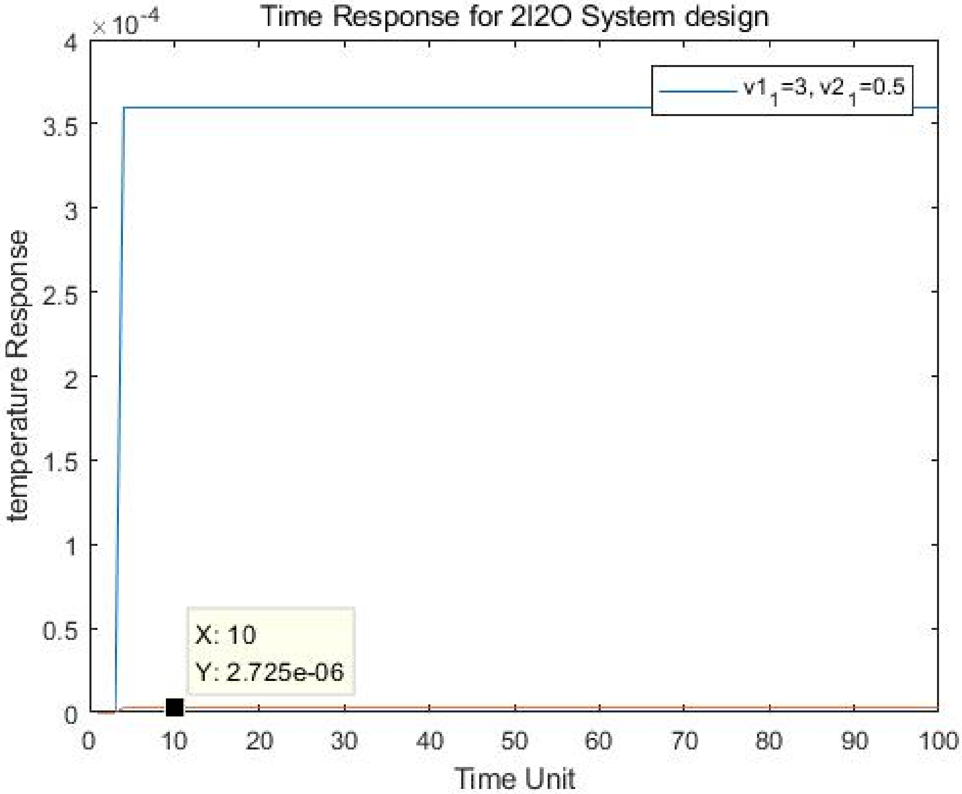

Figure 6-4 System Response for MIMO State-Space Model (v1=3, v2=0.5)

After examining the figure shown in Fig.6-3 and Fig 6-4, according to the two labelled points at the same time with the same input while retuning a different data, it is apparently that when change one of the inputs, both outputs are changed accordingly, which is not convenient for control system design.

6.1.3 pole placement design

Pole placement is designed accordance with the input fig.6-5.

In this research, used Matlab programs for computational experiment and control system operation simulation. As mentioned previously, in the specific model given input *v(t)* as:

$$v(t) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \tag{6.3.1}$$

Obtained state feedback gain matrix K as below,

$$\overline{K} = \begin{bmatrix} -3.24 & -24.8 \end{bmatrix} \tag{6.3.2}$$

Give the inputs as below,

$$\begin{cases} u_1 = 1 \\ u_2 = 1 \end{cases} \tag{6.3.3}$$

Therefore, proving a unit step response as input of the controller. With Matlab programming, the time response graph without control design in shown in Figure 6-4. Apparently, the system is somehow considered unstable.

After implementing the pole placement control, the time response has been under control significantly. The results represent that after allocating poles, the temperature change in 11 time units becoming stabled and the velocity of hot air in 8 time units becoming stable (as Figure 6-5 the labelled points shown).

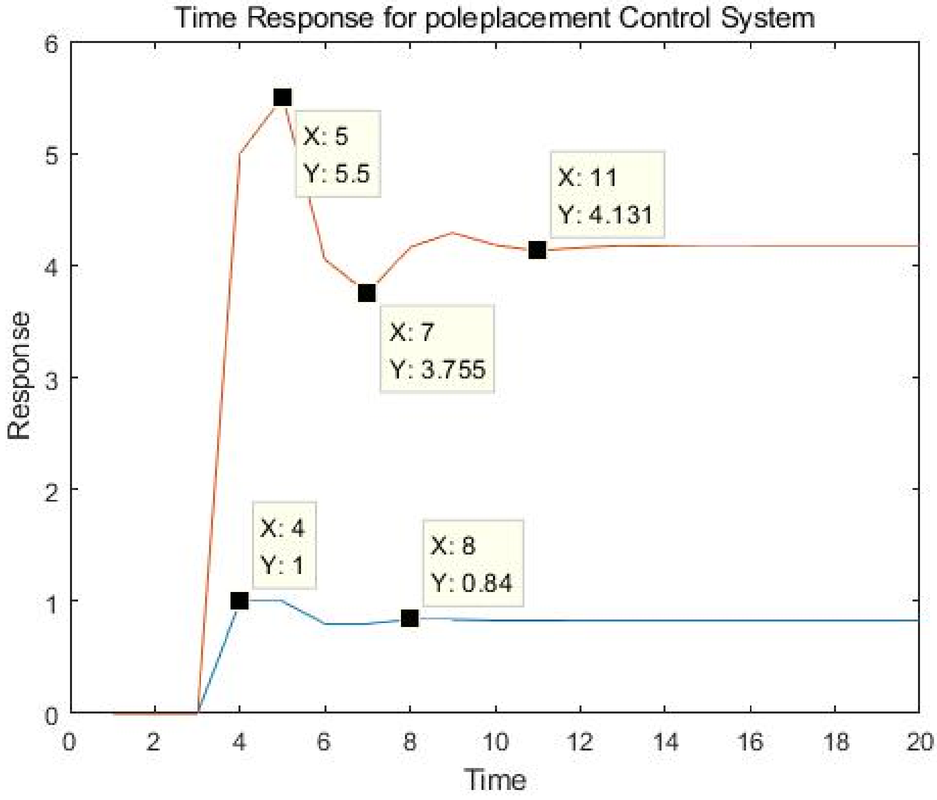

Figure 6-5 Time response for Pole Placement

However, the existing issues are the follows:

1. pole placement is designed accordance with desired output; if the desired output is changed, the whole process will start over again.

2. It shows a large steady error, although the overshoot occurred in temperature control is quite a desired performance.

3. The coupled nature between inputs and outputs are determined which requires a solution.

6.1.4 Time Response for decoupling design

Fig 6-6 shows the system response under different input values for temperature but the same input of pressure (which indicates the action of temperature increase). After the decoupling control, however, giving different values of reference inputs, respectively $v_1=3, v_2=4$ and $v_1=5, v_2=1$, the result, (compared in Fig.6-7), indicates that the effect of coupled inputs and outputs has been separated to non-interaction.

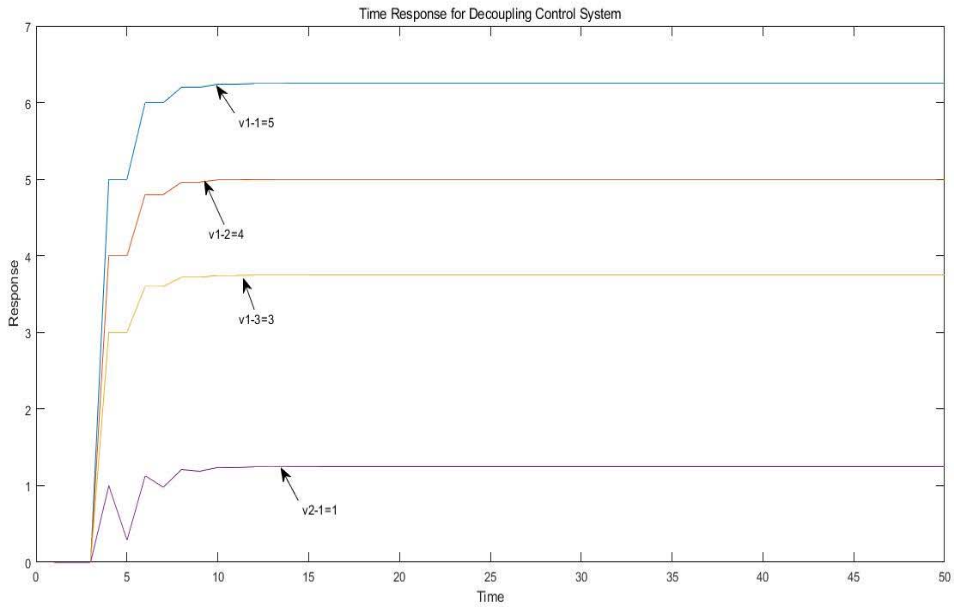

Figure 6-6 Time Response for different v1 values

In this case, *v1* is given three values, respectively 3, 4 and 5, and *v2* remains the same value of 1. Two of the conclusions are drawn as follows:

1. The decoupled nature has been loosed, that when *v1* is changed *v2* is remained, output *y2* will not be changed along with *v1*.

2. After decoupling control, performance of temperature keeps monotonous, but the pressure control returns a small damping performance with long term of fluctuation.

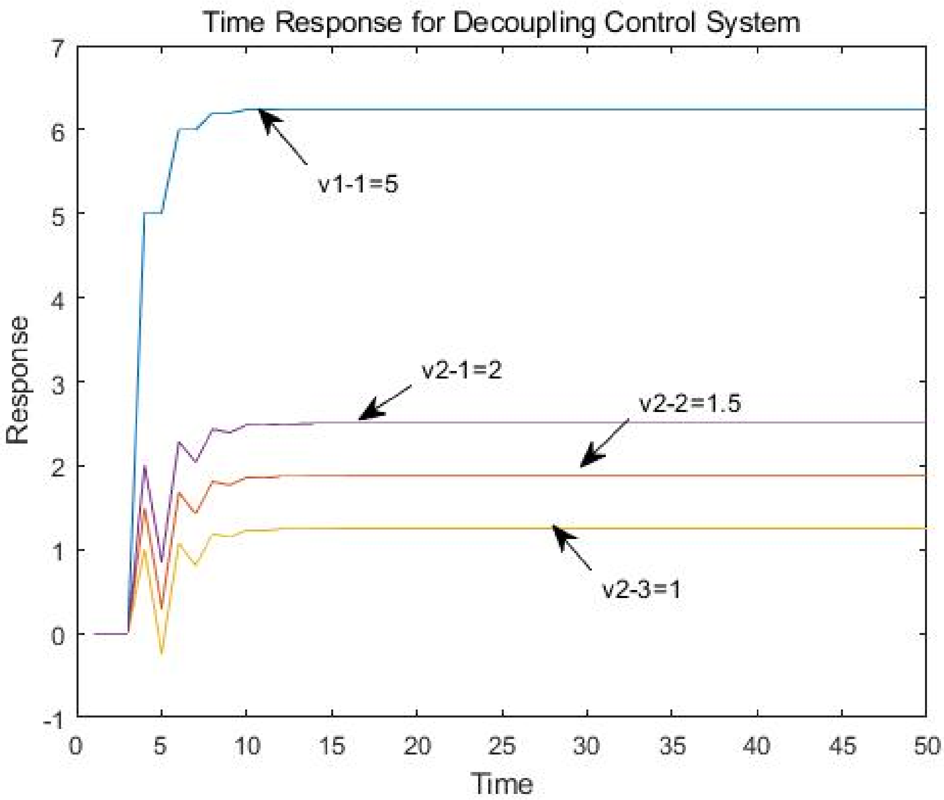

Figure 6-7 Time Response for different v2 values

Fig 6-7 shows the system response under different input values for pressure but the same input of temperature (which indicates the action of pressure change).

In this case, *v2* is given three values, respectively 1, 1.5 and 2, while *v1* remains as 5. Conclusions are drawn as follows:

The decoupled nature has been loosed, that when *v2* is changed *v1* is remained the same value, and output *y1* will not be changed along with *v2*. This is the proof that decoupling method works efficiently over this case.

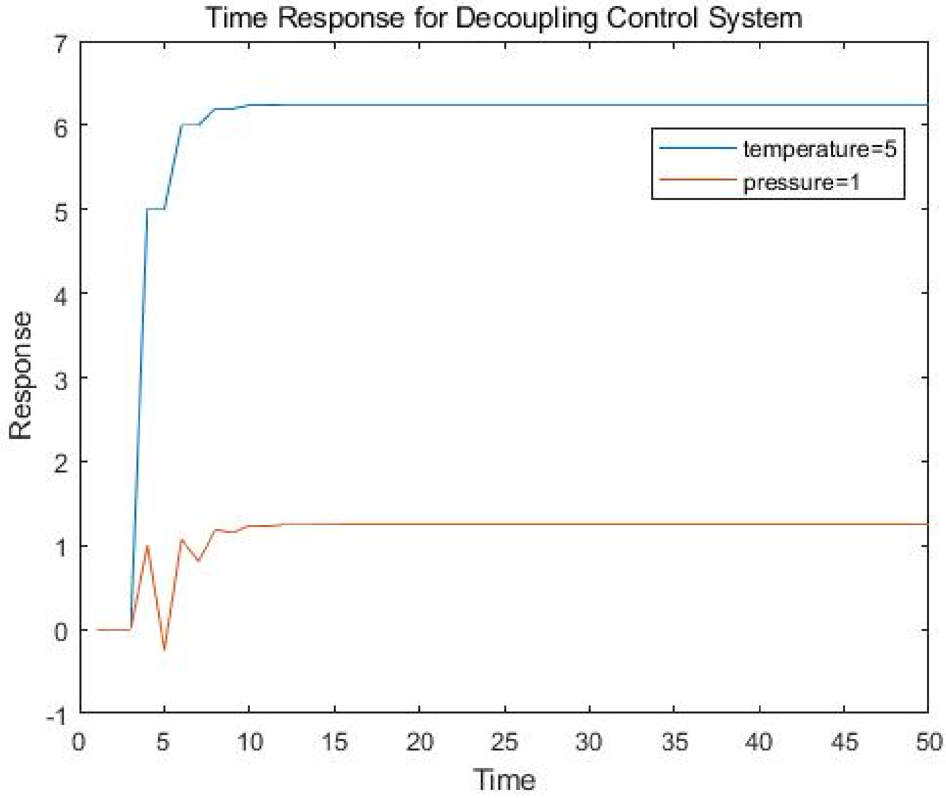

Figure 6-8 Time Response for Decoupling Control

Therefore, providing a unit step response as input of the controller, the time response for the representation is shown in Fig.6-8, where the upper curve represents the temperature change of hot air and the lower one represents r the velocity of hot air. Fig 6-8 shows the system response under pressure reference input of 1, and temperature input of 5. It leads to the following conclusions:

1. Decoupling control generates good control performance which is stable but it shows no efforts on eliminating the steady error.

2. However, it gives a longer response speed.

3. It generates damping performance for pressure, with over 50% of overshoot value, and 10 time units of settle time (slow as temperature control), which is not

a desired performance.

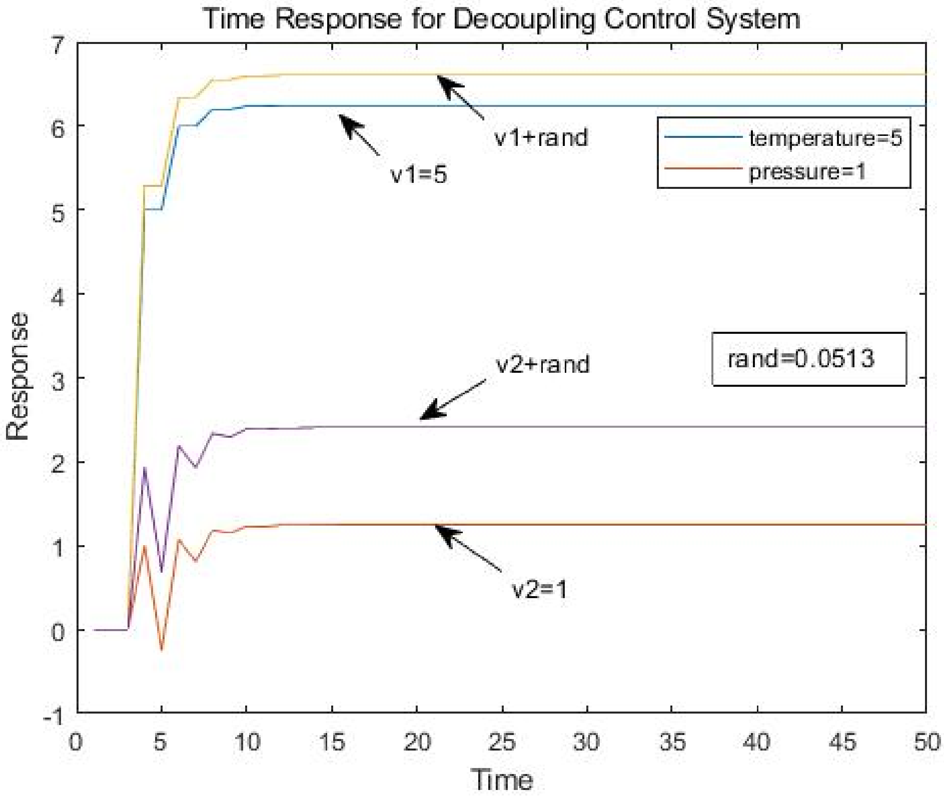

Figure 6-9 Decoupling control with noises

In order to validate the system's stability, random interferences are given to the two inputs. The results in figure 6-9 show the following conclusions:

1. Performances reveals a good resistance to interference, with acceptable stability.

2. With long settle time for about 12 time units.

3. The prominent damping performance (with about 50% overshoot) for pressure control is still not very satisfied.

6.2 Control System Performance

6.2.1 Mamdani-Temperature Control Performance

After given the reference input *V* of the system as v_{1_1}=0.5, v_{1_2}=0.8 and v_{1_3}=1, the simulation of mamdani fuzzy control system is achieved in MATLAB, and the simulation results are shown in Fig. 6-10.

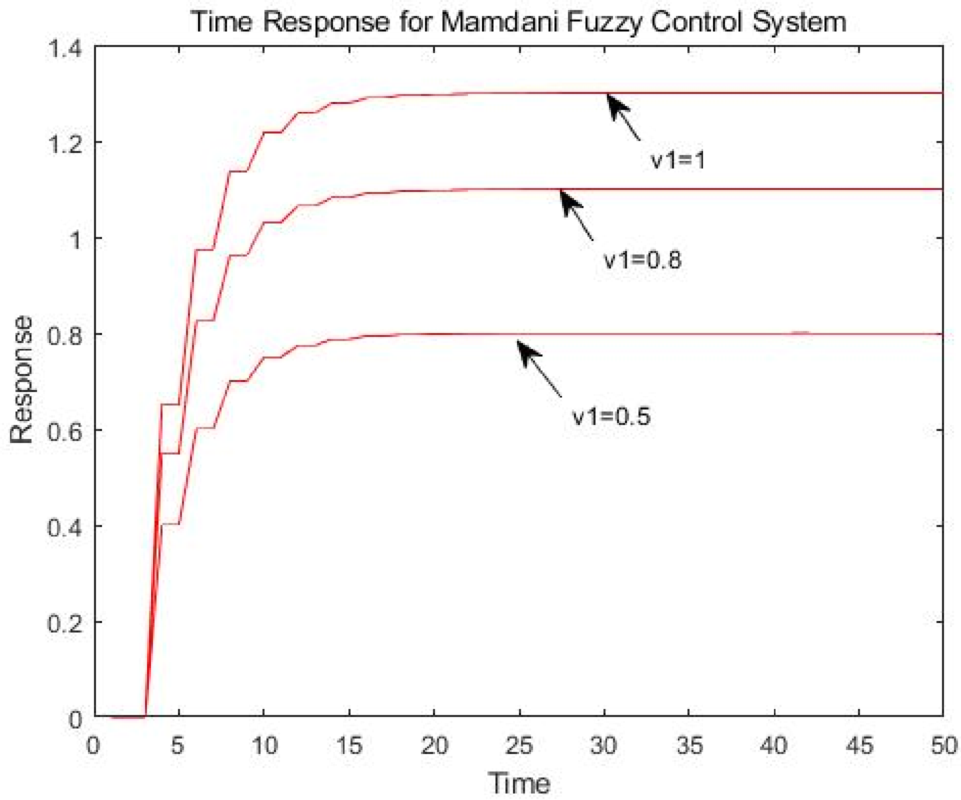

Figure 6-10 Mamdani type fuzzy control for temperature

According to Fig.6-10, the following conclusions can be drawn:

1. Simulation shows a stable control performance.

2. The settle down time is increased to about 15 time units which indicates a slower

stability.

3. There still exists some steady error.

However, such amount of steady error contributes partly to the method for defuzzification, which is the Maximum of Memberships. Such steady errors are acceptable, as the bottom line of temperature is strict limited.

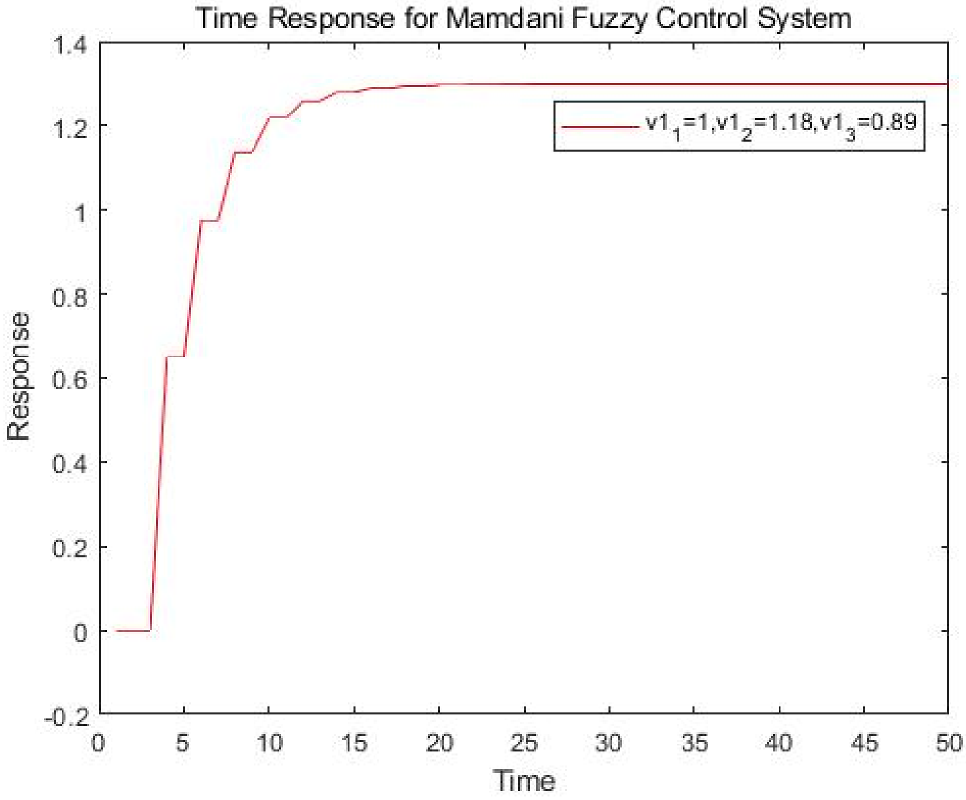

Figure 6-11 Mamdani type fuzzy control with noises

After given the interference to v_{1_1}=1, the simulation of Mamdani fuzzy control system is achieved in MATLAB, and the simulation results are shown in Fig. 6-11. From Fig 6-11 the following conclusions are drawn:

The result shows an extreme good robustness, given two interferences, indexes are almost overlapping on each other, and no exceeded steady errors are introduced to the design.

6.2.2 Type-2 Interval Fuzzy Control System Performance

As the consequence of type-2 fuzzy control of pressure control, time response of system with three step inputs of 1, 2 respectively is shown in Fig. 6-12. For comparison, time response of system with step input of 3 (which is the warning level of pressure change) is shown in Fig. 6-12. All the responses show a small fluctuation around 9^{th} time unit, and an even smaller fluctuation around 17^{th} time unit, while Fig. 6-12 shows a second slight fluctuation just after the first one, but none of the fluctuation exceed the warning level (5, -5). The steady error is almost 0, and the settling time is around 3 time units. The results show the design is successful under the assumption.

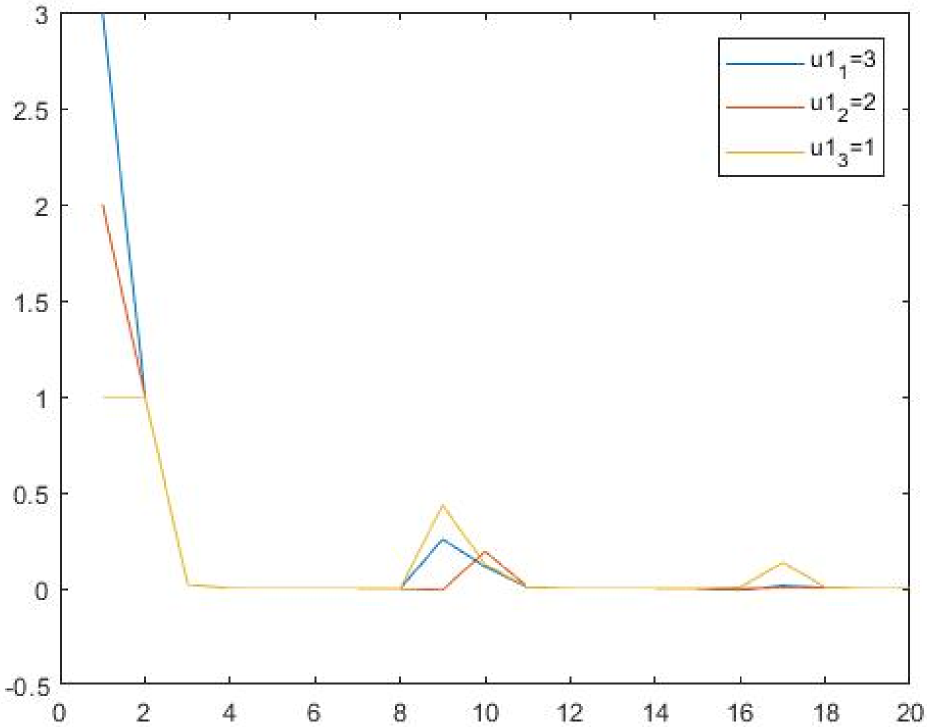

Figure 6-12 System Performance for IT-Type-2 Fuzzy Control

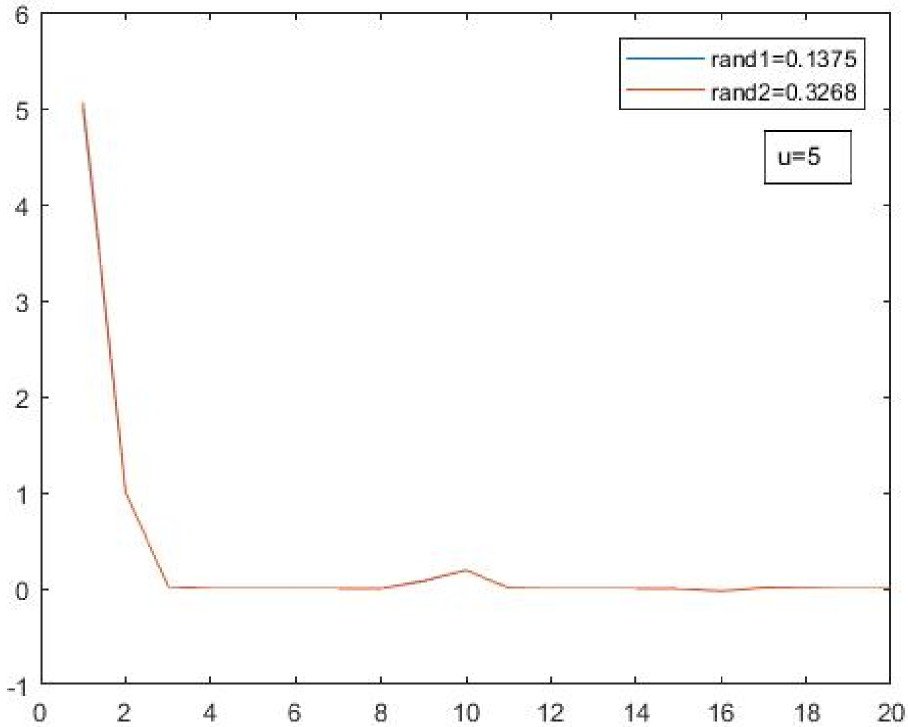

Figure 6-13 Time response for noises to u2=3

After given the two interferences to v_2=3, the simulation of interval type-2 fuzzy control system is achieved in MATLAB, and the simulation results are shown in Fig. 6-13. From Fig 6-13 the following conclusions are drawn:

The result shows an extreme good robustness, given two interferences, significant indexes are not changed, and no exceeded steady errors are introduced to the design.

6.3 Comparison of Performance

6.3.1 Comparison for temperature control

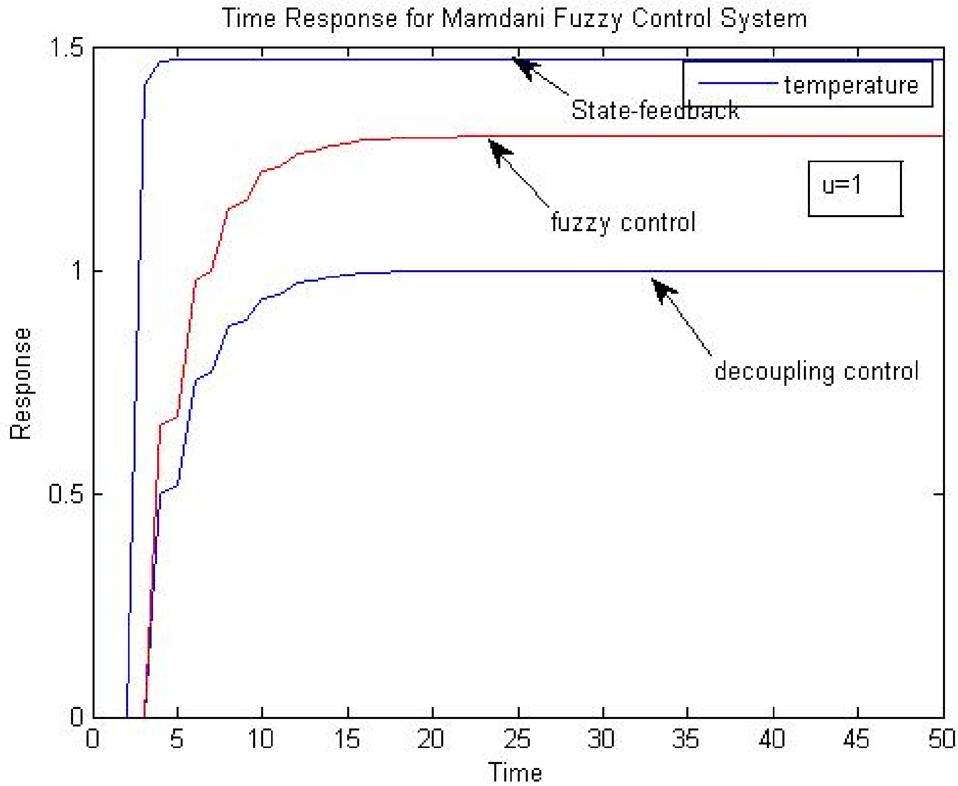

Figure 6-14 control system performance among state feedback, fuzzy control and decoupling

After implementing the method of state feedback, decoupling control and Mamdani fuzzy control, the time response has been improved significantly. The results represent that after decoupling control, rising time of system has increased (compared in Fig.6-14). After the decoupling control, however, giving different values of reference inputs, respectively $v_1=1$, $v_2=1.5$ and $v_1=1$, $v_2=2$, the result, (compare in Fig.6-14), indicates that the effect of coupled inputs and outputs has been separated to non-interaction.

Three control methods show a stable system performance when given reference at 1.

Among the results, state feedback method returns a very quick response, no damping ratio, but contains a very large steady error (about 50%). Fuzzy control provides a slower response than decoupling control's response with a smaller and acceptable steady error. It is partly because the small error is located in membership of PZ (steady state error=0.3 compared between Figure 6-8 and Figure 6-10 and the error change is also located in membership of ZO, so that the controller takes no further action, which should be eliminated in further design. Thirdly, after fuzzy control applied, the results show that more fluctuation has been introduced to the control system response.

6.3.2 Comparison for Pressure Control

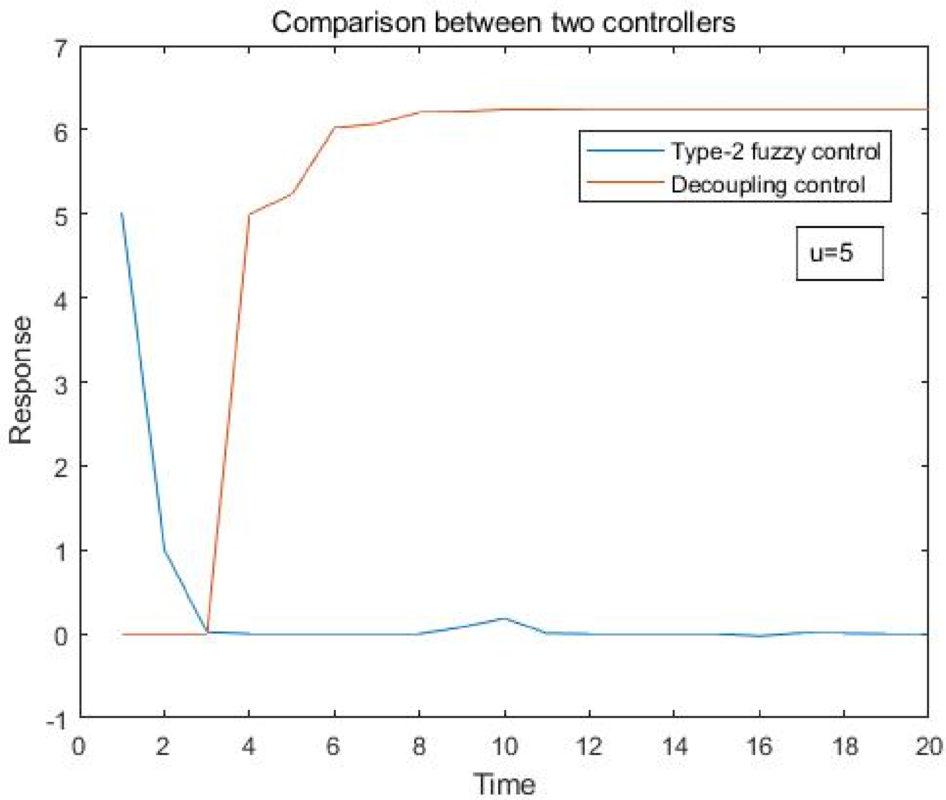

Figure 6-15 performance comparison between type-2 fuzzy control and decoupling control

After implementing the method of decoupling control and interval type-2 fuzzy control, the time response has been improved significantly. Only with state feedback control, the system is not stable. The results represent that after decoupling control, rising time of system has increased (compared with the unstable state feedback control in Fig.6-14). However, interval type-2 fuzzy control process the input as impulse, and it gives a response like an on-off switch. This makes more sense to reality, as most of pressure interference is fierce, fast, but won't last long for unknown reason. That's why, according to technicians, the plant sometimes shuts down for unclear reason. Therefore interval type-2 fuzzy control gives a more ideal control performance as expected.

6.4 Conclusions

In this chapter, all the simulation results are listed, illustrated and given cross comparisons. results are listed from two aspects, modelling and analysis, and control system performance. According to simulation results, the following conclusions can be drawn:

1. SISO for temperature reveals very stable performance with extreme large steady error, which explains the plant in practice would have a false high expected value.

2. MIMO for temperature and pressure seems like a stable one but with unacceptable steady error, as well as a large time delay. With the increase of input, performance shows a fierce damping nature. It explains why sometimes the practical plant will stop for some unknow reason.

3. MIMO with pole placement method is stable with damping nature in pressure control, which is not quite desired in this case. Additionally, as pole placement is a customized control method which is moved forward based on a specific output performance, the method is not so suitable for the following design.

4. MIMO with decoupling method shows very good control performance, where the steady error is almost eliminated from the SISO design. Meanwhile, the twisted inputs and outputs are unlocked by this method, which facilitates the consequent study.

5. Mamdani-type fuzzy control for temperature gives desired responses with a slightly floated steady error, and it shows a very good resistance against random interference.

6. Type-2 fuzzy control for pressure shows a required response, where all the interferences are treated as an impulse input, and the system gives an on-off reaction, indicates a very impressive robustness.

In conclusion, the design of fuzzy control returns a satisfied system performance with excellent robustness.

7 Conclusions and Future Work

The research and analysis work is summarised and the further development is directed and the conclusions are drawn in this chapter.

7.1 Conclusions

With the growing requirements of industrial systems for more accuracy while less energy consumption while suffering from complicated nature of the process, the spatially dynamic distributed systems are introduced to replace simple systems and soon become one of the popular research interests in system modelling and control area. Considering about complex systems with parameters, inputs or outputs distributed more than one dimension and cannot be ignored or simplified to a system represented by mathematical models conveniently, work in this thesis engages in enhancing the interests in this area to expand both theoretical research and practical applications. In the main, this thesis included the theoretical analysis and development, and the application simulation and demonstration with case studies.

The research starts from introduces the motivation of the present research based on a general review. For clarity purposes, in the beginning chapter it also provides the layout of the thesis and highlights its contributions. And then the research work has been illustrated in detail, consisting of background description and literature review of this research, and the content is discussed from four dimensions: 1). Spatially dynamic distributed systems as the object of this research. 2)State-space approach used for system analysis and modelling. 3)Fuzzy logic and fuzzy control adopted for control system design. 4)The Dyprygenation tunnel, which is the application case study.

In the part of spatially dynamic distributed systems, a general definition or description is given: a system with parameters, or outputs, or inputs spatially distributing, is categorised as a spatially dynamic distributed system. According to this description, in fact most of our systems, especially those commonly used in industry, are spatially dynamic distributed systems. However, for many reasons, in practical almost all the systems treated as lumped parameter ones, which will undoubtedly lose some precision. Sometimes such simplification is a feasible shortcut to deal with specific issues, but in some other situations it will expose the whole system to unknown risks, as the change of conditions. This section answers the question that why there's a need to study spatially dynamic distributed systems when there are already mature simplification approaches.

The consequent section is state space approach. In this chapter, state space approach is given for modelling, not only for its feasibility but also for its feasible superposition and the nature of involving many parameters / states. This is because the state space approach treat the parameters as state, thus the state is easily overlaid onto the state equations. As to this research, State-space approach suits the purpose of expanding SISO to MIMO, and taking another parameter into account. Therefore in this chapter the brief introduction is firstly made to reveal its characteristics and then the recent outcomes and development are reviewed; the results revealed that the work for 2 by 2 non-standard form of pole placement approach has not been studied very much. In this case, in this research the relevant work has been developed as well.

The third part is fuzzy logic and fuzzy control. It is employed for the design of the control systems for the sake that: on one hand, it can be established without a mathematical model, which would possibly introduce other facets affecting the precision of control system performance. In this chapter, two approaches are referred, Mamdani type and interval T-S type-2 fuzzy control, and also in this part the difference between a type-2 fuzzy control and a 3-D fuzzy design (though both of them are of three-dimensional nature) is also addressed.

The last section is the introduction and application of a Deprygenation Tunnel. As this term rare shows up in common use, it is well explained in this research including its description and application. It is a specific plant used mainly in biochemical pharmaceutical industry, and is a live, typical sample of a spatially dynamic distributed system. The consecutive conveying of vials (glass bottles) represents spatial distribution, while the physical structure of this plant is a design of a lumped parameter system.

After the background and relevant literature review has been introduced, the methodologies are introduced consequently, and the content is divided into two main parts, one is the part of modelling and system analysis, and the other is the control system design. As mentioned in Chapter 2, the state-space approach is employed for modelling and system analysis, and fuzzy control is used to design the control system. For modelling, to obtain a simple model that is closed to the mathematical model in practical use, the primary way of modelling is employed. In order to facilitate modelling, the relevant assumptions are made, and inputs, outputs and parameters are identified. And considered there are probably a coupled nature involved in this model, the pole placement control and decoupling control approach are used. Furthermore, in order to mount pole placement onto a non-standard-form state space model, an equivalent matrix transformation is designed.

After the theoretical study are introduced in detail, the modelling and simulation work are described clearly. First is modelling of plant, using state-space approach. Firstly is the plant modelling which firstly includes the analysis of this plant. This step includes the analysis of this plant which is to identify the inputs, outputs and states, also the relevant simplification and assumptions are figured out. Then this chapter follows the general process to establish a state-space model, defines two state variables, velocity of hot air, $\frac{d\Delta h}{dt}$, and ΔT_o, the tempeature discrepancy of vials.And then based on ***the principle of energy conservation***, the state equations are established. After the model is established, the coupled nature between inputs and outputs is proved, the relevant system analysis and control methods are implemented, respectively the pole placement and decoupling method. The designed state space model is non-standard form, therefore in the process of pole placement for the non-standard model, a matrix transform is designed to transfer the model to a standard form.

Based on modelling and system analysis, two control systems are respectively designed for temperature control and pressure control. Mamdani type fuzzy model is designed for temperature control. Based on two-dimensional process of error (Error and Error change), the control system is designed with two inputs. Therefore, for each input there is a fuzzification, after determine the fuzzy domain of the two inputs as well as the output, using scale factors respectively 0.3 and 0.4 to scale the real domain data to fuzzy domain inputs. Consequently, given two groups of language variables, the membership functions are determined for each input; establish the rule base according to experts' language and then the rule base is also determined with 8*7 (56) rules inside, accordingly the language variables for output are determined. When a real domain datum entered the control system, it will be firstly transferred a datum that a fuzzy system can identify, then will be inferenced according to the fuzzy rules. Therefore, in order to scale the fuzzy type output datum into a real domain datum that can be identified by actuators, the Maximum of Membership defuzzification method with a fuzzy type output datum.

An Interval type-2 fuzzy control has been designed in this Chapter for pressure control. There are two levels of fuzzy sets. The first level (primary level) is based on the input-output data, with similar "translation process" as Mamdani type; however, the secondary fuzzy sets are established based on limited work points spatially dynamic distributed, therefore the noise, or interference are considered as an impulse input added to the original step input.

Chapter 6 specifies all the simulation results, and the results are illustrated and given cross comparisons. results are listed from two aspects, modelling and analysis, and control system performance. According to simulation results, the following conclusions can be drawn:

- SISO for temperature reveals very stable performance with extreme large steady error, which explains the plant in practice would have a false high expected value.

- MIMO for temperature and pressure is not stable at all, with large time delay. With the increase of input, performance shows a fierce damping nature. It explains why sometimes the practical plant will stop for some unknow reason.

- MIMO with pole placement method is stable with damping nature in temperature control, which is not quite desired in this case. Additionally, as pole placement is a customized control method which is moved forward based on a specific output performance, the method is not so suitable for the following design.

- MIMO with decoupling method shows very good control performance, where the steady error is almost eliminated from the SISO design. Meanwhile, the twisted inputs and outputs are unlocked by this method, which facilitates the consequent study.

- Mamdani-type fuzzy control for temperature gives desired responses with a slightly floated steady error, and it shows a very good resistance against random interference.

- Type-2 fuzzy control for pressure shows a required response, where all the interferences are treated as an impulse input, and the system gives an on-off reaction, indicates a very impressive robustness.

7.2 Future Work

The research work in this thesis will have the following future directions:

1) Before state space modelling, a series of assumptions have been made, making the design job continuing easier but sacrificing some accuracy, while may lose some important information that would be critical to the modelling and control system design. In the future study, the relevant research needs effort to validate.

2) For pole placement, this study put effort forward to the MIMO type linear system, however, in fact, the object in this research is natured by dynamics and nonlinear parameters. Therefore, in the future study, the nonlinear issues shall be considered.

3) For the type-2 fuzzy control, as in this research, an embedded type-2 fuzzy set is developed to facilitate the design job, there are two ways to go further: on the one hand, the embedded type-2 fuzzy set can be expanded to a general type-2 fuzzy set who will have better adaptiveness (dealing with more uncertainties); on the other hand, the 3-D nature of type-2 fuzzy control can be popularized to language uncertainties, to enhance the system adapting with more than one operators. Furthermore, if the calculation can be reduced type-3 fuzzy sets may also be developed in the future.

Reference

Abbadi, A., Nezli, L. and Boukhetala, D. (2013), A nonlinear voltage controller based on interval type 2 fuzzy logic control system for multimachine power systems, *Electrical Power and Energy Systems* 45 (2013) 456 – 467.

Alvarez-Ramirez, J., Puebla, H. and Ochoa-Tapia, J.A (2001). Linear boundary control for a class of nonlinear PDE processes. *Systems & Control Letters*. 2001, 44: 395-403.

Bihlo, A. and Nave, J. C. (2012), Invariant discretization schemes for the heat equation, arXiv:1209.5028 [math-ph], 2012.9, last retrieved 05/06/2016.

Boubaker, O. and Babary, J. P. (2003), On SISO and MIMO variable structure control of nonlinear distributed parameter systems: application to fixed bed reactors. *Journal of Process Control*. 2003, 13: 729-737.

Butkovsky, A. G. (1961), Some approximate methods for solving problems of optimum control of distributed parameter systems, *Avtomat. i Telemekh.,* 1961, Volume 22, Issue 12, Pages 1565–1575.

Butkovsky, A.G. (1969), Distributed control systems. *Amsterdam, Elsevier*, 1969.

Calvettia, D. and Reichelb, L. (2003), Pole placement preconditioning.Linear Algebra and its Applications. 366 (1), 99-120.

Reference

Cassell, A. and Choi, C. (2012), Genetic Approach to Pole Placement in Linear State Space Systems, IEEE International Conference on Acoustics, Speech, and Signal Processing Conference Proceedings, 24, 2012.

Chai, T. Y. and Tong, S. C. (1999), Fuzzy direct adaptive control for a class of nonlinear systems, Fuzzy Sets and Systems, 103, pp. 379-387.

Chen, C.C. and Chang, H.C. (1992), Accelerated disturbance damping of an unknown distributed system by nonlinear feedback. *AIChE Journal*. 1992, 38(9): 1461-1476.

Chen, J. Y. (2001), Rule regulation of fuzzy sliding mode controller design: direct adaptive approach, *Fuzzy Sets and Systems*, 120, pp. 159-168.

Chinese Pharmacopoeia, 2010 Ed., Journal of Chinese Pharmacopoeia Commission 2010 (Chinese).

Christofides, P. D. (2001), Nonlinear and robust control of partial differential equation systems: methods and applications to transport-reaction processes. *Boston: Birkhäuser.* 2001.

Curtain, R.F. (1978), Infinite dimensional linear systems theory. *Berlin/Heidelberg, Springer Berlin/Heidelberg*, 1978.

Danieis, W.M., Jain, L.C., Mahajan, A., Forbes, S. and Puff, V. (1999), Multi-User Wireless Link for Real-time Video Transfer, *Proceedings of the Third International Conference on Knowledge-Based Intelligent Information Engineering Systems*, IEEE Press, USA, pp. 1-.4, 1999.

Dubois, D. and Prade, H. (1980), Fuzzy Sets and Systems: Theory and Applications. *New York: Academic*, 1980.

Dutton, K. (1998), Controllability and Observability in The Art of Control Engineering, 1998, pp. 307 - 316.

El-Hawary, M. E. (1998), Electric Power Applications of Fuzzy Systems, *Wiley-IEEE Press*, 1998.

Erdei, Z. and Borlan, P. (2011), Fuzzy logic control, *Carpathian Journal of Electrical Engineering*, vol. 5, no. 1, pp. 35-40, 2011.

Friedland, B. (2005). Control System Design - An Introduction to State-Space Methods, *United States: Dover Publications*. 337-369, 2005.

Galluzzo, M. & Cosenza, B. 2011, "Control of a non-isothermal continuous stirred tank reactor by a feedback–feedforward structure using type-2 fuzzy logic controllers", Information Sciences, vol. 181, no. 17, pp. 3535-3550.

Glowinski, R., Lions, J. L. and He, J. (2008), Exact and Approximate Controllability for Distributed Parameter Systems - A Numerical Approach, *Encyclopedia of Mathematics and its applications (No.117)*, Cambridge University Press, 2008.

Gueguen, C., Grenier, Y. and Giannella, F. (1980), Factorial Linear Modelling, Algorithms and Applications, ICASSP '80. *IEEE International Conference on Acoustics, Speech, and Signal Processing*, 618, 1980.

Hagras, H. A. (2004), A Hierarchical Type-2 Fuzzy Logic Control Architecture for Autonomous Mobile Robots, *IEEE Transactions on Fuzzy on Systems*, VOL. 12, NO. 4, 2004.8.

Heikkila, E. J. and Wang, Y. M. (2010), "Exploring the Dual Dichotomy within Urban Geography: An Application of Fuzzy Urban Sets", *Urban Geography*, 31:3, 406-421, 2010.

Hernandez Medina, M. D. and Mendez, G. M. (2006), Modelling and Prediction of the MXNUSD Exchange Rate Using Interval Singleton Type-2 Fuzzy Logic Systems, *2006 IEEE International Conference on Fuzzy Systems*, July 16-21, 2006.

Hoo, K.A, and Zheng, D. (2001), Low-order control-relevant models for a class of distributed parameter systems. *Chemical Engineering Science*. 2001, 56: 6683-6710.

Hussain, S. A. and Raju, M. P. (2010), Neuro-fuzzy system for medical image processing, Communication and Computational Intelligence (INCOCCI), *2010 International Conference at Erode*, pp. 382-385, 27-29 Dec. 2010.

Hwang, G. C. and Lin, S. C. (1992), A stability approach to fuzzy control design for nonlinear systems, *Fuzzy Sets and Systems*, 48, pp. 279-287.

Jain, L. C. and Martin, N.M. (Editors) (1999), "Fusion of Neural Networks, Fuzzy Systems and Evolutionary Computing Techniques: Industrial Applications", *CRC Press*, USA, 1999.

Jiang, X. H., Motai, Y. C. and Zhu, X. Q. (2005), Predictive fuzzy control for a mobile robot with non-holonomic constraints, Advanced Robotics, 2005. ICAR '05. Proceedings., *12^{th} International Conference*, Page(s): 58 – 63.

John, R., Mendel, J. M. and Carter, J. (2006), The Extended Sup-Star Composition for Type-2 Fuzzy Sets Made Simple[C], *IEEE International Conference on Fuzzy Systems*，2006：16-21．

Karnik, N. N. and Mendel, J. M. (2001), Centroid of a type-2 fuzzy set, *Information Sciences*, vol. 132, pp. 195 – 220, 2001.

Karnik, N. N., Mendel, J, M. and Liang, Q. L. (1992), Type-2 Fuzzy Logic Systems, *IEEE Transactions on Fuzzy Systems*, Vol.7, No. 6, 1999.12.

King, B. and Hovakimyan, N. (2003), An adaptive approach to control of distributed parameter systems. *Proceeding of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii USA. 2003, 5715-5720.

Koveos, Y. and Tzes, A. (2013), Resonant Fluid Actuator: Modelling, Identification, and Control, *IEEE Transactions on Control Systems Technology*, Vol. 21, Issue 3, 852-860, 2013.

Kravaris, C. and Arkun, Y. (1991), Geometric Nonlinear Control-An Overview, *Proceedings of 4th International Conference on Chemical Process Control*, Padre Island, TX. 1991, 477-516.

Li ,T. H. S., Hsiao, M. Y., Lee, J. Z. and Tsai, S. H. (2008), Interval Type 2 Fuzzy Sliding-Mode Control of a Unified Chaotic System, *2007 International Symposium on Nonlinear Dynamics (2007 ISND)*, Journal of Physics: Conference Series 96 (2008) .

Li, J. F. and Zhou, L. F. (2006), Improved Satisfactory Predictive Control Algorithm with Fuzzy Setpoint Constraints, *Intelligent Control and Automation, WCICA 2006,* the Sixth World Congress, Pages: 6247-6248, 2006.

Li, S. Y. and Xi, Y, G. (2000), Generalized Predictive Control with Fuzzy Soft Constraints, Fuzzy Systems, 2000, FUZZ IEEE 2000, T*he Ninth IEEE International Conference*, Volume 1, Page(s) 411-416, 2000.5.

Li, T. H. S. and Shieh, M. Y. (2000), Switching-type fuzzy sliding mode control of a cart-pole system, *Mechatronics*, Vol. 10, pp. 91-100, 2000.

Liang, Q. L. and Mendel, J. M. (1999), An Introduction to Type-2 TSK Fuzzy Logic Systems, *1999 IEEE International Fuzzy Systems Conference Proceedings* August 22-25, 1999, Seoul, Korea.

Liang, Q. L., Karnik, N. N. and Mendel, J. M. (2000), Connection Admission Control in ATM Networks Using Survey-Based Type-2 Fuzzy Logic Systems, *IEEE Trsactions on Systems, Man and Cybernetics-Part C: Applications and Views*, VOL. 30, NO. 3, August 2000.

Lions, J. L. (1971), Optimal control of systems governed by partial differential equations. *Berlin/Heidelberg, Springer Verlag*, 1971.

Liu, H. and Hussain, F. (2002), Chew Limtan Manoranjan Dash, Discretization: An Enabling Technique, Data Mining and Knowledge Discovery, 6, 393–423, 2002.

Ljung, L (2010), Perspectives on system identification, *Annual Reviews in Control* 34 (2010) 1–12.

Lu, S. Z, Wang, X. H., Yu, H. L. and Dong, H. J. (2012), Application of Fuzzy-PI Composite Control in Temperature Control of Decomposing Furnace, World Automation Congress (WAC), 2012, Page(s): 1 - 3 .

Lu, X. & Liu, M. (2016), Optimal Design and Tuning of PID-type Interval Type-2 Fuzzy Logic Controllers for Delta Parallel Robots, *International Journal of Advanced Robotic Systems*, vol. 13, pp. 96.

Mamdani, E. H. and Assilian, S. (1974), "An Experiment in Linguistic Synthesis with a Fuzzy Logic Controller", *Int. J. Man-Machine Studies*, 7, pp. 1-13, 1974.

Marconato, A., Sjoberg, J., Suykens, J.A.K. and Schoukens, J. (2014), Improved Initialization for Non-Linear State-Space Modelling, *IEEE Transactions on Instrumentation and Measurement*, Vol. 63, No.4, 972-980. 2014.

MathWorks, (2015). What Are State-Space Models, *http://uk.mathworks.com/help/ident/ug/what-are-state-space-models.html.* Last accessed 9 Aug 2015.

Mendel, J. M. (2001), Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. *Upper Saddle River, NJ: Prentice-Hall*, 2001.

Mendel, J. M. and John, R. I. (2002), Type-2 Fuzzy Sets Made Simple, *IEEE Transactions on Fuzzy Systems,* VOL. 10, NO. 2, APRIL 2002.

Mendel, J. M., John, R.I. and Liu, F. (2006), Interval Type-2 Fuzzy Logic Systems Made Simple, *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 6, pp. 808-821.

Mergner, S. (2009), Applications of State Space Models in Finance. *Göttingen, Germany: Universitätsverlag Göttingen*, 2009

Munro, N. (1979), Pole assignment, *Proceedings of the Institution of Electrical Engineers.* 126 (6), 549 (1979).

Naim, S. & Hagras, H. (2012), A hybrid approach for Multi-Criteria Group Decision Making based on interval type-2 fuzzy logic and Intuitionistic Fuzzy evaluation, *IEEE International Conference on Fuzzy Systems*, 2012

NASA. (2015), Newton's Second Law of Motion, *https://www.grc.nasa.gov/www/k-12/airplane/newton2.html.* Last accessed 18th Aug 2015.

Niewiadomski, A. (2010), On Finity, Countability, Cardinalities, and Cylindric Extensions of Type-2 Fuzzy Sets in Linguistic Summarization of Databases, *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 3, pp. 532-545, 2010.

Park, T. Y., Yoon, H. M. and Kim, O.Y. (1999), Optimal control of rapid thermal processing systems by empirical reduction of modes. Industrial & Engineering Chemistry Research. 1999, 38: 3964-3975.

Schouten, N. J., Salman, M.A. and Kheir N.A. (2002), Fuzzy logic control for parallel hybrid vehicles, *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 460-468, 2002.

Sepulveda, R., Castillo, O. and Melin, P. (2007), Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic. *Information Sciences.* 2007, 177(7):2023-2048.

Shen, D. M. (2015). The pole assignment for the regular triangular decoupling problem. *Automatica (Oxford)*. 53 (1), 208-215.

Siekmann, J., Thoma, M. and Carbonell, J.G. (2003), Decoupling Control, *Springer*, 2003.

Sira-Ramirez, H. (1989), Distributed sliding mode control in systems described by quasilinear partial differential equations. *Systems and Control Letters*. 1989, 13:117-181.

Soponariu, S. and Lupu, C. (2014), Temperature and Flow Decoupling Control for Air Heater Systems, *Journal of electrical and electronics engineering*, Vol. 7, 147-152, 2014.

Stevens, B. L. and Lewis, F. L. (2003), Aircraft Control and Simulation, 2nd Ed. *Hoboken, New Jersey: John Wiley & Sons*, Inc. ISBN 0-471-37145-9.

Takagi, T. and Sugeno, M. (1985), Fuzzy identification of systems and its applications to modelling and control, *IEEE Trans. Systems, Man, and Cybernetics*, Vol. 15, pp. 116-132, 1985.

Tan, W. W. and Kamal, D. H. (2006), On-line learning rules for type-2 fuzzy controller, in *Proc. IEEE Int'l Conf. on Fuzzy Systems*, Vancouver, Canada, July 2006, pp. 2530–2537.

The Physics Classroom (2015). Newton's Second Law of Motion, http://www.physicsclassroom.com/class/newtlaws/Lesson-3/Newton-s-Second-Law. Last accessed 18th Aug 2015.

Tsung-Chih, L., Hanleih, L. and Kuo, M. J. (2009), Direct adaptive interval type-2 fuzzy control of multivariable nonlinear systems[J]. *Engineering Applications of Artificial Intelligence*, 2009, 22(3): 420 – 430.

Turksen, B. (1993), Interval valued fuzzy sets and fuzzy connectives, *Journal of Interval Computations*, vol. 4, pp. 125–142, 1993.

U.S. Food and Drug Administration (2015). Bacterial Endotoxins/Pyrogens, http://www.fda.gov/ICECI/Inspections/InspectionGuides/InspectionTechnicalGuides/ucm072918.htm. Last accessed 30th Aug 2015.

Wang, L. X. and Mendel, J. M. (1992), Fuzzy basis functions, universal approximation, and orthogonal least-squares learning, IEEE Trans. on Neural Networks, vol. 3, pp. 807–813, 1992.

Wang, L., Zang, H. H. and Ning, Y. (2011), The Gas Water Heater Control System Design Based on Fuzzy Control, Electric Information and Control Engineering (ICEICE), 2011 International Conference, Page(s): 840 – 843.

Wang, M. L, Li, N. and Li, S. Y. (2008), Type-2 T-S Fuzzy Modelling for the dynamic systems with Measurement Noise, Fuzzy Systems, 2008. FUZZ-IEEE 2008, IEEE World Congress on Computational Intelligence, 2008.

Wang, M. L, Li, N. and Li, S. Y. (2010), Local Modelling Approach for Spatially Distributed System Based on Interval Type-2 T-S Fuzzy Sets, Ind. Eng. Chem. Res. 2010, 49, 4352–4359.

Wang, P. K. (1964), Control of distributed parameter systems. New York: Academic Press, 1964.

Wang, P., Li, N. and Li, S. Y. (2011), Interval Type-2 Fuzzy T-S Modeling for a Heat Exchange Process on CE117 Process Trainer, Proceedings of 2011 International Conference on Modelling, Identification and Control, Shanghai, China, June 26-29, 2011.

Wang, Y. Z., Zhu, Q. M. and Nibouche, M. (2015), State-Space Modelling and Control of a MIMO Depyrogenation Tunnel, 34th Chinese Control Conference and SICE Annual Conference 2015 (CCC&SICE2015), Hangzhou, China, 27-29/07/2015.

Wang, Y. Z., Zhu, Q. M. and Nibouche, M. (2015); Mamdani Type Controller Design for MIMO Systems with Case Study, International Conference of Identification, Modelling and Control in Tunisia, 18-20, Dec. 2015.

WikiBooks (2015), Control Systems/State Feedback, https://en.wikibooks.org/wiki/Control_Systems/State_Feedback. Last accessed 12th August 2015.

Wonham, W. M. (1967), On pole assignment in multi input controllable linear systems. IEEE Tram. AC-12, 660-665.

Woo, Z. W., Chung, H. Y. and Lin, J. J. (2000), A PID type fuzzy controller with self-tuning scaling factors, Fuzzy Sets and Systems 115 (2000) 321-326.

Wu, D. R. and Tan W. W. (2006), Genetic learning and performance evaluation of interval type-2 fuzzy logic controllers, Engineering Applications of Artificial Intelligence 19 (2006) 829–841.

Xie, X. and Zhang, S. (2002), A robust adaptive pole-placement controller without strictly positive real condition. International Journal of Adaptive Control and Signal Processing. 16 (1), 39-59, 2002.

Yu, G. and Hsiao, J. (2013), T-S fuzzy control of a model car using interval type-2 fuzzy logic system, IEEE, Neuro-computing, pp. 44, 2013.

Yu, G. R. and Kang, H. R. (2015), "T-S fuzzy control of a bidirectional converter", Networking, Sensing and Control (ICNSC), 2015 IEEE 12th International Conference at Taipei, pp 293-297, 9-11 April 2015.

Zadeh, L. A. (1965), Fuzzy sets, Information and Control Vol. 8, p. 338-353, 1965.

Reference

Zadeh, L. A. (1975), The concept of a linguistic variable and its application to approximate reasoning-1, Inform. Sci., vol. 8, pp. 199–249, 1975.

Zaher, M., Hagras, Hani., Khairy, Amr. and Ibrahim, M. (2010), A Type-2 Fuzzy Logic Based Model for Renewable Wind Energy Generation, 2010 IEEE International Conference on Fuzzy Systems (FUZZ), 2010.7.

Zaman, R. U., Khan, K. U. R. and Venugopal, A. R. (2014), Mamdani Fuzzy Control based adaptive gateway discovery for ubiquitous Internet access in MANET, India Conference (INDICON), 2014 Annual IEEE at Pune, page 1-5, 11-13 Dec. 2014.

Zhang, D. W, Han, Q. L and Jia, X. C. (2012), Tracking Control for Network-Based T-S Fuzzy Systems with Asynchronous Constraints, WCCI 2012 IEEE World Congress on Computational Intelligence Brisbane, Australia, FUZZ IEEE, 2012.6.

Zheng, D. and Hoo, K. A. (2004), System identification and model-based control for distributed parameter systems, Computers and Chemical Engineering. 2004, 28:1361-1375.

Zheng, Q., Chen, Z.Z. and Gao, Z.Q. (2009), A Practical Approach to Disturbance Decoupling Control, Control Engineering Practice, Vol. 17, No. 9, 1016-1025, 2009.

Zhu, Q. M. (2014), Lecture notes - Tool 2(Analytical) - Time Response Analysis, 2014.

Zhu, Y. G. (2011), Fuzzy Optimal Control for Multistage Fuzzy Systems, System, Man and Cybernetics, IEEE Transactions, Volume 41, Issue 4, Page(s): 964-975, 2011.8.

Zhang, X., Li, H. & Li, S.Y. 2008, "Analytical Study and Stability Design of a 3-D Fuzzy Logic Controller for Spatially Distributed Dynamic Systems", IEEE Transactions on Fuzzy Systems, vol. 16, no. 6, pp. 1613-1625.

Zhang, X., Jiang, Y. & Li, H. 2009, "3-d fuzzy logic controller for spatially distributed dynamic systems: A tutorial",2009 IEEE International Conference on Fuzzy Systems, pp. 854. 2009

Gu, F. & Yung, Y. 2013, "A SAS/IML program using the Kalman filter for estimating state space models", Behavior Research Methods, vol. 45, no. 1, pp. 38-53.

Appendix

Part 1 SISO-system modelling

```
% initialisation
length=50; %timulation length
X=[0 0]'; %initial stats
u=0.5; % step input
Y(1:length)=zeros;

A=[-1 0;-0.027 0];
B=[0.0012 0;0 0.000051];
C=[1 0];
D=0;
Ts=0.1;
I=[1 0; 0 1];
Ad=Ts*(A+I);
Bd=B*Ts;
% generate state and output sequences
for t=2:length
X(:,t)=Ad*X(:,t-1)+Bd*u;
Y(t)=C*X(:,t-1);
end

plot(Y)
title('SISO modelling-temperature');

legend('u=0.5, u=1, u=1.5');
xlabel('time unit');
ylabel('tempature unit')
```

Part 2 MIMO system modelling

```
%initialted at 22/10/2013
%Induction of SISO state feedback

%       .
%       x = Ax + Bu
%       y = Cx + Du
%
%with desired closed loop denominator:
%
%       s2 + 2*wns + wn2

clear


%The program started on 27/8/2013
% initialisation
length=100; %timulation length
X=[0;0];%initial state
%Y=zeros(2,length);
Y=[0;0];
u1=zeros(1,length); % plant input
u2=zeros(1,length);%plant input
u=[u1;u2];%plant input;
v=[1.5 0.5]';  %reference input

echo on
%System Parameters A,B,C,D

A=[-1 0;-0.027 0];
B=[0.0012 0;0 0.000051];
C=[1 0;0 1];
D=0;
Ts=0.1;
I=[1 0; 0 1];
Ad=Ts*(A+I);
Bd=B*Ts;

%syms  Mo R

%.     .
%y(t)=cx(t)=C*(A+B*Fs)*x(t)=C*B*H*v(t)
%Mo*C=(A+B*Fs)
%C*B*H=R
%==>Fs=inv(C*B)*(Mo*C-C*A)
%   H=inv(C*B)*R

%Mo=[0.5 0;0 0.5];
%R=[0.5 0;0 0.5];
%Fs=inv(C*Bd)*(Mo*C-C*Ad);
%H=inv(C*Bd)*R;
```

```
%.This is for state feedback and decoupling design

%y(t)=Mo*y(t)+R*u(t)=Mo*C*X(t)+R*u(t)
%X(t)=A*x(t)+B*u(t)
%u(t)=Fs*X(t)+Hv(t)
% generate state and output sequences
for t=2:length

    u(:,t)=X(:,t-1)+v;   %feedback
    X(:,t)=Ad*X(:,t-1)+Bd*u(:,t-1);%state
    Y(:,t)=C*X(:,t-1)%+%D*u(:,t-1); %output

end


plot(Y(1,:),'r');
hold on;
plot(Y(2,:),'b');
title('Time Response for 2I2O System design')
xlabel('Time Unit')
ylabel('temperature Response')
legend('v1_1=0.5, v2_1=1');
```

Part 3 System Design and Decoupling Control

```
%initiated at 22/10/2013
%Induction of 2I2O state feedback and Decoupling Control

%       .
%       x = Ax + Bu
%       y = Cx + Du
%
%with desired closed loop denominator:
%
%       s2 + 2*wns + wn2

clear


%The program started on 27/8/2013
% initialisation
length=30; %timulation length
X=[0;0];%initial state
%Y=zeros(2,length);
Y=[0;0];
u1=zeros(1,length); % plant input
u2=zeros(1,length);%plant input
u=[u1;u2];%plant input;
v=[1 4]';  %reference input

echo on
%System Parameters A,B,C,D

%for t=1:100;
    %A=[-0.6150 0; 3.0639*u2(t) 0];

%end
A=[-1 0;-0.027 0];
B=[0.0012 0;0 0.000051];
C=[1 0;0 1];
D=0;
Ts=0.1;
I=[1 0; 0 1];
Ad=Ts*(A+I);   %system discretization
Bd=B*Ts;

syms  Mo R %State Feedback

%.     .
%y(t)=cx(t)=C*(A+B*Fs)*x(t)=C*B*H*v(t)
%Mo*C=(A+B*Fs)
%C*B*H=R
%==>Fs=inv(C*B)*(Mo*C-C*A)
%   H=inv(C*B)*R
```

```
Mo=[0.5 0;0 0.5];
R=[0.5 0;0 0.5];
Fs=inv(C*Bd)*(Mo*C-C*Ad); %decoupling design
H=inv(C*Bd)*R;
%.
%y(t)=Mo*y(t)+R*u(t)=Mo*C*X(t)+R*u(t)
%X(t)=A*x(t)+B*u(t)
%u(t)=Fs*X(t)+Hv(t)
% generate state and output sequences
for t=2:length
    u(:,t)=Fs*X(:,t-1)+H*v;   %feedback
    X(:,t)=Ad*X(:,t-1)+Bd*u(:,t-1);%state
    Y(:,t)=C*X(:,t-1); %output
end

plot(Y');
title('Time Response for Decoupling Control System')
xlabel('Time')
ylabel('Response')
legend('v1=2.4;v2=4')
```

Part 4 Pole Placement Design

```
%initiated at 25/09/2014
%induction of system pole placement
%controllability judgement
%pole placement with damping ratio=0.5 and undammed natural
frequency wn =0.2


function m = controllable ([-1 0;-0.027 0],[0.0012 0;0 0.000051])
% Controllability determination
nc = rank(ctrb([-1 0;- 0.027 0],[ 0.0012 0;0 0.000051]));
if n==nc flagc='System is completely state controllable.'
m=1;
else flagc='System is not completely state controllable.'
m=0;
end
```

```
>> cam=ctrb([-1 0;-0.027 0],[ 0.0012 0;0 0.000051])
% Determine controllability matrix
cam =
1.0e+03 *
0.0141 0 -0.0870 0
0 0.3678 -5.7981 0
>> w=hankel([-1 0;-0.027 0],[10.0012 0;0 0.000051])
Warning: Last element of input column does not match first element
of
input row.
Column wins anti-diagonal conflict.
> In hankel at 27
```

```
w =
-1 -409.8408 0 0
-409.8408 0 0 0
0 0 0 0
0 0 0 367.7735
```

```
num=[-5002.9285]; % Obtain transfer function
den=[1 -1 0];
[a,b,c,d]=tf2ss(num,den);
```

```
m=controllble(a,b); % check the system controllability
if m==1 %fullrunk!controllability
fy=poly(a)
fq=conv([1 -0.0712],conv([1 -0.2449]))
w=hankel([fy(length(fy)-1:-1:2)';1)
cam=ctrb(a,b)
T=cam*w
i=length(fy):-1:2
diffa=-(fy(i)-fq(i))
K=diffa*inv(T)
else
message ('This system not controllable, cannot pole allocation')
end
```

Part 5 Mandani Fuzzy Control

```
%MIMO_program:FC_SI_main.m
% Initiated 14/08/2015

%model
clear

%Induction of 2I2O state feedback

%       .
%       x = Ax + Bu
%       y = Cx + Du
%
%with desired closed loop denominator:
%
%       s2 + 2*wns + wn2


% initialisation
length=30; %timulation length
X=[0;0];%initial state
%Y=zeros(2,length);
Y=[0;0];
u1=zeros(1,length); % plant input
u2=zeros(1,length);%plant input
u=[u1;u2];%plant input;
v=[1 4]';  %reference input
e=0;
ec=0;
uf2=v(2);
%System Parameters A,B,C,D
A=[-1 0; -0.0027 0];
B=[0.0012 0; 0 0.000051];
C=[1 0;0 1];
D=[1 0;1 0];

Ts=0.1;
I=[1 0; 0 1];
Ad=Ts*(A+I);
Bd=B*Ts;

syms  Mo R

%.     .
%y(t)=cx(t)=C*(A+B*Fs)*x(t)=C*B*H*v(t)
%Mo*C=(A+B*Fs)
%C*B*H=R
%==>Fs=inv(C*B)*(Mo*C-C*A)
%   H=inv(C*B)*R

Mo=[0.5 0;0 0.5];
R=[0.5 0;0 0.5];
```

```
Fs=inv(C*Bd)*(Mo*C-C*Ad);
H=inv(C*Bd)*R;
%Start Fuzzy Control
%###Determine nature domain, scale domain, calculate scale factors
for e, ec and u##########
DT=1;e0=e;
em=1;EM=6;Ke=EM/em;
ecm=1;ECM=6;Kec=ECM/ecm;
UM=7;um=0.3;Ku=um/UM;
U=[-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7];
%#########Calling Subprogram for fuzzy relationship maxtrix
R:F_Relation_2.m#################
[R,n,nE,nEC,nU,nfe,nfec,nfu,Me,Mec]=F_Relation_2;
%#################################################################
for k=1:length
    %############Calculate FC
output########################################
    %#########e Fuzzification#############################
        e=v(1)-Y(1,k);% obtain accurate value of e
    e1=round(e*Ke);%transfer value of e into scaled value
    if e1>EM;
        e1=EM;
    end
    if e1<-EM;
        e1=-EM;
    end
    if e1<0;
        j=e1+EM+1;
    end
    if (e1==0)&(e<0);
        j=7;
    end
    if (e1==0)&(e>0);
        j=8;
    end
    if e1>0;
        j=e1+EM+2;
    end

    %Obtain Fuzzy Language Value of e
    Fi=1;
    mfE=Me(1,j);
    for i=2:nE
        if Me(i,j)>mfE;
            Fi=i;
            mfE=Me(i,j);
        end
    end
    fe=Me(Fi,:);
    %##########ec Fuzzification###########################
    ec=(e-e0)/DT;
    e0=e;%obtain accurate value of ec
    ec1=round(Kec*ec);% transfer value of ec into scaled value
    if ec1>ECM;
        ec1=ECM;
```

```
    end
    if ec1<-ECM;
        e1=-ECM;
    end
    if ec1>-ECM&&ec1<ECM
        e1=ec1;
    end
    j=ec1+ECM+1;
    Fi=1;
    %Obtain Fuzzy Language Value of ec
    Fi=1;
    mfEC=Mec(1,j);
    for i=2:nEC
        if Mec(i,j)>mfEC;
            Fi=i;
            mfEC=Mec(i,j);
        end
    end
    fec=Mec(Fi,:);
    %continue;
    %################calling Fuzzy Decision-Making Subprogram for
control variable u ##########
    FU=F_Deduce_2(fe,fec,R,n,nfe,nfec,nfu);
    %Using LOM method
    nU=1;
    mFU=FU(1);
    for i=2:nfu
        if FU(i)>mFU;
            nU=i;
        else
            mFU=FU(i);
        end
    end

    uf1=Ku*U(nU)+v(1);
    uf=[uf1;uf2];
    %**************Simulation Calculation*******************
   %.
    %Y(t)=Mo*y(t)+R*u(t)=Mo*C*X(t)+R*u(t)
    %X(t)=A*x(t)+B*u(t)
    %u(t)=Fs*X(t)+H*v(t)
    % generate state and output sequences
    for t=2:length
        u(:,t)=Fs*X(:,t-1)+H*uf;   %feedback
        X(:,t)=Ad*X(:,t-1)+Bd*u(:,t-1); %state
        Y(:,t)=C*X(:,t-1)%+D*uf;  %output
    end
end
plot(Y')
title('Time Response for Mamdani Fuzzy Control System')
xlabel('Time')
ylabel('Response')
legend('v1=2.4;v2=4')
```

```
%F_Relation_2.c

%Fuzzy Relationship Calculation Subprogram F_Relation_2.c
function [R,n,nE,nEC,nU,nfe,nfec,nfu,Me,Mec]=func()
%##Define language variables for E:1=PB,2=PM,3=PS,4=-
O,5=+0,6=NS,7=NM,8=NB####
nE=8;%number of language variables for Fuzzy set E,which is also the
number of columns of control table
E=[8 7 6 5 4 3 2 1];
nfe=14;%Scale numbers of E
Me=[0 0 0 0 0 0 0 0 0 0 0.1 0.4 0.8 1.0;
    0 0 0 0 0 0 0 0 0 0.2 0.7 1.0 0.7 0.2;
    0 0 0 0 0 0 0 0.3 0.8 1.0 0.5 0.1 0 0;
    0 0 0 0 0 0 0 1.0 0.6 0.1 0 0 0 0;
    0 0 0 0 0.1 0.6 1.0 0 0 0 0 0 0 0;
    0 0 0.1 0.5 1.0 0.8 0.3 0 0 0 0 0 0 0;
    0.2 0.7 1.0 0.7 0.2 0 0 0 0 0 0 0 0 0;
    1.0 0.8 0.4 0.1 0 0 0 0 0 0 0 0 0 0];
%###Define language variables for
EC:1=PB,2=PM,3=PS,4=0,5=NS,6=NM,7=NB#####
nEC=7;% number of language variables for Fuzzy set EC,which is also
the number of columns of control table
EC=[7 6 5 4 3 2 1];
nfec=13;%Scale numbers of EC
Mec=[0 0 0 0 0 0 0 0 0 0.1 0.4 0.8 1.0;
    0 0 0 0 0 0 0 0 0.2 0.7 1.0 0.7 0.2;
    0 0 0 0 0 0 0 0.9 1.0 0.7 0.2 0 0;
    0 0 0 0 0 0.5 1.0 0.5 0 0 0 0 0;
    0 0 0.2 0.7 1.0 0.9 0 0 0 0 0 0 0;
    0.2 0.7 1.0 0.7 0.2 0 0 0 0 0 0 0 0;
1.0 0.8 0.4 0.1 0 0 0 0 0 0 0 0 0];
%###Define language variables for
U:1=PB,2=PM,3=PS,4=0,5=NS,6=NM,7=NB#####
nU=7;%  number of language variables for Fuzzy set U
U=[7 6 5 4 3 2 1];
nfu=15;%Scale numbers of U
Mu=[0 0 0 0 0 0 0 0 0 0 0 0.1 0.4 0.8 1.0;
    0 0 0 0 0 0 0 0 0 0.2 0.7 1.0 0.7 0.2 0;
    0 0 0 0 0 0 0 0.4 1.0 0.8 0.4 0.1 0 0 0;
    0 0 0 0 0 0 0.5 1.0 0.5 0 0 0 0 0 0;
    0 0 0 0.1 0.4 0.8 1.0 0.4 0 0 0 0 0 0 0;
    0 0.2 0.7 1.0 0.7 0.2 0 0 0 0 0 0 0 0 0;
    1.0 0.8 0.4 0.1 0 0 0 0 0 0 0 0 0 0 0];
%#######################Fuzzy Control Rules Consequents
Table########################
nfc=8;%%number of language variables for Fuzzy set E
mfc=7;%%number of language variables for Fuzzy set EC
UC=[1 1 2 2 2 3 4 4;1 1 2 2 2 3 4 4;1 1 2 3 3 4 5 6;1 2 3 4 4 5 6 7;
    2 2 3 4 5 5 6 7;2 3 4 5 6 6 7 7;3 4 5 6 6 7 7 7];
%##########################Calculation
R=ExECxU########################################
R=zeros(nfe*nfec,nfu);
for i=1:mfc
    for j=1:nfc
```

```
        %ExEC
        ie=E(j);
        iec=EC(i);
        for k=1:nfe
            for l=1:nfec
                if Me(ie,k)<Mec(iec,l)
                    Reec(k,l)=Me(ie,k);
                else
                    Reec(k,l)=Mec(iec,l);
                end
            end
        end
        %ExECxU
        iu=UC(i,j);
        n=0;
        for k=1:nfe
            for l=1:nfec
                n=n+1;
                for t=1:nfu
                    if Reec(k,l)<Mu(iu,t)
                        Reecu(n,t)=Reec(k,l);
                    else
                        Reecu(n,t)=Mu(iu,t);
                    end
                end
            end
        end
        for k=1:n
            for l=1:nfu
                if Reecu(k,l)>R(k,l)
                    R(k,l)=Reecu(k,l);
                end
            end
        end
    end
end
```

```
%2-Inputs Fuzzy Decision Subprogram F_Deduce_2.m
function FU=F_Deduce_2(fe,fec,R,n,nfe,nfec,nfu)
%########################Calculate
E¡ÁEC#####################################
n=0;
for i=1:nfe
    for j=1:nfec
        n=n+1;
        if fe(i)<fec(j)
            feec(n)=fe(i);
        else
            feec(n)=fec(j);
        end
    end
end
%######################## Calculate
(E¡ÁEC)¡£R################################
for l=1:nfu
    for i=1:n
        if feec(i)<R(i,l)
            fu(i)=feec(i);
        else
            fu(i)=R(i,l);
        end
        FU(l)=max(fu);
    end
end
```

```
%2-inputs fuzzy control table calculation:FC_MI_CTable.m
function FCU_T=FC_MI_CTable;
U=[-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7];
%############Calling Subprogram for fuzzy relationship matrix R:
F_Relation_2.m###############
[R,n,nE,nEC,nU,nfe,nfec,nfu,Me,Mec]=F_Relation_2;
%###########################Control Table
Calculation###############################
for i=1:nfe
    fE(i)=1;
    mfE=Me(1,i);
    for l=2:nE
        if Me(l,i)>mfE;
            fE(i)=l;mfE=Me(l,i);
        end
    end
    fe=Me(fE(i),:);
    for j=1:nfec
        fEC(j)=1;
        mfEC=Mec(1,j);
        for l=2:nEC
            if Mec(l,j)>mfEC;
                fEC(j)=l;
                mfEC=Mec(l,j);
            end
        end
        fec=Mec(fEC(j),:);
        FU=F_Deduce_2(fe,fec,R,n,nfe,nfec,nfu);

% Calling Fuzzy Decision Subprogram
        %LOM to determine the output
        mU=1;mFU=FU(1);
        for l=2:nfu
            if FU(l)>mFU;
                mU=l;
                mFU=FU(l);
            end
        end
        FCU_T(j,i)=U(mU);
    end
end
```

Part 6 Interval Type-2 Fuzzy Control

```
%initiated 08/15/2015

function twolevel_QTS
clc

xk=[5;0;3;0]
%initial input
xkk=[0;0;0;0]

p=[0.3;0.5;0.1;0.1]
%weight of each rules


xk1=xkk
xk2=xk1
xk3=xk2
xk4=xk3
xk5=xk4
xk6=xk5


u=[0.3;0.3;0.2;0.2]

uk = 0

M11=xk(1)*xk(1)/0.36
%membership degree calculation for out layer: M11, M12, M21, M22
if xk(3)==0
    M12=1
else
    M12=[0.8*sin(xk(3))-sin(0.8)*xk(3)]/xk(3)/(0.8-sin(0.8))
end
M21=1-M11

if xk(3)==0
    M22=0
else
    M22=[0.8*(xk(3)-sin(xk(3)))]/xk(3)/(0.8-sin(0.8))
end

T=0.05

A1=[0 1 1 0;1 2 0 0;1 0.36 0 0;0 0 1 0]
%out layer State Matrixes
A2=[0 1 0.0175 0;1 2 0 0;1 0.36 0 0;0 0 .0175 0]
A3=[0 1 1 0;1 2 0 0;1 0 0 0;0 0 1 0]
A4=[0 1 0.0175 0;1 2 0 0;1 0 0 0;0 0 0.0175 0]

B1=[1.36;0;0;0]
B2=B1
B3=[1;0;0;0]
```

```
B4=B3

C1 = [0 1 0 1.36;0 1 1 0]
C2 = C1
C3 = [0 1 0 1;0 1 1 0]
C4 = C3

D1 = [1;1]
D2 = D1
D3 = D1
D4 = D1

F1 = [4.9853 19.7671 -2.3501 -0.7059]
%primary membership function coefficient
F2 = [8.1979 31.7496 4.269 -2.2882]
F3 = [13 106 -37 -15]
F4 = [11.9651 78.8339 -21.3178 -8.0425]

for i=1:1:30
    x_axis=0.01*rand;
    % assume a fluctuation position randomly
    y_axis=0.01*rand;
% out layer1:
 if ((xk(1)<M11) && (xk(3)<M12))

xkk=p(1)*inner(A1,B1,C1,F1,xk,xk2,xk3,xk4,xk5,xk6,x_axis,y_axis,T,u)
     yk = C1*xk+D1*uk
 end
%out layer 2
if ((xk(1)<M11) && (xk(3)<M22))

xkk=p(2)*inner(A2,B2,C2,F2,xk,xk2,xk3,xk4,xk5,xk6,x_axis,y_axis,T,u)

    % x2kk=u(1)*x2kk1+u(2)*x2kk2+u(3)*x2kk3+u(4)*x2kk4
    yk = C2*xk+D1*uk
end

% out layer 3
if ((xk(1)<M21) && (xk(3)<M12))

xkk=p(3)*inner(A3,B3,C3,F3,xk,xk2,xk3,xk4,xk5,xk6,x_axis,y_axis,T,u)
    %x3kk = u(1)*x3kk1+u(2)*x3kk2+u(3)*x3kk3+u(4)*x3kk4
    yk = C3*xk+D1*uk
end
%out layer 4
if ((xk(1)<M21) && (xk(3)<M22))

xkk=p(4)*inner(A4,B4,C4,F4,xk,xk2,xk3,xk4,xk5,xk6,x_axis,y_axis,T,u)
    %x4kk=u(1)*x4kk1+u(2)*x4kk2+u(3)*x4kk3+u(4)*x4kk4
    yk = C4*xk+D1*uk
end
 y1(i)=yk(1)
 y2(i)=yk(2)
```

```
 %line(i,y1(i),'LineWidth',10,'Color',[.5 .5 .5])
 %line(i,y2(i),'LineWidth',20,'Color',[.9 .9 .9])


% xkk=p(1)*x1kk+p(2)*x2kk+p(3)*x3kk+p(4)*x4kk
% yk =p(1)*y1k+p(2)*y2k+p(3)*y3k+p(4)*y4k

  xk6=xk5;
  xk5=xk4;
  xk4=xk3;
  xk3=xk2;
  xk2=xk1;
  xk1=xk;
  xk=xkk;

   M11=xk(1)*xk(1)/0.36;
% determine which membership function is fired
   if xk(3)==0
      M12=1;
   else
      M12=[0.8*sin(xk(3))-sin(0.8)*xk(3)]/xk(3)/(0.8-sin(0.8));
   end
   M21=1-M11;
    if xk(3)==0
      M22=0;
    else
      M22=[0.8*(xk(3)-sin(xk(3)))]/xk(3)/(0.8-sin(0.8));
    end

end
i=1:1:20
plot(i,y2(i),'-b','LineWidth',2)
% plot(i,y2(i),'-r',i,y1(i),'-b','LineWidth',4)
grid


function
xkk=inner(Ai,Bi,Ci,Fi,xk,xk2,xk3,xk4,xk5,xk6,x_axis,y_axis,T,u)

%using T-S method to determine secondary memebership degree

%inner layer 1
     if  (0< x_axis <= T)
         if (0 <y_axis <= T)
              xkk=u(1)*(Ai*xk+Bi*Fi*xk2)
         end
         if (T <y_axis <= 2*T)
              xkk=u(1)*(Ai*xk+Bi*Fi*xk3)
         end
         if (2*T <y_axis <= 3*T)
              xkk=u(1)*(Ai*xk+Bi*Fi*xk4)
         else
              xkk=u(1)*(Ai*xk+Bi*Fi*xk2)
```

```
        end
    end
 %inner layer 2
    if  (T< x_axis <= 2*T)
       if (0 <y_axis <= T)
            xkk=u(2)*(Ai*xk+Bi*Fi*xk3)
       end
       if (T <y_axis <= 2*T)
            xkk=u(2)*(Ai*xk+Bi*Fi*xk4)
       end
      if (2*T <y_axis <= 3*T)
            xkk=u(2)*(Ai*xk+Bi*Fi*xk4)
      else
            xkk=u(2)*(Ai*xk+Bi*Fi*xk3)
      end
    end
%inner layer 3
if  (2*T< x_axis <= 3*T)
       if (0 <y_axis <= T)
            xkk=u(3)*(Ai*xk+Bi*Fi*xk4)
       end
       if (T <y_axis <= 2*T)
             xkk=u(3)*(Ai*xk+Bi*Fi*xk5)
       end
       if (2*T <y_axis <= 3*T)
             xkk=u(3)*(Ai*xk+Bi*Fi*xk6)
       else
             xkk=u(3)*(Ai*xk+Bi*Fi*xk4)
       end
   end
 %inner layer 4
   if ( x_axis >3*T)
       xkk=u(4)*(Ai*xk+Bi*Fi*xk)
   end
```