

Robust control for robotic manipulators with non-smooth strategy

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Abstract: In this paper, a novel non-smooth robust control approach is presented for robotic manipulators. By using decimal power rule in Lyapunov redesign methods, the conventional robust control for robot is improved. Our approach can achieve higher control precision with faster convergence speed. The formulations of estimating residual set and settling time have been initially established. The practical stability property is analyzed. An illustrative example is bench tested to validate the effectiveness of the proposed approach.

Keywords: Non-smooth control; robot control; robust control; practical stability.

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1 Introduction

Robotic manipulators have been extensively used in civilian industry (Brogr adh, 2007) and aerospace industry (Li *et al.*, 2010; Muelaner *et al.*, 2011). In these applications, robot control has become a key challenging issue (Hacioglu *et al.*, 2011; Hu *et al.*, 2012; Spong *et al.*, 2006). In regarding to the tracking error, it is only an ideal case to it to converge to zero. As a matter of fact, the request for a zero tracking error is unnecessary and unachievable. For example, most of robot control algorithms are asymptotically stable, which need infinite-time to stabilize tracking error to zero. In industrial applications, it is enough to stabilize tracking errors within a desired residual interval around zero in finite-time, which is called ‘‘practical stability’’ (Khalil, 2002). With the continuous improvements of industrial production, high precision control of robots requires smaller residual set and shorter settling time.

In a realistic scenario, perfect knowledge of dynamic model of manipulator can never ever be assumed. Indeed, model uncertainties are frequently encountered in robot control, such as unknown or changing payload, friction, backlash, flexible joints or robot parts for which only simplified dynamical models are available (Abdallah *et al.*, 1991). In general, neglecting the mentioned model uncertainties may cause significant performance degradation in terms of tracking accuracy and attainable velocity (Sage *et al.*, 1999; Umar *et al.*, 2014; Wang *et al.*, 2014; Xiao *et al.*, 2013). Consequently robust control has been brought in to deal with the uncertainties. Robust control has a fixed controller structure and is simple to implement. By using minimal information about the system such as bounds of uncertainty, robust control can deal with almost all of the mentioned uncertainties effectively (De Persis, 2009; Fiacchini *et al.*, 2010; Xu *et al.*, 2009). There have been many achievements on robust control for robotic manipulators (Spong *et al.*, 1987; Spong, 1992; Chiu *et al.*, 2004; Mauder, 2008; Bascetta *et al.*, 2010).

In general, robust control of robots is composed of three basic protocols. One is the nominal control, which is used to eliminate the known dynamics of robotic manipulators. The second is PD or PD like feedback control, which is used to stabilize the robot systems. And the third is a discontinuous control, which is used to cope with system uncertainty. Practically, the implementation of discontinuous controllers is characterized by the phenomenon of chattering. An effective method to avoid chattering is to approximate the discontinuous control law by a continuous one. This controller design philosophy is called Lyapunov redesign (Khalil, 2002). Due to the approximation, practical stability can be achieved instead of asymptotical stability. In conventional robot robust control, linear feedback control and/or linear general error are employed to design controller (Spong, 1992; Kim *et al.*, 2005). Though these algorithms can obtain satisfactory performance to some extent, high control gains are needed for high precision and fast convergence. High-gain control action of these classical controllers may deteriorate the performance in the presence of actuator saturations and neglected resonant dynamics.

Non-smooth control such as sliding mode control (Utkin *et al.*, 2009; Zhao *et al.*, 2009) and finite time control (Du *et al.*, 2011; Li *et al.*, 2011; Ou *et al.*, 2012; Zhao *et al.*, 2010)

have strong robustness. In light of Lyapunov redesign and non-smooth control principles, a new robot non-smooth robust control is proposed in this paper. Compared with existing robust control of robots (Spong *et al.*, 1987; Spong, 1992; Chiu *et al.*, 2004; Mauder, 2008; Bascetta *et al.*, 2010), the new approach can achieve smaller residual set and faster converging speed. The estimating method for residual set and settling time are initially presented with stability analysis.

The reset of this paper is organized as follows. In Section 2, the robotic manipulator dynamic model and its properties are presented for subsequent development. In Section 3, the main results of this paper are presented with corresponding theoretical analysis. In Section 4, illustrative examples are presented to validate the effectiveness of the proposed approach. Finally, in Section 5, some concluding remarks are given.

2 Robotic manipulator dynamic model

Consider a general n -link rigid robotic manipulator dynamic equation:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q, \dot{q}) + G(q) = \tau + d(t) \quad (1)$$

where q, \dot{q}, \ddot{q} are the joint position, velocity and acceleration vectors, respectively, $M(q) \in R^{n \times n}$ is the inertia matrix, $C(q, \dot{q})$ is the Coriolis and centrifugal matrix, $F \in R^{n \times n}$ is the diagonal matrix of viscous friction coefficients, $G(q) \in R^{n \times n}$ is the gravity torque vector, $d(t) \in R^n$ is the external disturbance vector, $\tau \in R^n$ is the control input vector.

Property 1: For all q , matrix $M(q)$ is symmetric and positive definite and satisfies: $\|M(q)\| \leq \bar{\mu}_M$, $\mu_m I \leq M(q) \leq \mu_M I$, I is an identity matrix with appropriate dimension, $\bar{\mu}_M, \mu_m, \mu_M > 0$.

Property 2: For all q and \dot{q} , $\|C(q, \dot{q})\| \leq \mu_c$, $\mu_c > 0$.

Property 3: For all q , $\|G(q)\| \leq \mu_g$, $\mu_g > 0$.

Property 4: For all q and \dot{q} , matrix $\Lambda_{\dot{q}}^{-1}(q, \dot{q})$ is always skew symmetric, that is, for a vector $x \in R^n$, there must be $x^T (\Lambda_{\dot{q}}^{-1}(q, \dot{q}) - (\Lambda_{\dot{q}}^{-1}(q, \dot{q}))^T) x = 0$.

In this study, $\|\cdot\|$ denotes Euclidean norm for vectors and induced norm for matrices. $\lambda_{\min}/\lambda_{\max}(A)$ denotes minimum/maximum eigenvalue for matrix $A^{n \times n}$.

Assumption 1: Dynamic equation (1) includes known parts and unknown parts, the following equations are satisfied:

$$M(q) = M_0(q) + \Delta M(q)$$

$$C(q, \dot{q}) = C_0(q, \dot{q}) + \Delta C(q, \dot{q})$$

$$F = F_0 + \Delta F$$

$$G(q) = G_0(q) + \Delta G(q)$$

By using Assumption 1, dynamic equation (1) can be rewritten as:

$$M_0(q)\ddot{q} + \dots = \tau + \xi(t) \quad (2)$$

where $\xi(t) \in R^n$ denotes system uncertainty, which is expressed as follows:

$$\xi(t) = -(\Delta M(q)\ddot{q} + \dots) + d(t)$$

3 Non-smooth robust control

Let q^d, \dot{q}^d be the desired joint position and velocity vectors, respectively. Define position error and velocity error as:

$$\begin{cases} e = q - q^d \\ \dot{e} = \dot{q} - \dot{q}^d \end{cases} \quad (3)$$

Before defining command vector, we introduce the following notation:

$$e_i^r = |e_i|^\alpha \text{sign}(e_i) \quad (4)$$

$$\dot{e}_i = \begin{cases} \delta |e_i|^{\alpha-1} \dot{e}_i & \text{if } |e_i| \neq 0 \\ 0 & \text{if } |e_i| = 0 \end{cases} \quad (5)$$

where $0 < \alpha < 1$, $\delta > 0$ is a small positive number, $i = 1, \dots, n$.

With reference to (3)-(5), command vector and its derivative are defined as:

$$\begin{cases} \dot{r} = \dot{e} \\ r = e \end{cases} \quad (6)$$

where Λ is diagonal positive definite matrices,

$$e^r = [e_1^r, \dots, e_n^r]^T, \quad \dot{e} = [\dot{e}_1, \dots, \dot{e}_n]^T$$

Then, general error vector and its derivative are defined as:

$$\begin{cases} r = \dot{e} \\ \dot{r} = \ddot{e} \end{cases} \quad (7)$$

Remark 1: The proposed general error is different from conventional linear general error, which is defined as $\hat{r} = \dot{e}$. Due to decimal power rule, it has terminal converging ability. Note that (5) is singular free (Man *et al.*, 1997).

In terms of definitions (6) and (7), dynamic equation (2) can be rewritten as:

$$M_0(q)\dot{r} + \dots = \tau + \xi(t) \quad (8)$$

where $h_0(q, \dot{q}, \ddot{q})$ represents the known parts of system dynamics, which is expressed as follows:

$$h_0(q, \dot{q}, \ddot{q}) = \dots$$

For dynamic equation (8), robot robust control algorithms can be designed. For example, by re-structuring the representation of robot model uncertainty, a robust controller is designed by using linear PD feedback control law (Bascetta *et al.*, 2010; Kim *et al.*, 2005). However, high gains are required to achieve high control precision and fast convergence in these approaches.

Assumption 2: The desired joint position trajectory q^d and its time derivatives \dot{q}^d, \ddot{q}^d are bounded smooth signals.

Assumption 3: External disturbance d is bounded by $\|d\| \leq D$, $D > 0$.

Assumption 4: Suppose control input is $\tau = \tilde{\tau}$. $\tilde{\tau}$ is a polynomial type control law. v is a saturation type control law. They will be designed in the following.

Assumption 5: $\tilde{\tau}$ can be bounded by the following polynomial:

$$\|\tilde{\tau}\| \leq c_0 + c_1 \|q\| + c_2 \|\dot{q}\|$$

where $c_0, c_1, c_2 > 0$ are positive real numbers.

Lemma 1: Consider the n -link rigid robotic manipulator in equation (1), if the control system uses a polynomial-type of controller, then system uncertainty can be bounded by the following inequality (Man *et al.*, 1997).

$$\|\xi\| \leq b_0 + b_1 \|q\| + b_2 \|\dot{q}\|$$

where $b_0, b_1, b_2 > 0$ are positive real numbers.

Assumption 6: According to Property 1-5 and Assumption 1-5, the following assumption is made:

$$\|\xi\| \leq \rho(q, \dot{q}, \ddot{q})$$

where $\rho(q, \dot{q}, \ddot{q}) = \kappa \|r\|$ and $0 < \kappa < 1$.

In light Lemma 1 and using Assumption 1-5, one can make Assumption 6 easily. It is applicable for most of robot control systems. Although the aforementioned assumptions show that the bound of system uncertainty ξ is control input related, only the controller structures are required in a priori. It will be validated in the stability analysis and simulation studies.

The non-smooth robust controller is designed as:

$$\tau = \tilde{\tau} \quad (9)$$

$$\tilde{\tau} = h_0(q, \dot{q}, \ddot{q}) - K_2 \text{sig}(r)^\beta \quad (9a)$$

$$v = \begin{cases} -\frac{\rho(q, \dot{q}, \ddot{q})}{1 - \kappa} \frac{r}{\|r\|} & \text{if } \|r\| \geq \varepsilon \\ -\frac{\rho^2(q, \dot{q}, \ddot{q})}{1 - \kappa} \frac{r}{\varepsilon} & \text{if } \|r\| < \varepsilon \end{cases} \quad (9b)$$

where $K_1, K_2 \in R^{n \times n}$ are diagonal positive definite matrices, $0 < \beta < 1$, $\varepsilon > 0$ is a small positive number.

For a vector $x \in R^n$, $\text{sig}(x)^\beta$ is defined as follows (Haimo, 1986):

$$\text{sig}(x)^\beta = \left[|x_1|^\beta \text{sign}(x_1), \dots, |x_n|^\beta \text{sign}(x_n) \right]^T$$

In control law (9), $-h_0(q, \dot{q}, \ddot{q})$ is nominal control used to eliminate known dynamic part of robotic manipulator, $-K_1 r - K_2 \text{sig}(r)^\beta$ is non-smooth feedback control used to stabilize the system, v is saturation control used to cope with system uncertainty.

Remark 2: Because decimal power rule is employed by general error r and feedback control law $-K_1 r - K_2 \text{sig}(r)^\beta$, the non-smooth robust control has strong terminal converging ability, that is, it can achieve higher precision and faster convergence speed.

The following two Lemmas are used in stability analysis (Yu *et al.*, 2005):

Lemma 2: Assume $a_1 > 0$, $a_2 > 0$ and $0 < c < 1$, the following inequality holds:

$$(a_1 + a_2)^c \leq a_1^c + a_2^c$$

Lemma 3: Suppose a_1, a_2, \dots and $0 < p < 2$ are all positive numbers, then the following inequality holds:

$$(a_1^2 + a_2^2 + \dots + a_n^2)^{1/2} \leq a_1 + a_2 + \dots + a_n$$

Theorem 1: For n -link rigid robotic manipulator (1), if general error is defined as (7) and control law is designed as (9), the closed loop system will be practically stable, that is, position error and velocity error will converge to the following residual sets in finite time T :

$$\Omega_r = \left\{ r \mid \|r\| \leq \Delta, \Delta = \sqrt{\frac{\mu_M}{\mu_m}} \left(\frac{\varepsilon}{4k_{\min}} \right)^{\frac{2}{\beta+3}} \right\}$$

$$\Omega_e = \left\{ e_i \mid |e_i| \leq (\Delta / \lambda_i)^{\frac{1}{\alpha}} \right\}$$

$$\Omega_{\dot{e}} = \left\{ \dot{e}_i \mid |\dot{e}_i| \leq \dots \right\}$$

$$T = t_0 + \frac{\mu_M l^2 - 2\varepsilon}{4k_{\min} \|\varepsilon\|^{\frac{\beta+3}{2}}}$$

where t_0 is the initial time, λ_i is the i th diagonal element of Λ , $i = 1, \dots, n$.

Proof: Consider the following Lyapunov function:

$$V = \frac{1}{2} r^T M(q) r \quad (10)$$

According to Property 1, V satisfies the following inequality:

$$\alpha_1(\|r\|) \leq V \leq \alpha_2(\|r\|) \quad (11)$$

where $\alpha_1(\|r\|) = \frac{1}{2} \mu_m \|r\|^2$, $\alpha_2(\|r\|) = \frac{1}{2} \mu_M \|r\|^2$. It is obvious that $\alpha_1(\|r\|)$ and $\alpha_2(\|r\|)$ are class K functions.

Differentiating V with respect to time along closed loop equation (8) gives rise to:

$$\dot{V} = -r^T h_0(q, \dot{q}, \ddot{q}) + \frac{1}{2} r^T \Lambda^{-1} \dot{\Lambda}^{-1} r + r^T C_0(q, \dot{q}) \quad (12)$$

By using Property 4 and substituting \tilde{v} into (12), one can get:

$$\dot{V} = -r^T K_1 r - r^T K_2 \text{sig}(r)^\beta + r^T v + r^T \xi(t) \quad (13)$$

$$\dot{V} = -\sum_{i=1}^n k_{1i} r_i^2 - \sum_{i=1}^n k_{2i} |r_i|^{\beta+1} + r^T v + r^T \xi(t) \quad (14)$$

where k_{1i} and k_{2i} , $i = 1, \dots, n$, are i th diagonal elements of K_1 and K_2 , respectively.

When $\|r\| \geq \varepsilon$, substituting control law v into (14) leads to:

$$\dot{V} = -\sum_{i=1}^n k_{1i} r_i^2 - \sum_{i=1}^n k_{2i} |r_i|^{\beta+1} - \frac{\|r\| \rho(q, \dot{q}, \ddot{q})}{1-\kappa} + r^T \xi(t) \quad (15)$$

$$\dot{V} = -\sum_{i=1}^n k_{1i} r_i^2 - \sum_{i=1}^n k_{2i} |r_i|^{\beta+1} - \frac{\|r\| \rho(q, \dot{q}, \ddot{q})}{1-\kappa} + \|r\| \|\xi(t)\| \quad (16)$$

Consider Assumption 6 we have:

$$\dot{V} = -\sum_{i=1}^n k_{1i} r_i^2 - \sum_{i=1}^n k_{2i} |r_i|^{\beta+1} \quad (17)$$

Let $k_{1\min} = \lambda_{\min}(K_1)$ and $k_{2\min} = \lambda_{\min}(K_2)$, then define $k_{\min} = \min\{k_{1\min}, k_{2\min}\}$. The following inequality holds:

$$\dot{V} = -k_{\min} \left(\sum_{i=1}^n r_i^2 + \sum_{i=1}^n |r_i|^{\beta+1} \right) \quad (18)$$

In light of Lemma 2 and Lemma 3, one can get the following inequality:

$$\dot{V} = -k_{\min} (\|r\|^2 + \|r\|^{\beta+1}) \quad (19)$$

Note that inequality $\|r\|^2 + \|r\|^{\beta+1} \geq 2\|r\|^{\frac{\beta+3}{2}}$ holds for all r . Accordingly, the following inequality is satisfied:

$$\dot{V} = -k_{\min} \|r\|^{\frac{\beta+3}{2}} \quad (20)$$

Let $\alpha_3(\|r\|) = 2k_{\min} \|r\|^{\frac{\beta+3}{2}}$. It is obvious that $\alpha_3(\|r\|)$ is a class K function. Apparently, as $\|r\| \geq \varepsilon$:

$$\dot{V} = -\alpha_3(\|r\|) < 0 \quad (21)$$

When $\|r\| < \varepsilon$, substitute control law v into (14), it yields:

$$\dot{V} = -\sum_{i=1}^n k_{1i} r_i^2 - \frac{\rho^2(q, \dot{q}, \ddot{q})}{1-\kappa} + \|r\| \|\xi\| \quad (22)$$

By taking into consideration Assumption 6, the following inequality is obtained:

$$\dot{V} = -\frac{\|r\|^2 \rho^2}{\varepsilon} + \|r\| \rho \quad (23)$$

Note that the term $-\|r\|^2 \rho^2 / \varepsilon + \|r\| \rho$ attains a maximum value $\varepsilon/4$ at $\|r\| \rho = \varepsilon/2$. Therefore:

$$\dot{V} = -\frac{\varepsilon}{4} \quad (24)$$

Inequality (24) is held whenever $\|r\| < \varepsilon$. On the other hand, when $\|r\| \geq \varepsilon$, \dot{V} satisfies:

$$\dot{V} = -\alpha_3 (\|r\|) + \frac{\varepsilon}{4} \quad (25)$$

Thus inequality (24) is satisfied irrespective of the value of $\|r\|$. Take $l > 0$, choose $\varepsilon < 2\alpha_3(\alpha_2^{-1}(\alpha_1(l)))$ and $\mu = \alpha_3^{-1}(\varepsilon/2) < \alpha_2^{-1}(\alpha_1(l))$. According to Lyapunov redesign method (Khalil, 2002), there must be:

$$\dot{V} = -\mu (\|r\|), \quad \forall \mu \leq \|r\| < l \quad (26)$$

Choose $\mu(\varepsilon) < \|r(t_0)\| < \alpha_2^{-1}(\alpha_1(l))$, $r(t_0)$ is the initial value of $r(t)$ at time t_0 , settling time and bounds of $r(t)$ can be estimated as T and Ω_r , respectively.

If $\|r\| \leq \Delta$, the following equation holds:

$$\dot{e}_i = -\lambda_i \text{sign}(e_i) = \phi_i, \quad |\phi_i| \leq \Delta \quad (27)$$

Equation (27) can be further written as follows:

$$\dot{e}_i = -\lambda_i |e_i|^\alpha \text{sign}(e_i) |e_i|^\alpha \text{sign}(e_i) = 0 \quad (28)$$

If $\lambda_i - \phi_i / |e_i|^\alpha \text{sign}(e_i) > 0$, equation (28) is still kept in the form of general error (7). Position error will converge to the following region:

$$\Omega_e = \left\{ e_i \mid |e_i| \leq (\Delta / \lambda_i)^{\frac{1}{\alpha}} \right\} \quad (29)$$

According to equation (29), velocity error will converge to the following region:

$$\Omega_v = \left\{ \dot{e}_i \mid |\dot{e}_i| \leq \Delta \right\} \quad (30)$$

Accordingly, position error and velocity error will converge to the residual sets Ω_e and Ω_v in a finite time T , respectively.

This completes the proof. \square

If we choose $\alpha = 1$, $\beta = 1$, $K = K_1 + K_2$, control law (9) will be smooth robust control (Khalil, 2002):

$$\hat{t} = \hat{t} \quad (31)$$

$$\hat{t} = \hat{t} \quad (31a)$$

$$\hat{v} = \begin{cases} -\frac{\hat{\rho}(q, \dot{q})}{1 - \kappa} \frac{\rho^2}{\|\hat{r}\|} & \text{if } \|\hat{r}\| \geq \varepsilon \\ -\frac{\hat{\rho}^2(q, \dot{q})}{1 - \kappa} \frac{\rho^2}{\varepsilon} & \text{if } \|\hat{r}\| < \varepsilon \end{cases} \quad (31b)$$

where ε and κ are same as equation (9), \dot{q} , \ddot{q} and \hat{r} are defined as:

$$\begin{cases} \dot{q} \\ \ddot{q} \\ \hat{r} \end{cases} = \begin{bmatrix} \dot{q} \\ \ddot{q} \\ \hat{r} \end{bmatrix}$$

Subject to control law (31), e_i and \dot{e}_i will converge to the following residual sets in finite-time.

$$\hat{\Omega}_{\hat{r}} = \left\{ \hat{r} \mid \|\hat{r}\| \leq \hat{\Delta}, \hat{\Delta} = \sqrt{\frac{\mu_m}{\mu_m}} \left(\frac{\varepsilon}{4k_{\min}} \right)^{\frac{1}{2}} \right\}$$

$$\hat{\Omega}_{\dot{e}_i} = \left\{ \dot{e}_i \mid |\dot{e}_i| \leq \hat{\Delta} / \lambda_i \right\}$$

$$\hat{\Omega}_{\hat{t}} = \left\{ \hat{t} \mid \|\hat{t}\| \leq \hat{\Delta} \right\}$$

$$\hat{T} = t_0 + \frac{\mu_m l^2 - 2\varepsilon}{4k_{\min} \|\varepsilon\|^2}$$

The stability analysis and estimating methods of control law (31) are similar to those of Theorem 1.

Remark 3: High precision control means smaller residual set and shorter settling time. For fair comparison, choose dimensionless position error and velocity error. This also means that $|r_i|$, $|\dot{r}_i|$ and ε far less than 1 in the steady state.

Because $0 < \beta < 1$, $\frac{2}{\beta+3} > \frac{1}{2}$ and $\frac{\beta+3}{2} < 2$, $\Delta < \hat{\Delta}$ and

$T_r < \hat{T}_r$. The above comparisons intuitively show that non-smooth robust control has smaller residual sets and shorter settling time than those of smooth robust control. This will be further validated by simulation results.

The controller design procedure is summarized as:

Step 1: Define position error, command vector and general error according to equation (3)-(7).

Step 2: Design control law as (9).

Step 3: Initially choose appropriate controller parameters according to the required control precision and settling time.

Step 4: Slightly retune the controller parameters using a trial and error method until the performance is satisfying.

4 Illustrative example

Two examples are given in this section. One is used to illustrate the effectiveness of the non-smooth general error, and the other is used to illustrate the effectiveness of the proposed non-smooth robust control for robotic manipulators.

4.1 Comparison study for general error

Consider a 2-second nonlinear system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -120t + u \end{cases}$$

Conventional smooth general error is defined as:

$$r = x_2 + x_1$$

The non-smooth general error is defined as:

$$r = x_2 + \lambda |x_1|^\alpha \text{sign}(x_1)$$

$$\dot{r} = \begin{cases} \lambda \alpha |x_1|^{\alpha-1} x_2 & \text{if } |x_1| \neq 0 \\ \lambda \alpha |x_1|^{\alpha-1} x_1 & \text{if } |x_1| = 0 \end{cases}$$

To fairly compare these general errors, a standard smooth robust control is designed by using Lyapunov redesign:

$$u = -x_r - r + v$$

$$v = \begin{cases} -\frac{\rho(t,x)}{1-\kappa_0} \frac{r}{|r|} & \text{if } |r| \geq \varepsilon \\ -\frac{\rho^2(t,x)}{1-\kappa_0} \frac{s}{\varepsilon} & \text{if } |r| < \varepsilon \end{cases}$$

where $\varepsilon = 0.02$, $\delta = 0.01$, $\rho = 0.12$, $\kappa_0 = 0.4$.

Note that linear PD type feedback control is used here. Under this control law, the different performances of two types of general errors can be revealed more clearly.

Figure 1 is the evolution of system state x_1 . Solid line is non-smooth general error and dashed line is smooth general error. It is obvious that the proposed non-smooth general error has faster converging speed and smaller residual set than those of smooth general error. Figure 2 shows the control input. From the simulation results, we can see that the proposed non-smooth general error has terminal converging power. This is consistent with the theoretical analysis.

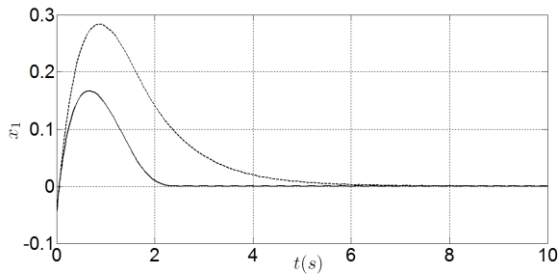


Figure 1 x_1 with respect to time

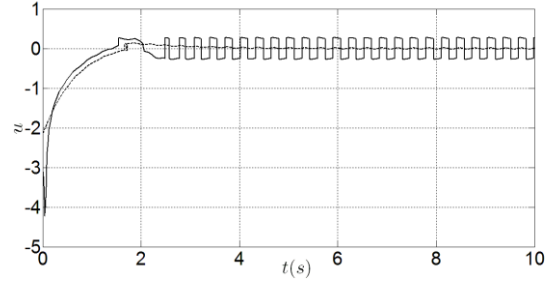


Figure 2 Control input u

4.2 Control performance of a robotic manipulator

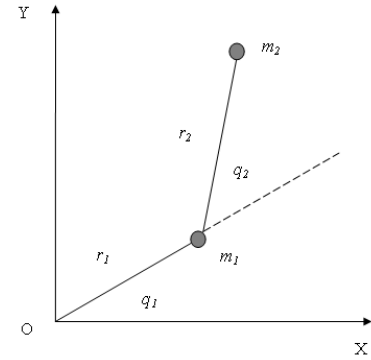


Figure 3 2-link robotic manipulator

Consider a 2-link rigid robotic manipulator (see Figure 3). Its dynamic equation is given as:

$$\begin{bmatrix} a_{11}(q_2) & a_{12}(q_2) \\ a_{12}(q_2) & a_{22}(q_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -b_{12}(q_2)\dot{q}_1 & \dots \\ 0 & b_{12}(q_2)\dot{q}_1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$a_{11}(q_2) = (m_1 + m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2 \cos(q_2) + J_1$$

$$a_{12}(q_2) = m_2r_2^2 + m_2r_1r_2 \cos(q_2)$$

$$a_{22}(q_2) = m_2r_2^2 + J_2$$

$$b_{12}(q_2) = m_2r_1r_2 \sin(q_2)$$

$$c_1(q_1, q_2) = (m_1 + m_2)r_1 \cos(q_2) + m_2r_2 \cos(q_1 + q_2)$$

$$c_2(q_1, q_2) = m_2r_2 \cos(q_1 + q_2)$$

The parameter values are chosen as: $r_1 = 1\text{m}$, $r_2 = 0.8\text{m}$, $J_1 = 5\text{kgm}^2$, $J_2 = 5\text{kgm}^2$, $m_1 = 0.5\text{kg}$ and $m_2 = 1.5\text{kg}$. The desired trajectories are given as:

$$q_1^d = 1.25 - (7/5)e^{-t} + (7/20)e^{-4t}$$

$$q_2^d = 1.25 + e^{-t} - (1/4)e^{-4t}$$

The initial values of system states are assumed to be: $q_1(0)=1.0$, $q_2(0)=1.5$, $\dot{q}_1(0)=0$ and $\dot{q}_2(0)=0$. External disturbances are assumed to be:

$$d_1 = 2\sin(t) + 0.5\sin(200\pi t)$$

$$d_2 = \cos(2t) + 0.5\sin(200\pi t)$$

The nominal values of m_1 and m_2 are assumed to be $m_{10} = 0.4\text{kg}$ and $m_{20} = 1.2\text{kg}$.

To validate the effectiveness of the proposed approach, non-smooth robust control and smooth control are compared.

Table 1 Controller parameters

| Controller | Parameters |
|---------------------------|--|
| Non-smooth robust control | $\Lambda = \text{diag}(1,1)$, $\alpha = 1/3$, $\delta = 0.01$, $K_1 = K_2 = \text{diag}(2.5, 2.5)$, $\beta = 1/3$, $\varepsilon = 0.02$, $\kappa = 0.4$, $b_0 = 1.5$, $b_1 = 1$, $b_2 = 1$ |
| Smooth robust control | $\Lambda = \text{diag}(1,1)$, $\alpha = 1$, $\delta = 0.01$, $K_1 = K_2 = \text{diag}(2.5, 2.5)$, $\beta = 1$, $\varepsilon = 0.02$, $\kappa = 0.4$, $b_0 = 1.5$, $b_1 = 1$, $b_2 = 1$ |

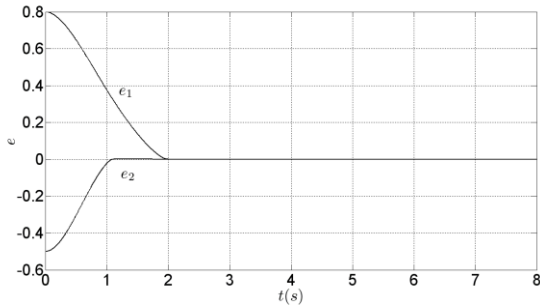


Figure 4 Position tracking error (non-smooth $\alpha = 1/3$, $\beta = 1/3$)

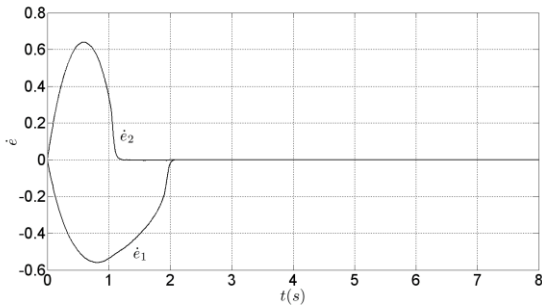


Figure 5 Velocity tracking error (non-smooth $\alpha = 1/3$, $\beta = 1/3$)

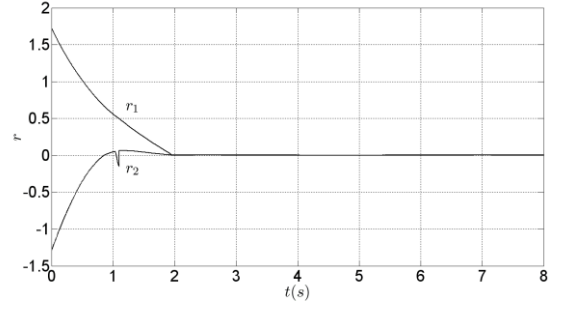


Figure 6 General error (non-smooth $\alpha = 1/3$, $\beta = 1/3$)

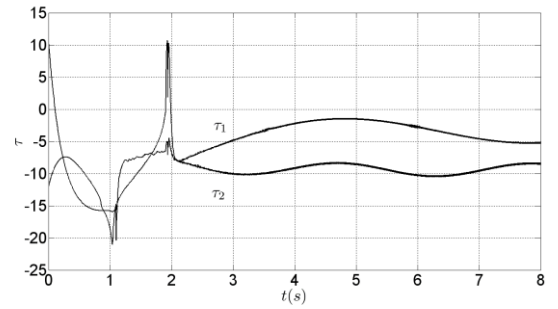


Figure 7 Control input (non-smooth $\alpha = 1/3$, $\beta = 1/3$)

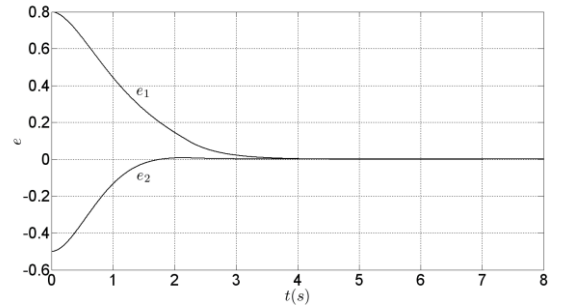


Figure 8 Position tracking error (smooth $\alpha = 1$, $\beta = 1$)

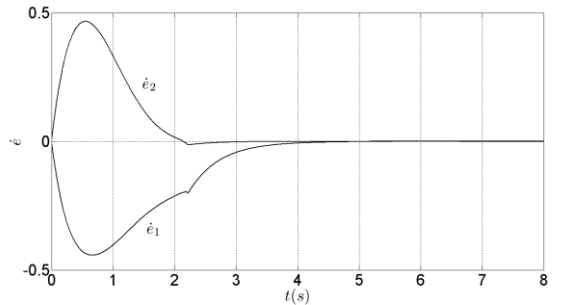


Figure 9 Velocity tracking error (smooth $\alpha = 1$, $\beta = 1$)

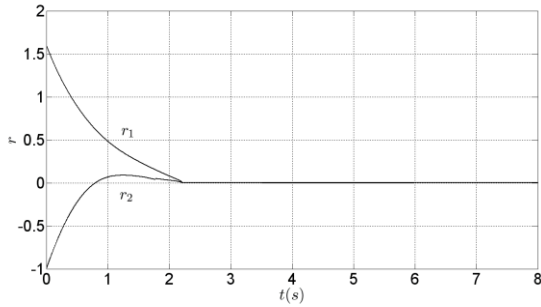


Figure 10 General error (smooth $\alpha = 1$, $\beta = 1$)

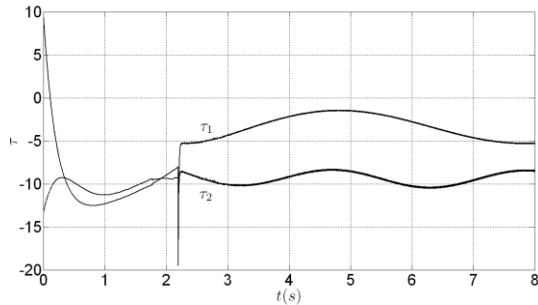


Figure 11 Control input (smooth $\alpha = 1$, $\beta = 1$)

Table 2 Residual set and settling time

| Controller | Setting time (s) | Residual set |
|--|------------------|-------------------------------------|
| Non-smooth robust control $\alpha = 1/3$ $\beta = 1/3$ | $t = 2.2523$ | $ e_1 \leq 7.0349 \times 10^{-5}$ |
| | $t = 1.9753$ | $ e_2 \leq 7.7315 \times 10^{-5}$ |
| | $t = 2.1250$ | $ \dot{e}_1 \leq 7 \times 10^{-4}$ |
| | $t = 1.2158$ | $ \dot{e}_2 \leq 7 \times 10^{-4}$ |
| | $t = 1.9597$ | $\ r\ \leq 0.0040$ |
| Smooth robust control $\alpha = 1$ $\beta = 1$ | $t = 4.2898$ | $ e_1 \leq 1.4000 \times 10^{-3}$ |
| | $t = 3.9908$ | $ e_2 \leq 1.5000 \times 10^{-3}$ |
| | $t = 4.9349$ | $ \dot{e}_1 \leq 0 \times 10^{-3}$ |
| | $t = 3.4946$ | $ \dot{e}_2 \leq 4 \times 10^{-4}$ |
| | $t = 2.2379$ | $\ r\ \leq 0.0045$ |

From Table 1, one can see that all the parameters of smooth robust controller are same as non-smooth robust controller except that $\alpha = 1$ and $\beta = 1$. This shows that the comparisons are fair.

Figure 4-Figure 7 are the performance of non-smooth robust controller. Figure 4 is the position tracking error. Figure 5 is the velocity tracking error. Figure 6 is the general error. Figure 7 the is control input (joint torque). From these simulation results, it can be seen that position

error and velocity error can converge to a small residual set in finite-time. The control input is bounded and continuous. This validates the effectiveness of the proposed approach. Figure 8-Figure 11 show the performance of smooth robust controller. To further compare these two approaches, Table 2 lists the residual sets of position errors, velocity errors and general errors with corresponding settling times of non-smooth robust controller and smooth robust controller, respectively. From Table 2, one can see that the proposed approach has smaller residual sets and shorter settling times. Note that Figure 5 and Figure 9 are control inputs of non-smooth and smooth robust control, respectively. From these two Figure s, we can see that the control inputs of these two approaches are similar. It means that non-smooth robust control can achieve higher precision with fast convergence speed without using high control gains. This is attributed to the non-smooth general error and feedback control.

5 Conclusions

Facing the challenging issue on dealing with uncertainty encountered in robotic manipulator systems. This paper has developed a new non-smooth robust control algorithm. The associated questions on stability analysis, estimating residual set and settling time have been properly addressed by using Lyapunov redesign method.

It is worthwhile noting that the paper has provided new solutions to robust control of robot. Compared with existing smooth robust algorithms, the proposed approach can stabilize robot tracking error into a smaller residual interval with a faster convergence speed. Hence, it can achieve high precision practical stability control.

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