Phase shift of the current under a single-phase short-circuit in the stator winding of a turbogenerator with double winding per phase

Zbigniew Fjałkowski · Hassan Nouri

Abstract An equivalent model of a faulty turbogenerator under a single-phase short-circuit in the stator winding is presented. The model is based on modified voltage–current equations for the positive-sequence component of the short-circuit current that takes into account the mutual spatial displacement of particular part of the faulty stator winding, the rotor magnetic asymmetry and the impact of the resultant reactance value of the unit transformer and the power network. The developed model also represents the effect of the mutual phase shift of the short-circuit current flowing in the relevant sections of the damaged stator winding. The mutual phase shift value of the short-circuit current (periodic component of basic frequency) is found to be dependent on the number of shorted turns in the stator winding. Also found is the influence of the load current value on the displacement angles between currents flowing in the particular part of the faulty stator winding. This phenomenon of phase shift of the short-circuit current could be used as an additional criterion of operation for turbogenerator protection systems.

Keywords Turbogenerator · Single-phase short-circuit · Phase shift of current

1 Introduction

The short-circuit current in the stator windings can many times exceed its rated value. It is dangerous for the generator and for the power system as well. Therefore, analysis of the short-circuit current with greater degree of accuracy is important irrespective of probability of their occurrence. Though the damaged turbogenerator should be turned off by the protection device as soon as this is possible. To achieve this, the adequate setting of protection devices, the accuracy in the calculation of the short-circuit current that includes the impact of various factors is important. Also important are the accurate measurement of data and selection of correct parameters for reliable and prompt activation of the turbogenerator protections. In this paper, attempt will be made to demonstrate that the mutual phase shift of the short-circuit current in spatially distributed coil of the stator windings can successfully be used as an additional data for fault detection.

Analysis of asymmetric internal short-circuits in electric machines is commonly performed by considering a particular complex short-circuit as a superposition of two (or a few) separate single-phase short-circuits. However, examination of a single-phase short-circuit in the turbogenerator stator winding with the use of symmetrical components method requires, above all the complete information on both the value and variation with time of the positive-sequence current components, which is the same requirement as for the short-circuit current calculation under symmetrical three-phase short-circuit. However for the damaged turbogenerator, the time varying parameters of the machine [5, 10] (X_d, X'_d, X''_d and X_q, X''_q), and magnetic as well as electric asymmetry of machines, both with salient and with latent poles need to be considered. It should be noted that accuracy of evaluation of the short-circuit current variation (positive-sequence component) for both transient and steady states determines the evaluation accuracy of the remaining sequence current components (i.e. negative- as well as zero-sequence). This in turn affects calculation of currents under any asymmetrical short-circuit in the turbogenerator stator windings.

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The short-circuit point divides the damaged phase winding into two parts which are offset relative to each other along the circumference of the stator. Thus, the winding axes of respective parts will be characterized with a correct and specified geometrical angle [1,8].

To increase calculation accuracy of the short-circuit currents, it is required to evaluate both the self-reactance and the mutual reactance for a particular section of the damaged windings as well as the mutual shift between magnetic fluxes (due to the short-circuit current in particular coil) and the mutual shift of resultant fluxes [2,8].

It should be noted that when taken into account the shift of magnetic axes of both parts of the faulted winding (in the R. Park's equations system [10]), one has to change the voltage notation (at the short-circuit point just before occurrence of the fault for both parts of the faulted winding) used in the voltage–current equations which describe (in the system of rotating axes dq) the distribution of the current flow under the three-phase symmetrical short-circuit [1]. Otherwise, the direct application of the method of symmetrical components for calculation of currents under single-phase internal short-circuits is not valid. Therefore to use the symmetrical components method for direct examination of any internal short-circuits in a turbogenerator, it is necessary to transform the previously specified voltage–current equations (for the three-phase symmetric short-circuits [2,3,9]) in such a way that only one strictly specified voltage of the positive-sequence component could be used. Such a transformation is described and recommended by Fjałkowski [1]. When taken into account the negative- and zero-sequence components, the analysis of the phase shifts of currents of fundamental frequency under single-phase internal short-circuits in the stator windings can be performed. Such the equivalent circuit model of the turbogenerator has been developed and discussed in the paper.

2 Simplifying assumptions

The analysis of a single-phase internal short-circuit in the turbogenerator (phase A) stator winding is performed for commonly used general simplifying assumptions regarding the circuit representation of phenomena, linearity of a magnetic and the electric circuit. A bipolar synchronous machine with a double-layer lap winding and a double winding per phase is selected for consideration. Purpose of research is to analyze phase shift change of the short-circuit current periodic component (fundamental frequency) as the function of the relative number of shorted turns of the stator winding (aperiodic component as well as double frequency component of the short-circuit currents are omitted). The detailed considerations are performed with additional assumptions as follows [1,2]:

- rotor angular speed during the short-circuit time is constant and equal to the synchronous value,
- electromotive force induced in the phase windings (due to rotor field) is sinusoidal,
- influence of higher harmonics of the magnetic flux in the air gap can be neglected,
- magnetic saturation of the turbogenerator material does not occur,
- resistance of the phase winding and the transient resistance at the short-circuit point is neglected,
- at the time of short-circuit the turbogenerator operates with the rated load,
- synchronous reactance values in the direct axis (d) and in the quadrature axis (q) are equal.

3 Developed mathematical model of the damaged turbogenerator

For a turbogenerator with double winding per phase, under single-phase short-circuit (in phase A) with a relative number of turns that is expressed as:

$$\alpha = \frac{n}{N}, \tag{1}$$

(where n number of shorted turns, N number of turns of one branch of the stator winding) one can distinguish three circuits, as illustrated in Fig. 1: the circuit between the turbogenerator neutral point 0 and the short-circuit point K, the circuit between the short-circuit point K and the turbogenerator phase terminal A, and the circuit made of the remaining undamaged parallel branch of the turbogenerator stator winding [6,7]. The remaining part of the circuit in Fig. 1 rep-

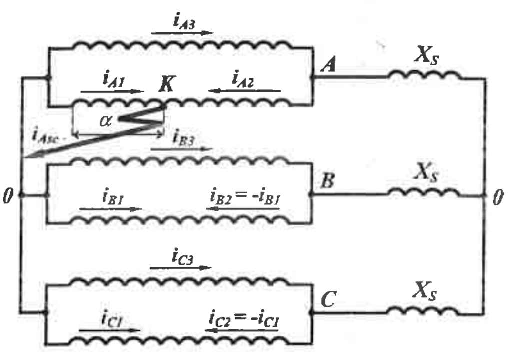

Fig. 1 Scheme of turbogenerator of two winding per phase (co-operating with the power system) at the single-phase (A) short-circuit in the stator windings, where i_{Ai}, i_{Bi}, i_{Ci} (i = 1, 2, 3)—the short-circuit currents in the windings of appropriate phase, i_{Asc}—the current at single-phase short-circuit point (in phase A), X_S—resultant reactance of the unit transformer and the power system

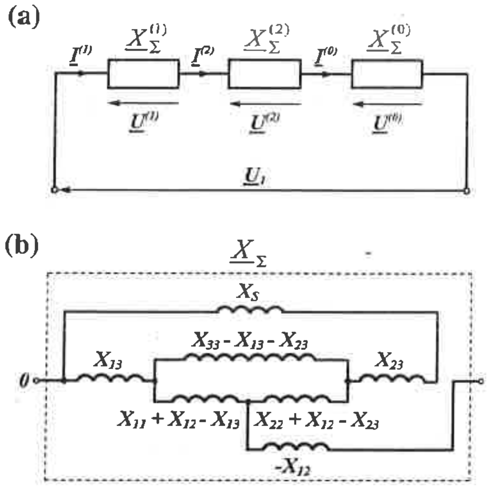

Fig. 2 The simplified equivalent scheme of the damaged turbogenerator (co-operating with the power system) under single-phase short-circuit in the stator winding using symmetric components (**a**); distribution of reactances of the damaged stator winding of the turbogenerator including the reactance of the power system (for respective sequence component of total reactance $\underline{X}_{\Sigma}$) (**b**)

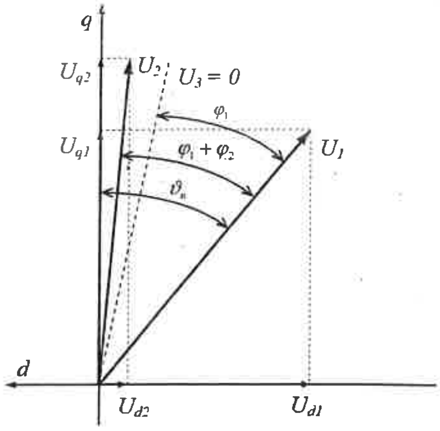

Fig. 3 A vector diagram of voltages U_1 (U_{d1}, U_{q1}) and U_2 (U_{d2}, U_{q2}) expressed in the $d\,q$ system under the three-phase symmetric short-circuit in stator windings, where ϑ_n nominal angle of load; φ_1, φ_2 angles of spatial shifts of axes of particular parts of the damaged stator winding related to the axis of winding of the undamaged branch

resents the combination of the unit transformer and the power network. In this paper, the considerations are limited to internal short-circuit case in the turbogenerator connected via a unit transformer (of equivalent reactance X_T) connected to a power network of infinite power (system reactance is equal to 0). Therefore, it is assumed that the resultant reactance denoted as X_S (seen from the generator terminals) is equal to reactance of the unit transformer:

$$X_S = X_T. \tag{2}$$

Hence, the asymmetrical short-circuit can be analyzed by means of the symmetrical components method. By considering the magnetic flux configurations for the positive-, negative- and zero-sequence components of a short-circuit current separately, a set of respective voltage–current equations of the damaged turbogenerator can be obtained [7,9]. Thus, the simplified equivalent scheme of the damaged turbogenerator can be deduced as in Fig. 2. It is composed of equivalent reactances expressed in complex notation for positive $\underline{X}_{\Sigma}^{(1)}$, negative $\underline{X}_{\Sigma}^{(2)}$ and zero-sequence $\underline{X}_{\Sigma}^{(0)}$ component which are connected in series and supplied with the voltage $\underline{U}_1$ at the short-circuit point at the time just before the fault occurrence.

3.1 Equivalent reactance for the positive-sequence component

Equations for the currents distribution under a three-phase short-circuit inside the turbogenerator stator winding are those of the positive-sequence components [9]. Therefore, the equivalent reactance for the steady state can be calculated as follows:

$$\underline{X}_{\Sigma}^{(1)} = \frac{\underline{U}_1}{\underline{I}_{sc(3f)}}, \tag{3}$$

where $\underline{I}_{sc(3f)}$ steady-state current at the three-phase symmetric short-circuit point.

To determine the current $\underline{I}_{sc(3f)}$, one has to solve the respective voltage–current equations of the system [1]. However, the available equations from the literature only concern with the simplified model of a synchronous machine having the concentrated windings [3,9]. Moreover, mutual spatial displacement of the particular part of the damaged winding (along the circumference of the stator) results in respective phase shift of voltages existing in these parts. With the application of the R. Park's transformation—the voltage and current quantities shifted in time for both damaged parts of the stator winding [1] can be expressed. Since the voltage at the short-circuit point (at the time just before the fault occurrence) is composed of two mutually shifted vectors (see Fig. 3 [1]), application of the symmetrical components method is impossible. To overcome this problem one has to introduce the respective transformation concept mentioned above. The voltage equations that affect the distribution of the short-circuit current of the positive-sequence component are found to be in the form shown below [1]:

$$\begin{aligned} &U_1 \cos(-\vartheta_n + \varphi_1) \\ &\quad = -\Big[X_{d11}\underline{I}_{d1}\cos(\varphi_1) \\ &\qquad + X_{q11}\underline{I}_{q1}\sin(\varphi_1)\Big]\overline{k}_1 + \Big[X_{d12}\underline{I}_{d2}\cos(\varphi_1) \end{aligned}$$

$$+X_{q12}\underline{I}_{q2}\sin(\varphi_1)\Big]\overline{k}_2 - X_{d13}\underline{I}_{d3}\cos(\varphi_1)$$
$$-X_{q13}\underline{I}_{q3}\sin(\varphi_1), \quad (4)$$
$$U_1\cos(-\vartheta_n+\varphi_1)$$
$$=\Big[X_{d21}\underline{I}_{d1}\cos(\varphi_2) - X_{q21}\underline{I}_{q1}\sin(\varphi_2)\Big]\overline{k}_1$$
$$-\Big[(X_{d22}+X_S)\underline{I}_{d2}\cos(\varphi_2)$$
$$-(X_{q22}+X_S)\underline{I}_{q2}\sin(\varphi_2)\Big]\overline{k}_2$$
$$+[X_{d23}\cos(\varphi_2)+X_S]\underline{I}_{d3} - X_{q23}\underline{I}_{q3}\sin(\varphi_2), \quad (5)$$
$$0=-X_{d31}\underline{I}_{d1}\overline{k}_1$$
$$+\Big[(X_{d32}+X_S\cos(\varphi_2))\underline{I}_{d2} - X_S\underline{I}_{q2}\sin(\varphi_2)\Big]\overline{k}_2$$
$$-(X_{d33}+X_S)\underline{I}_{d3}, \quad (6)$$
$$U_1\sin(-\vartheta_n+\varphi_1)$$
$$=\Big[X_{q11}\underline{I}_{q1}\cos(\varphi_1) - X_{d11}\underline{I}_{d1}\sin(\varphi_1)\Big]\overline{k}_1$$
$$-\Big[X_{q12}\underline{I}_{q2}\cos(\varphi_1)$$
$$-X_{d12}\underline{I}_{d2}\sin(\varphi_1)\Big]\overline{k}_2$$
$$+X_{q13}\underline{I}_{q3}\cos(\varphi_1) - X_{d13}\underline{I}_{d3}\sin(\varphi_1), \quad (7)$$
$$U_1\sin(-\vartheta_n+\varphi_1)$$
$$=-\Big[X_{q21}\underline{I}_{q1}\cos(\varphi_2) + X_{d21}\underline{I}_{d1}\sin(\varphi_2)\Big]\overline{k}_1$$
$$+\Big[(X_{q22}+X_S)\underline{I}_{q2}\cos(\varphi_2)$$
$$+(X_{d22}+X_S)\underline{I}_{d2}\sin(\varphi_2)\Big]\overline{k}_2$$
$$-\big[X_{q23}\cos(\varphi_2)+X_S\big]\underline{I}_{q3}$$
$$-X_{d23}\underline{I}_{d3}\sin(\varphi_2), \quad (8)$$
$$0=X_{q31}\underline{I}_{q1}\overline{k}_1 - \Big[(X_{q32}+X_S\cos(\varphi_2))\underline{I}_{q2}$$
$$+X_S\underline{I}_{d2}\sin(\varphi_2)\Big]\overline{k}_2 + (X_{q33}+X_S)\underline{I}_{q3}, \quad (9)$$

where 1 indicates quantities pertinent to the circuit formed between points 0 and K; 2, the circuit between the short-circuit point K and the turbogenerator phases terminal; 3, to undamaged branches respectively, U_1 amplitude of the voltage at the short-circuit point at the time just before the fault occurrence, derived from the formula:

$$U_1 = U_n\frac{\sin(\varphi_2)}{\sin(\varphi_1+\varphi_2)}, \quad (10)$$

where U_n amplitude of the rated voltage at the output terminals of the turbogenerator. The values of angles φ_1 and φ_2, for a bipolar turbogenerator, are equal to:

$$\varphi_1 = (1-\alpha)\frac{\pi}{6}, \quad \varphi_2 = \alpha\frac{\pi}{6}. \quad (11)$$

The current complex components $\underline{I}_{qi}$ and $\underline{I}_{di}$ in equations (4)–(9) of particular equivalent circuits of the stator ($i = 1, 2, 3$) are expressed as:

$$\underline{I}_{qi} = I_{qi}\underline{k}_i, \quad \underline{I}_{di} = I_{di}\underline{k}_i, \quad (12)$$

where I_{qi}, I_{di} short-circuit current components flowing in the equivalent circuits of the stator expressed in rectangular rotating axes $dq0$ system, $\underline{k}_i$ respective transformation factors (complex value) calculated from equations:

$$\underline{k}_1 = \cos(\varphi_1) + j\sin(\varphi_1),$$
$$\underline{k}_2 = \cos(\varphi_2) - j\sin(\varphi_2), \quad \underline{k}_3 = 1, \quad (13)$$

whereas $\overline{k}_i$ is adequate conjugate complex transformation factor value.

Note that Eqs. (4)–(9) represent the interdependent system that considers the following into account, i.e.:

- mutual spatial displacement (along the circumference of turbogenerator stator) of particular phase windings and their damaged parts,
- respective transformation relating current, voltage and reactance quantities of particular parts of phase windings to a common axis d and q,
- correct transformation from the $dq0$ system into the system of complex coordinates.

Taking into account the magnetic asymmetry of the machine along the d- and q-axis while shifting the defective parts of winding (along the periphery of the armature) has the effect of shifting currents.

The self- and mutual reactance of particular (equivalent) circuits of the stator (X_{dik} in the direct axis d and X_{qik} in the quadrature axis q) ($i = 1, 2, 3$, $k = 1, 2, 3$) in equations from (4) to (9) is related as follows:

$$X_{dik} = X_{dki} = X_{lik} + X_{adik}, \quad (14)$$
$$X_{qik} = X_{qki} = X_{lik} + X_{aqik}, \quad (15)$$

where X_{lik} leakage reactance and X_{adik}, X_{aqik} armature affected d- and q-axis reactances, respectively.

The leakage reactances X_{lik}, for particular equivalent circuit of the stator [in Eqs. (14) and (15)], are based on the simplest relationships developed by N. Margolin and A. Černin [3]:

$$X_{l11} = 2\alpha X_l, \quad X_{l22} = 2(1-\alpha)X_l, \quad X_{l33} = 2X_l \quad (16)$$
$$X_{l12} = X_{l13} = X_{l23} = 0, \quad (17)$$

where X_l leakage reactance of the turbogenerator stator winding.

Variation of the X_{adik} and X_{aqik} reactance values with number of shorted turns α, and the first harmonic of the magnetic flux due to the short-circuit current in one coil of the stator winding, can be described as [2]:

$$X_{ad11} = 4\sin^2(\alpha\pi/6)X_{ad},$$
$$X_{aq11} = 4\sin^2(\alpha\pi/6)X_{aq}, \tag{18}$$
$$X_{ad22} = 4\sin^2[(1-\alpha)\pi/6]\,X_{ad},$$
$$X_{aq22} = 4\sin^2[(1-\alpha)\pi/6]\,X_{aq}, \tag{19}$$
$$X_{ad33} = X_{ad}, \quad X_{aq33} = X_{aq}, \tag{20}$$
$$X_{ad12} = 4\sin(\alpha\pi/6)\sin[(1-\alpha)\pi/6]\,X_{ad},$$
$$X_{aq12} = 4\sin(\alpha\pi/6)\sin[(1-\alpha)\pi/6]\,X_{aq}, \tag{21}$$
$$X_{ad13} = 2\sin(\alpha\pi/6)X_{ad},$$
$$X_{aq13} = 2\sin(\alpha\pi/6)X_{aq}, \tag{22}$$
$$X_{ad23} = 2\sin[(1-\alpha)\pi/6]\,X_{ad},$$
$$X_{aq23} = 2\sin[(1-\alpha)\pi/6]\,X_{aq}, \tag{23}$$

where X_{ad}, X_{aq} turbogenerator total armature affected reactances in the d- and q-axes, respectively.

Determination of the short-circuit current components for the subtransient ($\underline{I}''_{dk}$, $\underline{I}''_{qk}$), transient ($\underline{I}'_{dk}$, $\underline{I}'_{qk}$) and steady ($\underline{I}_{dk}$, $\underline{I}_{qk}$) states are based on Eqs. (4)–(9). While the armature affected reactances [in the d- and q-axis, appearing in equations from (18) to (23)] for respective short-circuit states are expressed by the following relationships:

$$X''_{ad} = X''_d - X_l, \quad X''_{aq} = X''_q - X_l, \tag{24}$$
$$X'_{ad} = X'_d - X_l, \quad X'_{aq} = X'_q - X_l, \tag{25}$$
$$X_{ad} = X_d - X_l, \quad X_{aq} = X_q - X_l, \tag{26}$$

where X_d, X'_d, X''_d synchronous, transient and subtransient reactance in the direct d axis; X_q, X'_q, X''_q synchronous, transient ($X'_q = X_q$) and subtransient reactance in the quadrature q-axis, respectively.

Both the voltage–current Eqs. (4)–(9) and the mutual reactances (18)–(23) have been obtained taking into account the respective flux mutual shifts due to different location of faulty winding along the stator circumference. However, after appropriate transformation to distribute the generated fluxes along the d- and q-axis, the self- and mutual reactances of equivalent circuits are found to be related only to the basic turbogenerator reactance values along the direct—d (X_d, X'_d, X''_d) as well as the q—(X_q, X''_q) axis [4,6,10] irrespective of their space displacements by φ_1, φ_2 and $\varphi_1 + \varphi_2$ angle.

Assuming that the rated voltage in phase A is a sine wave

$$u_A = U_n \sin(\gamma - \vartheta_n), \tag{27}$$

where γ is the angle shift of the direct axis d of the rotor with respect to the phase A axis, what is expressed as:

$$\gamma = \omega_s t + \gamma_0, \tag{28}$$

where ω_s synchronous angular frequency, γ_0 initial angle shift for the time $t = 0$, respectively; thus, the voltage existing at the short-circuit point (in A phase) just before its occurrence is in the form [1]:

$$u_{A1} = U_1 \sin(\gamma - \vartheta_n + \varphi_1). \tag{29}$$

Introducing the complex notation $\underline{U}_{A1}$ Eq. (29) becomes

$$\underline{U}_{A1} = \underline{U}_1 e^{j\gamma}, \tag{30}$$

where

$$\underline{U}_1 = U_1[\cos(-\vartheta_n + \varphi_1) + j\sin(-\vartheta_n + \varphi_1)]. \tag{31}$$

The real and imaginary parts, of the voltage $\underline{U}_1$, represent respective values which force the equivalent short-circuit currents to flow, expressed in the complex coordinates system.

Having solved the system of equations from (4) to (9) and taking into account the flow direction of the short-circuit currents (as in Fig. 1), the periodic component for the steady state of the fundamental frequency (for the resultant current at the short-circuit point in A phase) is expressed as follows:

$$\underline{I}_{Asc(3f)} = \underline{I}_{sc(3f)} e^{j\gamma}, \tag{32}$$

where

$$\underline{I}_{sc(3f)} = \underline{I}_{q1} + \underline{I}_{q2} + j(\underline{I}_{d1} + \underline{I}_{d2}). \tag{33}$$

Hence, using Eqs. (3), (31) and (33), the equivalent reactance for the positive-sequence symmetric component of the resultant circuitry system due to the damaged turbogenerator together with the power network, for the steady-state period during the short-circuit, can be calculated from the following equation:

$$\underline{X}^{(1)}_{\Sigma} = U_1 \frac{\cos(-\vartheta_n + \varphi_1) + j\sin(-\vartheta_n + \varphi_1)}{\underline{I}_{q1} + \underline{I}_{q2} + j\left(\underline{I}_{d1} + \underline{I}_{d2}\right)}. \tag{34}$$

The subtransient and transient values for this component are easily obtained from the solution of (4)–(9) and (18)–(26).

3.2 Equivalent reactance for the negative-sequence component

To determine the equivalent reactance of the negative-sequence component of the system due to the damaged turbogenerator, one has to solve the following voltage–current equations [9]:

$$\underline{U}^{(2)} = jX^{(2)}_{11}\underline{I}^{(2)}_1 - jX^{(2)}_{12}\underline{I}^{(2)}_2 + jX^{(2)}_{13}\underline{I}^{(2)}_3, \tag{35}$$
$$\underline{U}^{(2)} = -jX^{(2)}_{21}\underline{I}^{(2)}_1 + j\left(X^{(2)}_{22} + X^{(2)}_S\right)\underline{I}^{(2)}_2 - j\left(X^{(2)}_{23} + X^{(2)}_S\right)\underline{I}^{(2)}_3, \tag{36}$$
$$0 = jX^{(2)}_{31}\underline{I}^{(2)}_1 - j\left(X^{(2)}_{32} + X^{(2)}_S\right)\underline{I}^{(2)}_2 + j\left(X^{(2)}_{33} + X^{(2)}_S\right)\underline{I}^{(2)}_3, \tag{37}$$

where $\underline{U}^{(2)}$ voltage drop across the equivalent reactance of the system, $\underline{I}_i^{(2)}$ components of the short-circuit currents in the equivalent circuits of the stator ($i = 1, 2, 3$), $X_S^{(2)}$ equivalent resultant reactance of the transformer–power network system which after respective transformations is expressed in the form [9]:

$$\underline{X}_\Sigma^{(2)} = j\frac{\left\|\begin{bmatrix} X_{11}^{(2)} & -X_{12}^{(2)} & X_{13}^{(2)} \\ -X_{21}^{(2)} & X_{22}^{(2)} + X_S^{(2)} & -X_{23}^{(2)} - X_S^{(2)} \\ X_{31}^{(2)} & -X_{32}^{(2)} - X_S^{(2)} & X_{33}^{(2)} + X_S^{(2)} \end{bmatrix}\right\|}{2\left(X^{(2)} + X_S^{(2)}\right)\left(X_{33}^{(2)} - X_{13}^{(2)} - X_{23}^{(2)}\right)}. \tag{38}$$

The self- and mutual reactances of particular equivalent circuits of the stator for the negative-sequence component in Eqs. (35)–(38) are not functions of time and can be specified by the expressions developed by N. Margolin and A. Černin as follows:

$$X_{11}^{(2)} = 2\alpha X_l + \alpha^2\left(X^{(2)} - X_l\right), \tag{39}$$

$$X_{22}^{(2)} = 2(1-\alpha)X_l + \left(1-\alpha^2\right)\left(X^{(2)} - X_l\right), \tag{40}$$

$$X_{33}^{(2)} = X_l + X^{(2)}, \tag{41}$$

$$X_{12}^{(2)} = X_{21}^{(2)} = \alpha(1-\alpha)\left(X^{(2)} - X_l\right), \tag{42}$$

$$X_{13}^{(2)} = X_{31}^{(2)} = \alpha\left(X^{(2)} - X_l\right), \tag{43}$$

$$X_{23}^{(2)} = X_{32}^{(2)} = (1-\alpha)\left(X^{(2)} - X_l\right), \tag{44}$$

where $X^{(2)}$ reactance of the turbogenerator for the negative-sequence component.

3.3 Equivalent reactance for the zero-sequence component

In a similar manner, the equivalent reactance for the zero-sequence component of the considered system can be obtained from the voltage–current equations expressed in the form of [9]:

$$\underline{U}^{(0)} = jX_{11}^{(0)}\underline{I}_1^{(0)} - jX_{12}^{(0)}\underline{I}_2^{(0)} + jX_{13}^{(0)}\underline{I}_3^{(0)}, \tag{45}$$

$$\begin{aligned}\underline{U}^{(0)} = &-jX_{21}^{(0)}\underline{I}_1^{(0)} + j\left(X_{22}^{(0)} + X_S^{(0)}\right)\underline{I}_2^{(0)} \\ &-j\left(X_{23}^{(0)} + X_S^{(0)}\right)\underline{I}_3^{(0)},\end{aligned} \tag{46}$$

$$\begin{aligned}0 = &\, jX_{31}^{(0)}\underline{I}_1^{(0)} - j\left(X_{32}^{(0)} + X_S^{(0)}\right)\underline{I}_2^{(0)} \\ &+j\left(X_{33}^{(0)} + X_S^{(0)}\right)\underline{I}_3^{(0)},\end{aligned} \tag{47}$$

where $\underline{U}^{(0)}$ voltage drop across the equivalent reactance of the system, $\underline{I}_i^{(0)}$ components of the short-circuit currents in the equivalent circuits of the stator ($i = 1, 2, 3$), $X_S^{(0)}$ equivalent reactance of the unit transformer together with the power network for the zero-sequence component, respectively, that after transformations takes form [9]:

$$\underline{X}_\Sigma^{(0)} = j\frac{\left\|\begin{bmatrix} X_{11}^{(0)} & -X_{12}^{(0)} & X_{13}^{(0)} \\ -X_{21}^{(0)} & X_{22}^{(0)} + X_S^{(0)} & -X_{23}^{(0)} - X_S^{(0)} \\ X_{31}^{(0)} & -X_{32}^{(0)} - X_S^{(0)} & X_{33}^{(0)} + X_S^{(0)} \end{bmatrix}\right\|}{2\left(X^{(0)} + X_S^{(0)}\right)\left(X_{33}^{(0)} - X_{13}^{(0)} - X_{23}^{(0)}\right)}. \tag{48}$$

Similarly, the self- and mutual reactances of particular equivalent circuits of the stator for the zero-sequence component, according to Eqs. (45)–(48), are not functions of time and they can be determined using the simplest relationships found by N. Margolin and A. Černin as shown below:

$$X_{11}^{(0)} = 2\alpha X^{(0)}, \quad X_{22}^{(0)} = 2(1-\alpha)X^{(0)}, \quad X_{33}^{(0)} = 2X^{(0)}, \tag{49}$$

$$X_{12}^{(0)} = X_{21}^{(0)} = 0, \quad X_{13}^{(0)} = X_{31}^{(0)} = 0, \quad X_{23}^{(0)} = X_{32}^{(0)} = 0, \tag{50}$$

where $X^{(0)}$ reactance of the turbogenerator for the zero-sequence component.

3.4 Currents under the single-phase short-circuit

Having determined equivalent reactances for positive $\underline{X}_\Sigma^{(1)}$, negative $\underline{X}_\Sigma^{(2)}$ and zero-sequence component $\underline{X}_\Sigma^{(0)}$ and knowing the voltage $\underline{U}_1$, at the short-circuit point just prior to the fault occurrence, one can calculate positive- $\underline{I}^{(1)}$, negative- $\underline{I}^{(2)}$ and zero-sequence $\underline{I}^{(0)}$ components of the current flowing through the single-phase short-circuit point. It is simply from the following relation:

$$\begin{aligned}\underline{I}^{(1)} &= \underline{I}^{(2)} = \underline{I}^{(0)} \\ &= U_1\frac{\cos(-\vartheta_n + \varphi_1) + j\sin(-\vartheta_n + \varphi_1)}{\underline{X}_\Sigma^{(1)} + \underline{X}_\Sigma^{(2)} + \underline{X}_\Sigma^{(0)}}.\end{aligned} \tag{51}$$

The respective voltage drop across the particular equivalent reactances is expressed as:

$$\underline{U}^{(1)} = \underline{X}_\Sigma^{(1)}\underline{I}^{(1)}, \quad \underline{U}^{(2)} = \underline{X}_\Sigma^{(2)}\underline{I}^{(2)}, \quad \underline{U}^{(0)} = \underline{X}_\Sigma^{(0)}\underline{I}^{(0)}. \tag{52}$$

The voltage drop due to the short-circuit current in Eqs. (4) and (5) is assumed to be the real part of the voltage drop $\underline{U}^{(1)}$, i.e. $\mathrm{Re}(\underline{U}^{(1)})$, while those responsible for the short-circuit currents distribution according to Eqs. (7) and (8) represent the imaginary part of the voltage drop $\underline{U}^{(1)}$, i.e. $\mathrm{Im}(\underline{U}^{(1)})$.

If asymmetry of the rotor is taken into account, the resultant reactance $\underline{X}_\Sigma^{(1)}$ varies in a nonlinear way on the voltage drop value $\underline{U}^{(1)}$. Therefore, one has to employ the iterative calculation to evaluate the symmetric positive-sequence components of the short-circuit currents for particular closed-circuits:

$$\underline{I}_i^{(1)} = \underline{I}_{qi} + j\underline{I}_{di}, \quad i = 1, 2, 3. \tag{53}$$

The remaining symmetrical components can be obtained from (51) for the currents and voltage drops from (52). Therefore, the steady-state short-circuit currents for the single-phase short-circuit in phase A can be determined as follows:

$$\underline{I}_{Ai} = \left|\underline{I}_i^{(1)} + \underline{I}_i^{(2)} + \underline{I}_i^{(0)}\right| e^{j(\gamma+\delta_i)}, \quad i = 1, 2, 3, \tag{54}$$

where δ_i the resultant phase shift angle of the current $\underline{I}_{Ai}$, that is expressed in form:

$$\delta_i = \arg\left(\underline{I}_i^{(1)} + \underline{I}_i^{(2)} + \underline{I}_i^{(0)}\right). \tag{55}$$

Taking into account the directions of the short-circuit currents (as in Fig. 1), the complex value of current at single-phase short-circuit point in phase A can be expressed as:

$$\underline{I}_{Asc} = \left|\underline{I}_1^{(1)} + \underline{I}_1^{(2)} + \underline{I}_1^{(0)} + \underline{I}_2^{(1)} + \underline{I}_2^{(2)} + \underline{I}_2^{(0)}\right| e^{j(\gamma+\delta_{sc})}, \tag{56}$$

where δ_{sc} the resultant phase shift angle of the current $\underline{I}_{Asc}$, it is expressed by the formula:

$$\delta_{sc} = \arg\left(\underline{I}_1^{(1)} + \underline{I}_1^{(2)} + \underline{I}_1^{(0)} + \underline{I}_2^{(1)} + \underline{I}_2^{(2)} + \underline{I}_2^{(0)}\right). \tag{57}$$

However, the short-circuit currents $\underline{I}_{Aisc}$ flowing in particular parts of the stator winding of the phase A (for linear magnetic and electric circuits) are composed of geometric sum of the short-circuit currents and the load current $\underline{I}_{Alo}$ before the short-circuit occurrence [1].

If the rated load current for the phase A is expressed as:

$$i_{Alo} = I_n \sin(\gamma - \vartheta_n - \varphi_n), \tag{58}$$

(where I_n amplitude of the rated current of the turbogenerator, φ_n nominal phase shift between rated load current and voltage at phase A, respectively), then the alternating-short-circuit current component in particular parts of the damaged winding under the single-phase short-circuit in phase A is described as:

$$\begin{aligned}\underline{I}_{Aisc} = \Big|&\underline{I}_i^{(1)} + \underline{I}_i^{(2)} + \underline{I}_i^{(0)} - (-1)^i 0.5 I_n \left[\cos(\vartheta_n + \varphi_n)\right.\\ &\left.- j\sin(\vartheta_n + \varphi_n)\right]\Big| e^{j(\gamma+\delta_{isc})}, \quad i = 1, 2, 3, \end{aligned}\tag{59}$$

where δ_{isc} the resultant phase shift angle of the current $\underline{I}_{Aisc}$, of the relation as:

$$\begin{aligned}\delta_{isc} = \arg\Big(&\underline{I}_i^{(1)} + \underline{I}_i^{(2)} + \underline{I}_i^{(0)}\\ &-(-1)^i 0.5 I_n \left[\cos(\vartheta_n + \varphi_n) - j\sin(\vartheta_n + \varphi_n)\right]\Big).\end{aligned}\tag{60}$$

Similarly complex values of both subtransient ($\underline{I}''_{Ai}$, $\underline{I}''_{Asc}$, $\underline{I}''_{Aisc}$) and transient ($\underline{I}'_{Ai}$, $\underline{I}'_{Asc}$, $\underline{I}'_{Aisc}$) short-circuit currents in particular part of the damaged stator winding can be evaluated. Their instantaneous values are expressed by the imaginary part $i = \mathrm{Im}(\underline{I})$.

4 The currents phase shift effect under internal single-phase short-circuit

To visualize the effect of the phase shift for different short-circuit currents the respective calculations are performed for the turbogenerator with the following given reactances expressed in per unit [3,9]):

$$X_d = X_q = 1.84, \quad X'_d = 0.29, \quad X''_d = X''_q = 0.198,$$
$$X_l = 0.147, \quad X^{(2)} = 0.198, \quad X^{(0)} = 0.095,$$
$$\cos\varphi_n = 0.8, \quad \vartheta_n = 0.61\,[\mathrm{rad}], \quad N = 12$$

And for the unit transformer of the reactances:

$$X_T^{(1)} = X_T^{(2)} = 0.11, \quad X_T^{(0)} = \infty$$

Furthermore, it is assumed that the initial phase of the voltage $\underline{U}_1$ is δ_u as expressed below:

$$\delta_u = \arg\left(\underline{U}_1\right), \tag{61}$$

the resultant phase shift angle λ_i of the respective short-circuit currents $\underline{I}_{Ai}$ (54) ($\underline{I}'_{Ai}$ and $\underline{I}''_{Ai}$ as well) with respect to the voltage $\underline{U}_1$, expressed as:

$$\lambda_i = \delta_u - \delta_i, \quad i = 1, 2, 3. \tag{62}$$

Similarly, the phase shift angle λ_{sc} of the current at the short-circuit point $\underline{I}_{Asc}$ (56) ($\underline{I}'_{Asc}$ and $\underline{I}''_{Asc}$ as well) can be determined as:

$$\lambda_{sc} = \delta_u - \delta_{sc}. \tag{63}$$

However, when taking into account the load current, the phase shift angles λ_{isc} of the short-circuit currents $\underline{I}_{Aisc}$ (59) ($\underline{I}'_{Aisc}$ and $\underline{I}''_{Aisc}$ as well) are equal to:

$$\lambda_{isc} = \delta_u - \delta_{isc}, \quad i = 1, 2, 3. \tag{64}$$

Two different scenarios are considered; the first one, when the reactance values of the turbogenerator for d and q axes are equal ($X_d = X_q$ and $X''_d = X''_q$) and the second one, when $X'_d \neq X'_q = X_q$. The effects of phase shifts due to the short-circuit currents passing through three parts of the stator winding are shown in Figs. 4, 5, 6 and 7 [with regard to the voltage existing at the short-circuit point just before its occurrence with angles different from 0.5π (rad)]. Analysis of the figures suggests that the mutual phase shifts have found to occur under the subtransient, transient and the steady states of the short-circuit. Under the transient state of the short-circuit, the angle λ_{sc} value is lower than 0.5π (rad) (Fig. 5). In general, the change of the phase shift angles λ_{isc} for different

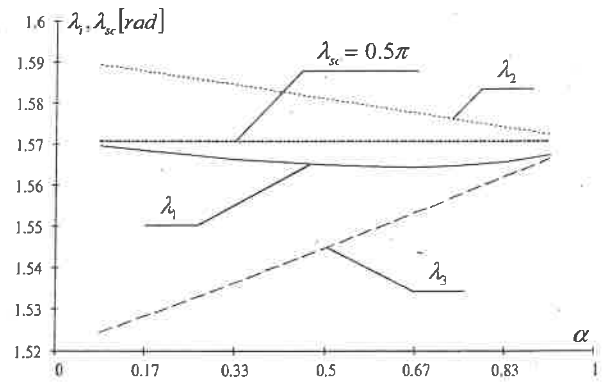

Fig. 4 Variation of the phase shift angle [between respective short-circuit currents $\underline{I}''_{Ai}$ (λ_i, $i = 1, 2, 3$) and the current at the short-circuit point $\underline{I}''_{Asc}$ (λ_{sc}) with respect to the voltage $\underline{U}_1$] under subtransient state (turbogenerator rated loaded and connected to the power network of infinite large power via the unit transformer of the relative reactance $X_T = 0.11$) with the relative number of the shorted turns α

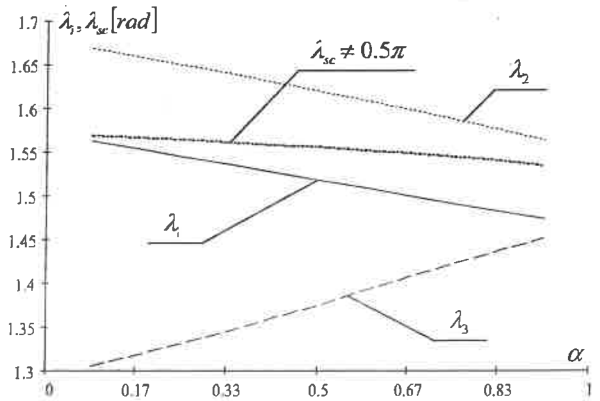

Fig. 5 Variation of the phase shift angle [between respective short-circuit currents $\underline{I}'_{Ai}$ (λ_i, $i = 1, 2, 3$) and the current at the short-circuit point $\underline{I}'_{Asc}$ (λ_{sc}) with respect to voltage $\underline{U}_1$] under transient state of the short-circuit (turbogenerator rated loaded and connected to the power network of infinite large power via the unit transformer with the relative reactance $X_T = 0.11$) with the relative number of the shorted turns α

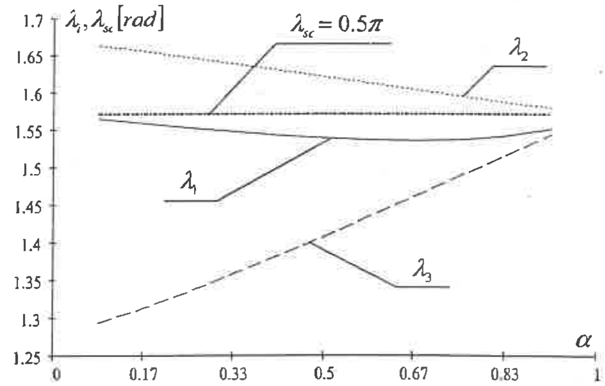

Fig. 6 Variation of the phase shift angle [between respective short-circuit currents $\underline{I}_{Ai}$ (λ_i, $i = 1, 2, 3$) and the current at the short-circuit point $\underline{I}_{Asc}$ (λ_{sc}) with respect to voltage $\underline{U}_1$] under steady state of the short-circuit (turbogenerator rated loaded and connected to the power network of infinite large power via the unit transformer with the relative reactance $X_T = 0.11$) with the relative number of the shorted turns α

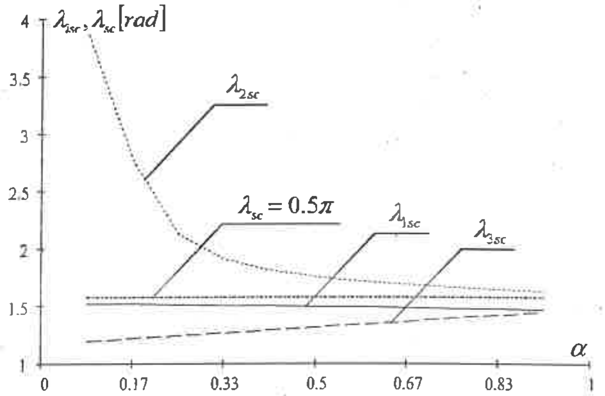

Fig. 7 Variation of the phase shift angle [between respective short-circuit currents $\underline{I}''_{Aisc}$ (λ_{isc}, $i = 1, 2, 3$) and the current at the short-circuit point $\underline{I}''_{Asc}$ (λ_{sc}) with respect to the voltage $\underline{U}_1$] under subtransient state (turbogenerator rated loaded and connected to the power network of infinite large power via the unit transformer of the relative reactance $X_T = 0.11$) with relative number of the shorted turns α

state of the short-circuit currents is similar. In this paper, only results of the calculation for the subtransient state of the short-circuit are presented (Fig. 7).

A tentative analysis of the results suggests that the phase shift variations depend not only on the relative number of shorted turns α of the stator winding, but also on the reactance value of the turbogenerator. The reactance value of the turbogenerator depends on the short-circuit state, and the short-circuit currents with regard to the voltage $\underline{U}_1$. Furthermore, the phase shift angles λ_1, λ_3 and λ_{1sc}, λ_{3sc} do not exceed 0.5π (rad). This reveals that the short-circuit currents lag the voltage $\underline{U}_1$ by angles smaller than 0.5π (rad). However, the phase shifts λ_2 and λ_{2sc} always are higher than 0.5π (rad), and consequently the appropriate currents lag the voltage $\underline{U}_1$ by angles higher than 0.5π (rad). For all considered cases the phase shift values λ_3, λ_{3sc} increase whereas λ_2 and λ_{2sc} decrease respectively when the damaged point (α) is shifted towards the turbogenerator phase terminals. Similarly the phase shift λ_{sc} value of the transient current at the short-circuit point $\underline{I}'_{Asc}$ and the voltage $\underline{U}_1$ depend on the relative number of shorted turns α of the stator winding. This effect is more pronounced only under the short-circuit transient state which is due to asymmetry of the rotor ($X'_d < X'_q = X_q$). However, for the short-circuits close to the neutral point of the turbogenerator, the phase shift λ_{sc} becomes closer to 0.5π (rad). Also with increase of the relative numbers of shorted turns α will result in this shift to get close to 1.53 (rad) (Fig. 5). For any other cases, the phase shift λ_{sc} has a constant value equal to 0.5π (rad) (Figs. 4, 6). On the contrary the phase shift angle λ_{2sc} due to the subtransient short-circuit current $\underline{I}''_{A2sc}$ with respect to the voltage $\underline{U}_1$ (Fig. 7) is changed significantly. This is more significant for the small number of shorted turns α. The λ_{2sc} value varies from 3.87 to 1.63 (rad) with the increase of the shorted turns number.

Having determined the phase shifts λ_i of the short-circuit currents $\underline{I}_{Ai}$ ($\underline{I}'_{Ai}$ and $\underline{I}''_{Ai}$) with respect to the voltage $\underline{U}_1$, it is not difficult to prove that the short-circuit currents in particular parts of the damaged stator winding are displaced in the phase from each other. The highest phase shift occurs between the currents $\underline{I}_{A2}$ and $\underline{I}_{A3}$ ($\lambda_2 - \lambda_3$, Fig. 6) as well as $\underline{I}'_{A2}$ and $\underline{I}'_{A3}$ ($\lambda_2 - \lambda_3$, Fig. 5). Its value is 0.37 and 0.36 (rad), respectively. These values are found for the short-circuits very close to the neutral point of the turbogenerator. While the smallest values are 0.04 and 0.11 (rad) that are found to occur for the short-circuit near the turbogenerator phase terminals. However, relatively smaller phase shift occurs between the currents $\underline{I}_{A2}$ and $\underline{I}_{A1}$, and $\underline{I}'_{A2}$ and $\underline{I}'_{A1}$. The largest phase shift ($\lambda_2 - \lambda_1$) between the aforementioned currents are 0.10 and 0.11 (rad), respectively, which appears to occur for the small relative number of shorted turns. While the smallest value of this phase shift is 0.03 and 0.09 (rad) respectively and takes place for the short-circuit at the turbogenerator phase terminals. Regardless of the relative number of shorted turns the constant value of 0.10 (rad) is characteristic for the shift between the currents $\underline{I}'_{A2}$ and $\underline{I}'_{A1}$ ($\lambda_2 - \lambda_1$, Fig. 5). On the other hand, the smallest phase shift occurs between currents $\underline{I}''_{A1}$ and $\underline{I}''_{A3}$, as well as $\underline{I}''_{A2}$ and $\underline{I}''_{A1}$ ($\lambda_1 - \lambda_3$ and $\lambda_2 - \lambda_1$, Fig. 4).

It is emphasized that if under the short-circuit analysis the spatial displacement of the damaged windings is omitted, the short-circuit currents are shifted only by an angle of 0.5π [3] with respect to the voltage at the short-circuit point just before the fault occurrence.

However, for the loaded turbogenerator, due to contribution of the load currents (which are superimposed onto the short-circuit currents) much higher values of the mutual phase shifts are observed. For the currents of the short-circuit, the smallest phase shift occurs between currents $\underline{I}''_{A1sc}$ and $\underline{I}''_{A3sc}$ ($\lambda_{1sc} - \lambda_{3sc}$, Fig. 7). The value of this phase shift changes together with increase of the relative number of shorted turns from 0.33 to 0.03 (rad). The largest phase shift occurs between currents $\underline{I}''_{A2sc}$ and $\underline{I}''_{A3sc}$ ($\lambda_{2sc} - \lambda_{3sc}$, Fig. 7). The value of this phase shift changes together with the increase in the relative number of shorted turns from 2.68 to 0.19 (rad). Here, the currents of the short-circuit $\underline{I}_{Aisc}$ ($\underline{I}'_{Aisc}$ and $\underline{I}''_{Aisc}$) are shifted from each other due to superposition of two phenomena: the superposition of the load current (in considered case of the winding with two branches on the phase, equal to $0.5\underline{I}_{Alo}$) on particular short-circuit currents $\underline{I}_{Ai}$ and the second phenomenon, existence of the shifts in phase between short-circuit currents $\underline{I}_{Ai}$ with respect to each other.

5 Conclusions

In the paper, the developed model of the turbogenerator under short-circuits that takes into account the electromagnetic field spatial dislocation is presented and discussed.

The field dislocation influences significantly the self- and mutual reactance values of respective circuits of the faulty stator winding as well as the voltages that effect short-circuit currents distribution.

On the basis of the theoretical results there exists different phase shift value between voltage and the short-circuit currents at the fault point. This effect becomes more evident under magnetic asymmetry of the turbogenerator rotor.

The phase shift different from 0.5π is seen, even when that both resistance of the stator winding and that of the short-circuit point are neglected. This suggests that the effect of angular shift can be successfully used in practice as an auxiliary criterion of operation for the protection of the turbogenerators.

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