

Some Extensions of the Conditional CAPM

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Abstract

The objective of this thesis is to consider some extensions of the CAPM and to investigate whether such extensions can offer a better explanation for the US average equity returns. This thesis focuses on four main extensions: (i) time-varying factor loadings; (ii) higher moments (coskewness and cokurtosis); (iii) time-varying risk premia; and (iv) conditional versions of the CAPM using individual assets.

Time-series and cross-sectional tests, conducted on portfolios sorted on market capitalization and/or book-to-market ratio, show no evidence in support of the CAPM. While the standard CAPM predicts that the risk premium should be positive and the intercept from a regression of expected returns on beta should be insignificant, the empirical evidence from the relatively simple models goes contrary to expectation. The use of time-varying betas with dynamic conditional correlations improves the performance of the CAPM, but does not confirm its validity. The introduction of coskewness and cokurtosis does not rescue the CAPM. In particular, the unconditional four-moment CAPM is rejected as coskewness and cokurtosis are not found to have additional explanatory power for the cross-section of returns of portfolios of stocks sorted on market capitalization and book-to-market. The conditional four-moment CAPM where coskewness and cokurtosis are obtained as counterparts of the covariance using dynamic conditional correlation is also rejected.

Time-varying risk premia, based on simple bull and bear regimes, combined with the conditional CAPM and the conditional four-moment CAPM, lead to interesting results. In particular, the hypothesis of time-varying risk premia is never rejected, and the conditional CAPM produces a positive beta premium.

The conditional CAPM and conditional four-moment CAPM are tested on individual assets. The results support the CAPM for individual stocks over the last 30 years. The four-moment CAPM seems to work especially well when the SMB factor is added to the model. All of the factors have the expected sign: beta demands a positive premium, coskewness a negative premium and cokurtosis a positive premium. Interestingly, SMB retains significance and has a positive risk premium. Small stocks tend to earn higher returns even after accounting for the comoments.

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Chapter 1

Introduction

1.0. Introduction

The aim of this thesis is to investigate several extensions of the Capital Asset Pricing Model of Sharpe (1964), Lintner (1965) and Mossin (1966) in an attempt to identify those factors for which investors require some compensation in terms of returns on their investment. Further, the thesis examines whether such extensions of the CAPM can provide a rational explanation for the existing empirical results that contradict the traditional model.

The theory of Asset Pricing deals with the relationship between risk factors and returns, and involves a search for what determines the returns observed in the market. Investors buy assets and sacrifice current consumption in the expectation of receiving a future benefit (return) that is expected to increase their future consumption. The magnitude of the expected or required return to investors should not only account for the time they postpone their consumption and the effect of the reduction in the purchasing power of their wealth due to inflation, but also for the risk or uncertainty related to the payoff of their investment. Since investors are risk averse, there is a positive relationship between risk and return, and the objective of asset pricing is to identify and measure the risk, and to investigate the relationship between this risk and asset returns.

The objective of Asset Pricing Models is to explain asset returns as a function of various risk factors. In particular, if the relationship between returns and risk factors is described by a linear function, as it usually is assumed to be, models are referred to as linear asset pricing models. Among the linear asset pricing models, the CAPM is the most famous and most widely employed, and the objective of this research is to explore extensions of this model.

In its elegant simplicity, the CAPM states that the excess return over the risk-free rate of an asset is a linear function of non-diversifiable risk only, and this is the market risk measured as the variance of the market portfolio return. The risk for which investors require some compensation is given by the contribution of the asset returns to non-diversifiable market risk, measured by beta.

Assuming that investors are risk averse, that is, they require some compensation for risk, and that the risk is measured by the variance of a fully diversified market portfolio, the CAPM suggests that the excess return of an asset is given by the contribution of the asset to the variance of the market portfolio times the risk premium, that is, the extra-return required for bearing additional market risk. Intuitively, the required risk premium - measured as the excess return of a broad market portfolio over the risk-free rate - depends on the magnitude of risk aversion that characterizes investors. The standard theory of finance predicts that asset returns should be positively related to asset risk, defined (in the CAPM case) as market beta. In other words, the expected excess return on any asset should be proportional to its market beta and only differences in market betas should explain differences in expected excess returns.

If the CAPM is a good approximation of the real world, at best we might investigate possible extensions that approximate reality even better, striking a balance between the theoretical simplicity and the methodological parsimony of the models on the one hand, and the degree of approximation to the real world on the other. However, there is significant empirical evidence in the existing literature that contradicts the CAPM. Thus, the investigation of possible extensions of the model, or of alternative asset pricing models, is primarily the result of failures of the CAPM.

The relationship between risk and return predicted by the CAPM is investigated in the literature by estimating either cross-sectional or time series regressions. In the time-

series regression based tests, monthly returns of asset returns are regressed on the monthly market excess return to investigate whether the intercepts (alphas) are significantly different from zero (Gibbons, Ross, and Shanken, 1989). In the cross-section regression based tests, in general, a two-pass methodology is applied in which (i) the monthly asset excess returns are regressed on the monthly market premium using time-series regressions to estimate the market betas and, (ii) the monthly asset excess returns are regressed on the estimated betas at each date (the Fama and MacBeth (1973) methodology) to estimate the expected market risk premium. The cross-section regression based tests, with the objective of investigating whether the differences in market betas can explain the differences in average returns across assets, are the main focus of this research.

The early tests of the CAPM gave some credence to the theory as they supported the linearity of the risk-return relationship and the positive relationship between returns and systematic risk, though the prediction that the market premium is equal to the historical average market return minus the risk-free rate was rejected.

Lintner (1965) finds an intercept larger than the risk-free rate and a weaker risk-return relationship than predicted. Black, Jensen, and Scholes (1972) argue that on the basis of a market beta (systematic risk), portfolios with higher systematic risk have a lower return than is predicted by the CAPM. Fama and MacBeth (1973) find a positive relationship between risk and return, but reject the hypothesis that the estimated market risk premium equals the average historical risk premium, and the hypothesis that the intercept equals the average risk-free rate.

For a long period, the CAPM was relatively successful in an empirical sense, and seemed to represent reality fairly well. However, in the 1970s, several tests of the CAPM showed that a large part of the variation in expected return is unrelated to the

market beta. Basu (1977) finds evidence that when stocks are sorted on earnings-price ratios, stocks with high E/P (earnings-price) ratios have higher future returns than predicted by the CAPM. There is a positive relationship between earnings-price ratios and returns. Banz (1981) documents a negative relationship between size and average excess returns. When stocks are sorted on market capitalization,¹ average returns on small stocks are higher than those predicted by the CAPM, whereas average returns on large stocks are lower than is forecasted by their betas. Furthermore, Bhandari (1988) finds that stocks with high leverage, measured as high debt-equity ratios,² have returns that are larger than is predicted by their market betas. Moreover, Stattman (1980) and Rosenberg et al. (1985) report that stocks with high book-to-market ratios³ have higher average returns than might be expected by their betas, meaning that stocks with higher future growth opportunities (high price-to-earnings ratios) tend to yield lower returns.

Most of the recent empirical research on the CAPM has been fuelled by the work of Fama and French (1992) who find a weak relationship between beta and returns for the period 1963-1990, and that size and book-to-market capture the cross-sectional variation in average stock returns associated with E/P and leverage, rejecting firmly the main tenet of the CAPM that stock returns are positively related to market beta, and that beta is the only variable that matters for the explanation of cross-sectional returns.

Contemporary thought, after many years of theoretical and empirical research into the CAPM, suggests that there are several anomalies, i.e. systematic empirical observations with a magnitude that cannot be explained by the model and that contradict the predictive ability of the model. There are portfolios of assets for which the relationship between beta and return is weak or even negative, and there are security characteristics

¹ Market price times number of shares outstanding.

² Ratio of book value of debt to the market value of equity.

³ The ratio of the book value of a common stock to its market value.

such as dividend yield, price-earnings, book-to-market, and market capitalization that can significantly better explain the differences in returns between such portfolios.

Some of the questions that naturally arise are: what are the economic risks or the risk factors underlying stock characteristics? What risk factors are not captured by the variance of the market portfolio return? What theoretical assumptions cause the model to be mis-specified? Is the variance an adequate measure of risk? Are there any important features of investor behaviour that the model does not encompass? These are some of the key questions that modern asset pricing theory attempts to answer. In the light of the empirical evidence contradicting a simple and appealing theoretical framework such as the CAPM, students of finance have embarked upon a challenging investigation of possible extensions of the traditional CAPM in the hope of finding rational explanations for its empirical failures. This thesis embarks in this direction and, beginning with the traditional CAPM model, attempts to address some of the questions arising from its empirical failures. In particular, the objective of this thesis is to consider possible extensions of the CAPM and to investigate whether such extensions can offer some explanation for the size and book-to-market anomalies.

1.1. Research questions

This thesis investigates whether certain extensions of the traditional CAPM can explain the cross-section of US stock average returns, and whether such extensions can in particular explain the size and book-to-market anomalies. The thesis focuses on four main additions to the traditional model: (i) the use of time-varying factor loadings obtained through multivariate GARCH and dynamic conditional correlations; (ii) the introduction of higher comoments of returns in addition to mean and variance, that is, using a Four-Moment CAPM with coskewness and cokurtosis; (iii) the assumption of time-varying risk premia, changing according to the regime of the market (where

regimes are assumed to follow a Markov Switching process); and (iv) testing the conditional CAPM and conditional Four-Moment CAPM on individual assets as opposed to portfolios of stocks sorted on a particular characteristic.

Specifically, the main research questions that this thesis will attempt to answer are as follows:

RQ1: Is a higher-moment CAPM, incorporating systematic skewness and kurtosis, capable of a better explanation of the cross-section of US average returns?

The CAPM is derived assuming that investors use a mean-variance criterion for their investment decision and that higher moments of the distribution of the market portfolio returns are irrelevant. Empirical evidence suggests that investors are more averse to large losses (extreme outcomes) and that returns are not normally distributed (see Kahneman and Tversky, 1979 and Taylor, 2005). Therefore, investors might be interested not only in the expected return and volatility of their portfolios, but also in the skewness and kurtosis of those portfolios.

Skewness is a measure of the asymmetry of returns whereas kurtosis is a measure of the extreme movements or outcomes. A distribution with a large kurtosis means that extreme outcomes are more likely. Investors fear large losses and they therefore dislike kurtosis, whereas they have a preference for positive skewness as it means that large positive returns are more likely than large negative returns.

Samuelson (1970) notices how the two-moment quadratic utility function can be a good approximation of the investor's utility function, but that higher moments of the distribution might be important. Rubinstein (1973) theorizes a relation between returns and higher moments of returns, and shows that the expected return of an asset is equal to the weighted sum of comoments. Horvath (1980) shows that risk averse investors

have a positive preference for mean and skewness and a negative preference for variance and kurtosis. Investors prefer higher expected returns and lower risk (volatility), but also prefer positive skewness as this means that large positive returns are more likely than large negative returns and dislike kurtosis as this means higher likelihood of extreme negative outcomes.

The CAPM is obtained assuming that investors, in their choice of how much to invest in different assets, maximize their expected utility by making a trade-off between the utility of consuming today and the utility of higher consumption in future. One of the main assumptions is that the utility function is, and can be defined by, the first two moments of the distribution of returns, that is, the mean and the variance. If the utility function is quadratic or the returns are elliptically distributed⁴, mean and variance are the only moments that affect the investment decision. However, not only does the empirical evidence show that asset returns exhibit skewness and large kurtosis, but it also shows that investors have a preference for positive skewness and an aversion to large kurtosis.

Since the traditional CAPM does not account for the way that asset returns covary with the variance and the skewness of the market portfolio, the risk might be under or overestimated, giving rise to the anomalies observed empirically. Some assets might yield higher returns because investors require higher compensation for kurtosis, whereas other assets might yield lower returns since they have positive skewness that is positively valued by investors.

This extension of the CAPM is not new to the finance literature as the first extension dates back to 1976 when Kraus and Litzenberger derived the three-moment CAPM in which the third moment of the distribution of returns (skewness) is included. The

⁴ The normal distribution is one of the elliptical distributions.

literature on the three-moment CAPM and the Four-Moment CAPM that incorporates, respectively, skewness, and skewness and kurtosis is now much more advanced. The higher-moment CAPM has been tested unconditionally by Kraus and Litzenberger (1976), Fang and Lai (1997), and Hwang and Satchell (1999), among others, and conditionally by Harvey and Siddique (2000), Dittmar (2002), and Fletcher and Kihanda (2005), among others. This line of research has gained new momentum in recent years for a number of reasons. First, the introduction and success of hedge funds, whose strategy adopts non-traditional assets such as derivatives where returns exhibit skewness, has created a problem in the performance evaluation of accounting for higher moments. Second, the widespread diffusion of Value-at-Risk and Extreme Value Theory, which focus on the probability of large losses, has contributed to the development of interest in the shape of the tails of the distribution of returns. In addition, market liberalization and globalization together with the development of investment funds have made investments in emerging markets more accessible, and emerging markets are characterized by asymmetric distributions of returns with long tails. Furthermore, technological advancements have resulted in fewer computational issues in an investment decision with multiple objectives (mean-variance-skewness-kurtosis). Finally, the observation that portfolios formed according to the characteristics that embody the main anomalies, such as size and book-to-market, exhibit a strong pattern in terms of coskewness and cokurtosis has prompted finance researchers to investigate whether these higher comoments with the market portfolio returns can find a rational explanation for the failures of the CAPM.

Kraus and Litzenberger (1976) document that when incorporating coskewness they find the intercept on average insignificant, a positive and significant premium for beta, and the market premium for coskewness to be significant and negative. Harvey and Siddique (2000) find that there is a non-negligible inverse relationship between

coskewness measures and average returns; more specifically, investors are willing to give up some returns for positive skewness.

Fang and Lai (1997) show a substantial improvement in explanatory power for the Four-Moment CAPM compared to the simple CAPM and the three-moment CAPM, and suggest that investors are compensated for systematic variance and kurtosis risk, and that they are willing to sacrifice some expected return for those assets that increase the systematic skewness of the market portfolio. Meanwhile, Hwang and Satchell (1999) estimate an unconditional Four-Moment CAPM for emerging markets, which represent a particularly interesting case since their distribution of returns exhibits skewness and kurtosis. They conclude that higher moments can add explanation to the returns of emerging markets, but not in a homogeneous fashion: for some countries the expected returns are better explained by beta and cokurtosis, whereas for others the expected returns are better explained by beta and coskewness. Dittmar (2002) finds that skewness and kurtosis provide a better explanation of the cross-section of average returns for industry portfolios and that the improvement is largely due to the higher moments of the distribution of human capital returns. Furthermore, Fletcher and Kihanda (2005) evaluate the performance of different unconditional and conditional asset pricing models. The results show that the Four-Moment CAPM reduces the pricing errors and that the conditional Four-Moment CAPM outperforms both the traditional and the three-moment CAPM.

The role of coskewness and cokurtosis conditional on market regimes and conditioning information has not yet been fully investigated. Moreover, there are few studies testing the conditional skewness premium and even fewer testing the conditional kurtosis premium. Finally, the choice of portfolios in many cases avoids the big challenges of the CAPM, that is, the size and book-to-market anomaly. This thesis attempts to address

this gap in the literature. The hypothesis tested here is that the standardized covariance and cokurtosis require a positive risk premium, whereas the standardized coskewness is associated with a negative risk premium, that is, investors are willing to forego some returns for positive coskewness.

RQ2: Can a conditional CAPM or a conditional Four-Moment CAPM with time-varying betas explain the cross-section of US asset returns and, hence, fix the empirical failures of the unconditional CAPM?

The CAPM is based on the assumption that beta and the risk aversion of investors are both constant over time, but this is clearly an unrealistic assumption. More realistically, it can be assumed that investors become more risk averse when the market is in a recession and less risk averse when investment opportunities improve in a period of economic growth. Therefore, risk premia should vary over the business cycle. This assumption seems to be supported by the variation in the credit risk spread. Moreover, the correlation between asset returns and risk factors may vary over time. Unconditional tests of the CAPM might lead to the result that the CAPM fails to explain the cross-section of average returns because such tests ignore time-varying parameters and conditioning information. However, conditional versions of the CAPM in which the market risk premium and beta are time-varying might be able to explain the cross-section of returns and the anomalies of the CAPM.

The conditional CAPM states that the expected excess return on any asset depends on the conditional beta multiplied by the market risk premium. However, the conditional beta is time-varying conditional on the information available to investors at a certain point in time. If market betas vary over the business cycle and the risk premium is time-varying as well, this time variation might explain the realized returns. In particular, investors will require higher returns for those stocks that are more sensitive to market

risk in the forecast of a downturn and they will prefer those stocks whose returns covary negatively with the economy as they are less risky.

From the 1970s onwards, empirical evidence mounted that returns are partly predictable, especially over a long horizon on the basis of variables such as the price-dividend ratio, the default spread, and the term spread. Therefore, asset pricing models should take into consideration that investors pay attention to some set of relevant information that forecasts future returns and future investment opportunities when making their investment decision.

An asset with higher sensitivity to certain risk factors when the risk premia for those risk factors are particularly high will demand higher returns, and capturing the dynamics of the correlation between betas and the risk premia will thus be crucial when testing the model.

Finance researchers are well aware of the importance of conditional models, and the literature on time-varying parameters is extensive. Among the seminal conditional models is the Jagannathan and Wang (1996) model which derives time-varying beta as a function of the default premium. Lettau and Ludvigson (2001) derive a conditional CAPM in which the conditioning variable is the ratio of consumption to wealth. Finally, Engle and Bali (2008) obtain a conditional ICAPM (Intertemporal CAPM) with time-varying betas, with a Multivariate GARCH with dynamic conditional correlations. The conclusion of these studies is that risk is indeed time-varying, and that only by capturing the conditional time variation can an asset pricing model improve its explanatory power of the cross-section of average returns.

Specifically, Jagannathan and Wang (1996) derive a conditional CAPM with human income and time-varying risk aversion, which is captured by introducing an additional beta with a time-varying risk premium, defined as a linear function of the default

spread. They document that their conditional model performs better than the three-factor model of Fama and French and that when the two factors of Fama and French are added to the “labour-CAPM” model, none of them is found to be statistically significant, whereas the premium associated with the time-varying beta and to the beta associated with the labour income return is significant, suggesting that the size effect might be a proxy for the risk associated with the return on human capital and beta instability.

The main problem for any conditional model is that in order to derive the time-varying parameters, some assumptions are required concerning the way risk premia change over the business cycle, and also concerning the precise set of conditional variables for consideration. The results of the test might be seriously affected by the variable chosen as the proxy for the dynamics of the risk premium. Lettau and Ludvigson (2001) derive a conditional CAPM in the stochastic discount factor approach using an instrumental variable to scale the factors in the discount factor to capture the time-variation of risk aversion. The results of the test on size and book-to-market portfolios provide evidence that the time-varying component of the intercept is not statistically different from zero, whereas the risk premia associated with the market return and the time-varying component of the market return are jointly significant, with a marked increase in the explanatory power of the model when human capital growth is included.

Engle and Bali (2008), using a multivariate GARCH with DCC that accounts for time-varying betas, pooling together the time series and cross-section of equity portfolios, document a significant positive risk-return relationship for size-sorted portfolios. This methodology is very innovative and its statistical derivation is an advantage compared with the problems raised by the arbitrary choice of conditioning information in other economic based models. This thesis extends the work of Engle and Bali to account for coskewness and cokurtosis and aims to provide a comprehensive investigation of the

conditional CAPM and conditional Four-Moment CAPM; using time-varying factor loadings for systematic covariance, coskewness and cokurtosis obtained through Multivariate GARCH with DCC.

RQ3: How do the CAPM and the Four-Moment CAPM perform under different regimes for the US equities market?

Recently, there has been a growing interest in the application of switching regimes models in asset allocation and asset pricing. Switching regimes models allow for time-varying parameters of the model according to the regime. There is compelling evidence in the existing literature that asset returns do not follow a single stochastic process, but that their returns are better captured by two or more regimes in which the correlation between the assets varies.

Guidolin and Timmermann (2002) introduce switching regimes with forecasting variables such as dividend yield, and find that risk premia change with regimes and with the investor's beliefs of the underlying state of the economy. Therefore, the market risk premium might be related to the sensitivity of equity returns to regimes, and cross-sectional average returns might be explained by the different sensitivities of categories of stocks to the regimes and to the predicting variables. These are important innovations that might represent a significant advancement in the field of asset pricing. The introduction of switching regimes and the analysis of the significance of each factor in the main asset pricing models for US equity returns is a largely unexplored field that could yield interesting results. The assumption in this thesis is that whilst the factor loadings change following a Multivariate GARCH process with dynamic conditional correlations, two different set of risk premia are required, one when the market is bullish and one when the market is bearish. The regimes are assumed to occur with certain probabilities, obtained through a Markov-switching regime model. This has the

advantage that the conditioning variables are not imposed. Rather than being exogenously determined, the regimes are determined by the data as suggested by the stochastic latent process of a random variable (the state or regime) which depends on an observable variable, that is, the market excess return in the case of this thesis.

The hypothesis tested in this thesis is that whilst a positive risk premium is associated with the more likely bullish regime, a negative realized risk premium is associated with the less likely bearish regime. However, the overall risk premium should be positive in order to rationally explain the cross section of average returns of US equity portfolios.

RQ4: Does the performance of the CAPM and the Four-Moment CAPM change when the models are tested using individual stocks rather than portfolios of stocks?

The common practice in testing asset pricing models is to build portfolios of stocks and then investigate the return-beta relationship in cross-sectional regressions. More recently, Ang, Liu, and Schwarz (2008) suggest that individual stocks lead to more efficient tests of whether the factors are priced. The common practice in empirical asset pricing tests, namely forming portfolios of stocks, has been motivated by the attempt to reduce estimation error in betas, as forming portfolios reduces idiosyncratic risk. However, Ang et al. argue that the reduction in the standard errors of the estimated betas does not lead to more precise estimates of the risk premia, as forming portfolios causes a lower dispersion in estimated betas and loss of information that results in higher standard errors in the premia estimates. Indeed, there is no theoretical reason why stocks should be grouped into portfolios, as the CAPM should be valid for individual assets too.

However, the following justifications can be made in favour of portfolio formation:

1. With portfolios, betas are more stable (Black, Jensen, and Scholes 1972);
2. Differences in average returns are more statistically significant, whereas individual returns are more volatile (Friend and Blume, 1970);
3. Stock characteristics (such as market capitalization and book-to-market ratio) are more stable in a portfolio, whereas individual stocks can migrate or change in nature throughout time (Cochrane, 2001);
4. Portfolios are easily tested using Dynamic Conditional Correlations, Generalized Method of Moments, Stochastic Discount Factor and other methodologies.

However, in the case of individual assets there is more dispersion in betas and therefore more information for the cross-sectional estimation of the risk premium, hence a more precise risk premium. Moreover, individual assets are more consistent with the assumption of a single period investment made by the CAPM, whereas testing asset pricing models on portfolios is more consistent with the testing of different investment strategies. Furthermore, portfolio formation might lead to a smoothing out of the cross-sectional behaviours of the assets, whereby for instance beta is particularly sensitive to extreme results which might be diluted in a portfolio (Kim, 1995). In addition portfolio formation can be exposed to the critique of data snooping biases (see Lo and MacKinlay, 1990). Therefore, in this thesis the CAPM and the Four-Moment CAPM are tested on individual US stocks to draw a comparison of the results of the tests involving such individual assets and the results obtained with tests conducted on portfolios.

1.2. Objectives and contributions of the research

In this section, the main objectives of the thesis and the relevant contributions offered here are discussed.

Objective 1: To evaluate the performance of the CAPM, both unconditionally and conditionally.

This is an important underpinning task to the sections that follow which focus on the central topic of this research, that is, the extension of the CAPM.

The accomplishment of this task requires the use of the principal methodology applied in testing asset pricing models: i) the test that the intercepts in the time series of portfolio returns are not significantly different from zero, by means of the Gibbons, Ross, and Shanken (1989) test; and ii) the Fama and MacBeth two-pass methodology to estimate the premium associated with the risk factors and whether the models can explain the cross-section of average returns. The conditional version with time-varying factor loadings is obtained using a Multivariate GARCH with dynamic conditional correlations as opposed to the simple rolling regression method. The main contribution of the thesis in this regard is that these tests are conducted for a more recent period than in the previous literature.

Objective 2: To test an extended version of the CAPM, which includes systematic skewness and kurtosis.

This task requires the derivation of a model which includes coskewness and cokurtosis and the two-pass methodology of Fama and MacBeth to test the significance of the higher moments for US equity portfolio returns. The major contribution of this thesis here is to assume time-varying sensitivity in the higher moments of the distribution of returns obtained using a Multivariate GARCH with dynamic conditional correlation. This is a technique introduced by Engle (2002), but not yet frequently applied to the Four-Moment CAPM in the existing literature. The main objective of this element of the thesis is to understand whether time-varying betas with dynamic conditional

correlations can have sufficient variability, and be significantly positively correlated with the risk premia for higher comoments, so that the model can explain the size and book-to-market anomalies.

The derivation of the model of this thesis in this case allows some significant innovations:

(i) the higher moment CAPM is derived such that the sum of the risk premium for all factors (beta, systematic coskewness and systematic cokurtosis) equals the market excess return; (ii) non-standardized coskewness is employed, as the market portfolio skewness might approach zero; (iii) the conditional coskewness and cokurtosis are estimated as counterparts of the conditional covariance using DCC GARCH; (iv) non-standardized coskewness is used so that the estimated or expected coefficient associated with skewness should be negative and independent of the sign of market skewness.

Objective 3: To introduce time variation in systematic risks (covariance, coskewness, cokurtosis).

This analysis is quite novel to asset pricing research and has the appealing feature that the parameters are not derived from a set of conditioning information whose choice might produce non-robust results and be exposed to the critique of finding the appropriate set of conditioning information actually observed and considered by investors, since the switch in regimes is determined by a latent stochastic process.

In particular, the assumption is made that there are two regimes, each with a probability that is returned by a Markov Switching process, and it is assumed that there are two different sets of risk premia in each regime. Whereas the factor loadings are still conditional and determined through a Multivariate GARCH, the risk premia are estimated in a panel data regression, and the average risk premia are calculated as the

average of the time series of the weighted average of the two risk premia where the weights are represented by the probability of being in each regime. The main objective of the research for this element is to investigate whether the further complication of time-varying factor loadings and time-varying risk premia can explain the cross-section of US average returns. The introduction of DCC GARCH, Markov Switching and Panel data together represents a novel approach in asset pricing.

Objective 4: To estimate a conditional version of the CAPM and Four-Moment CAPM using individual stocks as test assets.

As above mentioned, Ang et al (2008) argue that the reduction in the standard errors of the estimated betas does not lead to more precise estimates of the risk premia, but instead that forming portfolios causes a lower dispersion in estimated betas and a loss of information that results in higher standard errors in the premia estimates. Ang et. al show that the beta premium is positive and significant when using individual stocks for the test of the CAPM, whereas the construction of portfolios often results in a negative and insignificant beta premium. Following this result, this thesis investigates the performance of the Four-Moment CAPM when tested on individual assets following Avramov and Chordia (2005) who use individual assets as opposed to portfolios. However, in contrast to the previous authors, in this thesis the Four-moment CAPM is tested and augmented with the Fama and French three factors.

1.3. Structure of the thesis

This thesis is structured as follows. In Chapters 2, 3 and 4 the core theoretical and empirical literature review is introduced. In particular, in Chapter 2, the logic of the CAPM and its main tenets (the positive relationship between returns and systematic risk and the relevance of beta as a systematic measure of risk) are discussed, the tests of the

CAPM are presented comprehensively, based on time series regressions and a cross-section of average returns, and the empirical failures of the CAPM (the small size premium and the book-to-market premium, in particular) are also discussed.

The most important extensions of the CAPM are then introduced in Chapters 3 and 4. In Chapter 3, the rationale for the inclusion of higher order moments in the traditional CAPM is discussed. The chapter contains a detailed derivation of the Four-Moment CAPM and presents the relevant literature concerning the conditional and unconditional tests of the higher-moment CAPM.

Chapter 4 introduces the rationale for the conditional models and the problem of a time-varying beta, time-varying risk premium, and the predictability of asset returns. The most significant conditional models presented in the existing research (such as Lettau and Ludvigson's conditional CAPM, 2001 and Jagannathan and Wang's conditional CAPM, 1996) are then explained and critically discussed in terms of their implications.

In Chapter 5 the relevant econometric methodological techniques adopted in this thesis are presented and discussed, together with the traditional methodologies applied in asset pricing tests. A comprehensive overview of the main estimation methods such as time-series and cross-sectional regressions is offered, along with a discussion of the Gibbons, Ross and Shanken test. The chapter continues with a discussion of the methodology used to model time-varying parameters, that is, the Multivariate GARCH with dynamic conditional correlations, and switching regimes. In this chapter, the derivation of the Four-Moment CAPM that will be object of the investigation is presented, together with the main hypotheses to be tested and innovations. The chapter concludes with a discussion of the short-window regressions methodology used to estimate the CAPM and Four-Moment CAPM on individual stocks.

The data used in this thesis and the associated descriptive statistics are reported in Chapter 6. The chapter first discusses the portfolios formed according to market capitalization, book-to-market ratio, and double-sorted on both characteristics. The chapter then provides summary statistics on individual assets (US stocks). The two samples (portfolios and individual assets) will be used to test on one hand whether the extensions of the CAPM investigated in this thesis can explain the book-to-market and size premium, and on the other hand to analyse the results of the CAPM and the higher-moment CAPM when individual assets are considered in order to answer the final research question, i.e. what is the performance of the CAPM and of the Four-Moment CAPM when the models are tested on individual assets (stocks) instead of portfolios of stocks?

In Chapter 7 the results of the test of the unconditional and conditional CAPM with rolling regressions and with time-varying conditional correlations obtained with DCC are reported and the main anomalies to the CAPM, such as size and book-to-market are discussed. In the second part of the chapter, the results of the tests on the conditional extension of the CAPM (the Four-Moment CAPM) are discussed. This part of the chapter focuses on tests of the significance of the market beta and the higher moments (coskewness and cokurtosis) when the parameters of the model vary with a DCC GARCH approach.

In Chapter 8, a further assumption that risk premia change according to the market regime is introduced. In particular, time-varying risk premia together with the Markov switching regimes are introduced. The findings of a panel data estimation of the models under investigation on the 25 portfolios of stocks double-sorted on market capitalization and the book-to-market ratio are presented and discussed. The last part of the chapter

deals with the tests of the conditional CAPM and conditional Four-Moment CAPM for individual assets.

Finally, Chapter 9 summarises and readdresses the research questions of this thesis which taken together consider whether an extension of the CAPM can explain the size and book-to-market puzzles, and whether the CAPM and its extensions perform better or differently in the case of individual assets as opposed to portfolios of stocks. In particular, the extension of the CAPM consist of: firstly estimating a time-varying beta obtained with dynamic conditional correlations as opposed to constant betas or betas obtained with rolling regressions; secondly, including coskewness and cokurtosis in the single-factor CAPM; thirdly, questioning the assumption of a constant risk premium and introducing time-varying risk premia changing according to regimes obtained through a Markov Switching methodology; and finally, questioning the common practice of testing the CAPM on portfolios by testing the CAPM and the Four-Moment CAPM on individual assets. The chapter ends with a discussion of the major findings, their implications for asset pricing, and a discussion of possible opportunities for future research in this field.

Chapter 2

The CAPM: Theory and Evidence

2.0. Introduction

The aim of this thesis is to investigate possible extensions of the Capital Asset Pricing Model (CAPM) that have been proposed in modern Asset Pricing Theory and to examine whether these can explain the cross section of equity returns and the size and book-to-market anomalies.

Asset Pricing Theory studies the relationship between risk factors and returns, and involves the search for, and identification of, what determines the excess returns of financial assets observed in the market. In order to explain the excess returns of financial assets, several asset pricing models have been proposed in the literature. Among such models the CAPM is not only the most famous and most widely used, but also the model that has been the object of many tests and much debate among researchers.

Following the mean-variance framework of Markowitz, Sharpe (1964), Lintner (1965) and Mossin (1966) derived the CAPM based on two parameters, mean and variance, asserting that any asset expects a return in excess of the risk-free rate which should be proportional to its market beta. The beta here is a standardized measure of risk obtained as the covariance of the asset with the market portfolio divided by the market portfolio variance. Several studies such as those of Black, Jensen and Scholes (1972), Fama and MacBeth (1973), Friend and Blume (1970), among others, consistently find that the slope of the Security Market Line⁵ is lower than that predicted by the CAPM, and that the intercept is too large to reconcile with the historical risk-free rate. In particular,

⁵ The SML is obtained plotting the average excess returns on their respective betas and should theoretically have a positive slope.

Fama and French (1992) show that the cross section of returns cannot be explained by the market beta alone, stating that the CAPM remains problematic. The disappointing empirical performance of the CAPM, and in particular the discovery that certain stock characteristics such as book-to-market and market capitalization could explain more of the cross section of returns than the market beta alone, provided the foundations from which much of the recent research on asset pricing is built. In other words, researchers either try to identify new models that better explain the cross-section of asset returns or they try to revise the traditional CAPM to include more realistic assumptions.

In particular, the CAPM assumes that investors should agree on the expected returns and expected covariance of returns, have a quadratic utility function (i.e. they only care about mean and variance), and/or that returns should be elliptically distributed or normally distributed.

One possible solution proposed by some researchers to overcome the shortcomings of the CAPM is to introduce multifactor models such as the Chen, Roll and Ross model (1986), the Asset Pricing Theory of Ross (1976), the Intertemporal CAPM of Merton (1973), the Fama and French three-factor model (1993), the Liquidity CAPM of Pastor and Stambaugh (2003), and the Carhart four-factor model (1997) who extended the Fama and French three factor model adding a momentum factor based on previous stock performance. In these models, it is argued that a set of factors should better reflect the market risk that is not fully captured by the market beta alone. More recently, the introduction of conditional models with time-varying parameters has become the focus of research in this field. This latter branch of research suggests that the unconditional single factor model and the multifactor models fail to explain the cross-section of returns because they ignore conditioning information.

Finally, a further solution proposed is the extension of the CAPM to include higher moments of the distribution of returns. The traditional CAPM assumes that investors have preferences for the mean and the variance of portfolio returns, used as a proxy for wealth, and that the extra return of any asset over the risk-free rate is related to the systematic risk measured by the contribution of the asset to the variance of the diversified market portfolio. However, the variance is questionable as an adequate measure of risk given that it considers downside and upside volatility equally, which is unrealistic given that investors empirically exhibit a preference for positive skewness and show an aversion to large losses. Furthermore, the empirical findings show that the distribution of returns is not described solely by the mean and variance (returns are found to be both asymmetric and leptokurtic). Asset returns are better characterized by leptokurtic distributions with fatter tails and more likely to yield extreme outcomes than is predicted by the normal distribution. These findings have led researchers to investigate the possibility of incorporating higher moments of the distribution of returns into asset pricing models under the belief that the investor's portfolio choice depends not only on mean and variance, but also on preferences for skewness and kurtosis.

Together, this chapter and the following two chapters offer a comprehensive overview of the CAPM and the empirical failures of the traditional model, and should present in a systematic way some of the most important and advanced extensions of the CAPM and their rationale. Specifically, this chapter introduces the CAPM, its main tenets, and the most important empirical tests conducted on the CAPM.

The remainder of this chapter is divided into three sections. Section one discusses the CAPM and the relevant literature on the most important tests of the model itself. The literature review addresses the most important tests of the CAPM and discusses the main findings of those tests. After an early success of the model in explaining the cross-

section of average returns, including the Fama and MacBeth (1973) test, certain empirical anomalies emerged, and these will be object of discussion in the first section of this chapter. The second section contains an overview of multifactor models, especially the three-factor of Fama and French (1993). The final section concludes.

2.1. Theoretical and empirical background to the CAPM

2.1.1. The CAPM

The success of the CAPM rests on the easy and intuitive way in which it describes the relationship between risk and return in addition to its strong economic underpinnings. However, the empirical research challenges the main predictions (linear and exact relationship between beta and expected returns, and positive risk premium) of the model and leads financial researchers to question the causes of the failure of the CAPM, and to investigate possible extensions of this asset pricing model.

Before introducing the extant research on the tests of the CAPM and the possible extensions of the model which might explain empirical anomalies, the theoretical underpinning of the CAPM is briefly explained.

The CAPM is derived from the assumption that the distribution of one-period returns is normal and that investors are risk averse. The latter condition is obtained by assuming that the utility function of investors is quadratic. Under these assumptions, investors choose among portfolios on a mean-variance basis. Investors select portfolios that minimize the variance of returns, given the expected return, or maximize the expected return, given the variance. In modern portfolio theory, first outlined by Markowitz (1952), the only source of risk rewarded by the market is systematic risk, which is the contribution of each asset to the portfolio's total volatility. The idiosyncratic risk can be totally diversified away.

Markowitz is the first author to introduce the mean-variance trade-off and to show that the risk of an asset is described in terms of covariance of the asset returns with the market portfolio returns. Tobin (1958) introduces the important further assumption that investors can borrow or lend at the risk-free rate and shows that under this assumption the efficient frontier becomes a straight line tangent to the market portfolio return.

Together with the assumption of normally distributed returns, the risk is described in terms of standard deviation and under the assumption that investors have homogeneous expectations all investors will hold the same market portfolio, though in different quantities, and the risk-free asset. The contribution of any asset i to the standard deviation of the market portfolio is expressed as:

$$\frac{\text{cov}(R_i, R_m)}{\sigma(R_m)} \quad (2.1)$$

The problem of optimization for investors is represented by maximizing the utility function, that is, a function of the expected return and the standard deviation:

$$U = V(E(R_m), \sigma(R_m))$$

The expected return is given by the sum of the expected returns on an asset multiplied by the weights invested in each asset, where w_i are the weights of the risky assets and w_f is the weight of the riskless asset:

$$E(R_m) = \sum_{i=1}^N w_i E(R_i) + w_f R_f \quad (2.2)$$

and the standard deviation:

$$\sigma(R_m) = \sum_{i=1}^N w_i \frac{\text{cov}(R_i, R_m)}{\sigma(R_m)} \quad (2.3)$$

with the budget constraint:

$$\sum_{i=1}^N w_i + w_f = 1 \quad (2.4)$$

where $\beta_{i,m} = \frac{\text{cov}(R_i, R_m)}{\sigma^2(R_m)}$.

By using a Lagrangian method, the problem of the investor is solved so that the weights of any asset in the portfolio are chosen so that for any asset i :

$$E(R_i) - R_f = -\frac{d\sigma_m}{dE(R_m)} \beta_{i,m} \sigma(R_m) \quad (2.5)$$

where $-\frac{d\sigma_m}{dE(R_m)} > 0$ is the marginal rate of substitution between return and standard

deviation. Using the definition in (2.6)

$$\lambda_\beta = -\frac{d\sigma_m}{dE(R_m)} \sigma(R_m) \quad (2.6)$$

gives the usual CAPM

$$E(R_i) = \alpha + \lambda_\beta \beta_{i,m} \quad \forall i = 1, \dots, N \quad (2.7)$$

The intercept is the expected return on an asset whose return is uncorrelated with the market return, which is the risk-free rate or zero-beta asset return in the extension of Black (1972), i.e. the return on an asset which is uncorrelated with the market return. The market premium is a function of the risk aversion and the standard deviation of the market portfolio, i.e. the higher the uncertainty and the risk aversion, the higher is the demanded risk premium.⁶

⁶ It is worth noting that from the definition of the risk premium it follows that the risk premium depends on risk aversion $-\frac{d\sigma_m}{dE(R_m)}$ and volatility $\sigma(R_m)$. It follows that the risk premium for any asset i is a function of three factors: risk aversion, volatility and sensitivity to market risk $\beta_{i,m}$. Therefore, the risk premium of any asset i can depend on a conditional volatility, a conditional factor loading, and a conditional risk aversion. In this thesis, conditional volatility and conditional betas will be introduced with Multivariate GARCH dynamic conditional correlations, and time-varying risk aversion will be accounted for with the introduction of switching regimes.

The most important proposition of the Capital Asset Pricing Model is that in equilibrium, the expected return on a security is equal to the expected return on a risk-free asset plus the market risk premium times the security's market beta (sensitivity or exposure to the market risk, which is also known as systematic risk).

The tests of CAPM concern this basic proposition, which has the following implications:

1. The risk-return relationship across assets is linear and positive $\lambda_\beta > 0$;
2. The intercept in Equation 2.7 should be equal to the risk-free rate $\alpha = R_f$;
3. The only relevant measure of risk is $\beta_{i,m}$, that is, the cross-sectional spread of returns should be entirely explained by the difference in market betas, and not by further variables such as size and the book-to-market ratio;
4. The expected market premium, λ_β , and the long run average market premium should converge to the true market premium.

Both early and more recent tests of the CAPM focus on these predictions and employ cross-sectional or time series regressions in their methodologies. In other words, either they investigate whether the differences in betas can explain the differences in average returns across assets, or whether the market beta is sufficient to explain the historical variability of asset returns.

2.1.2. Tests of the CAPM

The CAPM has been the subject of many empirical tests, the results of which have cast doubt on the validity of the model itself. In particular, the CAPM predicts that the frontier of the portfolios plots as a straight line (the Security Market Line) with an intercept equal to the risk-free rate, (R_f), and a slope equal to the expected excess return

on the market, $E(R_m - R_f)$, and that in equilibrium the excess return on any asset i should be proportional to its sensitivity to the market portfolio excess return:

$$R_i = \alpha + \beta_i E[(R_m) - R_f] + \varepsilon_i \quad (2.8)$$

The early tests of the CAPM focus on the CAPM's prediction regarding the intercept and slope of the SML (the theoretical linear relationship between beta and returns). These tests attempt to examine whether the intercept corresponds to the historical average risk-free rate and whether the slope λ_β applied to the estimates of betas corresponds to the average market premium and is positive.

The first problem in testing the theory is that the expected risk premium is unknown. However, it is common practice to use realised excess returns instead of expected returns under the assumption of unbiased expectations. The approach is to use a two-pass procedure in which a time series of the asset returns on the market premium is undertaken to estimate the beta and, in the second step, a cross-section of average asset returns on the estimated betas of the assets is undertaken to estimate the market premium. The intercept of this cross-sectional regression should be the risk-free rate according to the CAPM, and the slope is the estimate of the expected market risk premium, which should be positive and not statistically different from the average market premium if expectations are unbiased.

The early tests for US stocks, such as Lintner (1965) and Douglas (1969), find that the intercept is larger than the risk-free rate as measured by the monthly return on the US Treasury bill, and that the coefficient of the beta is less than the market risk premium measured as the average return on a portfolio of US common stocks minus the Treasury bill rate. In these tests, the average returns of individual stocks are regressed against estimated betas. However, researchers soon realized that individual stocks have estimated betas with standard errors that are too large and that the market betas of

individual stocks are unstable as business risk changes considerably over time (Blume, 1970), and therefore it became common practice to use portfolios of stocks sorted according to the market beta or other characteristics thought to be related to higher returns instead of individual assets. Betas for portfolios tend in fact to be more stable and less subject to measurement errors as an effect of the diversification. However in building portfolios the dispersion of returns is reduced as a consequence.

Black, Jensen, and Scholes (1972), hereafter referred to as BJS, estimate a time-series of the monthly excess returns of a series of stock portfolios and a cross-sectional regression of the average excess return of a series of stock portfolios on estimated betas. The average returns of the portfolios are plotted against their respective betas. The results indicate that when portfolios are selected on the basis of market beta (systematic risk), portfolios with higher systematic risk exhibit a lower return than that predicted by the CAPM, whereas returns on the low beta portfolios are too high relative to their beta. In other words, low beta portfolios ($\beta < 1$) have a positive alpha, whereas high beta portfolios ($\beta > 1$) exhibit a negative alpha.

The results of the cross-sectional test of BJS indicate that hypotheses that the intercept is zero and that the estimated risk premium equals the historical market premium are rejected. In addition, evidence is found that the market risk premium is time-varying, and in particular it is steeper than hypothesised in the pre-war period and flatter than hypothesised in the post-war period.

Fama and MacBeth (1973), hereafter referred to as FM, propose one of the most widely used methodologies for testing the CAPM that investigates the prediction that there is a positive relationship between beta and average expected returns. FM consider a general extension of the Security Market Line equation:

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_i + \gamma_{2t}\beta_i^2 + \gamma_{3t}\sigma_i + \varepsilon_{it} \quad (2.9)$$

An exponential factor β_i^2 is included to test the hypothesis of linearity in the risk-return relationship, and the standard deviation of the residuals σ_i is included as well to test the proposition that beta alone captures the risk, and not other risk measures such as the individual standard deviations (idiosyncratic volatility). The predictions tested by FM are as follows:

- Linearity $E(\gamma_{2t}) = 0$. The squared beta should not be significant, ruling out the existence of a nonlinear relationship between risk and return;
- Insignificant idiosyncratic risk $E(\gamma_{3t}) = 0$. The standard deviation of the individual assets should not be significant, meaning that only systematic risk is rewarded by the market;
- Positive risk-return relationship $E(\gamma_{1t}) > 0$. The premium for the beta risk should be positive;
- Sharpe-Lintner Hypothesis $E(\gamma_{0t}) = R_{ft}$. The intercept should be approximately equal to the average risk-free rate.

In order to test these hypotheses, a two-pass technique is applied. In the first stage the betas for a set of portfolios sorted on beta are estimated with a 5-year time series rolling regression of the monthly excess returns over the monthly portfolio excess returns, and in the second stage a cross-sectional regression is conducted where the extra returns of portfolios are regressed on the estimated betas at each date as in Equation 2.10:

$$r_{pt} = \alpha_{0t} + \gamma_{1t}\hat{\beta}_{pt-1} + v_{pt} \quad (2.10)$$

This methodology allows for time-varying betas and accounts for cross-sectional correlation, which is not handled by the BJS methodology introduced above. The estimated alpha and market risk premium are obtained as the average of the monthly alpha and market risk premium (intercept and slope) from each period's cross-sectional

regression, and the standard deviation of the estimate is used as an estimator of the standard errors. Finally, the significance of the explanatory variables on the right hand side is tested with the t-test:

$$t(\hat{\gamma}_j) = \frac{\hat{\gamma}_j}{s(\hat{\gamma}_j)/\sqrt{T}} \quad (2.11)$$

The empirical findings show that:

- The non- β or idiosyncratic risk is not significantly different from zero and therefore no measure of risk except from beta seems to affect expected returns significantly;
- The linearity condition is not rejected in the data, as the coefficient for the squared beta is not statistically different from zero and therefore linearity is an acceptable assumption;
- A positive relationship is found between risk and return as the estimated risk premium from the cross-sectional regression is in general significantly positive, though lower than the historical average risk premium;
- There is evidence of time-varying risk premia;
- The empirical results lead to a rejection of the hypothesis that alpha is equal to the average risk-free rate. The values of $\overline{\hat{\gamma}_0 - R_f}$ are positive and significant.

Thus, the results of this pivotal test support the CAPM and show that investors do indeed require some compensation for systematic risk. However, soon after FM, different conclusions started to emerge in the academic literature.

Reinganum (1981) using ten beta-sorted portfolios shows that high beta stocks do not experience systematically higher returns than low beta stocks, and that estimated betas are not therefore an adequate measure of risk for which the market requires some compensation, or at least are not the only determinant of average returns. Furthermore,

Tinic and West (1984) notice that the relation between beta and returns changes month-by-month in a given year, and it is stronger in January than in the rest of the year. The model tested is reported in Equation 2.12:

$$\overline{R}_p = \gamma_0 + \gamma_1\beta_p + \gamma_2\beta_p^2 + \gamma_3\sigma(\varepsilon_p)\varepsilon_p \quad (2.12)$$

The findings show a positive and insignificant beta premium, a negative and insignificant coefficient for the squared beta, and a positive and significant coefficient for the unsystematic risk. Therefore, the results do not support the CAPM, and moreover suggest that idiosyncratic risk adds further explanation to the cross-section of average returns. Furthermore, TW find that when January alone is considered, the only coefficient statistically significant is the one associated with the unsystematic risk. When the rest of the year is analysed excluding January, the beta premium is positive (0.0178) and significant (with a t-statistic of 2.68), but contrary to the theoretical expectation of the CAPM, the squared beta is significant (with a t-statistic of -2.30) and has a negative coefficient (-0.0072), indicating the presence of nonlinearities in the relationship between returns and systematic risk for a large part of the year. Therefore, the results of TW indicate some non-linearities in the relationship between risk and return, that the market beta does not fully capture expected returns, and that there exists a January anomaly (as returns tend to be higher than suggested by the CAPM in January).

The main conclusion that can be drawn from the early tests of the CAPM is that the linearity of the risk-return relationship (with some exceptions, such as TW) and the positive sign of the beta premium seem to be accepted, though the prediction that the market premium is the expected (average) market return minus the risk-free rate is rejected. These early tests, therefore, seem to provide some evidence in favour of the CAPM. However, from the 1970s onwards, a long stream of empirical work

fundamentally has challenged the main predictions of the CAPM, and in particular challenge the theoretical corollary that only market beta is relevant in explaining the average return on assets.

2.1.3. Empirical anomalies

The second set of tests refer to the explanatory power of market betas: the CAPM suggests that differences in expected returns across securities and portfolios are entirely explained by differences in market beta and that other variables should not help to explain the cross-section of expected returns. However, the cross-section of average returns seems to be in large part related to factors other than the market beta (Banz (1981), Basu (1977), Fama and French (1992), for instance). In other words, there is strong evidence that the differences in returns across assets are not related to differences in market betas.

A way employed extensively to test the assumption that beta is the only explanatory variable of returns is to estimate time series regressions on a set of portfolios formed *ad hoc* and test the joint hypothesis that all of the intercepts are zero. The CAPM states that the average excess return on any asset should be equal to its market beta times the average excess return of the market portfolio. This implies that the intercept in the time series regression is zero. The empirical methodology requires sorting equities into portfolios on characteristics thought to be relevant in explaining the average returns. For instance, portfolios of small-cap stocks and portfolios of large-cap stocks should both have a alpha equal to zero, that is, only the differences in beta should explain the difference in average returns. If portfolios of small stocks had a positive alpha, then a strategy based on market capitalization such as purchasing small stocks and selling large stocks could help achieve an extra return without additional risk (no additional beta). According to the CAPM, the time series variability of asset returns should be

captured by the market beta alone, and the alpha, the residual unexplained by beta, should not be significantly different from zero.

In order to test the condition of the zero intercept in time series regressions, the Gibbons, Ross, and Shanken (1989) (hereafter known as GRS) test is used. In particular, GRS test the null hypothesis assumed by the CAPM that the joint intercepts in the time series regressions of the excess returns of the tested portfolios over the market portfolio excess returns are zero.

The null hypothesis to test the CAPM is that:

$$H_0: \alpha_i = 0, \quad \forall i = 1, \dots, N.$$

The test statistic for the hypothesis that all the pricing errors (all the alphas) are jointly equal to zero is obtained under the assumption of no autocorrelation and no heteroscedasticity, and GRS show that the statistic which is valid asymptotically has a finite-sample counterpart which is distributed as an F distribution, which is known as the Gibbons, Ross, and Shanken (1989) GRS test statistic:

$$\frac{T - N - 1}{N} \left[1 + \left(\frac{E_T(f)}{\hat{\sigma}(f)} \right)^2 \right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{N, T-N-1} \quad (2.13)$$

where $f_t = R_{mt} - R_{ft}$, $E(f)$ is the sample mean of the factor, and $\hat{\sigma}(f)$ is the sample standard deviation of the factor, and therefore the ratio $\frac{E(f)}{\hat{\sigma}(f)}$ can be considered as the Sharpe ratio⁷ for the factor (the market portfolio excess return for the CAPM). The $N \times 1$ vector of the intercepts is defined as $\hat{\alpha} = [\hat{\alpha}_1 \hat{\alpha}_2 \dots \hat{\alpha}_N]'$, and $\hat{\Sigma}$ denotes the estimated residual covariance matrix $E(\varepsilon_t \varepsilon_t')$, T is the number of observations and N the number of assets.

⁷ Expected return per unit of systematic risk.

GRS, in particular, test the CAPM for size and industry portfolios.

The results of the GRS test on the size portfolios show that higher beta portfolios earn lower returns (a negative alpha) over the period 1931-1965 than lower beta portfolios (a positive alpha), and yet does not lead to a rejection of the CAPM. However, the same test conducted over the industry portfolios results in a much larger F statistic, leading to the rejection of the model.

The interesting aspect to emphasise here is that the t-test on the single assets or portfolios often leads to a different result from the GRS test. In the case of the industry portfolios, the alphas are not distinguishable from zero due to the large standard errors though the CAPM is rejected using the GRS. In contrast, for the size portfolios, many alphas are significantly different from zero when considered individually, and nevertheless the CAPM is not rejected when the joint condition on multiple intercepts is tested.

From the 1970s onwards, the empirical research focuses extensively on the explanatory power of beta, though the empirical tests, conceived to investigate the prediction that only the market beta matters, firmly reject this prediction. Basu (1977) finds evidence that when stocks are sorted on earnings-price (E/P) ratios, stocks with high E/Ps have higher future returns than predicted by the CAPM. Therefore, not only do earnings-price ratios explain current returns but they also predict future returns. Basu forms five portfolios of stocks sorted on the E/P ratio. The results of the test show that the portfolio with high E/P ratios has positive alphas, whereas the portfolio with low E/P ratios has negative alphas. The findings support previous empirical results found by Black, Jensen, and Scholes (1972) and Friend and Blume (1970) that low beta portfolios earn higher returns than are predicted by their beta, whereas high beta portfolios earn lower returns than are predicted by their beta. The findings of Basu lead to a rejection of the

CAPM prediction that the only measure of risk is beta, as E/P ratios are significantly related to average returns.

The second setback for the CAPM comes from the research of Banz (1981) who documents that size can help to explain average returns when added to the market model. When stocks are sorted on market capitalization⁸, average returns on small stocks are higher than predicted by the CAPM, whereas average returns on large stocks are lower than predicted by their betas. Banz finds that there is a negative relationship between size and average excess returns and that this relationship is particularly strong for smaller companies. His study confirms the empirical evidence that additional factors to the market beta help to explain the cross-section of average returns. In fact, small stocks have higher risk-adjusted average returns than large stocks.

The usual Fama and MacBeth (1973) cross-sectional methodology is applied to estimate the beta premium and the size premium for portfolios double-sorted on size and beta, using a linear model that includes market size:

$$E(R_i) = \gamma_0 + \gamma_1\beta_i + \gamma_2 \left[\frac{(\phi_i - \phi_m)}{\phi_m} \right] \quad (2.14)$$

where ϕ_i is the market value of the stock i and ϕ_m is the average market portfolio value. The results for the overall period (1936-1975) show a negative coefficient (-0.052%) and significant premium (with a t-statistic of -2.92) for the size factor, which contradicts the CAPM assumption that no factor in addition to beta has marginal explanatory power.

Furthermore, Bhandari (1988) finds that stocks with high leverage, measured as high debt-equity ratios⁹ have returns that are larger than are predicted by their market betas.

⁸ Market price times the number of shares outstanding.

⁹ Book value of debt over the market value of equity.

The anomaly is that even if high leverage should be sensibly related to risk, this higher risk in the CAPM should be captured entirely by a higher market beta. Bhandari uses monthly cross-sectional regressions of stock returns against beta plus two additional factors, specifically size and the liability-to-equity ratio:

$$R_{it} = \gamma_0 + \gamma_1\beta_i + \gamma_2\text{LTEQ}_{it} + \gamma_3\text{DER}_{it} + \varepsilon_{it} \quad (2.15)$$

where

- LTEQ is the natural logarithm of market capitalization;
- DER is the liability-to-equity ratio: (book value of assets – book value of equity)/market capitalization.

The results of the cross-sectional regression for 27 portfolios triple-sorted on market capitalization, beta and the liability-to-equity ratio show that the beta premium is small but positive, though insignificant, the size coefficient is negative and significant thereby confirming a negative relationship between size and average returns, and the liability-to-equity ratio slope is positive and significant as expected by Bhandari. The results therefore do not support the CAPM. There is a weak relationship between beta and average returns and, in particular, not only size but also the liability-to-equity (leverage) ratio helps to explain the cross-section of average returns. Highly geared companies seem to be more prone to financial distress and this risk might not be adequately captured by beta. Furthermore, Stattman (1980) and Rosenberg et al. (1985) find that stocks with high book-to-market ratios¹⁰ have high average returns relative to their betas. Rosenberg et al. show that a portfolio long in stocks with high book-to-market ratios outperforms a portfolio long in low book-to-market stocks after ruling out other possible risk factors.

¹⁰ The ratio of the book value of a common stock to its market value.

Arguably the greatest setback for the CAPM comes from Fama and French (1992) (hereafter known as FF) who test explanatory power when size, the book-to-market ratio, leverage and the earnings-price ratio are included as variables in addition to the market risk factor. Specifically, FF find that two variables, size and the book-to-market ratio, are enough to capture the variation in average returns associated with market risk, leverage, the earnings-price ratio, size and the book-to-market ratio.

FF investigate the impact of different variables on the cross section of average returns. Starting with the size effect, as there is a positive relationship between size and the market beta and in order to capture the net effect of size on the cross-section of returns, they grouped US stocks first on size and then on the market beta estimated in the previous period (pre-ranking beta), and finally calculate the full-period post-ranking beta for every size- β portfolio and allocate this beta to each stock in the portfolio. The betas allocated to the stocks in each portfolio are then employed in the monthly cross-sectional regressions to estimate the market risk premia.

When stock portfolios are formed on size alone, the main prediction of the CAPM seems to be upheld: there is a positive relationship between average returns and beta. When portfolios are sorted on size and pre-ranking betas, the result is that average returns are strongly related to size, but weakly related (or unrelated) to market beta. In other words, when portfolios are sorted on size, there is a negative relationship between beta and size, so that the main tenet of the CAPM is upheld: higher betas produce higher returns since smaller size produces higher betas. When the portfolios are formed according to beta, the positive relationship between return and beta disappears. Finally, when, portfolios are sorted on size and then pre-ranking betas, there is a negative relation between size and average return, but no relation between beta and average return within size deciles.

In order to test whether the CAPM prediction is upheld or other variables are relevant to explain the cross-section of average returns, FF use cross sectional regressions of monthly stock returns on beta, size, leverage, the earnings-to-price ratio, and the book-to-market ratio either individually or in combination. First, a cross-sectional regression of stock returns on beta and size is conducted:

$$R_{it} = \alpha_t + \gamma_1\beta_{it} + \gamma_2\ln(ME)_{t-1} + \varepsilon_{it} \quad (2.16)$$

where:

- $\ln(ME)$ is the natural logarithm of the stocks' market capitalization (size).

The beta assigned to each stock is the post-ranking beta for the portfolio to which the stock belongs, and the average of the slopes from the monthly cross-sectional regressions is the estimate of the coefficient associated with each explanatory variable. The results of the test for the period July 1963 to December 1990 show that size is significantly priced (t-statistic of -3.41) and that there is a negative relationship between size and average returns (coefficient of -0.17), whereas the coefficient associated with beta is not significant (t-statistic of -1.21) and is slightly negative (-0.37), which is the opposite sign to that expected from theory.

When the Fama and MacBeth procedure is applied with beta alone on the right hand side as an explanatory variable, the coefficient is small but positive (0.15), that is, much lower than expected and it is insignificant (t-statistic of 0.46). Moreover, when in the cross-sectional regression other variables such as the book-to-market ratio, leverage, or the earnings-to-price ratio are included in the right hand side as explanatory variables, size still has a significant negative sign. These findings are evidence that size is a relevant variable for the explanation of cross-sectional returns and that there is a weak relationship between beta and average stock returns over the period 1963-1990.

FF also test the explanatory power of the book-to-market ratio and find that, when portfolios are sorted on this ratio, average returns increase systematically with the ratio with a large spread between the highest BE/ME portfolio and the lowest BE/ME portfolio, without a significant dispersion in the portfolios' beta. From these observations, FF run monthly cross sectional regressions of the stock returns on size and the book-to-market ratio, the so-called two-factor model:

$$R_{it} = \alpha_t + \gamma_1 \ln(ME)_{t-1} + \gamma_2 \ln(BE/ME)_{t-1} + \varepsilon_{it} \quad (2.17)$$

where:

- $\ln(ME)$ is the natural logarithm of the stocks' market capitalization (size);
- $\ln(BE/ME)$ is the natural logarithm of the book value to market value of equity.

When size and book-to-market are included in the cross-sectional regression, the book-to-market ratio does not replace size as the former has an average slope of 0.35 with a t-statistic of 4.44, and the latter has a coefficient of -0.11 with a t-statistic of -1.99. This confirms that both size and the book-to-market ratio are necessary in explaining the cross-section of average returns.

FF also analyse the effect of leverage and the E/P ratio and show that these variables become insignificant when size and book-to-market are included in the regression. In summary, they find evidence that two variables, the book-to-market ratio and size, are sufficient to capture the effect of leverage and E/P on cross-sectional average returns. Finally, FF estimate the premium associated with beta, size and the book-to-market ratio:

$$R_{it} = \alpha_t + \gamma_{1t} \beta_{it} + \gamma_{2t} \ln(ME_{it}) + \gamma_{3t} \ln(BE/ME_{it}) + \varepsilon_{it} \quad (2.18)$$

For the period 1963-1990, beta has a negative coefficient (-0.17), opposite in sign to the theoretical expectation but insignificant, size has a significant negative slope, whereas

the book-to-market ratio has a positive and significant coefficient. In conclusion, FF find that the relationship between beta and average return is very weak over the period 1963-1990 and that size and the book-to-market ratio capture the cross-sectional variation in average stock returns associated with E/P and leverage. Therefore, FF reject the main tenets of the CAPM that stock returns are positively related to the market beta and that beta is the only variable that matters for the explanation of cross-sectional returns.

A recent empirical test conducted by Kothari, Shanken, and Sloan (1995), hereafter referred to as KSS, gives rise to results which support the CAPM. KSS use the same period of observation as Fama and French, but with a full-period annualized beta. When regressing monthly returns on annualized estimated beta and average market capitalization, KSS cannot reject a significant annualized risk premium for beta of the order of 6-9%. The main conclusion is that by using annual betas KSS find that there is significant compensation for beta risk in portfolios formed on size and beta.

The choice of an annual beta might be adequate to overcome problems of seasonality in returns and non-synchronous trading, although the use of an annualized beta represents something of a 'brute force' construction in the 'desperate' attempt to rescue the CAPM. KSS also analyse the book-to-market ratio and argue that most of its effect might be due to survivorship bias in the COMPUSTAT database. However, Chan, Jegadeesh, and Lakonishok (1995) find that the relationship between the book-to-market ratio and average return for the missing firms on COMPUSTAT is as strong as for those firms included and therefore reject the explanation that the book-to-market anomaly is due to survivorship bias.

Even allowing that the beta premium is higher than that estimated by Fama and French, the fact remains that other factors apart from beta (such as size) help to explain cross-

sectional average returns. Fama and French (1996) test the CAPM using 100 portfolios sorted on size and beta and find that portfolios of small-cap stocks have a higher beta and earn higher returns than portfolios of large-cap stocks. The estimated risk premiums associated with beta and size are obtained with the monthly cross-section regressions of 100 Size- β portfolio returns over the monthly post-ranking betas:

$$R_{it} = \alpha_t + \gamma_{1t}\beta_{it} + \gamma_{2t}\ln(Me_{it}) + \varepsilon_{it} \quad (2.19)$$

The results show that the size premium is always negative and statistically significant, whereas the beta premium is positive and significant when beta is the only explanatory variable, but only slightly positive and insignificant when size is included as an explanatory variable. Thus as size has additional explanatory power for average returns, the CAPM is problematic.

Chan and Lakonishok (1993), hereafter referred to as CL, emphasize the sensitivity of the results to the precise time-period selected. CL use the Fama-MacBeth approach to test the CAPM over 10 portfolios of stocks sorted on beta. The results show a positive relationship between post-ranking beta and average returns, and that the slope estimated with the monthly cross-sectional regressions of the stock portfolio returns on the estimated betas is found to be positive and marginally significant for the overall period 1932-1991 and slightly positive and insignificant for the period 1962-1991.

Jegadeesh and Titman (1993) identify a new anomaly: the momentum anomaly. The momentum anomaly is based on the observation that portfolios with good returns over the prior twelve months (winners) tend to do well for the next few months, whereas portfolios with poor returns in the last twelve months (losers) tend to do badly for the next few months. In other words, there is a continuation pattern.

Before the numerous empirical findings which contradict the CAPM, one of the possible defences of the model is the impossibility of using the theoretical market portfolio in practice. Roll (1977) argues that because the tests use proxies rather than the true market portfolio, one does not know whether the CAPM is valid or not, and this is Roll's famous critique. Roll argues that the CAPM might be rejected not because it is false but because the market portfolio proxy is not efficient. However, Stambaugh (1982) tests the CAPM using a broad range of market portfolios and finds that the volatility of market returns is mainly due to the volatility of stock returns, so that the results do not depend much on the proxy used for the market portfolio.

2.1.4. Conditional tests of the CAPM

A further group of tests of the CAPM have focused on the effect of beta conditional on the positive or negative trend of the market. These studies argue that the weak relationship documented between beta and returns is due to the fact that the CAPM is tested on realized excess returns, whereas the theory is based on ex-ante expected excess or required returns. In particular, periods of positive realized returns can be offset by periods of negative realized returns, resulting in a weak relation between systematic risk and return.

Pettengill et al. (1995), by using a methodology that accounts for the conditional relationship between beta and realized returns, find a systematic and significant relationship between risk and return in up and down markets. When excess returns are positive, the CAPM predicts the usual positive relation between beta and returns. However, when excess realized returns are negative, the CAPM predicts an inverse relation between beta and return. Pettengill et al. estimate the beta premium separately in the case of positive and negative market risk premia, and test the hypotheses that the risk premiums are statistically significantly different from zero and, respectively, positive in up markets and negative in down markets.

Specifically, they suggest splitting the returns sample into upmarket and downmarket periods. The upmarket and downmarket are first defined as months with positive or negative market excess returns, respectively. Having estimated betas from a first pass, Pettengill et al. define a conditional CAPM as:

$$R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t} \cdot D_{Bull,t} \cdot \beta_i + \hat{\gamma}_{2t} \cdot (1 - D_{Bull,t}) \cdot \beta_i + \varepsilon_{it} \quad (2.20)$$

where $D_{Bull} = 1$ if the realised excess return is positive, and 0 otherwise. It is worth noting that the model is estimated for each t , that is, there are T cross sectional models, yielding T risk premia. These are then split into two samples depending on whether we are in an upmarket regime or a downmarket regime. Pettengill et al. propose a conditional relationship between beta and realised returns as follows:

In the upmarket:

$$H_0: \bar{\gamma}_1 = 0,$$

$$H_a: \bar{\gamma}_1 > 0.$$

and in the downmarket

$$H_0: \bar{\gamma}_2 = 0,$$

$$H_a: \bar{\gamma}_2 < 0.$$

A systematic conditional relationship between beta and realized returns is confirmed if the null hypotheses are rejected in both cases.

The estimation for the overall period shows a positive risk premium in an up market of 3.36% and a negative risk premium in a down market of -3.37%.

Both the estimated risk premia are highly statistically significant (t-statistics of 12.61 and -13.82, respectively). It is interesting to compare this result with the unconditional test of the CAPM for the overall period, which shows a small positive (0.50%) and significant beta premium (t-statistic of 2.30).

Moreover, the upmarket risk premia appear to be consistent with the downmarket risk premia in terms of magnitude (ignoring the sign). Put differently, there is evidence of a symmetric relation between risk and return during positive and negative sub-periods which according to the authors would uphold a systematic conditional relationship between returns and beta. Specifically, Pettengill et al. propose an unconditional test, on the basis that the positive risk premium should on average be greater than the negative risk premium.

Specifically, they propose the test

$$H_0: \bar{\gamma}_1 + \bar{\gamma}_2 = 0$$

$$H_a: \bar{\gamma}_1 + \bar{\gamma}_2 \neq 0$$

using a two-population t-test. However, as Freeman and Guermat (2006) show, this test is not well specified since the sum is different from zero under both the null and alternative hypotheses. More importantly, the sum of the two average premia does not reflect the probability (or frequency) with which the up and down event takes place. The conclusion of Pettengill et al. is that when the CAPM is tested separately in up and downmarkets the relationship between beta and return is quite strong.

The methodology followed by Pettengill et al. can be criticized for several reasons. Firstly, the test is based on realized returns, but this hypothesis has no economic grounding and is almost meaningless as far as risk-return is concerned as asset pricing models are stated in terms of expected or required returns, not realized returns. Secondly, the model is tested conditionally on the market ex-post return, which means that the assumption is made indirectly that investors can exactly forecast whether the future excess returns will be positive or negative. Finally, the test does not consider the information included in current returns, since it simply distinguishes between positive and negative returns, without any consideration of the current state of the economy at

each date. Nevertheless, the test is useful as it shows that the CAPM does not hold unconditionally, though it might hold conditionally. The approach of this thesis will depart from the work of Pettingill et al. in so far as the regimes will be determined endogenously by a Markov Switching process.

As noted earlier, the common practice in testing asset pricing models is to build portfolios of stocks and then investigate the return-beta relationship in cross-sectional regressions. More recently, Ang, Liu, and Schwarz (2008), hereafter referred to as ALS, suggest that individual stocks lead to more efficient tests of whether the factors are priced. The common practice in empirical asset pricing tests to form portfolios of stocks has been motivated by the attempt to reduce the estimation errors in the betas, as forming portfolios reduces the idiosyncratic risk. However, ALS argue that the reduction in the standard errors of the estimated betas does not lead to more precise estimates of the risk premia, rather that forming portfolios causes a lower dispersion in estimated betas and loss of information that result in higher standard errors in the premia estimates. They find that the annualized beta premium is positive (5.24%) and significant when using individual stocks for the test, whereas the construction of portfolios often results in a negative and insignificant beta premium. Interestingly, ALS remind us that when cross-sectional regressions are used to estimate the risk premia, there is no theoretical reason why stocks should be grouped into portfolios, as the CAPM theory should be valid for individual assets too.

Furthermore, with individual assets there is more dispersion in betas and therefore more information for the cross-sectional estimation of the risk premium, hence a more precise risk premium (Kim, 1995). Moreover, focusing on individual assets is more in line with the assumption of a single period investment made by the CAPM.

In summary, considering the history of empirical tests of the CAPM, there is (at times contentious) extensive evidence of shortcomings in the prediction that market betas suffice in explaining expected returns. These results lead financial researchers to investigate possible extensions of the CAPM which are capable of giving a rational explanation of the anomalies. One of the possible solutions to overcome the problems of the CAPM is to introduce a multifactor model in which the asset returns are a linear function of a set of risk factors that can capture that risk which is unexplained by the market factor. In particular, in the next section the Fama and French (1993) three-factor model is presented.

2.2. Multifactor models

The empirical failures of the CAPM pave the way for more complicated asset pricing models. Several models have been introduced on the basis that multiple variables are needed in order to fully capture the variation of returns. Indeed, the CAPM is based on some unrealistic assumptions, such as the assumption that investors care only about the mean and variance of distributions of one-period portfolio returns. Arguably, investors are also interested in how their portfolio returns covary with their labour income and future investment opportunities. It is worth pointing out that the CAPM is based on the assumption of a one-period investment, but the tests of the CAPM generally use multi-period data. Therefore it is possible that even if the single CAPM holds, in the multi-period setting other factors than beta are priced.

According to the Intertemporal CAPM (I-CAPM) of Merton (1973), the marginal value of individual wealth is affected by several factors, and not only by stock market returns. The theory suggests that investors require a higher return for those assets that do badly in periods of financial slowdown, and they require lower returns for the assets that represent a hedge in periods of economic downturn. Merton shows that the demand for stocks is affected by changes in the investment opportunity set. The expected return on

any asset is therefore better described as the sum of an excess return accounting for systematic risk and an excess return accounting for the unfavourable shifts in the investment opportunity set which depend on the correlation of asset returns with the changes in the investment opportunity (or the covariance of the assets with the hedging portfolio). In other words, Merton assumes, more realistically, that investors wish to minimize the volatility of their consumption over time. This leads to an intertemporal asset pricing model in which not only is there the riskless fund and the risky market fund, but there is also a hedging fund against unfavourable changes in the state of the economy, which is tantamount to saying that three funds are required to span the mean-variance efficient frontier. Merton's model is therefore a conditional multifactor asset pricing model in which an asset's risk depends on its covariance with the market portfolio and the covariance with a hedging portfolio.

Following the intuition that the risk cannot entirely be accounted for by the volatility of the market portfolio returns, the main result of the academic search for alternative asset pricing models is represented by multifactor models. Multifactor models assume that the stochastic process generating asset returns can be represented as a linear function of k factors of risk:

$$R_i = \alpha + \sum_i^k \beta_i \lambda_i + \varepsilon_i \quad \text{for } i=1 \text{ to } k \quad (2.21)$$

In order to apply these models it is necessary to identify the risk factors and to estimate the β coefficients of the sensitivity of the asset to the risk factors and the λ risk premia for changes in the risk factors. Examples of the risk factors might include inflation, the growth in gross domestic product, changes in interest rates and the oil price, among others, such as in Chen, Roll, and Ross (1986) or Aretz et al. (2007).

The basic assumption of the multifactor model is that there are many risk factors that affect returns, unlike in the case of the CAPM where the only relevant risk factor is the

covariance of the asset with the market portfolio (market beta). However, the theory does not help to identify the factors. As in the case of the CAPM, the multifactor framework assumes that idiosyncratic risk can be diversified away and that the return on a zero-systematic-risk portfolio is the risk-free rate. The major difference is that the CAPM defines the risk as the beta on the market portfolio, whereas in the multifactor approach the risk is identified by several factors. The CAPM has the additional practical advantage of identifying the risk factor (the excess return on the market portfolio), whereas the multifactor approach requires the specification of the risk factors.

The inability to identify the risk factors is a serious limitation to the implementation of the multifactor models. In practice, three different approaches have been used. The first includes the use of a microeconomic factor model, by adding some characteristics of the stocks to the market portfolio. One example is the Fama and French (1993) three-factor model that makes use of size and the book-to-market ratio. A second example in the literature involves the use of macroeconomic factors, such as the model proposed by Chen, Roll and Ross (1986); however this model is beyond the scope of this thesis. Finally, a third approach to identify the factors is using factor analysis to extract the principal components that describe the variance of the market portfolio as unobservable latent variables. In the next paragraph, the Fama and French three-factor model is discussed.

2.2.1. The Fama and French three-factor model

Fama and French (1993) develop a multifactor model which makes use of size and book-to-market ratio hedging portfolios of stocks that has become the second most used model after the CAPM. These portfolios are zero investment portfolios of stocks long in stocks with small market capitalization and high book-to-market and short in stocks with big market capitalization and low book-to-market. It is important to say that the

returns of the hedging portfolios are not proper risk factors but should indirectly reflect latent variables that produce non-diversifiable risks not captured by the market beta. FF start from the observation that small-capitalized and value stocks have higher historical average returns than large-capitalized and growth stocks. Furthermore, they show evidence that the CAPM is not capable of capturing the abnormally high returns due to the small-cap and value effect. The conclusion they reach in 1992 is that size and the book-to-market ratio do a good job at explaining the cross-section of average returns on US stocks for the period 1963-1990.

Fama and French (1993), henceforth FF93, employ a time-series regression methodology to investigate the existence of some common risk factors able to explain the average returns of stocks. The idea is to conduct a test of the CAPM by means of two null hypotheses. The first hypothesis is that size and the book-to-market ratio should be insignificant in a time series regression in addition to the market factor and therefore an analysis is conducted of the coefficients of these factors and of their statistical significance together with an analysis of the model R-squared. The second hypothesis is that a good asset pricing model should explain the time series of returns and leave intercepts not statistically different from zero, a test which requires the GRS statistic.

The observation that small-cap stock portfolios outperform large-cap stock portfolios and that high book-to-market stock portfolios outperform low book-to-market stock portfolios leads FF (1993) to introduce a three-factor model with the market portfolio and two other factors: SMB (the return of a portfolio of small capitalization stocks minus the return of a portfolio of large capitalization stocks) and HML (the return of a portfolio of stocks with high book-to-market ratios minus the return of a portfolio of stocks with low book-to-market ratios).

Algebraically, the proposed model is represented by:

$$R_i - R_f = \alpha + \beta_{i,m}(R_m - R_f) + \beta_{i,smb} SMB + \beta_{i,hml} HML + \varepsilon_i \quad (2.22)$$

The model states that the expected excess return of any portfolio is explained by the sensitivity of its return to the market portfolio excess return, to the SMB, and to the HML premium.

The results of time series regressions for 25 portfolios of stocks formed on size and the book-to-market ratio show that size and the book-to-market ratio can account for most of the differences in average returns across portfolios, whereas the market factor accounts for the difference in return between stocks and one-month Treasury bills. When the excess returns are regressed on the market factor alone, the intercepts are large, especially for small portfolios and high book-to-market ratio portfolios, and the R-squared approaches 70%. Once the size and book-to-market mimicking portfolios are introduced, the R-squared increases to above 90%, the intercepts are reduced towards zero, and the SMB and HML variables have coefficients that are significantly different from zero. Interestingly, once the SMB and HML factors are included in addition to the market factor, the market betas of every portfolio are very close to 1, meaning that the cross-sectional differences in average returns are captured by the two additional factors, as already suggested by FF93 following the cross-sectional test of 1992. Dempsey (2013) comments that in the three-factor model every asset has the market return as a base plus or minus an element depending on the exposure to the book-to-market and size mimicking portfolios.

In summary, FF93 find that the three-factor model captures much of the variation in cross-sectional average returns for portfolios formed on size, the book-to-market ratio, and other price ratios that cause problems for the CAPM, and that stocks have some common risk factors synthesized by the market factor, SMB and HML.

Fama and French (1996) test whether the three-factor model can explain the anomalies documented in the empirical literature, such as size, the earnings/price ratio, the cash flow/price ratio, the book-to-market ratio, past sales growth, reversal, and momentum. The findings suggest that the three factors can capture all of these anomalies except momentum. Another important anomaly for the CAPM, that of reversal, has been outlined by DeBondt and Thaler (1985). When portfolios are sorted on long past performance, low past returns portfolios have high future returns and high past returns portfolios have low future returns. In other words, long-term returns have a tendency to revert around the mean. Fama and French show that when portfolios are formed on past returns, the three-factor model can explain this anomaly as long-term past losers behave like distressed firms and produce large coefficients on SMB and HML.

However, when portfolios of stocks are formed using recent past returns (momentum) from three to twelve months prior to portfolio formation, the three-factor model fails to explain the momentum effect. In other words, recent losers have larger slopes on SMB and HML, but they continue to lose in the few months following, whereas recent winners have smaller slopes for SMB and HML, but continue to win in the few months following. This anomaly works in the opposite direction of a reversal and therefore cannot be explained by the three-factor model. This continuation effect is called momentum.

Cochrane (2001) notices that the two factors, SMB and HML, act as hedging portfolios against state-dependent risks of interest to investors and that Fama and French seem to have identified a combination of the broad index market portfolio and two additional portfolios which approximates more closely the mean-variance efficient portfolio.

2.2.2. Size and the book-to-market ratio

Although the three-factor model has been very successful and is now widely used in empirical research, it has a theoretical limitation, that is, the small-minus-big (SMB) and high-minus-low (HML) factors of returns are not themselves state variables of relevance to investors as they only indirectly capture certain systematic risks, but it is not clear what the underlying risks are. This leads researchers to use other multifactor models to examine the underlying risk factors determining the returns and investigate what economic risk factors small and value effects are rewarded for.

The extant financial literature points out that the risk captured by the book-to-market ratio is concerned with financial distress risk. Firms that are considered riskier and with poor forecasts of performance are characterised by low share prices, high book-to-market ratios and have higher expected returns than firms expected to be less risky and more stable. This is the conclusion invoked by Fama and French (1996). However, behaviourists believe that the value effect is linked to an overreaction of investors to stocks with high book-to-market ratios. They contend that value stocks are more sensitive to good and bad times and investors overreact to business cycles by pricing growth stocks too high and value stocks too low. When the overreaction is corrected, the result is an extra return for value stocks and a lower return for growth stocks. This view is advocated by DeBondt and Thaler (1987), Lakonishock, Schleifer, and Vishny (1994), and Haugen (1995). Therefore the value premium would be attributable mainly to overconfidence in investors who in a bull market tend to be too optimistic about small-growth stocks and tend to require too low risk premia for these stocks.

Banz (1981) suggests that small firms are less desirable for investors since there is a lack of information on them compared with the case for larger firms, given that there seems to be a positive relationship between information and size which makes smaller companies riskier. Chan, Chen, and Hsieh (1985) argue that the size effect accounts for

the default risk calculated as the difference between the monthly returns on low and high-grade corporate bonds. In addition, Petkova and Zhang (2005) find that there is a positive relation between change in the term structure slope and HML, suggesting that HML could be a proxy for term structure risk.

Kim (1995) highlights that the errors-in-variables problem incorporated in the estimation of beta in the two-pass methodology of Fama and MacBeth can lead to underestimating the beta risk price and overestimating the significance of size and the book-to-market ratio. Using an errors-in-variables correction, EIV, Kim shows a positive and significant risk premium for beta and a lower risk premium for size, although the latter still remains significant.

An important explanation for the small size premium lies in the observation that small stocks are more thinly traded than large stocks. Liquidity is an important characteristic that investors consider in their investment decision, and can be defined as the ability to trade large quantities of assets quickly, at a low transaction cost and with a small effect on price. Investors should consider their returns net of transaction and illiquidity costs and should require a higher return for more illiquid assets. Moreover, liquidity is time-varying and it becomes particularly important in downturns when investors fear holding illiquid stocks that can lock them in to substantial losses. Investors fear distressed stocks with high illiquidity and low demand. Therefore, liquidity seems to be one of the candidates to explain the anomalies of the CAPM, particularly the value and size premium.

The liquidity CAPM is an augmentation of the traditional model to incorporate a liquidity factor. Amihud and Mendelson (1986), hereafter referred to as AM, show that assets characterized by larger bid-ask spreads yield higher returns: there is a positive

relationship between bid-ask spread (a measure of illiquidity and transaction costs) and returns.

Datar, Naik and Radcliffe (1998) find evidence confirming that liquidity is inversely related to returns, that is, more liquid assets tend to yield lower returns. They use a different measure of liquidity defined as the turnover rate (the number of shares traded divided by number of shares outstanding).

Pastor and Stambaugh (2003) find that stocks that are more sensitive to an aggregate measure of liquidity yield higher returns than less liquidity-sensitive stocks even after adjusting for size, book-to-market and momentum. Moreover, the liquidity betas are found to be significant for most size-ranked portfolios: small stocks tend to have higher liquidity betas.

Amihud (2002) further investigates the effect of illiquidity on asset returns. He constructs an illiquidity measure for a stock as the yearly average of the daily ratio of absolute stock returns to dollar volume:

$$ILLIQ_{iy} = \frac{1}{D_{iy}} \sum_{i=1}^{D_{iy}} \frac{|R_{iyd}|}{VOLD_{iyd}} \quad (2.23)$$

where D is the number of trading days in year y , $VOLD$ is trading volume in dollars and R_{iyd} is the return of stock i in day d .

Illiquid assets are therefore characterized by a strong effect of the traded volume on price, meaning that the change in absolute return per dollar of trading volume is large. In other words, an asset is illiquid if its return per dollar of volume trading changes significantly.

Amihud estimates an augmented CAPM that incorporates the illiquidity factor for a cross-section of stocks traded on the NYSE in the period 1963-1997 and he finds a positive and significant coefficient associated with illiquidity. Therefore the conclusion is that stocks with higher sensitivity to illiquidity yield higher average returns.

Moreover, a strong negative correlation between illiquidity and size is documented suggesting that the size premium is partly an illiquidity premium.

Acharya and Pedersen (2004) investigate the relationship between liquidity and returns and in particular the channels through which liquidity can affect returns. In their liquidity-augmented CAPM the return of an asset depends on the covariance of its returns with the market portfolio (beta), the covariance of its returns with its own liquidity, the covariance of its own liquidity with aggregate market liquidity, and the covariance of its own returns with aggregate market liquidity. The liquidity premium is therefore decomposed into three different components reflecting three different types of liquidity risk. They find that returns increase with illiquidity and that the difference between the returns of the most illiquid and most liquid portfolio is largely explained by a liquidity premium due to liquidity sensitivity to market portfolio returns. Investors fear that an asset might become illiquid when the market portfolio return is low and are therefore willing to pay a premium for holding an asset that is liquid when the market return is low.

Liu (2006) develops a liquidity-augmented CAPM and documents that this two-factor model captures the size and book-to-market anomalies better than the three-factor model of Fama and French for US stocks over the period 1960-2003. Using a time series regression approach for a cross-section of stocks, the R-squared of the model is superior to that of the CAPM and the three-factor model and the abnormal return (alpha) of the small and high book-to-market portfolios are not significant.

Liu employs a measure of liquidity which accounts for continuity of trading and trading speed and that is defined as the standardized turnover-adjusted number of zero daily trading volumes over a period of 12 months:

$$LM_{12} = \left[N_{0,12} + \frac{1/(12\text{-month turnover})}{Deflator} \right] \times 252 \quad (2.24)$$

where $N_{0,12}$ is the number of zero daily volumes in the prior 12 months. The rationale is that an illiquid asset is characterized by frequent absence of trading.

Liu defines the liquidity factor LIQ to use in the time series with the monthly market excess return $R_{mt} - R_{ft}$ as the difference in the monthly return between a low-liquidity portfolio and a high-liquidity portfolio in the same fashion as the size and book-to-market premia of Fama and French. The liquidity premium is found to be positive with an average of 0.75% over the period of investigation and to be negatively related with the market factor, suggesting that liquidity can be considered a state variable that captures distress risk. Interestingly, moreover, the liquidity factor is slightly correlated with the size factor, meaning that when using this definition of liquidity the liquidity premium does not coincide with the small size effect.

The following model is estimated:

$$R_{it} - R_{ft} = \alpha_i + \beta_{i,m}(R_{mt} - R_{ft}) + \beta_{i,L}LIQ_t + \varepsilon_{it} \quad (2.25)$$

The findings suggest that whereas the CAPM cannot explain the small size premium, the liquidity-augmented CAPM does it. Moreover, when portfolios are sorted by book-to-market, neither the CAPM nor the three-factor model of Fama and French can explain the value premium, but the two-factor model can. Interestingly, the loadings on the liquidity factor increase with book-to-market and decrease with size, meaning that liquidity might capture both the anomalies related to these characteristics. Small and value stocks are characterized by a higher factor loading in the liquidity factor.

Akbas, Boehmer, Genc and Petkova (2010) estimate a conditional liquidity-augmented CAPM and claim that a time-varying liquidity premium is capable of explaining a large part of the value premium. They use a similar approach to Liu (2006) in that they also employ a liquidity factor defined as the return on a zero-cost mimicking portfolio long in illiquid stocks and short in liquid stocks. However, the loadings on the risk factors and the market premia are allowed to vary over time according to the business cycle as

described by certain conditioning variables. Investors tend to prefer liquid assets in bad times and this leads to a flight-to-quality that affects value and growth stocks. The findings suggest that value stocks are riskier than growth stocks as in bad times their sensitivity to liquidity increases and is higher than the sensitivity of growth stocks. In bad times the liquidity premium is higher than in good times and the combined effect of a larger exposure to the risk and a higher risk premium might explain the value premium.

The recent findings in liquidity-augmented asset pricing models confirm that there is an important link between liquidity and the size premium and that the time-varying nature of liquidity risk might enhance the explanation of the value premium itself.

Anthonisz and Putnins (2013) test a liquidity CAPM in which the liquidity premium is separated between upside and downside liquidity risk. They find that stocks sensitive to illiquidity risk in a downmarket tend to yield higher returns on average. This is due to the asymmetric behaviour of liquidity. Investors appear to ask for higher risk premia for holding stocks that are more prone to liquidity shocks when the market becomes illiquid and negative. The model is innovative in so far as it takes into account not only asymmetric preferences of investors for upside and downside beta risk but also for asymmetric liquidity risk.

Chordia, Subrahmanyam, and Anshuman (2001) find that there is a negative relationship between expected returns and volatility of trading activity over and above the effect of size, book-to-market and momentum in US for the period 1966-1995. Brennan, Chordia and Subrahmanyam (1998) find that there is a negative relationship between turnover (trading volume in dollar) and equity returns.

Zhang (2005) suggests that value stocks are riskier than growth stocks as they are less flexible and have more unproductive (underutilized tangible) assets in a recession when

the price of risk is higher. Furthermore, Hahn and Lee (2005) find a significant negative relationship between changes in default probability and SMB, suggesting that SMB could be a proxy for default risk.

Vassalou and Xing (2004) compute default risk for individual firms using Merton's (1974) option pricing model and find that the size effect is caused by default risk. Vassalou (2004) sorts stocks on the default measure into five portfolios and notices that over the period 1971-1999 the high default probability portfolio earns a monthly return which is 0.53% higher than that for the low default probability portfolio. Moreover, she provides evidence that value and small stocks have a higher default probability than growth and big stocks.

Perhaps one of the most interesting insights into the size and book-to-market ratio anomalies has been offered by Campbell and Vuolteenaho (2004), hereafter referred to as CV, who explain the size and value anomalies, decomposing the stock's beta into a cash flow beta and a discount rate beta and showing that small and value stocks are characterized by higher cash flow betas than large and growth stocks. Therefore, the small and value premia might be explained by greater sensitivity to cash flow risk. Risk averse investors should ask higher premia for stocks that covary with market cash-flow related news than for stocks that covary with discount rate related news, as the fall in returns due to the rise in the discount rate (cost of capital) should be partially compensated by the better scenario for future returns. Therefore, beta can be decomposed into a "bad" beta and a "good" beta, which is less "worrisome" for investors. CV show that growth stocks have a higher beta which is not compensated by higher returns because that riskiness is mostly made up of the discount rate beta for which a lower return is required.

Chen, Novy-Marx and Zhang (2011) propose an alternative three-factor model to explain the cross-section of equity returns. The two additional factors are obtained as per the SMB and HML factors of Fama and French. Two mimicking portfolios are built as the difference in returns between a portfolio of high profitability stocks and a portfolio of low profitability stocks, where profitability is defined as ROE, and the difference in returns between a portfolio of low investment stocks and a portfolio of high investment stocks, as follows:

$$E(R_i - R_f) = \beta_{MKT}^i E[MKT] + \beta_{INV}^i E[r_{INV}] + \beta_{ROE}^i E[r_{ROE}] \quad (2.26)$$

where MKT is the market return in excess of the riskfree rate, r_{INV} is the difference in returns between a portfolio of stocks with high level of investments (Investments-to-Assets) and a portfolio of stocks with low level of investments, r_{ROE} is the difference in returns between a portfolio of stocks with high profitability as return on equity and a portfolio of stocks with low profitability.

The two additional factors have positive average returns over the period 1972-2010, i.e. stocks with lower investments tend to give higher returns and more profitable stocks tend to earn higher returns. The investment related factor plays more or less the same role as the value factor in the model as firms with a higher growth potential invest more and earn lower returns, whereas the second factor is mainly due to the low profitability of small-growth stocks. The model is tested with the GRS test, and although the intercept for small-growth portfolios is reduced the model is still rejected. The main problem with this model is that it simply claims to explain the returns but does not provide any explanation of the risk factors that should be rewarded by the market. In other words, the model seems more a tautology than a real asset pricing model.

Adrian and Rosenberg (2008) find that market volatility commands a negative risk premium in the cross-section of US stocks, and interpret market volatility as a predictor

of future deterioration in investment opportunities. They decompose equity returns volatility into a short-run and a long-run component where the former correlates highly with market skewness and the latter correlates highly with the business cycle and industrial production. Their model is quite interesting as an extension of the ICAPM of Merton (1973). When volatility is stochastic, the returns of stocks are not just determined by the covariance with market returns but also with the covariance with the determinants of stochastic market wide volatility. The model with market returns, and short-run and long-run components of stochastic volatility outperforms the Fama and French three-factor model. The authors find that on average, given that the sensitivity to volatility components are negative, volatility is positively rewarded. In particular they argue that the short-run component of volatility can explain much of the cross-sectional returns of value stocks as growth stocks load positively on the short-run volatility which is considered a hedge to market skewness.

Ang et al. (2006) find that for individual stocks idiosyncratic volatility is negatively rewarded by the market and that exposure to aggregate market wide volatility helps to explain returns above systematic beta risk, in contradiction with financial theory. Specifically, stocks more exposed to market wide volatility, extracted from the VIX index, yield low average returns that are not explained by beta, size, book-to-market or momentum factors.

Iqbal et al. (2008) find that for an emerging market characterized by thick tails of the distribution of returns, an unconditional Fama and French three-factor model augmented with cokurtosis can outperform the conditional version of the CAPM and of the simple three-factor model. They recognize that ...*“in emerging markets the frequency of extreme observations is considerably higher which results in thicker tails as indicated by high kurtosis values”*. They use a stochastic discount factor approach with GMM and include a cubic market factor for kurtosis and compare it with a conditional CAPM and

three-factor model with dividend yield and trading volume as conditioning variables, among others. The period tested is 1992-2006 for 101 stocks listed on the Pakistani stock market forming 16 portfolios sorted on size and book-to-market.

Finally, Zhang (2010) offers a different explanation for the value premium based on skewness. He argues that small-growth stocks performed poorly during the 1980s and 1990s and that they were mainly high-tech firms for which rational investors expected low returns that were accepted in exchange for positive skewness. A skewness premium is introduced, obtained as the difference in returns between a portfolio of stocks with low skewness and a portfolio of stocks with high skewness. The results show that whereas size has a negative effect on returns, the book-to-market ratio has a positive effect on returns, and idiosyncratic skewness has a negative effect on returns. The intercepts (pricing errors) are reduced when the additional skewness premium is added to the market premium.

2.3. Conclusion

The Capital Asset Pricing Model of Sharpe (1964), Lintner (1965) and Mossin (1966) is an elegant and a theoretically sound model that should normally be expected to address the question of what drives asset returns successfully. According to the model the expected return of any asset in excess of the risk-free rate is given by the expected market portfolio excess return times the asset's beta (sensitivity to the market portfolio volatility). Differences in returns across assets should be entirely explained by the extent to which their returns covary with the returns of the market portfolio, and no additional variables should help to explain the cross-section of average returns.

In the first part of the literature review, after introducing the CAPM, the most important empirical tests of that model have been presented. In general, these tests are based on time-series or cross-sectional regressions. In other words, their aim is either to ascertain

whether the market beta is sufficient to describe the time-series variability of returns or to determine whether the differences in returns can be explained by differences in beta. Time-series regression-based models test the null hypothesis that the intercept equals zero for each asset. As discussed in this chapter, the joint null hypothesis is tested with the GRS test. On the other hand, cross-sectional tests are based on monthly regressions of the monthly excess returns of any tested assets on their estimated beta, and the main hypotheses tested are that the intercept should not be statistically different from zero, whereas the coefficient associated with beta (estimated market premium) should be positive and statistically significant. Among the cross-sectional tests, the most influential has been the Fama and MacBeth (1973) test in which in the first stage a time series regression is applied to estimate market betas and in the second stage a cross-sectional regression of the monthly excess returns on the estimated betas is conducted to estimate the monthly risk premium. Whilst FM find results to support the CAPM, they find a positive risk-return relationship, though an intercept larger than is theoretically expected. However, the following tests outline several anomalies that the CAPM cannot explain. Banz (1981) identifies the size anomaly: smaller firms tend to have higher returns than larger firms and moreover the difference is not explained by their beta. Bhandari (1988) illustrates the leverage anomaly: firms with higher debt-to-equity ratios are riskier than is indicated by their beta, and tend to have higher returns than are predicted by the model. Basu (1977) identifies the P/E ratio anomaly: firms with a higher P/E ratio tend to have lower returns than firms with a lower P/E ratio and these differences in returns are left unexplained by the differences in betas. Stattman (1980) and Rosenberg et al. (1985) give evidence of the book-to-market anomaly: stocks with higher book-to-market ratios, commonly denominated as value, yield higher returns than stocks with lower book-to-market ratios, in spite of often having lower betas.

Fama and French (1992) show that book-to-market and size can encompass the other anomalies and add to the explanation of the cross-section of average returns. In particular, FF93 show that the market beta can explain the difference between the returns of stocks from other asset categories such as bonds, but that the differences in returns among stocks are better described by the different market capitalizations and book-to-market ratios of the stocks.

Confronted with the empirical failures of the CAPM, financial researchers have tried to develop some alternative models or to amend the CAPM (extensions of the CAPM). In one approach, researchers have developed multifactor models that add variables to the market portfolio in order to better capture the risk not entirely explained by the market beta. Among these models, the CRR (1986) macroeconomic multifactor model and the FF93 three-factor model are pivotal. Since its development, the three-factor model of Fama and French with two additional variables, size and the book-to-market ratio, has rapidly gained favour among practitioners as an alternative to the CAPM. Indeed, in a time-series regression, the additional factors augment explanatory power, reaching an R-squared of around 90%, and the alphas of the portfolios are significantly reduced. However, it must be noted that the three-factor model does not perform very well cross-sectionally or when portfolios are built according to different criteria than size and the book-to-market ratio. The main limitation of the model is that the two additional variables are deemed to reveal some indirect risk, i.e. high book-to-market stocks and small capitalisation stocks are riskier than is predicted by their beta alone, but it is not exactly clear what the underlying risk is. While for the small size anomaly there seems to be a consensus towards a liquidity risk and a default risk, the book-to-market effect remains a moot point. High book-to-market stocks are characterized by a lower beta and nevertheless yield higher returns when the market is negative.

Chapter 3

The Three- and Four-Moment CAPM: Literature Review

3.0. Introduction

One of the main highlights of the previous chapter is the empirical failures of the traditional CAPM, and the various attempts made to revive it. Existing research has investigated possible extensions of the CAPM. One particularly interesting line was to incorporate higher moments of the distribution of returns. This is the main focus of this chapter.

The CAPM is derived by Sharpe (1964), Lintner (1965) and Mossin (1966) under Markowitz's assumptions of elliptical distributed returns and/or a quadratic utility function. However, there is evidence that the distribution of returns deviates from normality and is characterized by larger kurtosis and skewness, a phenomenon known as the 'stylized facts'. As a consequence, the inclusion of skewness and kurtosis in the investor's portfolio choice is warranted. In other words, whereas the traditional CAPM is based solely on the mean and variance, systematic risk would be better defined by including skewness and kurtosis as well. As in the two-parameter CAPM, systematic risk is given by the contribution of any asset to the variance of the market portfolio. However, in the extension of the CAPM, systematic risk also includes the contribution of the assets to the skewness and kurtosis of the market portfolio. Therefore, by ignoring these additional components of systematic risk, the traditional CAPM might overestimate or underestimate the risk, and thus might not be able to explain the cross-section of the average returns of assets which differ from each other in terms of skewness and kurtosis.

Before introducing the three- and Four-Moment CAPM, the rationale for the inclusion of higher moments is discussed.

3.1. The Non-Normality of Returns

The traditional CAPM requires that either the utility function is quadratic or that the returns are normally/elliptically distributed. If the utility function is quadratic, investors only consider mean and variance when making their portfolio decisions, and if the returns are normally distributed then skewness and kurtosis are irrelevant as returns are symmetric. Empirical findings reject the assumption of normality (see Taylor, 2005), suggesting instead that the distribution of returns is asymmetric and leptokurtic, i.e. distributions of returns are long-tailed with extreme outcomes which are more likely than is predicted by the normal distribution, and with large negative returns which are more likely than large positive returns. Therefore, it is argued that investors not only care about mean and variance, but also about the skewness and kurtosis of their investment portfolio, where skewness measures the asymmetry of the distribution and kurtosis refers to the shape of the tails of the distribution of returns.

The theoretical preference of economic agents for positive skewness (see Horvath, 1980) and aversion to large losses (see Kahneman and Tversky, 1979), on one hand, and the non-normality of the distribution of assets returns, on the other hand, provides the motivation to incorporate such stylized facts into the derivation of the asset pricing models. The literature concerning the non-normality of asset returns and the preference for higher moments is both extensive and compelling.

The extension of the CAPM to include higher moments has been investigated since the early 1970s by Arditti and Levy 1975 who point out that if the utility function of investors is characterized by non-increasing absolute risk aversion then investors have a preference for positive skewness. Samuelson (1970) considers the two-moment utility

function as a good approximation of reality, but maintains that higher moments of the distribution can improve the approximation. Furthermore, Rubinstein (1973) suggests a link between returns and the higher moments of returns, and that the expected return of an asset is equal to the weighted sum of the co-moments. Further, he argues that, in assuming a cubic utility function and homogeneous subjective probability beliefs, any asset's excess return becomes a weighted function of covariance and coskewness, where the weights are determined by the degree of risk aversion.

Moreover, Horvath (1980) shows that risk averse investors under the assumptions of a positive marginal utility of wealth, non-decreasing absolute risk aversion, and preferences strictly in one direction, have a positive preference for all odd moments including mean and skewness and a negative preference for all even moments including variance and kurtosis, which implies that:

$$\begin{cases} U'(W) > 0 \\ U''(W) < 0 \\ U'''(W) > 0 \\ U^{IV}(W) < 0 \end{cases}$$

Therefore, in order to maximize the expected utility of wealth, risk-averse investors prefer higher returns, smaller variance, higher skewness and lower kurtosis.

Furthermore, Simkowitz and Beedles (1978) argue that returns are not normally distributed and that securities exhibit positive skewness, although skewness is quickly diversified away in portfolios. However, Singleton and Wingender (1986) find that US stock returns consistently exhibit positive skewness between 1961 and 1980, though their findings lead them to suggest that skewness, either at the portfolio or single security level, is not persistent over time and therefore investors cannot use ex-post skewness to predict future skewness. Finally, the excess of kurtosis in the distribution of returns has also been documented by Fama (1965) and Officer (1972), among others. In

summary, there is extensive evidence for the non-normality of returns and aversion to large losses that motivates extensions of the traditional investment decision framework to higher moments.

The aversion to extreme losses is also documented by Kahneman and Tversky (1979) in prospect theory: investors attribute more weight to losses than to gains. This possible extension of the CAPM is not new in the financial literature as the first extension dates back to 1976 when Kraus and Litzenberger derived the three-moment CAPM in which the third moment of the distribution of returns is included. However, this line of research gained new momentum for a number of reasons. First, the introduction and success of hedge funds whose strategy adopts non-traditional assets such as derivatives, and whose returns exhibit skewness, has urged for methods of performance evaluation that can account for higher moments. Secondly, the widespread diffusion of Value at Risk and Extreme Value Theory that focus on the probability of large losses has contributed to the development of an interest in the shape of the tails of the distribution of returns. In addition, market liberalization and globalization together with the development of investment funds have made investments in emerging markets more accessible, and emerging markets are characterized by a distribution of returns with long tails which is characterized by asymmetry. Furthermore, technological advancements have resulted in fewer computational issues in the investment decision with multiple objectives (mean-variance-skewness-kurtosis). Finally, the observation that portfolios formed according to the characteristics that give rise to the main anomalies, such as those relating to size and book-to-market, exhibit a strong pattern in terms of coskewness and cokurtosis has prompted financial researchers to investigate whether these higher co-moments with market portfolio returns can find a rational explanation in the failures of the CAPM.

In particular, the distribution of returns and the shape of its tails have become a matter of concern for investors and regulators in recent years, following episodes such as the Asian financial crisis in 1997, the high-tech bubble burst in 2000, the financial crisis related to subprime lending and CDOs starting in 2008, and the more recent sovereign debt crisis.

3.2. The Four-Moment CAPM

The empirical findings from studies such as Taylor (2005) suggest that the distribution of returns is characterized by skewness and excess kurtosis. The distribution of returns exhibits extreme positive or negative returns more often than would be predicted by the normal distribution. The Four-Moment CAPM is the traditional CAPM augmented to account for systematic skewness and systematic kurtosis. In order to include skewness and kurtosis, the expected value of the utility function of wealth, W , of an economic agent is expanded up to the fourth-order Taylor series:

$$\begin{aligned}
U(W_{t+1}) &= U(\bar{W}_{t+1}) + U'(\bar{W}_{t+1})(W_{t+1} - \bar{W}_{t+1}) \\
&\quad + \frac{1}{2} U''(\bar{W}_{t+1})(W_{t+1} - \bar{W}_{t+1})^2 + \frac{1}{3!} U'''(\bar{W}_{t+1})(W_{t+1} - \bar{W}_{t+1})^3 \\
&\quad + \frac{1}{4!} U''''(\bar{W}_{t+1})(W_{t+1} - \bar{W}_{t+1})^4 + o(W_{t+1}^4)
\end{aligned} \tag{3.1}$$

where \bar{W}_{t+1} is the expected value of wealth at time $t+1$, and $o(W_{t+1}^4)$ refers to higher orders than the fourth in the Taylor series that are not considered in the model. By taking the expectation, it obtains:

$$\begin{aligned}
E_t(U(W_{t+1})) &= U(\bar{W}_{t+1}) + \frac{1}{2} U''(\bar{W}_{t+1})\sigma_{w,t}^2 + \frac{1}{3!} U'''(\bar{W}_{t+1})s_{w,t}^3 \\
&\quad + \frac{1}{4!} U''''(\bar{W}_{t+1})k_{w,t}^4 + o(W_{t+1}^4)
\end{aligned} \tag{3.2}$$

If the investor's utility function has to satisfy properties such as the decreasing marginal utility of wealth and non-increasing absolute risk aversion then investors must show an

aversion to standard deviation, a preference for positive skewness, and an aversion to kurtosis.

In this section, a formal derivation of the Four-Moment CAPM is reported, following the approaches of Jurczenko and Maillet (2010) and Athayde and Flores (1997), in a way that conveys a more intuitive derivation of the model. Each investor wants to maximize her expected utility $V_k(\cdot)$ which is assumed concave and increasing with the expected portfolio return, concave and decreasing with variance, concave and increasing with skewness, and concave and decreasing with kurtosis. Therefore, the expected utility is a function of the first four moments is given as follows:

$$E[U_k(R_p)] = V_k[E(R_p), \sigma^2(R_p), s^3(R_p), k^4(R_p)] \quad (3.3)$$

with:

$$V_k^{(1)} = \frac{\partial V_k(\cdot)}{\partial E(R_p)} > 0, V_k^{(2)} = \frac{\partial V_k(\cdot)}{\partial \sigma^2(R_p)} < 0, V_k^{(3)} = \frac{\partial V_k(\cdot)}{\partial s^3(R_p)} > 0$$

$$\text{and } V_k^{(4)} = \frac{\partial V_k(\cdot)}{\partial k^4(R_p)} < 0$$

where $R_p = W_F/W_0$ is the gross return of the individual portfolio, or the portfolio held by agent k , and W_0 and W_F are, respectively, the initial and final wealth of the agent.

If the investor splits her wealth into N risky assets and a riskless asset, the first four moments of the portfolio return are given by:

$$\begin{cases} E(R_p) = w_{p0}R_f + E\left[\sum_{i=1}^N w_{pi}R_i\right] \\ \sigma^2(R_p) = E\left\{[R_p - E(R_p)]^2\right\} \\ s^3(R_p) = E\left\{[R_p - E(R_p)]^3\right\} \\ k^4(R_p) = E\left\{[R_p - E(R_p)]^4\right\} \end{cases} \quad (3.4)$$

with the constraints:

$$\begin{cases} R_p = w_{p0}R_f + \sum_{i=1}^N w_{pi}R_i \\ \sum_{i=1}^N w_{pi} = (1 - w_{p0}) \end{cases} \quad (3.5)$$

where R_i is the gross rate of return on the risky asset i , and R_f is the rate of return on the riskless asset. This system can be written in vectorial notation following Diacogiannis (1994) and Athayde and Flores (1997) as:

$$\begin{cases} E(R_p) = w_{p0}R_f + w_p'E \\ \sigma(R_p) = w_p'\Omega w_p \\ \gamma(R_p) = w_p'\Sigma(w_p \otimes w_p) \\ k(R_p) = w_p'\Gamma(w_p \otimes w_p \otimes w_p) \end{cases} \quad (3.6)$$

with

$$w_p'\mathbf{1} = (1 - w_{p0})$$

where w_p' is the transposed vector of the investor's weights of risky assets; w_p is the vector of the N risky assets in the portfolio p ; E is the vector of the expected returns of risky assets; Ω is the variance-covariance matrix of the N risky asset returns; Σ is the skewness-coskewness matrix of the N risky asset returns; Γ is the kurtosis-cokurtosis matrix of the N risky asset returns; Ωw_p is the vector of covariance between the asset returns and the portfolio return; $\Sigma(w_p \otimes w_p)$ is the vector of coskewness between the asset returns and the portfolio return; $\Gamma(w_p \otimes w_p \otimes w_p)$ is the vector of cokurtosis between the asset returns and the portfolio return; \otimes is the Kroenecker operator and $\mathbf{1}$ is the vector of ones.

By maximizing the utility function, the first order condition is given by:

$$\begin{aligned} \frac{dV(\cdot)}{dw_p} &= V_k^{(1)}(E - R_f \mathbf{1}) + 2V_k^{(2)}\Omega w_p + 3V_k^{(3)}\Sigma(w_p \otimes w_p) \\ &+ 4V_k^{(4)}\Gamma(w_p \otimes w_p \otimes w_p) = 0 \end{aligned} \quad (3.7)$$

In order to move from the individual to the market equilibrium condition, the usual assumptions that individuals have the same probability beliefs and utility functions with the same cautiousness parameter are invoked.

Therefore the market equilibrium condition becomes:

$$\begin{aligned} \frac{dV(\cdot)}{dw_m} = & V_k^{(1)}(E - R_f 1) + 2V_k^{(2)}\Omega w_m + 3V_k^{(3)}\Sigma(w_m \otimes w_m) \\ & + 4V_k^{(4)}\Gamma(w_m \otimes w_m \otimes w_m) = 0 \end{aligned} \quad (3.8)$$

which can be rewritten as:

$$E - R_f 1 = \left[-\frac{2V_k^{(2)}}{V_k^{(1)}} \right] \Omega w_m + \left[-\frac{3V_k^{(3)}}{V_k^{(1)}} \right] \Sigma(w_m \otimes w_m) + \left[-\frac{4V_k^{(4)}}{V_k^{(1)}} \right] \Gamma(w_m \otimes w_m \otimes w_m) \quad (3.9)$$

Let define the marginal rate of substitution between, respectively, standard deviation and return, skewness and return, and kurtosis and return as:

$$\theta_1 = \left[-\frac{2V_k^{(2)}}{V_k^{(1)}} \right] > 0$$

$$\theta_2 = \left[-\frac{3V_k^{(3)}}{V_k^{(1)}} \right] < 0$$

$$\theta_3 = \left[-\frac{4V_k^{(4)}}{V_k^{(1)}} \right] > 0$$

and recalling that:

$$E \left\{ \begin{array}{l} \Omega w_m = \beta_{im} \sigma_m^2 \\ \Sigma(w_m \otimes w_m) = \gamma_{im} \gamma_m^3 \\ \Gamma(w_m \otimes w_m \otimes w_m) = k_{im} k_m^4 \end{array} \right. \quad (3.10)$$

and defining:

$$\begin{cases} \lambda_\beta = \theta_1 \sigma_m^2 \\ \lambda_s = \theta_2 \gamma_m^3 \\ \lambda_k = \theta_3 k_m^4 \end{cases} \quad (3.11)$$

The equilibrium condition for any asset i is given by:

$$E(R_i) - R_f = \lambda_\beta \beta_{im} + \lambda_s \gamma_{im} + \lambda_k k_{im} \quad (3.12)$$

The covariance, coskewness and the cokurtosis can be seen as the contribution of the asset i to the variance, skewness and kurtosis of the portfolio. The variance, skewness and kurtosis of the portfolio return can be written as a weighted average of the coskewness and cokurtosis between the N risky asset returns and the portfolio return.

Therefore, covariance, coskewness and cokurtosis are obtained as follows:

$$\begin{cases} \beta_{ip} = \frac{E\{[R_i - E(R_i)][R_p - E(R_p)]\}}{E\{[R_p - E(R_p)]^2\}} \\ \gamma_{ip} = \frac{E\{[R_i - E(R_i)][R_p - E(R_p)]^2\}}{E\{[R_p - E(R_p)]^3\}} \\ k_{ip} = \frac{E\{[R_i - E(R_i)][R_p - E(R_p)]^3\}}{E\{[R_p - E(R_p)]^4\}} \end{cases} \quad (3.13)$$

with $i = 1, \dots, N$; these measures can be thought of as the covariance between the asset return and the volatility, and the covariance between the asset return and the skewness of the portfolio p . Therefore, an asset which exhibits positive coskewness tends to perform best when the portfolio is more volatile and is therefore considered safer, whereas an asset which exhibits positive cokurtosis tends to have larger losses when the portfolio becomes more volatile and it is therefore riskier.

The variance, skewness and kurtosis of the portfolio return can be written as a weighted average of the covariance, coskewness and cokurtosis between the N risky asset returns and the portfolio return:

$$\begin{cases} \sigma_p = \sum_{i=1}^N w_i E\{[R_i - E(R_i)][R_p - E(R_p)]\} \\ \gamma_p = \sum_{i=1}^N w_i E\{[R_i - E(R_i)][R_p - E(R_p)]^2\} \\ k_p = \sum_{i=1}^N w_i E\{[R_i - E(R_i)][R_p - E(R_p)]^3\} \end{cases} \quad (3.14)$$

For completeness, a different derivation of the Four-Moment model is presented in the following part.

The agent's portfolio problem is to decide how much of her wealth to invest in the riskless asset and in the N risky assets in order to maximize her expected portfolio excess return subject to variance, skewness and kurtosis by maximizing the Lagrangian under the usual budget constraint $w_p'1 = (1 - w_{p0})$:

$$L = V(E(R_p), \sigma(R_p), \gamma(R_p), k(R_p)) - \lambda(w_p'1 + w_{p0} - 1)$$

and the first order conditions for a maximum $\frac{dL}{dw_p}$ are:

$$(R_i) - R_F = -\frac{dV_{\sigma(R_p)}}{dV_{E(R_p)}} \beta_{iP} \sigma_P - \frac{dV_{\gamma(R_p)}}{dV_{E(R_p)}} \gamma_{iP} \gamma_P - \frac{dV_{k(R_p)}}{dV_{E(R_p)}} k_{iP} k_P \quad (3.15)$$

$dV_{\sigma(R_p)}/dV_{E(R_p)}$, $dV_{\gamma(R_p)}/dV_{E(R_p)}$ and $dV_{k(R_p)}/dV_{E(R_p)}$ equal the investor's marginal rates of substitution between expected return and standard deviation, between expected return and skewness, and between expected return and kurtosis, respectively. In other words, the investor's portfolio contains riskless and risky assets in proportions such that the expected excess return on each risky asset equals the sum of her marginal rate of substitution between mean and variance times the market beta (contribution to the portfolio's standard deviation), plus her marginal rate of substitution between mean and skewness times the asset's marginal contribution to the portfolio's skewness (coskewness divided by skewness), plus her marginal rate of substitution between mean

and kurtosis times the asset's marginal contribution to the portfolio's kurtosis (cokurtosis divided by kurtosis).

In order to move from the individual solution to the investment decision problem, and then to the market general equilibrium, the assumption is made that agents have homogeneous expectations and the same risk aversion parameter in the utility function. Such a condition is necessary to obtain the optimum risky portfolio whose weights are the same as those of the market portfolio m . In such a case, investors will hold a fraction of the riskless portfolio and a fraction of the market portfolio according to their preference for mean-variance-skewness-kurtosis. Under these assumptions, the generic portfolio p becomes the market portfolio m .

For any asset i , the equilibrium expected excess return is written as:

$$E(R_i) - R_F = -\frac{dV_{\sigma(R_m)}}{dV_{E(R_m)}}\beta_{im}\sigma_m - \frac{dV_{\gamma(R_m)}}{dV_{E(R_m)}}\gamma_{im}\gamma_m - \frac{dV_{k(R_m)}}{dV_{E(R_m)}}k_{im}k_m \quad (3.16)$$

Let define:

$$\theta_1 = -(dV_{\sigma(R_m)}/dV_{E(R_m)}) > 0$$

$$\theta_2 = -(dV_{\gamma(R_m)}/dV_{E(R_m)}) < 0$$

$$\theta_3 = -(dV_{k(R_m)}/dV_{E(R_m)}) > 0$$

then Equation 3.16 can be rewritten as:

$$E(r_i) - R_f = \theta_1\beta_{im}\sigma_m + \theta_2\gamma_{im}\gamma_m + \theta_3k_{im}k_m \quad (3.17)$$

and defining:

$$\begin{cases} \lambda_\beta = \theta_1\sigma_m \\ \lambda_\gamma = \theta_2\gamma_m \\ \lambda_k = \theta_3k_m \end{cases} \quad (3.18)$$

The Four-Moment CAPM obtains as:

$$E(R_i) - R_f = \lambda_\beta \beta_{im} + \lambda_s \gamma_{im} + \lambda_k k_{im} \quad (3.19)$$

It follows that for every security i , the expected excess return can be written as a linear function of the three co-moments of the asset returns with the market portfolio β, γ , and k :

$$E(R_i) - R_f = \lambda_\beta \frac{E(r_i \cdot r_m)}{E(r_m^2)} + \lambda_s \frac{E(r_i \cdot r_m^2)}{E(r_m^3)} + \lambda_k \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} \quad (3.20)$$

where:

$$\begin{cases} \beta_{im} = \frac{Cov(r_i, r_m)}{\sigma^2(r_m)} = \frac{E(r_i \cdot r_m)}{E(r_m^2)} \\ \gamma_{im} = \frac{Cos(r_i, r_m)}{s^3(r_m)} = \frac{E(r_i \cdot r_m^2)}{E(r_m^3)} \\ k_{im} = \frac{Cok(r_i, r_m)}{k^4(r_m)} = \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} \end{cases} \quad (3.21)$$

where $r_i = R_i - E(R_i)$. The coefficients λ_β, λ_s and λ_k are interpreted as systematic risk market premia. The sign of the coefficients depends on the θ s, and therefore λ_β should be strictly positive, λ_s should have the opposite sign of $s^3(R_m)$ as $\theta_2 < 0$, and λ_k is strictly positive since θ_3 is positive.

Theoretically, when testing the Four-Moment CAPM, a positive risk premium is expected for beta as investors require a higher return for higher systematic beta risk. As for gamma, if the market portfolio returns have negative skewness, investors should prefer assets with lower coskewness and dislike assets with high coskewness, whereas if the market portfolio returns have positive skewness, investors prefer assets with high coskewness that become more valuable, and theoretically a negative coefficient for λ_s is expected as investors are willing to forego some return for positive skewness. However, in the model set out in this thesis, coskewness will not be standardized so that the

expected sign for the coefficient will only be negative, i.e. independent of the skewness of the market portfolio.

Finally, a positive risk premium is expected for systematic kurtosis as investors require higher compensation for assets with more likely extreme outcomes. In particular, for the market portfolio, which has a beta, gamma and kappa equal to 1, the following relation obtains:

$$[E(R_m) - R_f] = (\lambda_\beta + \lambda_s + \lambda_k) \quad (3.22)$$

The Four-Moment CAPM nests the case of the three-moment CAPM of Kraus and Litzenberger (1976) when $\lambda_k = 0$ and the case of the traditional CAPM when also $\lambda_s = 0$. Kraus and Litzenberger introduce the three-moment CAPM, arguing that the systematic skewness is a fundamental market risk and is therefore a priced risk factor. Their model is obtained if the fourth moment is ignored. The specification of the three- and Four-Moment CAPM is particularly useful when the assumption of the normality of returns is violated, i.e. returns are asymmetric and the distribution of returns presents fat tails. Under such circumstances, the assumptions that lead to the mean-variance CAPM are obviously violated.

3.3. Tests of the Higher-moment CAPM

Kraus and Litzenberger (1976) (henceforth KL) test the three-moment CAPM for portfolios of stocks double-sorted on beta and systematic coskewness over the period 1936-1970, using the Fama and MacBeth methodology. The model is reported in Equation 3.23:

$$r_{it} = \alpha + \lambda_1 \beta_{it} + \lambda_2 \gamma_{it} + \varepsilon_{it} \quad (3.23)$$

Their findings give an intercept that is, on average, insignificant and negative, a positive and significant premium for beta (which is, incidentally, larger than the beta premium

obtained when beta is the only explanatory variable), and a market premium for gamma which is significant and negative, consistent with expectations as the market skewness is positive over the testing period. Interestingly, high beta portfolios show higher coskewness and this evidence leads the authors to suggest that when systematic skewness is incorporated into the CAPM, the anomaly of high beta portfolios having lower returns than predicted by their market beta, and of low beta portfolios having higher returns than predicted theoretically by their market beta, might be reconciled.

However, this first test of the three-moment CAPM must be put into context. In 1976 researchers knew very little about size and book-to-market anomalies. The most important anomaly for the CAPM was at that time the fact that low beta portfolios had higher risk-adjusted returns than high beta portfolios. Hence, it might be that the three-moment CAPM can actually provide an explanation for the beta anomaly (for the period until 1976), but this does not mean that the model can solve the size and book-to-market anomalies.

Further to their early tests of the three-moment model, KL interpret the failures of the CAPM as a consequence of the misspecification of the traditional model due to the exclusion of systematic skewness. Unfortunately, subsequent empirical research is less supportive of the model. Friend and Westerfield (1980) (henceforth FW) extensively test the three-moment model with results that partly contradict the findings of KL. The tests cover individual stocks and portfolios of stocks sorted on a pre-ranking beta, and coskewness using several market portfolio proxies, though the results are not particularly sensitive to the specific market proxy used. Therefore, the differences between the findings of Friend and Westerfield and KL have to be ascribed mainly to the different time periods examined. For individual stocks, the intercept estimated from

the cross-sectional regressions is significantly different from zero for the overall period 1952-1976, whereas the coskewness coefficient is negative, but insignificant.

When portfolios are used, the intercept is positive and significant, whereas beta and coskewness are positive, but insignificant.

The conclusion of FW is that the findings of KL are not robust, although there is some evidence of the importance of coskewness in certain periods.

The three-moment CAPM has been tested using the Generalized Method of Moments (GMM) by Lim (1989). This is an interesting test as the GMM does not impose strong assumptions regarding the distribution of asset returns, and therefore it might be more appropriate than a cross-sectional OLS test. Lim tests the model on 10 portfolios whose beta and coskewness have equal rank, and estimates the parameters with a system GMM over 10 non-overlapping 5-year periods from 1933 to 1982. The results show that the model is not rejected by the Hansen J-test¹¹ for the whole period at the 10% level, and that the model is not rejected in 6 out of the 10 subsamples. The coskewness premium is found to be negative for the overall period as expected theoretically since the market skewness is positive, and the beta premium is found to be positive. The conclusion of Lim is that investors prefer coskewness when market returns are positively skewed, dislike coskewness when market returns are negatively skewed, and that the three-moment CAPM improves its performance over a long horizon.

The three-moment CAPM has been further investigated by Harvey and Siddique (2000) (hereafter HS). HS start from the observation that small-sized stock portfolios and

¹¹ Once the parameters have been estimated by GMM, the Hansen J-test can be used to evaluate the goodness of fit of a model. The test statistic which measures whether the pricing errors are too big is given by the pricing errors divided by the variance-covariance matrix of the errors and it is asymptotically distributed as a *chi-squared* distribution: $TJ_T = T[g_T(b)' \hat{S}^{-1} g_T(b)] \sim \chi^2(\#moments - \#parameters)$. If the test statistic is greater than the critical value, the model is rejected as the pricing errors are too large.

winner momentum portfolios are in general characterized by higher negative skewness and that they might therefore yield a higher return as they incorporate a coskewness premium. HS test a conditional version of the three-moment model on several portfolios built according to size, industry, size and book-to-market, book-to-market, and momentum. For every portfolio a time-series regression of monthly returns on the market index excess return and coskewness is conducted from 1963 to 1993. The conditional coskewness is defined in several ways:

1. A direct measure of coskewness is constructed as:

$$\hat{\beta}_{SKD_i} = \frac{E[\epsilon_{i,t+1}\epsilon_{M,t+1}^2]}{\sqrt{E[\epsilon_{i,t+1}^2]E[\epsilon_{M,t+1}^2]}} \quad (3.24)$$

where $\epsilon_{i,t+1} = r_{i,t+1} - \alpha_i - \beta_i(r_{M,t+1})$ is the residual from the regression of the monthly portfolio excess return on the market excess return. A negative $\hat{\beta}_{SKD}$ means that the asset adds negative skewness to the portfolio and is therefore riskier and should have a higher expected return;

2. Regressing the asset return on the square of the market excess return:

$$r_{it} = \beta_i(R_M - R_f)^2 \quad (3.25)$$

3. As the beta of every asset with the spread between the portfolio with the most negative coskewness (S^-) and the portfolio with the most positive coskewness (S^+), in the manner of Fama and French's SMB and HML; this beta is called β_{SKS} . The excess returns on ($S^- - S^+$) are used as a coskewness premium and are found to be 3.60 per cent over the period 1963-1993.
4. As the beta in the regression of asset returns on the excess returns on the S^- portfolio. This measure is called β_{S^-} .

Their results show that there is a positive relationship between negative coskewness and average returns which leads HS to test whether the inclusion of measures of conditional

coskewness in the three-factor model of Fama and French can improve the pricing errors compared to the GRS test that assumes that all the intercepts considered jointly are insignificant. Their results show that the inclusion of coskewness reduces the F-statistic significantly.

Finally, HS investigate the effect of augmenting the three-factor model of Fama and French with coskewness. The procedure is applied first by including in the model β_{SKD} as a measure of coskewness and then β_{SKS} as a measure of coskewness, though the results are similar. The second model is reported in Equation 3.26:

$$r_i = \gamma_0 \beta_{iM} + \gamma_1 \beta_{iSMB} + \gamma_2 \beta_{iHML} + \gamma_3 \beta_{iSKS} + \varepsilon_i \quad (3.26)$$

Interestingly, all of the coefficients are significant with a positive beta premium, a positive SMB premium, a positive coskewness premium and a negative premium for HML. HS conclude that coskewness can help explain the cross-section of average returns and partly explain momentum, the book-to-market effect, and the size effect.

There are two points worth noting here: (i) HS start their research from the empirical observation that small size and high book-to-market portfolios have a clear pattern in terms of skewness (high negative skewness), and (ii) methodologically, they introduce an important coskewness premium as per the SMB and HML of FF, that is, the spread between the portfolio with the most negative coskewness (S^-) and the portfolio with the most positive coskewness (S^+). As for the first point, the size premium seems to have disappeared for the US market and the pattern of skewness is not robust over time. As for the second point, this is the best approach methodologically unless assuming that the market portfolio is on the mean-variance-skewness efficient frontier, or that there are instrumental portfolios uncorrelated with the market portfolio. A possible limitation of this methodology is, however, that coskewness and cokurtosis are estimated and therefore there might be an error-in-variables problem.

Kumar (2005) examines investor preference for skewness and finds that institutional investors exhibit a preference for systematic skewness, but an aversion to idiosyncratic skewness. Smith (2006) investigates the role of conditional coskewness in the cross-section of US average returns. He finds that coskewness helps explain average returns and that investors require a premium for coskewness when the market's skewness is negative, and are willing to sacrifice a more substantial premium when the market's skewness is positive. In other words, investors have a preference for systematic skewness and an aversion to negative skewness, though the desire for skewness when market skewness is positive is stronger than the aversion to skewness when the market's skewness is negative. This is an interesting example of time-varying risk premia. The traditional CAPM and the three-factor CAPM are tested by modelling the conditional moments of market returns as linear functions of the conditioning variables. The conditional model examined by Smith is analogous to the three-moment CAPM of Kraus and Litzenberger (1976), but with time-varying parameters:

$$E_{t-1}[r_{i,t}] = \beta_{i,t-1}\mu_{1,t} + \gamma_{i,t-1}\mu_{2,t} \quad (3.27)$$

where

$$\beta_{i,t-1} = \frac{Cov_{t-1}(r_{i,t}, r_{m,t})}{\sigma_{m,t}^2} \quad (3.28)$$

and

$$\gamma_{i,t-1} = \frac{Coskew_{t-1}(r_{i,t}, r_{m,t})}{\kappa_{m,t}^3} \quad (3.29)$$

are, respectively, the conditional beta and the conditional gamma of asset i , and $\mu_{1,t}$ and $\mu_{2,t}$ are the price of the beta risk and the price of gamma risk.

Given that investors are risk averse and prefer positive skewness, the beta risk price should be positive and the price for gamma risk should have the opposite sign of the

market skewness: when the market is negatively skewed, $\mu_{2,t}$ should be positive, and when the market is positively skewed, $\mu_{2,t}$ should be negative. This asymmetry between the gamma price of risk and market skewness is accounted for by an indicator variable:

$$\mu_{2,t} = Z_{t-1}^T \mu_2 + \delta 1_{\{Z_{t-1}^T \kappa\}} \quad (3.30)$$

where $1_{\{Z_{t-1}^T \kappa\}}$ is an indicator variable taking the value of one when the conditional skewness of the market is positive, and zero otherwise. The beta premium, the market conditional variance, and the market conditional skewness are modelled as a function of conditioning variables (Z_{t-1}^T):

$$\begin{cases} \mu_{1,t} = Z_{t-1}^T \mu_1 \\ \sigma_{m,t}^2 = (Z_{t-1}^T \sigma_m)^2 \\ \kappa_{m,t}^3 = Z_{t-1}^T \kappa_m \end{cases} \quad (3.31)$$

where Z_{t-1}^T is the vector of the conditioning variables at time $t-1$.

Therefore, the market premium can be obtained as the sum of the beta premium and the gamma premium:

$$\mu_{m,t} = Z_{t-1}^T \mu_1 + Z_{t-1}^T \mu_2 + \delta 1_{\{Z_{t-1}^T \kappa\}} \quad (3.32)$$

The model is estimated with GMM on industry portfolios using the S&P500 index return, the dividend price ratio on the S&P500 index, the term spread, the default spread, and the T-bill rate as conditioning variables. The results show that expected returns are positively related to the dividend price ratio and the default spread, and negatively related to the term spread and the T-bill rate, suggesting that expected returns follow the business cycle, and that the set of conditioning variables describes the business cycle well. Most importantly, in the test, Smith rejects the traditional CAPM using the Hansen J-statistic at the 5% level of significance, whereas he does not reject the three-moment CAPM.

As predicted, the price for beta risk is found to be positive and significant, the premium for coskewness is positive as the market skewness is negative but insignificant, the indicator variable is significant, meaning that coskewness premium is not unconditional and the coefficient is negative as expected, and the model is significant at the 5% level.

The premium associated with conditional coskewness is estimated to be just 1.80% when the market skewness is negative, but -7.87% when the market skewness is positive. When the market is positively skewed, investors are willing to give up a larger return to hold assets that increase positive skewness, whereas when the market is negatively skewed, investors require a higher return to hold assets that increase negative skewness. The former effect is larger in magnitude than the latter effect.

Harvey and Siddique (2000) measure the coskewness premium as the return on the negative coskewness portfolio minus the return on the positive coskewness portfolio, and find an average return of 3.60% in the period 1963-1993, but an average annualized return of roughly 5% when the market skewness is positive, and only 2.81% when the market skewness is negative, confirming the findings of Smith (2006). In summary, investors are more attracted to positive skewness than they are averse to negative skewness.

Furthermore, Smith tests the conditional CAPM, the conditional three-factor model, the conditional three-moment model, and the conditional three-factor model augmented with coskewness. The third model with covariance with the market, coskewness with the market, and covariance with the three-factor model additional factors is specified as:

$$\begin{aligned}
 E_{t-1}[r_{i,t}] &= Cov_{t-1}(r_{i,t}, r_{m,t})\lambda_{1,t} + Coskew_{t-1}(r_{i,t}, r_{m,t})\lambda_{2,t} \\
 &+ \sum_{j \in \{smb, hml\}} Cov_{t-1}(r_{i,t}, f_{j,t})\lambda_{j,t}
 \end{aligned} \tag{3.33}$$

The model states that the expected return on an asset i depends on the premium for covariance, the premium for coskewness, and the premia for the other risk factors. The results of the four conditional models (the traditional CAPM, the three-moment CAPM, the three-factor Fama and French model, and the three-factor Fama and French plus coskewness model) show that the traditional CAPM is rejected at the 5% level, even though the price for the covariance risk is positive and significant. For the three-factor model, the covariance premium remains significant and positive and the model is rejected. More interestingly, the three-moment CAPM and the FF plus coskewness model cannot be rejected. Moreover, when coskewness is added to the FF model, the coefficients associated with SMB and HML become negative. Smith's results therefore appear to support the proposition that conditional coskewness has some ability to explain expected returns and that it might partly capture the SMB and HML factors, and therefore give a rational explanation for the CAPM anomalies. Perhaps, distressed stocks that have higher loadings on SMB and HML factors have also negative coskewness and therefore require a coskewness premium.

Friend and Westerfield (1980) investigate the results of the three-moment CAPM in up and downmarkets for both individual stocks and portfolios of stocks. The results are strikingly similar to those reported by Pettengill et al. (1995), *i.e.* a segmented significant relationship between beta and returns.

The results show that the market premium for beta (which is significant in both up and downmarkets) is roughly the same in absolute value, and is of the sign hypothesized by Pettengill et al., that is, positive in upmarkets and negative in downmarkets, with a negative premium for coskewness, especially in upmarkets.

3.4. Tests of the Four-Moment CAPM

As demonstrated in Section 3.2., a Four-Moment CAPM involves including systematic kurtosis in the three-moment CAPM. The literature relating to this model is now quite rich both for the unconditional and the conditional version. Fang and Lai (1997) (hereafter FL) test the Four-Moment CAPM, including systematic kurtosis on 27 portfolios triple-sorted on beta, coskewness, and cokurtosis. The test is conducted over three 5-year non-overlapping periods: 1974-1978, 1979-1983, and 1984-1988, and a cross-sectional regression is conducted to estimate the coefficients of the Four-Moment CAPM associated with the measure of systematic risk. FL show that the cubic market model is consistent with the Four-Moment CAPM. The cubic market model assumes that the excess return of any asset is generated by the following model:

$$R_{it} = \alpha_0 + \beta_i R_{mt} + \gamma_i R_{mt}^2 + k_i R_{mt}^3 + \varepsilon_{it} \quad (3.34)$$

where R_{it} and R_{mt} are excess returns of a generic asset i and of the market portfolio respectively, $R_{mt}^2 = (R_{mt} - E(R_{mt}))^2$ is the de-measured market excess return to the power of two and $R_{mt}^3 = (R_{mt} - E(R_{mt}))^3$ is the de-measured market excess return to the power of three. The beta, coskewness, and cokurtosis of the portfolios are estimated using time series regressions of the cubic market model, and then cross-sectional regressions of the average excess returns on the estimated moments are conducted to estimate the market risk premia:

$$R_i - R_f = \lambda_\beta \hat{\beta}_i + \lambda_s \hat{\gamma}_i + \lambda_k \hat{k}_i \quad (3.35)$$

Their results show a substantial improvement in the R-squared for the Four-Moment CAPM compared to the two- and three-moment CAPM for the subsamples. Most importantly, the risk premia for beta and cokurtosis are significant and positive for all the three subsamples, whereas the coskewness results are significant in two of the subsamples and have the opposite sign of the market skewness (negative when market

skewness is positive, and positive when market skewness is negative), as expected. In summary, the results suggest that investors are compensated for systematic variance and kurtosis risk, and that they are willing to sacrifice some expected return for those assets that increase the systematic skewness of the market portfolio.

However, some observations are warranted here. First, the portfolios are sorted on beta, coskewness, and cokurtosis, which is sensible only if one assumes that these factors are the main drivers of returns. Second, the test is based on a short period (1974-1988) with three non-overlapping five year subsamples, and it has already been determined that the pattern in higher moments is not very robust over time. Third, the sensitivity to the three higher moments of the market portfolio returns is estimated with a time series regression of a cubic model, not as bivariate covariances, and lastly the test spans only 15 years of time.

The model applied in this thesis is specified differently from Fang and Lai (1997) and introduces time-varying DCC betas. Moreover, the model is applied to 25 portfolios double sorted on size and book-to-market for a long period of time and on individual assets.

Athayde and Flores (1997) (hereafter AF) derive and test a Four-Moment CAPM for the Brazilian stock market. AF introduce three portfolios, z_2 , z_3 , and z_4 , with, respectively, zero coskewness and zero cokurtosis, zero covariance and zero cokurtosis, and zero covariance and zero co-kewness with the market portfolio

The model is obtained as:

$$E(R_i) - R_f = \beta_i \frac{E(R_{z_2}) - R_f}{\beta_{z_2}} + \gamma_i \frac{E(R_{z_3}) - R_f}{\gamma_{z_3}} + k_i \frac{E(R_{z_4}) - R_f}{k_{z_4}} \quad (3.36)$$

where the formula states that the expected excess return of any security is given by its beta, gamma, and delta times, respectively, the premium for a unit of covariance, coskewness, and cokurtosis risk.

The model is estimated with a GMM approach over the period 1996-1997. AF test the traditional CAPM, the three-moment model of KL, the CAPM plus kurtosis model, and the Four-Moment CAPM. The four models are not rejected using the Hansen J-test, and the J statistics are less significant when the higher moments are included, especially in the case of skewness. However, it should be noted that this approach requires the existence of the instrumental portfolios and it is highly complex when the number of the moments and the number of the assets to test increases. Moreover, since the test concerns a single emerging market (Brazil) and a very short time period, the conclusion can hardly be generalized.

Hwang and Satchell (1999) estimate an unconditional Four-Moment CAPM for emerging markets. Emerging markets represent a particularly interesting case as the distribution of returns exhibit skewness and kurtosis and are therefore a text book case in which the mean-variance analysis of the traditional CAPM appears inadequate. HS estimate and test the traditional CAPM, the three-moment KL model, and the Four-Moment CAPM, among other models, for 17 emerging markets over the period 1985-1997 with a GMM approach. The results of the GMM estimation show that the traditional CAPM is rejected on the basis of the Hansen J-test for Latin America countries alone, even though the R-squared is never particularly high for the other emerging countries. When the three-moment KL model is estimated, the model is rejected for the Latin American countries but the R-squared statistics substantially increase, except for Asia. Although the market prices of beta and coskewness have the signs expected from theory (respectively, positive and of the opposite sign to the market

skewness – as coskewness should have opposite sign to the market skewness), they are not significant.

When the Four-Moment CAPM is estimated, the R-squared increases markedly and the model is not rejected for any of the groups. Nevertheless, none of the coefficients are significant except for the beta premium and cokurtosis for the countries clustered in the group Latin America, and moreover the signs are not consistent with theory. Despite this, the higher-moment CAPM seems preferable given the high significance of the systematic cokurtosis for most of the emerging markets, especially for Latin America. The final conclusion is that higher moments can add explanatory power to the returns of emerging markets, but not in a homogeneous fashion: Latin American market returns appear better explained by beta and cokurtosis, Asian market returns by a Four-Moment CAPM, and the remaining countries' returns by beta and coskewness.

In summary, the research of HS supports the benefits of including higher moments, but it is argued that the focus on the R-squared statistic and the Hansen J-test is perhaps overstated, whereas the role of the sign/significance of the risk premia is to some extent overlooked. The statistical rejection of normality for the returns, the existence of skewness and the excess of kurtosis lead the authors to favour the Four-Moment CAPM more than the actual estimate and significance of the risk premia for the higher moments. Furthermore, the failure of the CAPM for emerging markets might be due to the existence of peculiar country risk or other factors not captured by the model, as recognized by HS.

One of the most interesting papers on the higher-moment CAPM has been written by Dittmar (2002) using a stochastic discount factor approach. He introduces a conditional Four-Moment CAPM in which the coefficients are time-varying. As the conditional moments can be derived as a function of the set of information available at time t (the

conditioning information), the discount factor implied by the conditional Four-Moment CAPM is:

$$m_{t+1} = \delta'_0 Z_t + \delta'_1 Z_t R_{W,t+1} + \delta'_2 Z_t R_{W,t+1}^2 + \delta'_3 Z_t R_{W,t+1}^3 \quad (3.37)$$

In order to force the preference restrictions implied by decreasing absolute risk aversion, that is, a dislike for variance and kurtosis and a preference for skewness, the final model is rewritten as Equation 3.38:

$$m_{t+1} = (\delta'_0 Z_t)^2 - (\delta'_1 Z_t)^2 R_{W,t+1} + (\delta'_2 Z_t)^2 R_{W,t+1}^2 - (\delta'_3 Z_t)^2 R_{W,t+1}^3 \quad (3.38)$$

The trick of squaring the coefficients is used in order for them to always be positive, so that the signs of the term in the stochastic discount factor are imposed by the preferences. Two models are implemented: (i) a model with the equity market index as a market proxy for wealth, and (ii) a model including human labour wealth following Jagannathan and Wang (1996) who argue that human capital is an important component of wealth that can improve the explanation of the cross-section of average equity returns. The second model incorporating the return on human labour, defined as the growth rate in labour income, is given by:

$$m_{t+1} = (\delta'_0 Z_t)^2 - (\delta'_1 Z_t)^2 R_{W,t+1} + (\delta'_2 Z_t)^2 R_{W,t+1}^2 - (\delta'_3 Z_t)^2 R_{W,t+1}^3 - (\delta'_1 Z_t)^2 R_{l,t+1} + (\delta'_2 Z_t)^2 R_{l,t+1}^2 - (\delta'_3 Z_t)^2 R_{l,t+1}^3 \quad (3.39)$$

The stochastic discount factor therefore has three terms in the market portfolio and three terms up to the cubic term for the human labour wealth. The instrument set Z_t is given by r_{mt} , the excess return on the market index, by dy_t , the dividend yield on the market index, by ys_t , the spread on interest rates between three-month T-bills and one-month T-bills, and by tb_t , the return on one-month T-bills. The coefficients are then estimated using a GMM methodology by imposing the moment condition and using industrial portfolios:

$$E[(1 + R_{t+1})m_{t+1}|Z_t] - 1_N = 0 \quad (3.40)$$

For the first model (the market portfolio alone), the quadratic term is significant at the 10% level, but not at the 5% level, whereas the cubic term is insignificant. However, the terms all become significant, the linear and cubic term are negative as expected, and the quadratic term is positive as expected, and the pricing errors are drastically reduced when human capital is included in the second model of the Four-Moment CAPM. Moreover, the improvement in the model becomes large when human capital and the higher moments of the distribution of returns are both included.

Furthermore, Dittmar augments the Four-Moment CAPM to incorporate the SMB and HML factors of Fama and French. The results show that when the higher moments of the distribution and human capital are included in the model, SMB and HML become insignificant. Therefore, he concludes that the Four-Moment CAPM can better describe the cross-section of average industrial portfolio returns than the three-factor model of Fama and French.

The conclusion that can be drawn from this study is that skewness and kurtosis allow a better explanation of the cross-section of average returns for industry portfolios, and that the improvement is largely due to the higher moments of the distribution of human capital returns. Therefore, this work seems to support the inadequacy of the proxy used for the market portfolio, and supports the introduction of the higher moments of the distribution of returns for a more comprehensive measure of aggregate wealth.

Dittmar's test leads to further observations. First, the model is highly meaningful economically as some predictive economic variables are used as conditioning information to capture the dynamics of risk aversion, and as the coefficients associated with the higher moments are restricted in order to satisfy the preferences for the higher moments. Second, the test is based on industry portfolios which might present a

limitation as there is not a particular anomaly related to industry portfolios, and it is well known that the three-factor model of Fama and French does not perform well for industry portfolios. Finally, the model considers human labour as an important source of wealth, and in agreement with the results of Jagannathan and Wang (1996), suggests a broader proxy for the market portfolio. The model tested in this thesis differs from Dittmar as its tests are conducted on the ME and BM portfolios which are known to be the more difficult to price and because the beta representation is used as opposed to the stochastic discount factor. Moreover, the techniques used to obtain time-varying sensitivities are different as the thesis does not provide a linear function of macroeconomic variables, but instead a DCC Garch, and tests individual assets.

Christie-David and Chaudry (2001) implement the Four-Moment CAPM to investigate the cross-section of the returns of futures contracts. Three models are tested: the traditional CAPM, the three-moment CAPM with coskewness, and the Four-Moment CAPM with cokurtosis. The moments are found to be significant in all the three specifications of the model and the R-squared increases with number of moments included in the model. Most importantly, the coefficients associated with the moments are significantly different from zero and have a sign consistent with theoretical expectations – the beta premium is positive, the coskewness premium is negative, and cokurtosis has a positive premium.

The results are consistent with the findings of Scott and Horvath (1980) who suggest that in the presence of risk aversion and a preference for positive skewness, investors dislike kurtosis. In the period examined, market skewness is positive and the premium is negative, since investors sacrifice some return for positive skewness, and the premium for kurtosis is positive, as investors have an aversion to kurtosis.

Fletcher and Kihanda (2005) evaluate and compare the performance of different unconditional and conditional asset pricing models in the stochastic discount factor framework for UK stocks returns for the period 1975-2001 using the Hansen-Jagannathan (1997) distance measure of how well the models price the most mispriced assets. Seven specifications of the stochastic discount factor, both conditional and unconditional, are examined: the CAPM, the three-moment CAPM, the Four-Moment CAPM, the CAPM with labour income, the three-moment CAPM with labour income, the Four-Moment CAPM with labour income, and the Fama and French three-factor model. The results show that the Four-Moment CAPM reduces pricing errors and that conditional models outperform their unconditional versions. In particular, the conditional Four-Moment CAPM outperforms both the traditional and the three-moment CAPM. However, when testing the conditional Four-Moment CAPM with labour income, all of the explanatory variables are insignificant. The conditional Four-Moment CAPM performs very poorly when using a set of size-sorted portfolios, suggesting that the apparent success of the model is due solely to the very large number of variables estimated (the problem of overfitting).

Furthermore, Harvey (2000) shows that coskewness, kurtosis and idiosyncratic skewness are especially significant in the explanation of returns in emerging markets for which the distribution of returns most significantly deviates from normality. Tan (1991) tests the three-moment CAPM on a set of mutual funds and finds an intercept significantly greater than zero and a positive but insignificant estimated risk premium for beta. The coefficient for coskewness is unexpectedly negative in a period of negative skewness for the market portfolio's returns, though it is not statistically significant. Therefore, the findings do not support the three-moment CAPM, at least when explaining mutual funds returns.

Ranaldo and Favre (2003) investigate the three- and Four-Moment CAPM for hedge fund returns and conclude that coskewness is particularly relevant for the cross-section of hedge funds returns (hedge funds are known to be characterised by an asymmetry of returns as a consequence of the complicated dynamic strategies they employ, and their use of derivatives). Furthermore, Conrad, Dittmar and Ghysels (2008) find that asset returns are strongly related to volatility, skewness and kurtosis. Their results show that: (i) stocks with high idiosyncratic volatility earn lower returns, i.e. beta and not idiosyncratic volatility is the relevant measure of risk; (ii) assets with more negative skewness earn higher returns in subsequent months than assets with higher positive or lower negative skewness; and (iii) portfolios with higher kurtosis earn higher returns. These findings seem to confirm the usefulness of the Four-Moment CAPM but also leave the way open for models that argue the importance of idiosyncratic skewness and kurtosis. It is also worth noting that tests of the Four-Moment CAPM are particularly suitable for the returns of hedge funds or derivatives, given the strategies used by hedge funds which adopt derivatives and the non-linear payoffs of derivatives.

In summary, incorporating the third and fourth moment of the distribution of returns in the traditional CAPM gives rise to evidence that systematic skewness and kurtosis may play some role in the investment decision of investors, but the results do not provide conclusive evidence that the CAPM can explain the main anomalies affecting asset pricing.

Hasan and Kamil (2013) use a higher-moment CAPM with coskewness, cokurtosis, market capitalization and book-to-market for equity returns for Bangladeshi stocks and find that coskewness and cokurtosis are, respectively, negatively and positively related to returns at the 10% level of significance.

Lambert and Hubner (2010) extend the CAPM to include coskewness and cokurtosis premia obtained as mimicking factor portfolios and test whether the four-moment CAPM can explain the time series and cross-section of US equity returns over the period 1989 to 2008. Specifically, the authors build three hedge portfolios mimicking the reward implied by covariance, coskewness and cokurtosis risk as per the F&F factors and Carhart's momentum factor. They apply a conditional sorting methodology by which portfolios are first sorted on two comoments and finally on the third comoment, to be priced in turn, to measure the spread in return due to the third comoment alone. The authors test the model at time series on portfolios sorted on two dimensions and find high a R-squared statistic and insignificant pricing errors. When testing an augmented three-factor model of Fama and French with coskewness and cokurtosis on the 25 portfolios sorted on size and book-to-market, they find that low book-to-market portfolios are positively related to high cokurtosis risk and small sized portfolios are positively related to low coskewness, that is, they are more exposed to coskewness risk. At cross-section they find that the augmented model reduces the pricing errors and has a higher R-squared statistic than the single CAPM. Even if overall there is no sign of significance of the higher comoments, when the sample is split as in Pettengill et al. (1995) they find that in up markets coskewness and cokurtosis have a positive risk premium, whereas covariance has a negative risk premium. Cokurtosis has a negative risk premium in down markets. When the model is augmented with the Carhart's four-factor model (1997), they find that cokurtosis and SMB are significant and positive in upmarkets, and HML is significant and positive in down markets together with a non linear cokurtosis premium that is negative, and a non linear covariance premium that is positive.

Kostakis et al. (2011) test the higher-moment CAPM for UK stocks for the period 1986-2008 and find that coskewness demands a negative risk premium whereas stocks with

higher cokurtosis yield higher returns on average. In particular, coskewness and cokurtosis have additional explanatory power to covariance risk, size, book-to-market and momentum factors. They construct zero cost hedge portfolios based on estimated coskewness and estimated cokurtosis. The spread in returns between the portfolio with most negative skewness and the portfolio with most positive skewness is large and positive and the same holds for the spread in return between the portfolio with most positive cokurtosis and the portfolio with lowest cokurtosis, confirming the expectations for risk-averse investors that dislike negative skewness and large kurtosis (negative or extreme outcomes). The alpha or unexplained return of portfolios with negative coskewness and positive cokurtosis is not eliminated in a time series after controlling for size, value and the momentum factor (the Carhart model). When portfolios are sorted on coskewness and cokurtosis, the joint hypothesis that the alphas equal zero is strongly rejected and further the zero cost hedge strategies yield significant positive returns which are not explained by size, value or momentum.

Kostakis et al. test then the Carhart model and the four-moment CAPM augmented with the two factors obtained as monthly spread between the low and high coskewness portfolio and high and low cokurtosis portfolio over ten portfolios sorted on coskewness and ten portfolios sorted on cokurtosis. None of the original factors are found to be significant in explaining the cross-sectional returns of the portfolios, whereas coskewness and cokurtosis maintain a significant coefficient as expected. In particular, there is a negative relationship between beta and returns for portfolios sorted on cokurtosis.

Young et al. (2010) use a higher-moment CAPM in which the moments are estimated using the daily data on the S&P 500 option index. They find that stocks with high exposure to change in implied market volatility and market skewness yield lower returns on average, whereas stocks with higher sensitivity to kurtosis yield higher

returns on average. Investors appear to be willing to forego returns of 3-5% annually for a positive contribution to the market portfolio skewness. This negative return does not appear to be explained by size, value or momentum, and is robust to changes in assets or periods of time.

Heaney et al. (2012) test whether coskewness and cokurtosis are priced for in US equity returns over the period 1963-2010, splitting the sample into the subsamples 1963-1990 and 1991-2010 and using individual assets as opposed to portfolios. They find little evidence that the higher moments are priced and show that these are encompassed by size and book-to-market factors. In particular, size tends to eliminate the significance of cokurtosis, which incidentally is found to be unexpectedly negatively rewarded, and coskewness varies over time. Size and value are the only factors which are statistically significant, whereas momentum appears to be priced only in the period 1963-1990 and to be particularly sensitive to the inclusion of very small sized firms.

Teplova and Shutova (2011) use a four-moment CAPM for the Russian stock market. They compare the CAPM, the downside CAPM and four-moment CAPM unconditionally and conditionally for the pre-crisis period 2004-2007 and the crisis period 2008-2009. Whereas the unconditional CAPM performs poorly, the conditional four-moment CAPM appears to have relatively better performance. In particular, the factors are all insignificant when the unconditional four-moment CAPM is tested. When the model is tested conditional on the sign of the market return, the four-moment CAPM improves in down markets with an R-squared statistic of 36%, beta and cokurtosis are negatively rewarded and coskewness has a positive coefficient on average. Furthermore, the authors conclude that coskewness is negatively related to returns in the crisis period and positively related to returns when the financial market is stable.

Galagedera and Maharaj (2004) find a positive relationship between beta, cokurtosis and returns in up markets and a negative relationship in down markets, where the up-

and down markets are defined as in Pettengill et al. (1995) for the Australian equity market. They find no support for the unconditional higher-moment CAPM, as none of the factors are significant in the unconditional specification.

Javid and Ahmad (2008) test the unconditional and conditional four-moment CAPM in which the higher comoments are obtained from an autoregressive process as in Harvey and Siddique (2000) for the Karachi stock exchange. They find that coskewness is priced unconditionally and is conditionally important for the cross-section of Pakistani equity returns and it has an opposite sign to the market skewness as expected, whereas cokurtosis and beta show little evidence of reward from the market.

Chung et al. (2006) find that the first 10 co-moments can eliminate the significance of the size and book-to-market factors.

Kapadia (2006) suggests that small-growth stocks in US tend to have lower returns because investors are willing to forego returns for positive skewness. The findings suggest that stocks with high volatility have lower returns because of the higher skewness. Kapadia uses a measure of cross-sectional skewness as opposed to time-series coskewness and gives evidence that small-growth stocks and highly volatile stocks tend to load highly on the cross-sectional skewness.

Agarwal, Bakshi, and Huij (2008) find that the cross-section of returns of hedge funds is significantly related to higher comoments, especially for hedge funds that adopt equity-orientated styles. They also find that the time series of returns is significantly related to investable higher moment factors. Their proxies for volatility, skewness and kurtosis are extracted from S&P 500 index options. When hedge funds are sorted on one of the higher moments, the authors notice a monotonically increasing alpha which is evidence that the higher moments can indeed explain the returns of the hedge funds with some investment styles more affected than others. In general, they find a positive sign for the volatility factor and a negative coefficient for skewness and kurtosis.

Moreno and Rodriguez (2009) analysing the returns of US mutual funds between 1962-2006 find that coskewness is a priced risk factor for mutual funds and that risk adjusted alphas from models that do not consider coskewness can be totally reclassified in a model of performance evaluation that acknowledges the role of coskewness; in particular positive alphas under the simple CAPM can become negative alphas under the higher-moment CAPM. The coskewness factor is obtained as a tradable factor as per SMB and HML using the definition of coskewness obtained by Harvey and Siddique (2000):

$$\gamma_i = \frac{E(\varepsilon_{it-1}\varepsilon_{Mt-1}^2)}{\sqrt{E(\varepsilon_{it-1}^2)E(\varepsilon_{Mt-1}^2)}} \quad (3.41)$$

where $\varepsilon_{it+1} = (R_{it+1} - R_{ft+1}) - \alpha_{i0} - \alpha_{i1}(R_{Mt+1} - R_{ft+1})$ are the residuals from a single factor market model and $\varepsilon_{Mt} = (R_{Mt} - \overline{R_M})$ is the residual of the excess market return over its mean.

The findings show that funds which invest in stocks with more negative coskewness tend to yield higher average returns.

Doan et al. (2008) analyse the higher-moment CAPM in Australia and the US and find that Australian stock returns are sensitive to the higher moments, that indeed the higher moments can add explanation to returns not explained by the Fama and French factors, and that size has a significant impact on the higher moments of Australian stocks. The authors augment the Carhart's model to include conditional coskewness and cokurtosis obtained as in Harvey and Siddique (2000) using daily returns as opposed to monthly returns. Specifically they find a negative relationship between size and coskewness in Australia, but limited evidence in the US. They also find that size is inversely related to cokurtosis and book-to-market is positively related to cokurtosis in the US. The test is difficult to generalize given the use of only 15 years of data in the US and just 6 years

of data in Australia, and given the use of the S&P 500 index in the US which is more biased towards large stocks and more is likely to underestimate the role of skewness. Coskewness appears more relevant for Australian stocks whereas cokurtosis is more relevant for US stocks. The authors suggest that this difference is due to the different industrial composition of the two markets, with more technological companies with high volatility in the US. In summary, they find that the time-series of returns for 25 portfolios double-sorted on size and book-to-market is improved by the introduction of two higher-moments premia, but that even in the presence of the higher comoments SMB, HML and momentum retain significance. The test is interesting but the downside is that it is based on time series regressions and constructing the higher-comoments premia as per SMB and HML in Fama and French using zero cost hedge portfolios or mimicking factor portfolios as opposed to the use of cross-sectional regressions and given the difficulty of estimating the higher comoments.

3.5. Conclusion

The main extension of the CAPM analysed in this thesis is the higher-moment CAPM. The CAPM is obtained under the assumption that investors only care about the trade-off of the mean and variance of their portfolio returns, whereas stylized facts show that investors should care also about the skewness and kurtosis of their portfolio returns. Skewness measures the asymmetry of the distribution of returns, whereas kurtosis measures the peakedness of the distribution and reflects the shape of the tails of the distribution. Returns are not normally distributed but are leptokurtic with asymmetry and are characterised by extreme outcomes (especially negative outcomes) which are more likely than is predicted by the normal distribution (thick tails). As a consequence investors should have a rational preference for positive skewness and an aversion to kurtosis. Positive skewness refers to the situation in which extreme positive outcomes are more likely than extreme negative returns (a longer right tail in the distribution),

whereas negative skewness refers to the case of a longer left tail in the distribution with extreme negative outcomes more likely than extreme positive outcomes.

The higher-moment CAPM is an augmentation of the CAPM to include the third and fourth moment of the distribution of returns (skewness and kurtosis). The risk of an asset might be underestimated or overestimated using the market beta alone, as an asset might contribute to the negative skewness or to the kurtosis of the market portfolio. In this case, investors will ask for compensation not only for the covariance risk, but also for the coskewness and cokurtosis risk.

Since the first three-moment CAPM of Kraus and Litzenberger (1976) was proposed, several extensions of the model have been introduced, though with mixed results. In recent years, interest in the higher-moment CAPM has gained new momentum especially in view of: (i) the progressive liberalization of markets; (ii) more popular investment in emerging markets which are strongly characterized by a non-normality of returns; (iii) an interest in the shape of the tails, especially the left tail of the returns distribution with methodologies such as Value at Risk and Extreme Value Theory; and (iv) the success of hedge funds whose strategy adopts non-traditional assets such as derivatives, and whose returns exhibit skewness.

After the positive results of the seminal work of Kraus and Litzenberger (1976), who find that investors seem to give up some returns for those stocks that contribute to positive skewness, Friend and Westerfield (1980) fail to confirm such results when studying a different time period. Harvey and Siddique (2000) document that investors seem to prefer positive coskewness, and that they are willing to forego some returns for

portfolios of stocks with positive coskewness, whereas they require higher returns for portfolios of stocks with negative coskewness.

Fang and Lai (1997) introduce the Four-Moment CAPM and find evidence that investors require compensation for covariance and cokurtosis, but that they accept lower returns for positive coskewness. In one of the most significant papers on the higher-moment CAPM in the stochastic discount factor framework, Dittmar (2002) shows that a conditional Four-Moment CAPM augmented to include the return on human capital is not rejected when tested on industry-sorted portfolios of US stocks. The interest in the higher moments of the distribution of returns is partly due to the success of hedge funds, whose management, in an attempt at timing the market and creating non-linear payoff strategies with derivatives leads to asymmetric returns. Another reason for the interest in the higher moment CAPM is attributable to the success of measures of risk such as Value at Risk and Extreme Value Theory that focus on aversion to large losses. Moreover, the observation that the distribution of returns, especially for emerging markets and portfolios related to the main asset pricing anomalies, exhibits skewness and kurtosis has led researchers to investigate the role played by higher moments in the explanation of average returns. A further issue concerns the estimation methodology for the parameters of the Four-Moment model. The assumption is made that the market portfolio is the optimal portfolio as investors have utility functions with the same parameter for cautiousness, or, alternatively, instrumental portfolios are introduced, but this latter solution relies on the existence of such instruments. The violation of the normality assumption may require estimation methods such as the GMM approach, but in that case there remains the problem of increased complexity and a large number of moments.

Chapter 4

Conditional Models

4.0. Introduction

In Chapter 3 a higher-moment extension of the CAPM was introduced. This chapter deals with a further extension of the CAPM, namely the conditional asset pricing models. The objective of this chapter is to present a comprehensive overview of the conditional models in which beta is time-varying. The assumptions of constant beta and constant risk premium appear to be too strong. In the real world, investors are likely to require higher expected returns to hold stocks when their risk aversion is higher and the economic outlook appears bleak. Conversely, in an economic boom, investors require lower returns to invest in stocks. This pattern is also observed in the bond markets (Fama and French, 1989).

A conditional CAPM might hold even if the CAPM does not hold unconditionally (see Jagannathan and Wang, 1996). If, for example, small stocks are riskier and more sensitive to market portfolio returns when risk aversion is higher, and investors require a larger risk premium to hold stocks, the conditional correlation between beta and the risk premium might explain the small size premium. The same reasoning goes for the value premium. Value stocks might be more sensitive to the market portfolio in times of distress, recession or economic downturn, see Cochrane 2001. The main problem with conditional models, as is evident from the literature discussed in this chapter, is how to model the time-variation of the parameters.

4.1. Conditional asset pricing models

A relatively recent field of research in asset pricing considers the introduction of conditional models with time-varying parameters. This branch of research suggests that the unconditional CAPM or multifactor models fail to explain the cross-section of returns because they do not account for conditioning information and time-varying risk aversion (i.e. conditional betas and time-varying risk premia). Until the 1970s, returns were thought to be unpredictable and asset prices were thought to follow a random walk (Cochrane, 2001). However, over time evidence began to suggest that stock returns are at least in part predictable, especially in the long horizon. Therefore, it is reasonable to think that investors consider using a set of predicting variables in their investment decision. If investors can partly predict the state of the economy and future investment opportunities, their preference for stocks should change over time. After all, this is the way that portfolios are managed in real life. But unconditional tests or the unconditional version of asset pricing models ignore the way in which portfolios of stocks are actually managed.¹² The conditioning idea is not new. It was advanced by Merton (1973) who suggested that, besides a component of market risk, investors consider a hedging component for the changes in the investment opportunity set.

The assumption that the expected risk premium is constant and equal to the average historical risk premium is a strong assumption in the tests of the CAPM. Economic outlooks change from one period to another. The supply and demand of risky investments are also time-varying. Finally, investor's attitudes towards risk also change through time.

Another problem that is related to conditioning is the use of realized returns as proxy for the expected returns. Although little can be done about the unknown expected risk

¹² This is especially true in a multi-period setting, whereas the traditional CAPM is by nature framed in a single-period setting.

premium, at least an approximation of reality can be attempted through conditional models.

Elton (1999) criticizes the use of realized returns as a proxy for expected returns. The same view is shared by Brav et al. (2005) who use expected returns obtained from Value Line analysts' forecasts and find a negative relationship between expected return and market capitalization. In other words, investors would require ex-ante higher returns for holding small stocks and therefore the size anomaly would not be an anomaly any longer.

Pettengill et al. (1995) show that conditional tests of the CAPM based on the upward or downward trend of the market can give more support to the CAPM. However, conditional versions of the CAPM based simply on positive and negative market returns are a gross simplification of reality. Investors do not know exactly when markets will be upward or downward, and thus they can at best only make predictions on the basis of the information available. That is why conditional asset pricing models, in which risk-aversion and therefore risk premia are conditional on macroeconomic and microeconomic information, are economically more interesting as they not only reproduce the way in which investors actually behave, but they have solid economic underpinnings without which an asset pricing theory becomes vacuous (Cochrane, 2001).

One of the most important fields of research concerning asset pricing models focuses on the role of macroeconomic variables and firm characteristics to capture information about the time-varying beta and risk premia. Conditional asset pricing models allow time-varying risk aversion and loading coefficients for risk factors. Static or unconditional models in which risk premia and betas are constant over time ignore important risk dynamics. Risk aversion is sensibly linked to the business cycle and

therefore different risk factors have different impacts over time, in turn demanding a different risk premium.

There are at least two common problems for conditional models: (i) the identification of the set of predictive variables; and (ii) how to model the dynamics of the risk premium and beta over time. One widely used way of incorporating the time variation of beta in conditional models in the literature is to express beta as a linear function of those instrumental variables which help to capture time-varying risk aversion, with examples given in Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Dittmar (2002). Another important approach to time-varying betas is given by the models that explicitly tackle the time-varying parameters using multivariate GARCH with dynamic conditional correlations, as in Engle (2002) and Engle and Bali (2008). This last approach will be used in this thesis. This thesis proposes a number of extensions to the traditional CAPM, and uses time-varying betas as well as time-varying risk premia in the conditional versions of the CAPM and four-moment CAPM. However, although important in its own right, modelling the time variation in betas per se is not the main focus of this thesis. Nevertheless, it may be useful to briefly review some alternative techniques that have been employed in the literature on the subject of time-varying beta modelling.

A simple technique, which is also employed in this thesis, is the short-window regression approach such as the 60-month rolling regression of Fama and MacBeth (1973) or the shorter window rolling regression used by Lewellen and Nagel (2006). The main rationale for this approach is that, in a short window, beta should vary very little and therefore a short-window should allow an estimation of the time variation in beta without much bias.

Another approach discussed in the literature is the use of macroeconomic variables to arrive at a time varying beta. Here, beta is assumed to be a linear function of certain macroeconomic variables such as interest rates, the credit spread, and the consumption-to-income ratio. Examples of studies employing this approach include Ferson and Harvey (1999) and Lettau and Ludvigson (2001). In some studies, microeconomic variables have also been used. These include firm level characteristics such as E/P and B/M, which are employed in Bauer et al. (2008) and Avramov and Chordia (2005). The main reason why this methodology has not been employed in this thesis is that the beta modelling is sensitive to the conditioning variables chosen by the researcher. Given this lack of robustness, a simpler but robust approach such as the DCC Multivariate GARCH is preferred.

An additional alternative approach adopted in the literature is the use of state-space models in which beta is an unobservable latent variable modelled as an autoregressive model of first order, that is, an AR(1) or a random walk model, estimated through a Markov Chain Monte Carlo (MCMC) approach or a Kalman filter approach (Durbin and Koopmans, 2001). Examples of this technique are found in Ang and Chen (2007), Adrian and Franzoni (2009) and Jostova and Philipov (2004). However, these techniques are particularly complicated and need intensive computational power. The author recognizes however that this methodology might represent a good alternative to the DCC GARCH and that might be worth applying it in future research.

The most recent alternative approach in the empirical literature is the use of a realized beta which is based on realized or estimated variances and covariances, estimated using intra-period or high frequency data (Andersen et al., 2003). Although promising, this technique suffers from a lack of data availability as it requires data over very long time periods as well as data of high frequency. The advantages of the approach are, however,

that no assumptions of conditioning variables are needed, and that high frequency data are usually richer in information than less frequent data. This is quite important as higher moments are difficult to estimate with precision.

Another approach that has been considered in the literature but not employed in this thesis is the use of a Markov switching regime for beta, according to which beta would have different sets of parameters within different regimes (see Guidolin and Timmermann, 2002). In this thesis the choice has been made that whereas the risk premia can change with the regime, given that risk aversion is changing according to the set of information available to investors at each point in time, betas are assumed to follow a dynamic process described by the DCC Multivariate GARCH, i.e. betas change over time but do so more smoothly than is the case in a Markov switching regime.

Finally, this thesis employs a DCC GARCH approach. Since its development, the GARCH model has been used to estimate conditional betas that can be obtained by a multivariate GARCH. However, the problem is that with multiple assets the multivariate GARCH suffers from the curse of dimensionality. To overcome this problem, one solution is to use constant correlations as discussed in Bollerslev (1990), or in more advanced developments to employ dynamic conditional correlations estimated as conditional variances (DCC GARCH) as discussed in Engle (2002). The DCC GARCH allows for a degree of parsimony and flexibility that the constant correlations do not permit.

Overall, the complexity and multitude of techniques and approaches available ultimately drive the researcher towards choosing those approaches that are more appropriate to his or her research skills and data availability. Given the time and resource constraints faced by researchers, difficult and sometimes imperfect choices are often necessary.

4.2. Time-varying risk premia

As mentioned in the previous section, there is evidence today that asset returns are partly predictable, especially at longer horizons (Cochrane, 2001). Moreover, it is now acknowledged that asset price variations, once thought to be mainly due to the expectations about dividends, are instead the result of changes in the expectations of future returns. Therefore, changes in expected returns are the main drivers of asset returns and not changes in the expectations of future dividends. Furthermore, several variables such as the price/dividend ratio, the default spread, and the term spread, among others, have been shown to predict stock market returns well, and all these variables are related to the business cycle, suggesting that expected returns and risk premia vary over the business cycle, or more specifically that investors require higher risk premia to hold stocks in a recession and lower risk premia in a boom.

Berkman et al. (2011), for instance, find that crisis risk is positively correlated with variables such as dividend yield and the earnings-to-price ratio.

There is evidence that the dividend yield and, in general, other fundamentals-to-price ratios can predict returns in the long run (see Cochrane 2001). High price-dividend ratios (or interchangeably low dividend yield) are mainly related to lower expected excess returns and lower risk premia. The important conclusion is that a ratio such as price/dividend can help to predict future returns. When the price/dividend ratio is high, required returns are lower and expected future returns are lower, whereas when the price/dividend ratio is low, the required risk premia are higher and future returns are expected to be higher.

Fama and French (1988) show that past returns can predict future returns and that returns are negatively serially correlated, which means that positive past returns are followed, on average, by negative returns, and negative past returns are followed by a

series of positive future returns: this is the reversal anomaly. In summary, there is evidence that risk premia vary slowly over the business cycle and that the prices, and in particular price multiples, reveal market expectations of returns.

Ferson and Harvey (1991) study the relationship between the predictability of returns and changes in expected risk premia. Asset pricing models assume that expected returns or average returns should depend on an asset's sensitivity (beta) to risk factors. For each risk factor there is a risk premium, the market price for unit of beta, that is, the sensitivity to the factor. It follows that predictable variations in returns are due either to changes in the betas or to changes in the risk premia. Ferson and Harvey argue that most of the variation in the predictability of returns is due to changes in the risk premia and not betas. In particular, risk premia are higher in a recession when risk aversion and the marginal utility of wealth are high, and they are lower when the market is characterized by low risk aversion and a low marginal utility of wealth. The authors find that a large part of the predicted variation of expected returns is found to result from a change in the risk premia and not from a change in betas. This result has important implications for time-varying asset pricing models as it suggests that, more than time-varying betas, time-varying risk premia are the drivers of average returns. Therefore, the authors conclude that a constant beta model might not be a bad approximation of the sensitivity to systematic risk as long as the time-variation in risk premia is modelled. In particular, time-varying risk premia will be estimated in this thesis through the adoption of switching regime techniques.

4.3. Conditional asset pricing: a review of existing literature

In this section the literature concerning key conditional asset pricing models is reviewed. In general, conditional models vary with the solution used to model time-varying parameters and the choice of the parameters that are time-varying. One of the

most prominent conditional models is developed by Jagannathan and Wang (1996) (hereafter referred to as JW), who derive a conditional CAPM with human labour income and time-varying risk aversion, captured by introducing an additional beta with a time-varying risk premium, defined as a linear function of the default spread.

JW show that a conditional CAPM can be written as an unconditional model with two factors in which the average returns on an asset are a linear function of the average expected unconditional beta and a measure of beta instability (the time-variation of beta). Their conditional CAPM is thus derived as in Equation 4.1:

$$E[R_{it}|\Omega_{t-1}] = \gamma_{0t-1} + \gamma_{1t-1}\beta_{it-1} \quad (4.1)$$

where β_{it-1} is the conditional beta of asset i , γ_{0t-1} is the conditional expected return on a riskless portfolio, and γ_{1t-1} is the conditional market risk premium. Taking the unconditional expectation of the conditional CAPM, the model reduces to Equation 4.2:

$$E[R_{it}] = \gamma_0 + \gamma_1\bar{\beta}_i + Cov(\gamma_{1t-1}, \beta_{it-1}) \quad (4.2)$$

where γ_0 is the average risk-free rate, γ_1 is the expected market premium and $\bar{\beta}_i$ is the expected beta.

Equation 4.2 is very important as it shows that only when the covariance between the conditional beta and the conditional market risk premium is zero, does the conditional CAPM reduce to the unconditional version. Therefore, if the conditional CAPM holds, this does not imply that the unconditional version holds and this is the reason why the failure of the unconditional CAPM does not necessarily mean that the conditional CAPM cannot hold.

The market beta is decomposed into two orthogonal components: the expected beta $\bar{\beta}_i$ and the measure of beta instability $\vartheta_i(\gamma_{1t-1} - \gamma_1)$ which depends on the time-varying risk premium:

$$\beta_{it-1} = \bar{\beta}_i + \vartheta_i(\gamma_{1t-1} - \gamma_1) + \epsilon_{it-1} \quad (4.3)$$

where ϑ_i measures the sensitivity of the conditional beta to the market risk premium.

Assuming that the expected beta and the sensitivity of the beta to the market risk premium can only be inferred from the way that the asset returns react to the market returns and to changes of the market risk premium, the market beta and the premium beta or beta-instability risk can be defined as in Equations 4.4 and 4.5 respectively:

$$\beta_i = \frac{\text{Cov}(R_{it}, R_{mt})}{\text{Var}(R_{mt})} \quad (4.4)$$

$$\beta_i^\gamma = \frac{\text{Cov}(R_{it}, \gamma_{1t-1})}{\text{Var}(\gamma_{1t-1})} \quad (4.5)$$

and the conditional model is then written as an unconditional model with two orthogonal components:

$$E[R_{it}] = a_0 + a_1\beta_i + a_2\beta_i^\gamma \quad (4.6)$$

Equation 4.6 states that the conditional asset returns are a linear function of the conditional market beta and that the conditional model can be written as an unconditional model with two betas.

In the JW model, the conditional risk premium is derived as a linear function of the default premium, the difference between BAA- and AAA-rated bonds, which seems to be a sensible variable to measure the business cycle as the credit spread generally increases in a recession and when there is more risk aversion. Therefore, the conditional risk premium is modelled as a linear function of the credit spread:

$$\gamma_{1t-1} = \alpha_0 + \alpha_1 R_{t-1}^{prem} \quad (4.7)$$

where $R_{t-1}^{prem} = BAA_{t-1} - AAA_{t-1}$.

Hence, the prem-beta, or the sensitivity of beta to the market risk premium, is defined as in Equation 4.8:

$$\beta_i^{prem} = \frac{Cov(R_{it}, R_{t-1}^{prem})}{Var(R_{t-1}^{prem})} \quad (4.8)$$

JW also include a measure of the return on human capital in the model, obtained as the growth rate in the per capita labour income:

$$R_t^{labor} = \frac{L_t - L_{t-1}}{L_{t-1}} \quad (4.9)$$

and with the labour-beta defined as in Equation 4.10

$$\beta_i^{labor} = \frac{Cov(R_{it}, R_t^{labor})}{Var(R_t^{labor})} \quad (4.10)$$

the Premium-Labour model, the conditional CAPM of JW obtains as in Equation 4.11:

$$E[R_{it}] = \alpha_0 + \alpha_1 \beta_i^{vw} + \alpha_2 \beta_i^{prem} + \alpha_3 \beta_i^{labor} \quad (4.11)$$

Therefore, the unconditional expected return on any asset i is a linear function of its value-weighted beta (market beta), the prem-beta (time-varying beta), and the labour-beta (covariance with the labour income return), where the combination of the value-weighted beta and the prem-beta forms the market beta.

JW test and compare three models on 100 portfolios of stocks sorted first on size and then on pre-ranking beta: (i) the static CAPM; (ii) the conditional CAPM without human capital; and (iii) the conditional CAPM with human capital, the so-called P-L model.

The models are estimated both with the cross-sectional regression procedure of Fama and MacBeth and with GMM, and are then compared in terms of R-squared and Hansen-Jagannathan distance. The first model to be estimated is the static CAPM:

$$E[R_{it}] = \alpha_0 + \alpha_{vw} \beta_i^{vw} \quad (4.12)$$

The R-squared of the model in the cross-sectional procedure is found to be very low, at 1%, and the market premium associated with the beta is slightly negative and

insignificant. Even when the model is estimated with the GMM, the model is firmly rejected, indicating that the pricing error is significantly different from zero. Interestingly, when the variable size, defined as the logarithm of market capitalization, $\ln(ME_i)$, is added to the model, the R-squared improves to 57% and the coefficient of size is found to be negative and statistically significant, thus confirming the role played by size in the cross-section of average returns as outlined by Banz (1981).

The second model to be tested is the conditional CAPM with time-varying betas, but without human capital:

$$E[R_{it}] = \alpha_0 + \alpha_{vw}\beta_i^{vw} + \alpha_{prem}\beta_i^{prem} \quad (4.13)$$

The R-squared improves to 29%, the premium for time-varying risk is positive and significant, but the model is still rejected when estimated with GMM.

Finally, in the P-L model, the conditional CAPM with human capital, is tested:

$$E[R_{it}] = \alpha_0 + \alpha_{vw}\beta_i^{vw} + \alpha_{prem}\beta_i^{prem} + \alpha_{labor}\beta_i^{labor} \quad (4.14)$$

The R-squared is 55% and the market premium associated with the labour-beta is significant and positive. When the model is estimated using the GMM, the HJ distance is 0.62 with a p -value of 19%, and the model is not rejected by the data. Most interestingly, when the variable size is added to the model, it is found to be insignificant. The model appears, therefore, to capture the size effect. However, some of the results are less comforting as the premium associated with the value-weighted beta is negative and the intercept is positive and significant. Therefore, even if the model performs better than the static version of the CAPM, the results show that some restrictions on the coefficients are required to test the economic significance of the model, and that there is some factor not captured by the model.

JW also add the two Fama and French factors (SMB and HML) to the P-L model and notice that neither of them is statistically significant, whereas the premia associated with the instability of beta and with beta-labour are significant. This result leads them, perhaps optimistically, to suggest that HML and SMB might be a proxy for the risk associated with the return on human capital and beta instability. In summary, the JW study shows that a conditional model is needed to capture the dynamics of risk premia and the time-variation of beta. Moreover, the results lead the researcher to understand that the size effect is related to the dynamics of the risk premia.

The main problem for any conditional model is that in order to derive the time-varying parameters, certain assumptions are required concerning the way risk premia change over the business cycle and concerning the choice of the set of conditional variables. The results of the test might be significantly affected by the variable chosen as a proxy for the dynamics of the risk premium.

Arguably, the most powerful economics-based conditional model has been proposed by Lettau and Ludvigson (2001) (hereafter referred to as LL). LL assume that the stochastic discount factor used to discount future payoffs is a linear function of factors with time-varying parameters, and that the conditional discount factor which prices the assets depends on the conditional covariance of asset returns with the risk factors. One way to model this conditional relationship is to derive the conditional model as an unconditional model in which the payoffs have been rescaled using some instrumental variables to account for conditioning information. The advantage of this methodology is that it avoids the complicated derivation of the conditional moments (Cochrane, 2001).

LL suggest using the consumption to aggregate wealth ratio, cay , as an instrumental variable to describe the state of the economy and predict future returns. In the literature other variables are used to predict future returns such as the dividend-price ratio.

However, LL argue that there are at least three reasons why *cay* should be preferred: (i) whereas the dividend-price ratio focuses only on the stocks component of wealth, *cay* refers to a more comprehensive concept of aggregate wealth; (ii) consumption can be seen as the stream of cash flow coming from wealth in the same way as dividends can be thought of as the cash flow paid by stocks; and (iii) the choice for *cay* is supported by the findings of Jagannathan and Wang (1996) that human capital in addition to the market portfolio can improve the explanatory power of the CAPM.

The model is obtained in the stochastic discount factor (SDF) approach as an unconditional model in which the factors are rescaled by the instrumental variables. Starting with the usual moment condition:

$$1 = E_t[m_{t+1}(1 + R_{i,t+1})] \quad (4.15)$$

and assuming that the discount factor is a linear function of the risk factor (market portfolio return):

$$m_{t+1} = a_t + b_t R_{m,t+1} \quad (4.16)$$

a set of instrumental variables, z_t , is chosen to scale the factors in the model in order to capture the time-variation of risk aversion:

$$a_t = a_0 + a_1 z_t \quad \text{and} \quad b_t = b_0 + b_1 z_t \quad (4.17)$$

By substitution, the stochastic discount factor is given by Equation 4.18:

$$m_{t+1} = (a_0 + a_1 z_t) + (b_0 + b_1 z_t) R_{m,t+1} \quad (4.18)$$

Therefore, the conditional model can finally be written as an unconditional multifactor model:

$$1 = E[(a_0 + a_1 z_t) + (b_0 + b_1 z_t) R_{m,t+1}](1 + R_{i,t+1}) \quad (4.19)$$

In the model, the payoffs are scaled by the instrumental variable, cay . If the consumption-to-wealth ratio is high, investors expect either high returns on wealth or low consumption growth in the future. Therefore, the insight is that when cay is low, excess returns are expected to fall, and when cay is high, excess returns are expected to rise. LL show that cay really helps to predict future returns as when the returns of the stock market index are regressed on the lagged instrumental variable, cay , the R-squared increases with the horizon, reaching 18% at four quarters.

The final model obtained by rescaling the factors is given in Equation 4.20:

$$E(R_{i,t+1}) = E(R_{0,t}) + \beta_{zi}\lambda_z + \beta_m\lambda_m + \beta_{mzi}\lambda_{mz} \quad (4.20)$$

where $\lambda_{m,t+1} = R_{m,t+1}$, and the instrumental variable is $z_t = \widehat{cay}_t$.

It is worth noting that as the number of instrumental variables increases, the number of factors unavoidably increases as well. Therefore, it is critical in such a methodology to identify with parsimony the predictive variables. In their paper, LL test the unconditional version of the model in its beta representation:

$$E(R_{i,t}) = \alpha + \beta_1 \cdot R_{m,t} + \beta_2 \cdot R_{m,t} \cdot cay_{t-1} \quad (4.21)$$

The conditional CAPM is tested using portfolios of stocks sorted on size and the book-to-market ratio, and the estimated risk premium associated with beta in the unconditional version is found to be insignificant and negative with an extremely low R-squared, confirming the poor performance of the CAPM at explaining the cross section of average returns of portfolios of stocks sorted on size and the book-to-market ratio. When the test is conducted on the conditional model, the time-varying component of the intercept is found not to be statistically different from zero, whereas the risk premia associated with the market return and the time-varying component of the factor are significant when jointly considered. The R-squared increases from 1% to 31% and

the R-squared reaches 75% when the human capital growth variable is added to the model. The conclusion of LL is that the risk of an asset depends on the conditional correlation of asset returns with market portfolio returns. It must be noted however that LL use quarterly rather than monthly data, and that the derivation of the variable *cay* is quite complicated and the result of an econometric estimation.

More recently, Bauer, Cosemans and Scotchman (2008), hereafter referred to as BCS, investigate the performance of a conditional three-factor model compared to its unconditional version. BCS model the time variation in betas as a linear function of instrumental variables following the methodology of Jagannathan and Wang. The betas are modelled as a linear function of instrumental variables which capture the risk aversion and help to predict future returns:

$$\beta_{it} = \gamma_{i0} + \gamma_{i1}Z_{it} \quad (4.22)$$

where Z_{it} are the instrumental variables used to capture the time variation. Therefore the conditional three-factor model becomes:

$$R_{it+1} = \alpha_{i0} + (\gamma_{i0} + \gamma_{i1}Z_{it})FF_{t+1} + \varepsilon_{it+1} \quad (4.23)$$

where FF_{t+1} are the three factors used by Fama and French.

The unconditional and conditional models are tested over 25 portfolios of pan-European stocks, sorted on size and the book-to-market ratio using time-series regressions for each of the 25 size-book-to-market portfolios excess returns on the three factors:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \delta_i SMB_t + \varphi_i HML_t + \varepsilon_{it} \quad (4.24)$$

The instrumental variables are: default spread (difference of yield between BAA- and AAA corporate bonds), size, the book-to-market ratio, and the interaction between the default spread, size and the book-to-market ratio. Interestingly, when time series regressions are conducted for each portfolio on the conditional factors, the R-squared

increases with time-varying betas, and the null hypothesis that betas are time-varying, that is, the coefficients of the interaction term between instrumental variables and factors are significant, is accepted.

In a second stage, a cross-sectional regression of the portfolio net returns (monthly returns minus risk-adjusted returns) is conducted to estimate whether the coefficients of a set of additional variables such as size, book-to-market, and momentum factors are significant. In these cross-sectional regressions the instrumental variables should be insignificant. The results show that the intercept and size become insignificantly different from zero, whereas the momentum effect remains significant. Theoretically, incorporating some conditioning information should make variables such as size, book-to-market and momentum insignificant if these anomalies are mainly due to investors relying on these variables in the real world.

The conclusion is that a conditional version of the three-factor model, when betas are allowed to vary over the business cycle as a linear function of instrumental variables, outperforms the static unconditional version and can explain the size and the book-to-market effects, but not momentum.

Bodurtha and Nelson (1991), hereafter referred to as BN, derive a conditional CAPM in which the expected risk premium, the variance, and the covariance are all time-varying. BN model the conditional variance of the market return and covariances using an autoregressive conditional heteroscedastic model with three lags, assuming that past observations may help to predict future variance and covariance, and an expected risk premium using an autoregressive process with three lags:

$$\begin{cases} E(u_{mt}^2 | J_{t-1}) = \gamma_0 + \sum_{j=1}^3 \gamma_j u_{mt-j}^2 \\ E(u_{it} u_{mt} | J_{t-1}) = \alpha_{i0} + \sum_{j=1}^3 \alpha_{ij} u_{it-j} u_{mt-j} \\ E(r_{mt} | J_{t-1}) = \pi_0 + \sum_{j=1}^3 \pi_j r_{mt-j} \end{cases} \quad (4.25)$$

They then test the conditional CAPM over five size portfolios of US stocks over the period 1926-1985 with the GMM. The results show that the CAPM with a constant beta is rejected and that the null hypothesis that the market risk premium is constant is also rejected.

One of the natural barriers to the CAPM is that the covariance matrix of asset returns is time-varying. Bollerslev, Engle and Wooldridge (1988), hereafter BEW, estimate a CAPM with time-varying covariances using a multivariate GARCH for T-bills, bonds and stocks. The conditional covariance matrix is allowed to vary over time following a GARCH process, where the mean and covariance expectations are updated every period on the basis of the last change in returns. The model assumes that the expected returns depend on the conditional covariances with the market portfolio (Equation 4.26) and that the covariances depend on their own past values and shocks (Equation 4.27):

$$y_t = b_i + \delta \sum_j \omega_{j,t} h_{ij,t} + \varepsilon_{i,t} \quad (4.26)$$

$$h_{ij,t} = \gamma_{ij} + \alpha_{ij} \varepsilon_{i,t-1} \varepsilon_{j,t-1} + \beta_{ij} h_{ij,t-1} \quad \text{for } i, j = 1, \dots, N \quad (4.27)$$

The model is tested over the period 1959-1984 for quarterly returns and the results show a positive and significant risk premium for the market covariance, supporting the conditional covariance in preference to the unconditional covariance as a better measure of risk. In summary, the findings suggest that covariance is time varying and that risk is best approximated by conditional covariance rather than unconditional covariance. Conditional covariance is in fact positively priced in the test conducted by BEW.

4.4. Conditional CAPM with Dynamic Conditional Correlations

From the discussion above, it is evident that one of the major problems in testing a conditional CAPM is the modelling of time-varying conditional betas. Whereas tests of the unconditional CAPM with a constant beta produce an insignificant and often

negative risk premium, the conditional CAPM might hold if the time-varying betas are highly correlated with the risk premium. In such a case, the time variation might explain the anomalies faced by the unconditional CAPM. For instance, if small stocks had a higher beta when investors require higher returns, the size anomaly might be explained. One of the most important studies on the conditional CAPM is Bali and Engle (2008), hereafter referred to as BE. They find that a conditional CAPM, where the conditional covariances are obtained from a multivariate GARCH-in-mean model, and with dynamic conditional correlation (DCC), succeeds in explaining the cross-sectional average return of size, book-to-market and industry portfolios. The study concerns the conditional version of the CAPM and ICAPM of Merton (1973), where the conditional CAPM simply states that the expected excess return on a risky asset depends on its conditional time-varying covariance with the market portfolio excess return:

$$E[R_{i,t+1}|\Omega_t] = \frac{E[R_{m,t+1}|\Omega_t]}{\text{var}[R_{m,t+1}|\Omega_t]} \cdot \text{Cov}[R_{i,t+1}, R_{m,t+1}|\Omega_t] \quad (4.28)$$

where the expected variance and covariances at time $t+1$ are time-varying, conditional on the set of information available at time t . BE investigate the risk-return trade-off for the CAPM with time-varying covariances. The conditional version of the CAPM is therefore given by:

$$E[R_{i,t+1}|\Omega_t] = \alpha_i + \theta \cdot \text{Cov}[R_{i,t+1}, R_{m,t+1}|\Omega_t] \quad (4.29)$$

and the conditions in order to test the theory are that: (i) the covariance risk premium θ is positive and constant across portfolios or assets; and (ii) all of the alphas are jointly not different from zero.

The model of BE is innovative as: (i) they use a cross section of time series for equity portfolios; and (ii) the conditional covariances are calculated using a multivariate GARCH with dynamic conditional correlations. BE find that both conditions are

necessary to obtain a positive estimate of the risk premium. The model is tested for four groups of ten portfolios formed on size, book-to-market, momentum, and industry over the period 1926-2009, and the hypothesis that all the intercepts are equal to zero is tested. The results show a significant positive coefficient of risk aversion for all of the four groups of portfolios and the null hypothesis of the CAPM for size, book-to-market, and industry portfolios is not rejected (the alphas are not significantly different from zero and the risk premium is positive). BE also show that when the CAPM is tested unconditionally on a cross-section of portfolios, the estimated market premium is insignificant and for the industry portfolios even becomes negative. Moreover, the null hypothesis of jointly zero intercepts is rejected. Therefore, the GARCH model with DCC produces time-varying conditional betas that covary significantly with the market risk premium to explain most of the asset pricing anomalies (momentum excluded). This thesis uses also the other higher moments (coskewness and cokurtosis) and introduces a time-varying risk premium according to the market regime, extending the model of Bali and Engle.

4.5. Conditional models and time-varying betas

Another interesting path of research is represented by the inclusion of a learning Bayesian process in the conditional models. Franzoni and Adrian (2005), hereafter referred to as FA, start from the observation that the market betas of value and small portfolios have strongly decreased over time, whereas the market betas of growth and large companies have increased. FA argue that investors infer the factor loadings from the history of the betas according to a Bayesian process, that is, they gradually revise their expected beta on the basis of the most recent information available, considering also the long-run betas, and that this expected beta, resulting from the learning process, determines the expected returns. The learning process leads to an expected ex-ante beta that might be different from the ex-post realized beta. This is particularly true for

portfolios of value and small stocks for which changes in beta have historically been highly volatile. Thus, the higher average returns of value and small stocks that have experienced decreasing betas might be the result of high expected betas resulting from a learning process in which the high betas of the past are still taken into consideration by investors who only gradually revise their expectations in the light of the recent realized beta.

FA test the model on 25 portfolios formed according to size and the book-to-market ratio over the period 1963-2004, but allow investors to start learning about the long-run beta from 1926. The results show that the betas of value stocks increase in particular in a recession. Therefore one explanation for the value premium is the increasing riskiness of value stocks in a recession, an explanation consistent with the distress-related rational explanation given by Fama and French. The estimated expected beta and risk premium show that the estimated betas are higher than the realized betas for value and small stocks because of the high betas of the past. It is however worth pointing out that one of the weaknesses of this methodology is the fact that the high estimated betas for value and small stocks are obtained assuming that the learning process starts from 1926. In other words, the results are sensitive to the starting point chosen for the Bayesian learning process. Furthermore, in the 1940s betas were at their highest, and in addition, the assumption of mean reversion is crucial to producing a high estimate of beta even when the realized betas are lower.

Ang and Chen (2007), hereafter referred to as AC, argue that the unconditional CAPM cannot explain the book-to-market anomaly for the period 1963 to 2001, but that looking at the long period 1929-2001 there is no statistical evidence of any anomaly. This is due to the difference between the first period 1929-1963 and the second period 1963-2001. The former period is characterized by high betas for value stocks which can rationally explain the cross-section of average returns for book-to-market sorted

portfolios, whereas the latter period is characterized by much lower betas for the value portfolios which cannot explain their high average return. Therefore, the book-to-market anomaly is limited to this second period, whereas over the long run the unconditional CAPM can explain the cross-section of average returns.

AC introduce a conditional CAPM model with conditional betas, time-varying market risk premia, and stochastic systematic volatility. In this model the conditional betas follow an endogenous AR(1) latent process, the market risk premium follows a mean-reverting latent process, and the market excess return has a conditional market risk premium and stochastic systematic volatility, i.e. it follows a Brownian motion.

The conditional betas of value stocks obtained by Ang and Chen are estimated to vary from 0.5 to 3.00, whereas the growth stocks betas are close to 1. Therefore, the conditional betas are highly time-varying and, being positively correlated with the market risk premium, they can explain the book-to-market anomaly during the period 1963-2001.

Unlike Jagannathan and Wang (1996), and Lettau and Ludvigson (2001), who argue that betas change over the business cycle in a way that might explain the cross-section of average returns, Lewellen and Nagel (2006) are sceptical on the role of conditional models to explain asset pricing anomalies. They argue that although a conditional CAPM might hold even when the unconditional version does not, in order to explain the large alphas on small, value and winner momentum portfolios, conditional time-varying betas should vary much more than empirically observed, and should covary positively with the expected risk premium or market volatility. Using a short window time-series regression approach which allows the researcher to bypass the problem of identifying the conditioning variables, LN show that the conditional betas, although time-varying,

are not large enough and are not so positively correlated with the expected risk premium as to explain the large pricing errors of the unconditional CAPM.

Instead of using cross sectional regressions, they use time series regressions. Under the assumption that beta does not change much over short periods of time, short window time series regressions (quarterly) are used to estimate the conditional beta and conditional alpha of portfolios long on small stocks and short on large stocks (SMB), portfolios long on high book-to-market stocks and short on book-to-market stocks (HML), and portfolios long on winner momentum stocks and short on loser momentum stocks (mom). Should the CAPM hold conditionally, the conditional alphas should on average be insignificant. They find that this is not the case, and that moreover the conditional alphas are not much different from the unconditional alphas.

In summary, the betas simply do not vary enough over time to explain the large alphas when the betas are estimated with short windows time series regressions. However, although their methodology is very interesting, it must be noted that they focus on time series regressions to assess the role of a time-varying beta, whereas in this thesis the interest is mainly in explaining the cross-sectional differences in returns. Further, they use realized returns, not an expected risk premium and this might be the reason why they find a negative correlation between conditional betas and the market premium.

Avramov and Chordia (2005) test whether conditional forms of the CAPM can explain market anomalies, allowing beta to vary with stock characteristics such as market capitalization, book-to-market, but also macroeconomic variables related to the business cycle. The authors use cross-sectional regressions of risk-adjusted returns of individual stocks, as opposed to gross returns, over market characteristics and momentum. Whereas the factor loadings are time-varying in the first regression, the risk adjusted returns are regressed at cross section on size, book-to-market, momentum and these

variables should be insignificant in the second step regression, that is they should not have any residual information once the time-variation of beta has been adequately captured.

Specifically, assuming that returns are generated by a conditional version of a k -factor model:

$$R_{it} = E_{t-1}(R_{it}) + \sum_{k=1}^K \beta_{ikt-1} f_{kt} + \varepsilon_{it} \quad (4.30)$$

where $E_{t-1}(R_{it})$ is the conditional expectation of returns of stock i at time t , R_{it} , beta is the conditional sensitivity to the factors, and f_{kt} are the risk factors at time t .

The expected return is modelled as:

$$E_{t-1}(R_{it}) = R_{ft} + \sum_{k=1}^K \beta_{ikt-1} \lambda_{kt-1} \quad (4.31)$$

where λ_{kt-1} is the risk premium for factor k .

The risk-adjusted return is therefore given by:

$$R_{it}^* = R_{it} - R_{ft} - \sum_{k=1}^K \hat{\beta}_{ikt-1} F_{kt} \quad (4.32)$$

where beta has been estimated in the time series regression and $F_{kt} = f_{kt} + \lambda_{kt-1}$ is the sum of the factor innovation and factor risk premium.

In the cross-sectional regression, the adjusted returns are regressed on a firm's characteristics:

$$R_{it}^* = c_{ot} + \sum_{m=1}^M c_{mt} Z_{mit-1} F_{kt} + \varepsilon_{it} \quad (4.33)$$

where Z_{mit-1} is the value of characteristic m for security i at time $t-1$.

Under exact pricing, the vector of characteristics should be insignificant. This hypothesis is tested using the Fama and MacBeth (1973) test.

Several models, from the CAPM to the three-factor model of Fama and French and the liquidity model of Pastor and Stambaugh, are tested on individual assets from 1964 to 2001. The findings show that the conditional three-factor model of Fama and French is the model that provides the best results in terms of explanation of size and book-to-market. Even if time-varying factor loadings improve the explanatory power of the asset pricing models, momentum is never explained or captured. Interestingly, the variable momentum loses significance when the risk-adjusted returns are purged of a time-varying alpha related to the business cycle. The problem is that this time-varying risk factor is unobservable or not identified. The unscaled or conditional CAPM performs very poorly whereas the conditional three-factor model of Fama and French can capture size and book-to-market characteristics when the factor loadings are conditioned on size, book-to-market, and the default spread. However, it can be argued that there is no risk factor based reason for conditioning the beta on size or book-to-market. Most of the positive results seem to be due to the time-variation and business cycle relation between time-varying beta and time-varying SMB.

Jostova and Philipov (2004) propose a stochastic and mean reverting beta and show that modelling beta in this way outperforms GARCH, rolling regressions, constant beta and other conditional specifications of beta. Beta so obtained has some favourable properties such as persistence, randomness and time-variation. The stochastic beta is obtained by Bayesian rules in line with the argument that investors adapt to the new information through learning and the parameters are obtained from a Markov Chain Monte Carlo approach which provides exact finite sample inference without problems of lack of convergence. The use of a stochastic beta in the cross-section of individual asset returns

allows the estimation of a positive risk premium and 23% of the cross-sectional variation in asset returns. Beta is assumed to be mean reverting around the unconditional beta α_p :

The return of any asset p is obtained as in a single factor model:

$$R_{pt} = \beta_{pt}R_{Mt} + \sigma_p \varepsilon_{pt} \quad (4.34)$$

and beta follows a mean-reverting process AR(1):

$$\beta_{pt} = \alpha_p + \delta_p(\beta_{pt-1} - \alpha_p) + \sigma_{\beta_p} v_t \quad (4.35)$$

where σ_p and σ_{β_p} are, respectively, the standard deviation of the portfolio return and of the conditional beta and ε_{pt} and v_t are the stochastic components.

They use individual US stocks from 1964 to 2003 and show that a stochastic beta can explain the cross-section of equity returns and make stock characteristics insignificant, unlike a GARCH beta, a firm-characteristic conditional beta, rolling regression betas, or macroeconomic conditional betas.

Morana (2009) implements a conditional CAPM as in Jagannathan and Wang (1996) with realized betas using daily data over the period 1965-2005, and shows that his model can explain 63% of the cross-sectional variation of returns of the 25 size/BM portfolios of Fama and French. Morana estimates the realized beta at high frequency (daily) given that some literature (Andersen et al., 2003, for example) suggests that high frequency data are more informative for the estimation of higher moments and therefore also of beta given that this is the ratio of covariance to variance. Realized covariances and variances at high frequency tend to be a good estimator of integrated covariances and variances. By considering jointly the hypothesis of the zero alpha of Jensen and the cross-sectional coefficient R-squared, Morana suggests that the JW model with realized

beta outperforms the CAPM and the F&F models. However, it must be noted that the market premium is found to be negative.

Andersen et al. (2003) argue for a conditional CAPM in which beta is obtained as a high-frequency realized beta inheriting the time-variation and persistence characteristics of variance and covariances as statistically observed. The main point is that at high frequency the estimate of the higher moments will be more precise or consistent to the true integrated beta as the frequency tends to zero and beta appears to be less volatile and less persistent than the covariances and variance used in the ratio.

In particular, by the theory of quadratic variation, the sum of the squared of intra-period returns approximates closely to the integrated volatility as the frequency becomes higher.

Therefore the estimated or realized market variance for the h -period is:

$$\hat{\sigma}_{M,t,t+h}^2 = \sum_{j=1}^{[h/\Delta]} r_{M,t+j\Delta}^2 \quad (4.36)$$

and the estimated or realized covariance of individual asset i with the market:

$$\hat{\sigma}_{iM,t,t+h} = \sum_{j=1}^{[h/\Delta]} r_{i,t+j\Delta} r_{M,t+j\Delta} \quad (4.37)$$

The realized beta is the simple ratio:

$$\hat{\beta}_{i,t,t+h} = \frac{\hat{\sigma}_{iM,t,t+h}}{\hat{\sigma}_{M,t,t+h}^2} \quad (4.38)$$

and this beta converges to the true beta for high frequency.

4.6. Regime switching and asset pricing

In recent years, the literature on asset pricing and, in particular, asset allocation, and the decision of how much weight to give to different categories of assets in the investment portfolio, has been strengthened by the introduction of switching regimes, for instance Ang and Bekaert (2004). The distribution of returns seems to follow a different stochastic process characterized by different moments across different regimes. This finding suggests the possibility of using switching regimes in conditional asset pricing models.

Guidolin and Timmermann (2002), hereafter referred as GT, identify two states or regimes for the UK stock market: (i) a bull regime with low volatility and high average returns; and (ii) a bear regime with high volatility and low average returns. GT suggest that investors are affected in their decision regarding what type of stocks to hold in their portfolio, and in what proportion, by their beliefs regarding the state of the economy. After all, it may be reasonable to assume that investors require a time-varying risk premium according to the states of the economy and investment opportunities, the latter of which also depend on their investment horizon. In fact, GT notice that changing regimes are less relevant for long-horizon investors and much more relevant for short-term investment decisions.

GT find that when the probability of being in a bear regime is perceived to be high, investors require higher risk premia, whereas when the probability of a prolonged bull regime is perceived to be high, investors require lower compensation for taking the risk. Interestingly, GT introduce not only regimes in the investment decision but also a predictive variable of the underlying state of the economy: the dividend yield. In fact, low dividend yields anticipate lower returns and are associated with a lower proportion of stocks in portfolios, whereas periods with high dividend yields anticipate higher returns and are associated with a larger proportion of investment in stocks.

The conclusion that can be drawn from the work of GT is that risk premia change over time with regimes and with the beliefs regarding the state of the economy. Interestingly, not only can the market risk premium be explained by the sensitivity of stock investments to regimes, but also cross-sectional average returns might be explained by the different sensitivity of categories of stocks to the regimes and to the predicting variables.

4.7. Conclusion

This chapter discusses a further important extension of the CAPM that involves conditional models. The assumption that beta and the risk premium required by investors are constant is a strong one. Investors seem to require different returns at different stages in the business cycle (higher in the trough and lower in the boom phase). Returns appear to be predictable and certain variables such as price multiples, the default spread, the term spread, among others, have some predictive power. Therefore, investors might actually consider some conditioning variables when building their expectations and taking their investment decisions. Conditional models commence with the idea that risk aversion is time-varying and that the market beta is in reality time-varying. If the sensitivity of an asset's returns to the risk factor increases when the risk premium demanded by investors is higher (typically in downturns of the market), this high correlation might explain why some stocks earn higher average returns in spite of lower unconditional betas.

The main problem of conditioning a model is how to model the time-variation of beta. Jagannathan and Wang (1996) introduce a conditional CAPM in which the time-variation of beta is obtained as a linear function of the default spread. Their model captures well the size anomaly. Lettau and Ludvigson (2001) condition the CAPM to an economic variable, *cay*, the consumption to wealth ratio. Their model seems to be able to explain quite well the cross-section of average quarterly returns for portfolios of

stocks sorted on size and book-to-market. Another solution to condition the static model requires scaling the asset returns by the conditioning variables in the stochastic discount factor framework or using managed portfolios. Dittmar (2002) uses a conditional Four-Moment CAPM in the stochastic discount factor, whereas Aretz et al. (2007) use managed portfolios and test them together with the unmanaged portfolios (for a discussion of this methodology see Cochrane, 2001, pp.133).

A further solution is to derive the conditional beta using dynamic conditional correlations and the multivariate GARCH. Engle and Bali (2008) use this methodology and show that portfolios of stocks sorted on size, the book-to-market ratio and other variables, excluding momentum, have average returns that can be explained by their conditional CAPM. The main criticism of conditional models is that some conditioning variables have to be identified to derive time-variation. Unfortunately, the results seem to depend on the variables used, whilst conditional models are often more successful as they have many variables to estimate, thereby giving rise to a problem of over-fitting. In the next chapter the methodology used in asset pricing tests in the existing literature, and in this thesis in particular, will be discussed.

Chapter 5

Methodology

5.0. Introduction

This chapter focuses on how to estimate the parameters of asset pricing models and how to test the models themselves. In particular, the methodology applied in the literature and in this research to test the unconditional and conditional asset pricing models under investigation is introduced.

This chapter starts with the description of the two basic types of asset pricing tests: time series regressions and cross-sectional regressions. Time series regression tests focus on the ability of the model to explain the historical variability of returns and a particularly important test for time series is the Gibbons, Ross, and Shanken test (1989), henceforth GRS. Asset pricing tests based on the cross sectional variation in returns try to explain what risk factors explain the difference in returns across stocks, and in particular two such tests are described: the cross-sectional regression of Black, Jensen, and Scholes (1972) and the Fama and MacBeth (1973) methodology with rolling regressions. The difference between unconditional and conditional asset pricing models is then introduced and the chapter will then focus on the time-varying beta and the techniques used in this research to derive time-varying parameters: including: (i) the multivariate GARCH with DCC following Engle (2002) for the time-varying factor loadings, and (ii) Markov switching regimes for the time-varying risk premia.

The last section of the chapter introduces the empirical tests of the CAPM and the Four-Moment CAPM using individual US assets. Individual assets show a larger dispersion in betas and this might be the reason why the estimates of the risk premia can be more precise. In particular a short-window regression methodology will be used, in which the factor loadings will be estimated through short-window time-series regressions of 24

months and the expected returns (average return of each window) will be regressed at cross section over the factor loadings to estimate the risk premia.

5.1. The time series regression methodology for modelling equity returns

Time-series regressions can be used in asset pricing for tradable factors, i.e. portfolios containing tradable assets and expressed in the form of returns. In time-series tests, monthly excess returns are regressed on the factors (market portfolio excess return in the CAPM) to estimate the intercept and the slope of the assets, and a model is then judged in relation to how well it describes the historical variability of returns, in other words, how small and insignificant the residuals are.

If the assets tested are all expressed in excess returns and the factors are excess returns as well, such as in the case of the CAPM and the three-factor model, it is possible to conduct time-series regressions of the asset's excess returns on the factors f :

$$R_{it} - R_{ft} = \alpha_i + \beta' f_{it} + \varepsilon_{it} \quad t = 1, 2, \dots, T \quad i = 1, 2, \dots, N \quad (5.1)$$

where β' is a vector of slopes, and f_{it} is a vector of k factors. The estimates of the factor risk premiums in a time-series regression are simply the sample mean of the factors themselves. For example, the market risk premium is simply the average return for the market in excess of the risk free rate:

$$E[\hat{\lambda}] = \overline{R_M - R_f}$$

In a time series regression the main implication of the asset pricing models is that the intercepts are zero. Thus, all the N intercepts should be jointly insignificantly different from zero. In other words, the alphas should be zero and should not be related to any attributes of a stock. Alternatively, all the variability of returns should be explained by beta.

If portfolios based on characteristics such as the P/E ratio, market capitalization or B/M ratio are added to the market beta on the right hand side of the time series regression, their coefficients (betas) should be found to be insignificant. The inclusion of additional variables on the right hand side of the regression is often obtained using tradable portfolios of stocks which are long on one attribute (high P/E ratio) and short on the opposite of this attribute (low P/E ratio). These portfolios are thought to mimic some risk factors and are therefore also known as mimicking portfolios. There are numerous examples of tests in the literature based on this methodology. In particular, the three-factor model of Fama and French (1993) is based on time series regressions. More recently, many tests based on Generalised Method of Moments such as Jagannathan and Wang (2002), Cochrane (2005), Lim (1989) and Dittmar (2002) among others, are based on time series approach.

Another way to test the CAPM using a time series approach, and to indirectly examine whether additional attributes can add explanation to returns, requires the building of portfolios of stocks according to one attribute (P/E ratio as in Basu, 1977, for instance) and then conducting a time series regression of the portfolio returns on the portfolio beta: here the unexplained return (alpha) should be insignificant. Moreover, the intercepts of all of the assets should be jointly zero. This implication can be tested using the Gibbons, Ross and Shanken test. For instance, if portfolios of stocks with small capitalization have a positive alpha, this is evidence against the CAPM, because the extra return should be explained by beta alone.

5.2. The cross-sectional regression methodology

The second type of asset pricing test is conducted in the cross-section. Assuming that some factors such as the market beta can explain the difference in average returns across assets, as the CAPM does, one can run a cross-sectional regression of the average

excess returns on the estimated betas (or sensitivities/loadings) of the relevant factors (with only one beta if the model is the CAPM) to see whether they are significantly priced or rewarded by the market:

$$E(R_i) = \alpha_i + \hat{\beta}_i' \lambda \quad i = 1, 2, \dots, N \quad (5.2)$$

The parameters $\lambda = (\lambda_1, \dots, \lambda_k)'$ are estimated via OLS.

The implications of the asset pricing models here focus on the significance and the sign of the risk premiums $\hat{\lambda}$, and on the hypothesis that the pricing errors are zero. The first implication is tested with a t-test. The second implication (i.e. the intercepts) can be tested using a t-test as in Fama and MacBeth (or Black, Jensen and Scholes) and involves testing the time average of the intercepts (or a single intercept in BJS). Alternatively, one can test the zero intercept implication through the following statistic (see Cochrane 2001, pp. 237):

$$\hat{\alpha}' \text{cov}(\hat{\alpha})^{-1} \hat{\alpha} \sim \chi_{N-1}^2 \quad (5.3)$$

The CAPM states that only differences in betas can explain differences in returns across stocks or portfolio of stocks and that no additional explanatory power can be gained by adding other variables. If characteristics such as the P/E ratio, size or the B/M ratio are added to the market beta then their coefficients should not be significant.

Examples of such tests using characteristics which found anomalies include Banz (1981) who included size, Bandhari (1988) who introduced leverage on the right hand side of the regression, and Rosenberg et al. (1985) who use the B/M ratio. Fama and French (1992) extensively employed a cross-sectional regression and concluded that size and book-to-market are the best explanatory variables for asset returns.

5.3. The Gibbons, Ross and Shanken (1989) (GRS) test

Although this is essentially a time series test, it is included separately because of its importance in the literature. Time series regressions are first estimated for each asset or portfolio and then the null hypothesis that the intercepts are jointly different from zero is tested. Since the CAPM states that the expected excess return of any asset i is linear in beta:

$$E(R_i) = \beta_i E(R_M) \quad (5.4)$$

realized excess returns can be used to test the model in its time series form:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \quad \text{for } i = 1 \text{ to } N \quad (5.5)$$

The implication of the CAPM is that all intercepts α_i , which represent the pricing errors for every single portfolio, are zero. In other words the following hypothesis is tested:

$$H_0: \alpha_1 = \alpha_2 = \dots = \alpha_N = 0$$

The hypothesis that all the pricing errors (all the alphas) are jointly equal to zero is obtained under the assumption of no autocorrelation and no heteroskedasticity, scaling the $\hat{\alpha}$ regression coefficients by their variance-covariance matrix $\hat{\Sigma}$ as follows:

$$T \left[1 + \left(\frac{E(f)}{\sigma(f)} \right)^2 \right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim \chi_N^2 \quad (5.6)$$

where $f_t = R_{mt} - R_{ft}$, $E(f)$ is the sample mean of the factor and $\sigma(f)$ is the sample standard deviation of the factor, and therefore the ratio $\frac{E(f)}{\sigma(f)}$ can be considered as the Sharpe ratio¹³ for the factor (the market portfolio excess return for the CAPM). The $N \times 1$ vector of the intercepts is defined as $\hat{\alpha} = [\hat{\alpha}_1 \hat{\alpha}_2 \dots \hat{\alpha}_N]'$, and $\hat{\Sigma}$ denotes the estimated residual covariance matrix $E(\varepsilon_t \varepsilon_t')$.

¹³ Expected return per unit of systematic risk.

If the alphas in Equation 5.6 are not statistically different from zero then the χ^2 statistic will be close to zero. The GRS test rejects an asset pricing model when the statistic is too large, that is, greater than the critical value for a *chi*-squared distribution with N degrees of freedom. This statistic which is valid asymptotically has a finite-sample counterpart which is distributed as an F distribution, known as the Gibbons, Ross, and Shanken (1989) GRS test statistic:

$$\frac{T - N - 1}{N} \left[1 + \left(\frac{E(f)}{\hat{\sigma}(f)} \right)^2 \right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{N, T-N-1} \quad (5.7)$$

If there is more than one factor in the asset pricing model, the test statistic will have a different formula to that given above. The intercepts and the coefficients of the risk factors are determined by OLS time-series regressions. However, the second term of the test statistic is different, as an estimate of the variance-covariance matrix of the factor excess returns must be introduced:

$$\frac{T - N - K}{N} \left[1 + E(f)' \hat{\Omega}^{-1} E(f) \right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{N, T-N-K} \quad (5.8)$$

where T = number of observations,

N = number of assets,

K = number of factors,

$$\hat{\Omega} = \frac{1}{T} \sum_{t=1}^T [f_t - E(f)] [f_t - E(f)]',$$

$$\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T \varepsilon_t \varepsilon_t'.$$

The GRS procedure, therefore, requires the estimation of the alphas using time series regressions and then the building of the quadratic statistic and comparing it with the critical value.

In summary the methodology can be described as follows: run a time series regression of the excess returns on the market portfolio excess returns for each portfolio; store the residuals; calculate the variance-covariance matrix of the residuals; calculate the sample mean (expected value) of the factors; calculate the variance-covariance matrix of the factors; and compute the GRS statistic. If the statistic is greater than the critical value then reject the model.

This methodology has some limitations. First, it cannot be used when there are non-tradable assets for which the researcher is forced to conduct a two-pass regression in order to estimate the related risk premium. Secondly, the test might reject the model even if this produces very small alphas (because of excessively small standard errors), or if a small number of portfolios are mispriced (because of excessively large pricing errors). Third, the test can lead to contradictory results against the cross-sectional regression, that is, where a model may not be rejected by GRS but is rejected in the cross-sectional methodology. Finally, and most importantly, the test does not answer the real question of asset pricing theory, which is why different portfolios yield different returns and not what factors explain the historical variability of returns. For these reasons, cross-sectional regressions should be considered as the most important approach and it is the approach pursued in this research.

5.4. Black Jensen and Scholes (BJS) single cross-section

The second methodology to be introduced is the Black, Jensen and Scholes (1972), henceforth BJS, approach to testing the CAPM. This is a test based on cross-sectional regressions and can easily be extended to test a multifactor asset pricing model. Given the asset pricing model which states that average excess returns should be proportional to the exposure to the market portfolio, a two-step methodology can be used to test this.

First, the coefficients of sensitivity to the market are obtained with time-series regressions for each asset:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \quad \text{for } t = 1, 2, \dots, T \text{ and } i = 1, 2, \dots, N \quad (5.9)$$

In this way, a full sample beta is estimated from the time-series regressions. The second step is to run a single cross-sectional regression of asset average excess returns on the estimated betas in order to estimate the factor risk premia:

$$\bar{R}_i = a_0 + \hat{\beta}_i \lambda \quad \text{for } i = 1, 2, \dots, N \quad (5.10)$$

The cross-sectional regression results in an estimate of an intercept and the risk premium (lambda). The null hypotheses to test for the validity of the model are that: 1) the intercept is not statistically different from zero; and 2) the estimated risk premium is positive and significant:

$$\begin{cases} a_0 = 0 \\ \lambda > 0 \end{cases}$$

However, the BJS approach does not correct for the cross-sectional correlation nor for the fact that the betas are estimated (the error in variable problem). The solution for the fact that the betas are estimated has been provided by Shanken (1992). The correct asymptotic standard errors for lambda and alpha can be obtained by introducing the Shanken correction:

$$\sigma^2(\hat{\lambda}) = \frac{1}{T} [(\beta' \beta)^{-1} \beta' \Sigma \beta (\beta' \beta)^{-1} (1 + \lambda' \Sigma_f^{-1} \lambda) + \Sigma_f] \quad (5.11)$$

where the factor $(1 + \lambda' \Sigma_f^{-1} \lambda)$ is the correction term. With k factors, the covariance matrix $\sigma^2(\hat{\lambda})$ is of dimension $k+1$. $\Sigma = E(\varepsilon_t' \varepsilon_t)$ is the $N \times N$ covariance matrix of the residuals from the N time series regressions (from which the N betas were obtained), and β is an $N \times k$ matrix of regressors. Finally, $\Sigma_f = E[(f_t - \mu)'(f_t - \mu)]$ is the $k \times k$ variance covariance matrix of the factors. The main problem with the Shanken

correction is that it requires estimates of two variance covariance matrices. This seems contradictory as the correction is proposed to correct for the fact that the betas are estimated, though itself uses estimates. However, the correction can be substantial, especially for asset pricing models that use risk factors with small variance, such as the consumption CAPM related models.

While the error in variable can be partially resolved through the Shanken correction, the cross-sectional correlation is not resolved under the BJS approach. Fama and MacBeth (1973), however, offer a solution to this problem.

5.5. The Fama and MacBeth methodology

The most popular approach to testing the CAPM is the Fama and MacBeth (1973) methodology. The methodology is known as a two-pass regression. In the first step a time series regression for assets or portfolios of stocks is run to estimate the beta, and in the second step a cross-sectional regression of portfolio returns is run on the estimated beta in order to produce an estimate of the price of risk. In contrast to BJS, the cross-sectional regressions are conducted at each period of time, in this case monthly. The alphas and the prices of risk estimated each month are then averaged over time and the hypothesis that alpha is zero (insignificant) and the price for risk is positive (and approximately equal to the average historical risk premium) are tested on the basis of the distribution of alpha and the price for risk so obtained.

The methodology can be described as follows:

1. In the first step a rolling time series regression is conducted with the stock's excess return on the left hand side and the market portfolio's excess return over and above the risk-free rate (monthly T-bills) on the right hand side to obtain the market beta. Thus, for each stock i , we have $T-60$ time series regressions (based on rolling regressions of 60 months);

$$R_{i,\tau} - R_{f,\tau} = \alpha_i + \beta_i(R_{M,\tau} - R_{f,\tau}), \quad \tau = t - 59, t - 58, \dots, t \quad \text{for each } t=60, \dots, T$$

2. In the second step, the monthly excess returns of the portfolio are regressed against the estimated ex-ante betas (the previous month's betas). The cross-sectional regression is conducted for each date and produces an estimate for the intercept and for the market risk premium:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1}\lambda_t \quad i = 1, 2, \dots, N \quad \text{for each } t=60, \dots, T$$

It is worth emphasizing that the Fama and MacBeth methodology accounts for the predictive role of the market beta, since the monthly excess returns are regressed over the betas estimated in the previous period. However, the assumption is made that the expected return is replaceable by the realized return and this is one of the main criticisms of this cross-sectional methodology.

The Fama and MacBeth procedure, originally introduced for the purpose of testing the CAPM, can be extended to multifactor models without much added complication. Once the average alpha and lambda are obtained, their sample standard deviation can be used in a t-test. The logic of this procedure is that each month, or each period, represents a sample from which alpha and the market risk premium are estimated and therefore the variation in the estimates over time allows the researcher to derive the variation across samples (see Cochrane, 2001).¹⁴

$$\bar{\lambda} = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_t, \quad \bar{\alpha} = \frac{1}{T} \sum_{t=1}^T \hat{\alpha}_t \quad (5.12)$$

$$s^2(\hat{\lambda}) = \frac{1}{T-2} \sum_{t=1}^T (\hat{\lambda}_t - \bar{\lambda})^2, \quad s^2(\hat{\alpha}) = \frac{1}{T-2} \sum_{t=1}^T (\hat{\alpha}_t - \bar{\alpha})^2 \quad (5.13)$$

¹⁴ Although we use T for simplicity, the sample size is actually T-60.

To test the significance of the market risk premium, (i.e. whether it is zero), a simple t-test is advocated by Fama and MacBeth:

$$t(x) = \frac{x}{s(x)/\sqrt{T}} \quad (5.14)$$

where x is a sample mean. When the t-statistic is greater than the critical value the null hypothesis that $x = 0$ can be rejected. In addition, the Fama and MacBeth methodology corrects for the cross-sectional correlation problem which a single cross-sectional regression cannot do.

In summary, the Fama and MacBeth methodology can be described as follows:

1. Estimate the betas of the portfolios using a rolling time series regression;
2. Run a cross-sectional regression at each date to estimate the intercept and the risk premium;
3. Average the cross-sectional estimates to get the estimate of the intercept and the market risk premium;
4. Use the standard deviation of the cross-sectional regression estimates in order to estimate the standard errors of the intercept and of the market risk premium;
5. Use a t-test that the intercept is zero and that the risk premium is positive.

5.6. Conditional models

The conditional CAPM states that the expected excess return on any stock is given by the risk premium multiplied by the conditional beta, a time-varying beta dependent on the set of information, Ω_t , available at each point in time. The conditional beta is expressed as the conditional covariance between the market portfolio return and the asset return, divided by the conditional market variance:

$$\beta_{i,t+1} = \frac{\text{cov}[R_{i,t+1}, R_{m,t+1} | \Omega_t]}{\text{var}[R_{m,t+1} | \Omega_t]} \quad (5.15)$$

One of the major problems in the specification of conditional models is how to model the time-varying parameters. In other words, the problem is deciding what the set of relevant information, Ω_t , is, and the functional form relating conditional variances and covariances to the conditioning information. The solution presented here is that suggested by Engle (2002). The conditional variances and covariances can be estimated using multivariate GARCH models with dynamic conditional correlations. In order to understand this methodology we briefly introduce the basic building blocks of the GARCH models.

5.6.1. GARCH (1,1)

The GARCH model developed by Bollerslev (1986) and Taylor (1986) allows the estimation of the conditional variance of returns one-period ahead as a function of its previous lags:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (5.16)$$

The estimated variance can be seen as a function of the long-term average value captured by α_0 , the past volatility lag ε_{t-1}^2 and the fitted variance from the previous period σ_{t-1}^2 . As long as $\alpha + \beta < 1$ the conditional variance is mean reverting around the long-run average (unconditional variance).

When there are several assets to be tested as in asset pricing models, a multivariate GARCH model is required to account for conditional variances and covariances. In particular, as the CAPM requires the researcher to estimate the market beta, the covariance between an asset's excess returns and the market portfolio excess returns must be estimated.

5.6.2. Multivariate GARCH models with Dynamic Conditional Correlations (DCC)

Multivariate GARCH models are similar in nature to their univariate GARCH counterparts. One way to estimate conditional time-varying betas is to employ the multivariate GARCH with dynamic conditional correlation as introduced by Engle (2002).

The DCC is a multivariate development of the generalised autoregressive conditional heteroscedasticity (GARCH) models which allows us to obtain dynamic correlations and covariances. This is particularly suitable for the Four-Moment CAPM since it requires bivariate covariances. These models are estimated in two steps: (i) the conditional variances are estimated with univariate GARCH; and (ii) the conditional correlations are estimated using a multivariate model. Since the estimates are obtained using pairs, the DCC for two variables is briefly outlined below.

The DCC proposed by Engle (2002) is given by the following equations

$$y_t = \mu_t + \Sigma_t^{1/2} z_t \quad (5.17)$$

$$\Sigma_t = D_t R_t D_t \quad (5.18)$$

$$D_t = \text{diag}(\sigma_{11,t}^{1/2} \dots \sigma_{kk,t}^{1/2}) \quad (5.19)$$

The positive definite matrix of (pseudo) correlations is given by

$$Q_t = (1 - \alpha - \beta)R + \alpha u_{t-1} u_{t-1}' + \beta Q_{t-1} \quad (5.20)$$

where $u_t = (u_{1t} \dots u_{kt})'$, $u_{it} = (y_{it} - \mu_{it}) / \sqrt{\sigma_{ii,t}}$ and R is the unconditional covariance matrix of u_t .

Engle (2002) proposes the following estimator of the correlation matrix

$$R_t = (\text{diag} Q_t)^{-1/2} Q_t (\text{diag} Q_t)^{-1/2} \quad (5.21)$$

A typical element of Q_t is given by

$$q_{ij,t} = \bar{\rho}_{ij}(1 - \alpha_1 - \alpha_2) + \alpha_1 q_{ij,t-1} + \alpha_2 u_{it-1} u_{jt-1} \quad (5.22)$$

where $\bar{\rho}_{ij}$ is the unconditional covariance (correlation) between u_{it} and u_{jt} .

A typical element of the correlation estimator R_t is

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} \quad (5.23)$$

To recover conditional covariances, these conditional correlations are simply multiplied

by the conditional standard deviations, i.e. $\text{cov}_{ij,t} = \rho_{ij,t} \sqrt{\sigma_{ii,t} \sigma_{jj,t}}$.

5.6.3. CAPM with Multivariate GARCH DCC betas

The CAPM with dynamic conditional correlations (DCC) can be defined as:

$$E[R_{i,t+1}] = \alpha_{i,t} + \lambda_t \cdot \frac{\sigma_{im,t+1}}{\sigma_{m,t+1}^2} = \alpha_{i,t} + \lambda_t \cdot \frac{\rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}}{\sigma_{m,t+1}^2} \quad (5.24)$$

where, λ_t , the premium related to the conditional beta, and with the variance for the i -th asset and the market portfolios return are defined as simple GARCH(1,1) models:

$$E[\varepsilon_{i,t+1}^2] = \sigma_{i,t+1}^2 = \gamma_0^i + \gamma_1^i u_{i,t}^2 + \gamma_2^i \sigma_{i,t}^2 \quad (5.25)$$

$$E[\varepsilon_{m,t+1}^2] = \sigma_{m,t+1}^2 = \gamma_0^m + \gamma_1^m u_{m,t}^2 + \gamma_2^m \sigma_{m,t}^2 \quad (5.26)$$

The covariance between asset i and market portfolio returns is given by

$$E[\varepsilon_{i,t+1} \varepsilon_{m,t+1}] = \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \quad (5.27)$$

where $\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}$ is the conditional correlation obtained as the conditional covariance divided by the product of the conditional standard deviations, with mean-reverting dynamic conditional correlations around the unconditional correlation $\bar{\rho}_{im}$,

and the conditional covariance is mean-reverting around the unconditional correlation $\bar{\rho}_{im}$:

$$q_{im,t+1} = \bar{\rho}_{im} + a_1(u_{i,t} \cdot u_{m,t} - \bar{\rho}_{im}) + a_2(q_{im,t} - \bar{\rho}_{im}) \quad (5.28)$$

An example of this methodology is given in Engle and Bali (2008), where the authors test a conditional ICAPM with DCC covariances.

5.7. Markov Switching Regimes

Previous research has generally assumed that the risk premia are constant (Ferson and Harvey, 1999, discuss this issue). However, there is evidence that the price of risk is time varying and changes with the economic and political environment, such as the business cycle (Ferson and Harvey, 1997; Cochrane, 2001). Higher returns are required in a recession while lower returns are required in an expansion. In this study a conditional model is employed in which the risk premia change over the states of the world but are constant throughout the regimes. The regime is obtained through a Markov switching model which has the advantage that the conditioning variables are not imposed nor are the regimes exogenously determined, but they are determined by the data as suggested by the stochastic latent process of an unobservable variable, the random state variable which depends in this study on the market excess return.

Conceptually, asset pricing models with switching regimes are a particular version of conditional models with parameters that vary across regimes but remain constant throughout the regime. However, these models have the methodological quality to define the regime endogenously without using arbitrary conditioning variables.

It is useful to introduce therefore the Markov switching regimes methodology in general. A switching regime allows for a set of parameters in one model to depend on the regime or state assumed by an observable or a latent unobservable variable. The

basic idea behind regime-switching models is that the parameters of the model or the data generation process can change across different states. Therefore, the model can have two or more different specifications according to these states.

The simplest way to account for regime-switching is to have regimes which change when an observed variable is above or below a certain threshold. However, in Markov-switching models the state variable is an unobservable latent variable, a random variable, following a Markovian process, and the transition probability, or probability to migrate from one regime to another, is expressed through a probability matrix. At each date, there exists a probability that the process is in regime 1 and a probability that it is in regime 2. A Markovian process means that the probability that the state variable equals some value today depends solely on the most recent value of the state variable:

$$P\{s_t = j | s_{t-1} = i, s_{t-2} = k, \dots\} = P\{s_t = j | s_{t-1} = i\} = p_{ij} \quad (5.29)$$

In general, let y_t be a vector of observed variables (dependent variables) and x_t a vector of observed explanatory variables. Let Ω_t be the vector containing all the observations through date t :

$$\Omega_t = \{y_t, y_{t-1}, \dots, y_{t-n}, x_t, x_{t-1}, \dots, x_{t-n}\} \quad (5.30)$$

Let the process depend on regime $s_t = j$ at date t , then the conditional density of y_t is given by

$$f(y_t | s_t = j, \Omega_{t-1}; \theta) \quad (5.31)$$

and θ is the vector of parameters characterizing the conditional density.

Assuming that the state evolves over time, following a Markov process with transition probabilities collected in the transition probability matrix P :

$$P = \begin{bmatrix} p_{11} & p_{21} & \dots & p_{N1} \\ p_{12} & p_{22} & \dots & p_{N2} \\ \vdots & \vdots & \vdots & \vdots \\ p_{1N} & p_{2N} & \dots & p_{NN} \end{bmatrix} \quad (5.32)$$

where p_{ij} is the probability of moving from state i to state j .

Assuming a linear model as follows:

$$y_t = \beta x_t + \varepsilon_t$$

The conditional density functions of the residuals (assumed to come from different stochastic processes with a different mean and standard deviation) according to the regime i will be:

$$\begin{cases} f(y_t | \Omega_{t-1}, s_t = 1; \theta) = \frac{1}{\sqrt{2\pi\sigma_1}} \exp\left(\frac{-(y_t - \beta_1 x_t)^2}{2\sigma_1}\right) \\ \vdots \\ f(y_t | \Omega_{t-1}, s_t = n; \theta) = \frac{1}{\sqrt{2\pi\sigma_n}} \exp\left(\frac{-(y_t - \beta_n x_t)^2}{2\sigma_n}\right) \end{cases} \quad (5.33)$$

where s_t denotes the regime, and Ω_{t-1} denotes the information set available at time t , and the likelihood function is given by the sum of the probability-weighted state densities across the possible states:

$$\text{Log}L(\{x_t\}; \theta) = \sum_{t=1}^T \log f(y_t | x_t, \Omega_{t-1}; \theta) \quad (5.34)$$

$$\log f(y_t | x_t, \Omega_{t-1}; \theta) = \sum_{i=1}^n \log f(y_t | \Omega_{t-1}, S_t = i; \theta) \text{Prob}(S_t = i | \Omega_{t-1}; \theta) \quad (5.35)$$

where $\text{Prob}(S_t = i | \Omega_{t-1}; \theta)$ are the conditional probabilities of state i at time t given the information set at time $t-1$. The conditional state probabilities are obtained recursively as:

$$\begin{aligned} & \text{Prob}(S_t = j | \Omega_{t-1}; \theta) \\ &= \sum_{i=1}^n \text{Prob}(S_t = j | S_{t-1} = i; \Omega_{t-1}; \theta) \text{Prob}(S_{t-1} = i | \Omega_{t-1}; \theta) \end{aligned} \quad (5.36)$$

In words, the probability to be in state j at time t is given by the sum of weighted conditional probabilities of being in state j given that the previous period was in all possible states. The conditional probability is obtained by the Bayesian rule:

$$\begin{aligned} Prob(S_{t-1} = i | \Omega_{t-1}; \theta) \\ = \frac{Prob(S_{t-1} = i | \Omega_{t-2}; \theta) \times f(y_t | \Omega_{t-2}, S_t = i; \theta)}{\sum_{i=1}^n f(y_{t-1} | \Omega_{t-2}, S_{t-1} = i; \theta) \times Prob(S_{t-1} = i | \Omega_{t-2}; \theta)} \end{aligned} \quad (5.37)$$

The parameters are estimated by maximum likelihood using the Expected Maximization algorithm. This algorithm is an iterative technique that starts from a given set of parameters $\theta^{(0)}$, estimates the probabilities, the parameters, and the variance, and leads to a new set of parameters $\theta^{(1)}$, and continues iteratively until the likelihood reaches the maximum, a point where it does not change given a small range or criterion of convergence.

5.8. Methodology and models estimated in this study

5.8.1. The models

Two models are tested in this study either unconditionally or conditionally: the CAPM and the Four-Moment CAPM. The models are briefly recalled here.

Model 1: The CAPM:

$$E(R_i) - R_f = \alpha + [E(R_m) - R_f] \beta_{i,m} \quad \forall i = 1, \dots, N \quad (5.38)$$

and defining the risk premium $\lambda_\beta = E[(R_m) - R_f]$ and substituting into Equation 5.38, the following equation is obtained:

$$E(R_i) - R_f = \alpha + \lambda_\beta \beta_{i,m} \quad (5.39)$$

The hypotheses tested are that the intercept is zero and the risk premium is positive and approximately equal to the average historical market excess return:

$$\begin{cases} \alpha = 0 \\ \lambda_\beta > 0 \\ \lambda_\beta = [(R_m) - R_f] \end{cases} \quad (5.40)$$

Model 2: The Four-Moment CAPM:

$$E(R_i) - R_f = \lambda_\beta \frac{E(r_i \cdot r_m)}{E(r_m^2)} + \lambda_s \frac{E(r_i \cdot r_m^2)}{E(r_m^3)} + \lambda_k \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} \quad (5.41)$$

The model states that an asset average excess return is a function of the risk premium for the first three moments: systematic covariance, systematic coskewness and systematic cokurtosis. The Four-Moment CAPM is obtained as follows. As $E(r_m^3)$ might be zero (the distribution of the market portfolio can be symmetric), to avoid divisions by zero the model is represented as:

$$\begin{aligned} E[(R_i) - R_f] &= \lambda_* \frac{E(r_i \cdot r_m)}{E(r_m^2)} + \lambda_s E(r_i \cdot r_m^2) + \lambda_k \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} \\ &= \lambda_* \beta_i + \lambda_s s_i + \lambda_k k_i \end{aligned} \quad (5.42)$$

where λ_s is the premium for coskewness, rather than standardized coskewness.

For the market portfolio:

$$E[(R_m) - R_f] = \lambda_* + \lambda_s s_m + \lambda_k \quad (5.43)$$

which implies $\lambda_* = E[(R_m) - R_f] - \lambda_s s_m - \lambda_k$. Substituting λ_* in Equation 5.43, the final model obtains:

$$E[(R_i) - R_f] = \lambda_\beta \beta_i + \lambda_s (s_i - s_m \beta_i) + \lambda_k (k_i - \beta_i) \quad (5.44)$$

where $\lambda_\beta = E[(R_m) - R_f]$. This model has the advantage that it nests the CAPM. The main hypotheses to be tested are that the price of beta is positive and equal to the market risk premium, the premium for (excess) coskewness is negative, the premium for (excess) cokurtosis is positive.

$$\begin{cases} \lambda_\beta > 0 \\ \lambda_s < 0 \\ \lambda_k > 0 \end{cases} \quad (5.45)$$

The advantage of this formulation is that it can be compared with the standard CAPM. Since the CAPM is a special case of our Four-Moment CAPM, we are able to progress with two distinct tests. First, we can test directly the economic hypothesis that investors optimise in mean-variance-skewness-kurtosis space rather than just in mean-variance space. Given the moment estimation problems (error in variable problems), and, more crucially, the fact that average returns are poor proxies for expected returns, we would not be surprised that our four moment CAPM will not be supported by the data, as in the standard CAPM. This leads to the second interesting question, namely, whether the extended CAPM has higher predictive power compared with the standard CAPM.

5.8.2. Conditional tests of the CAPM

The simple procedure proposed by Pettengill et al. (1995) involves splitting the sample into upmarket and downmarket periods. The upmarket and downmarket are first defined as months with positive or negative ex post market excess returns, respectively. Having estimated betas from a first pass, Pettengill et al. define a conditional CAPM as:

$$R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t} \cdot \delta_t \cdot \beta_i + \hat{\gamma}_{2t} \cdot (1 - \delta_t) \cdot \beta_i + \varepsilon_{it} \quad (5.46)$$

where $\delta_t = 1$ if the realised excess return is positive, and 0 otherwise. It is worth noting that the model is estimated for each t , that is, there are T cross sectional models, yielding T risk premia. These are then split into two samples depending on whether we are in an upmarket regime or a downmarket. Pettengill et al. propose a conditional relationship between beta and realised returns as follows.

In the up market:

$$H_0: \bar{\gamma}_1 = 0,$$

$$H_a: \bar{\gamma}_1 > 0.$$

and in the down market:

$$H_0: \bar{\gamma}_2 = 0,$$

$$H_a: \bar{\gamma}_2 < 0.$$

A systematic conditional relationship between beta and realized returns is confirmed if the null hypotheses are rejected in both cases. However, this hypothesis has no economic grounding and is almost meaningless as far as risk return is concerned. Asset pricing models are stated in terms of expected or required returns, not realised returns. Another problem with this hypothesis is that the scale of average conditional risk premia is not taken into account. For example, suppose the market is split equally into up and down. In that case, what does a ‘significant’ average $\bar{\gamma}_1 = 1\%$ versus a significant average $\bar{\gamma}_2 = -10\%$ mean? According to the proposed test by Pettengill et al. the conditional relationship holds because the two null hypotheses are rejected. However, in reality, the market does not reward investment at all (regardless of systematic risk). In the 50% cases where the investor ‘wins’ he earns an average 1%, but loses 10% on average on the remaining 50% cases, thus a disastrous investment proposition.

Pettengill et al. then propose an unconditional test, on the basis that the positive risk premium should on average be greater than the negative risk premium. Specifically, they propose the test:

$$H_0: \bar{\gamma}_1 + \bar{\gamma}_2 = 0$$

$$H_a: \bar{\gamma}_1 + \bar{\gamma}_2 \neq 0$$

using a two-population t-test. However, as Freeman and Guermat (2006) show, this test is not well specified since the sum is different from zero under both the null and alternative hypotheses. More importantly, the sum of the two average premia does not reflect the probability (or frequency) with which the up and down event takes place. For example, suppose in a thousand day sample, a high average return, say 10%, is expected to occur in two days only, while a small but negative return, say -1%, occurs in the 998

remaining days. A test of averages, as the one above, does not take into account the frequency of losses and will show a significant (positive) test statistic provided the returns on each state are not too volatile. In other words, while by definition expected returns are the sum of outcomes weighted by their respective probabilities, the Pettengill et al. test simply assumes equal weighting (as if the probabilities of being in an up and downmarket were equal).

In this thesis a new conditional test based on the probability of being in one of the two states is proposed. To motivate the test, suppose an investment opportunity where the investor is paid a return $\hat{\gamma}_1\beta_i$ if he wins (upmarket) and $\hat{\gamma}_2\beta_i$ if he loses (downmarket). Suppose the probability of a win is p . The expected excess return is therefore:

$$E(R_t) = (p\hat{\gamma}_1 + (1 - p)\hat{\gamma}_2)\beta_i \quad (5.47)$$

The investor will not invest if the expected excess return is negative or zero. For positive risk (beta), that means $p\hat{\gamma}_1 + (1 - p)\hat{\gamma}_2 > 0$. For any investment to be viable, the wins must be either large or highly frequent or both, compared with the losses. Returning to the conditional test of the CAPM, assuming that the states are not known with certainty, each period t , returns are generated by the up state with probability p_t and down with one minus that probability.

$$R_{it} = \hat{\gamma}_{0t} + (p_t\hat{\gamma}_1 + (1 - p_t)\hat{\gamma}_2)\beta_i + \varepsilon_{it} \quad (5.48)$$

The expectation is that:

$$E(R_{it}) = E(\hat{\gamma}_{0t}) + E(p_t\hat{\gamma}_1 + (1 - p_t)\hat{\gamma}_2)\beta_i \quad (5.49)$$

Therefore, a simple test can be devised by looking at the average of the beta (conditional) slope, namely:

$$H_0: E(\Gamma_t) = 0$$

$$H_a: E(\Gamma_t) > 0$$

where $\Gamma_t = p_t\hat{\gamma}_1 + (1 - p_t)\hat{\gamma}_2$.

It is important to note that the proposed conditional test has the standard unconditional CAPM as a special case. The unconditional test of the CAPM obtains when the downmarket risk premium is equal to the upmarket risk premium. In that case:

$$E(R_{it}) = E(\hat{\gamma}_{0t}) + E(p_t \hat{\gamma}_1 + (1 - p_t) \hat{\gamma}_1) \beta_i = E(\hat{\gamma}_{0t}) + E(\hat{\gamma}_1) \beta_i \quad (5.50)$$

A strong implication of the unconditional test is that both up and down premiums must be not only equal but also positive. This is clearly an advantage of the conditional test, as it allows the two premiums to be of different size and sign. We can also test for the unconditional CAPM by testing the equality and positivity of the two risk premiums.

Another implication of the Pettengill et al. proposed test is that a proper testing procedure will yield the standard unconditional test. The main reason is that the up and down states are strictly related to the market realised returns. Suppose the market is positive for A months and negative for B months. In that case the weighted average of the up and down premiums will be simply the grand average of all premiums. In the case of this study, $p=A/A+B$ and thus:

$$\bar{\Gamma} = \frac{A}{A+B} \bar{\gamma}_1 + \frac{B}{A+B} \bar{\gamma}_2 = \frac{1}{A+B} \left(\sum_{t=1}^A \hat{\gamma}_{1t} + \sum_{t=1}^B \hat{\gamma}_{2t} \right) \quad (5.51)$$

This is the same as Fama and MacBeth test. Thus, a proper application of the conditional test when the states are assumed to be known with certainty leads to the standard unconditional test.

The state probabilities are obtained from Markov switching models applied to the market return. However, there is a problem with estimating the risk premia. Because these are estimated from cross sectional regressions this is only one beta but two parameters. Therefore a time series of risk premia as in Fama MacBeth cannot be obtained. Instead, panel data models are used to obtain estimates of the two premia, and then the statistical test on Γ_t is undertaken. Specifically, given β_i and p_t , a panel regression can be run:

$$R_{it} = \gamma_0 + \gamma_{12}p_t\beta_i + \gamma_2\beta_i + \varepsilon_{it} \quad (5.52)$$

where $\gamma_{12} = \gamma_1 - \gamma_2$. Once $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are obtained, the test as described above can be undertaken (i.e. conduct a time series test on the mean of $\Gamma_t = p_t\hat{\gamma}_1 + (1 - p_t)\hat{\gamma}_2$ as in Fama MacBeth).

The above proposed model is a conditional model in which the risk premia change over two states of the world, but are constant throughout the regimes. The regimes are assumed to occur with certain probabilities, obtained through a Markov-regime switching model. This has the advantage that the conditioning variables are not imposed. Rather than being exogenously determined (as in Pettengill et al.), the regimes are determined by the data as suggested by the stochastic latent process of a random variable (the state or regime) which depends on an observable variable, that is, the market excess return in the case of this study.

The conditional Four-Moment CAPM is a simple extension of the CAPM. Namely,

$$R_{it} - R_f = \Gamma_{\beta t}\beta_i + \Gamma_{st}(s_i - s_m\beta_i) + \Gamma_{kt}(k_i - \beta_i) + \varepsilon_{it} \quad (5.53)$$

where $\Gamma_{\beta t} = p_t\lambda_{\beta 1} + (1 - p_t)\lambda_{\beta 2}$, $\Gamma_{st} = p_t\lambda_{s1} + (1 - p_t)\lambda_{s2}$, and $\Gamma_{kt} = p_t\lambda_{k1} + (1 - p_t)\lambda_{k2}$.

The regime is determined by the market excess return following the following stochastic process:

$$R_M - R_f = \mu_{Mi} + \sigma_{Mi}\varepsilon_i \quad (5.54)$$

The coefficients, μ_{Mi} , and σ_{Mi} , $i = 1,2$ take one of two values, depending on the regime, and ε_i is a random disturbance assumed to be normally distributed.

5.8.2.1. Estimating State Probabilities

Two states, bull and bear markets, are assumed which is the simplest Markov process. Full details on the estimation procedures can be found in Hamilton (1989). Here the procedure adopted in this study is briefly outlined.

To specify how the state evolves over time, it is assumed that the state transition probabilities follow a first-order Markov chain. Let $p_{11} = Prob(S_t = 1|S_{t-1} = 1)$ be the probability of staying in state 1, and $p_{12} = Prob(S_t = 1|S_{t-1} = 2)$ the probability of moving from state 2 to state 1. At any given period, t , the probabilities and the likelihood functions are calculated recursively as follows:

$$\pi_{t|t-1} = p_{11}\pi_{t-1|t-1} + p_{12}(1 - \pi_{t-1|t-1}) \quad (5.55)$$

$$LogLik_t = \log\{\pi_{t|t-1}f_1(R_{mt}|\Omega_{t-1}, \theta) + (1 - \pi_{t|t-1})f_2(R_{mt}|\Omega_{t-1}, \theta)\} \quad (5.56)$$

Then the updated probabilities are obtained from the likelihood function

$$\pi_{t|t} = \frac{\pi_{t|t-1}f_1(R_{mt}|\Omega_{t-1}, \theta)}{\pi_{t|t-1}f_1(R_{mt}|\Omega_{t-1}, \theta) + (1 - \pi_{t|t-1})f_2(R_{mt}|\Omega_{t-1}, \theta)} \quad (5.57)$$

The parameters of the model are estimated through the maximum likelihood method. Let θ be the vector of parameters in the likelihood function. The conditional density functions of the residuals (assumed to come from different stochastic processes with a different mean and different standard deviation) according to the regime i will be:

$$f_i(R_{M,t}|\Omega_{t-1}, \theta) = \frac{1}{\sqrt{2\pi\sigma_{it}}} \exp\left(\frac{-(R_{M,t} - \mu_{m,it})^2}{2\sigma_{it}}\right) \quad (5.58)$$

where $i = 1, 2$ denotes the regime and Ω_{t-1} denotes the information set available at time t . The filtered probabilities are estimated using the EM algorithm of Hamilton (1989). With these probabilities, pooled or panel regressions will be used to obtain estimates of the risk premia for each state. These give estimates of $\Gamma_{\beta t}$ for the CAPM, and Γ_{st} and Γ_{kt} for the Four-Moments CAPM. These can be treated like time series of

(unconditional) risk premia which can be tested with a simple t-test. The expected signs of these conditionally based premia are the same as in the case of the standard models.

5.9. Short-window regression, conditional alphas and conditional betas

In this thesis the CAPM and the Four-Moment CAPM are tested on individual assets using an extension of the short-window regression methodology used by Lewellen and Nagel (2006), hereafter referred as LN. Specifically, LN, under the assumption that beta does not change much over a short period of time, use short-window time series rolling regressions (quarterly, semi-annually, and annually) of (daily, weekly, monthly) portfolios excess returns on market excess return to estimate the conditional alpha and conditional beta for size, B/M, and momentum portfolios and then test whether the average conditional alpha is insignificantly different from zero.

Similarly, in this thesis a two-step method is used. First, short-window (24 months) time series regressions of monthly individual asset excess returns over the market excess return are used to estimate conditional betas in the CAPM, and rolling short windows (24 months) are used to estimate conditional betas, coskewness, and cokurtosis in the Four-Moment CAPM. The average excess return of the individual stocks over the short windows is assumed to be the expected excess return of the individual stocks.

In a second step, the average excess return of individual stocks calculated over the window used to estimate the factor loadings are regressed at cross section over the conditional factor loadings (calculated over the same time window as in the first step) to estimate the risk premia.

The monthly conditional risk premia are then treated as time series. Specifically, the time series average of the risk premia is the risk premium and its significance is tested using a t-test as in the Fama and MacBeth methodology.

The methodology can be described as follows:

- *For the CAPM:*

1. In the first step a rolling time series regression is conducted with the stock's excess return on the left hand side and the market portfolio's excess return over and above the risk-free rate (monthly T-bills) on the right hand side to obtain the market beta. Thus, for each stock i , we have $T-24$ time series regressions (based on rolling regressions of 24 months);

$$R_{i,\tau} - R_{f,\tau} = a_i + \beta_i(R_{M,\tau} - R_{f,\tau}), \quad \tau = t - 23, t - 22, \dots, t \quad \text{for each } t=24, \dots, T$$

2. The stock's excess returns are then averaged over the same period used to estimate the beta as in step 1:

$$E(R_{i,t} - R_{f,t}) = \frac{\sum_{\tau=t-23}^t (R_{i,\tau} - R_{f,\tau})}{24}$$

3. The stock's average excess returns are regressed against the estimated betas (24-month window betas). The cross-sectional regression is conducted for each month and produces an estimate for the intercept and for the market risk premium:

$$E(R_{i,t} - R_{f,t}) = \alpha_t + \hat{\beta}_{i,t} \lambda_t \quad i = 1, 2, \dots, N \quad \text{for each } t=24, \dots, T$$

The alphas and the prices of risk (lambdas) estimated each month are then averaged over time and the hypotheses that alpha is zero (insignificant) and the price for risk is positive (and approximately the average historical risk premium) are tested on the basis of the distribution of alpha and the price for risk so obtained.

$$\bar{\lambda} = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_t, \quad \bar{\alpha} = \frac{1}{T} \sum_{t=1}^T \hat{\alpha}_t \quad (5.59)$$

$$s^2(\hat{\lambda}) = \frac{1}{T^2} \sum_{t=1}^T (\hat{\lambda}_t - \bar{\lambda})^2, \quad s^2(\hat{\alpha}) = \frac{1}{T^2} \sum_{t=1}^T (\hat{\alpha}_t - \bar{\alpha})^2 \quad (5.60)$$

To test the significance of the market risk premium, (i.e. whether it is zero), a simple t-test is used:

$$t(x) = \frac{x}{s(x)/\sqrt{T}} \quad (5.61)$$

where x is a sample mean. When the t-statistic is greater than the critical value, the null hypothesis that $x = 0$ can be rejected.

- *For the Four-Moment CAPM:*

1. In the first step conditional beta, coskewness and cokurtosis are estimated over the 24-month short-window as (for simplicity let $R_j^e = R_j - R_{f,t}$) :

$$\begin{cases} \beta_{i,t} = \frac{C \hat{\sigma} v[R_i^e, R_m^e]}{C \hat{\sigma} v[R_m^e, R_m^e]} = \frac{\sum_{\tau=t-23}^t [R_{i,\tau}^e - \bar{R}_{i,\tau}^e][R_{m,\tau}^e - \bar{R}_{m,\tau}^e]}{\sum_{\tau=t-23}^t [R_{m,\tau}^e - \bar{R}_{m,\tau}^e]^2} \\ \gamma_{i,t} S_{m,t} = C \hat{\sigma} v[R_i^e, (R_m^e)^2] = \sum_{\tau=t-23}^t [R_{i,\tau}^e - \bar{R}_{i,\tau}^e][R_{m,\tau}^e - \bar{R}_{m,\tau}^e]^2 \\ k_{i,t} = \frac{C \hat{\sigma} v[R_i^e, (R_m^e)^3]}{C \hat{\sigma} v[R_m^e, (R_m^e)^3]} = \frac{\sum_{\tau=t-23}^t [R_{i,\tau}^e - \bar{R}_{i,\tau}^e][R_{m,\tau}^e - \bar{R}_{m,\tau}^e]^3}{\sum_{\tau=t-23}^t [R_{m,\tau}^e - \bar{R}_{m,\tau}^e]^4} \end{cases} \quad (5.62)$$

$$\tau = t - 23, t - 22, \dots, t \quad \text{for each } t=24, \dots, T$$

2. The stock's excess returns are then averaged over the same period used to estimate the factor loadings as in step 1:

$$E(R_{i,t} - R_{f,t}) = \frac{\sum_{\tau=t-23}^t (R_{i,\tau} - R_{f,\tau})}{24}$$

3. The stock's average excess returns are regressed against the estimated factor loadings (24-month window betas). The cross-sectional regression is conducted for each month and produces an estimate for the intercept and for the market risk premium:

$$E(R_{i,t} - R_{f,t}) = \alpha + \lambda_\beta \hat{\beta}_{i,t} + \lambda_s \widehat{\gamma_{i,t} S_{m,t}} + \lambda_k \hat{k}_{i,t}$$

As in the case of the CAPM, the alphas and the prices of risk for each comoment (lambdas) estimated each month are then averaged over time, and the hypothesis that alpha is zero (insignificant), and the price for beta and cokurtosis is positive, and the price of coskewness is negative, are tested on the basis of the distribution of alpha and

the prices for risk so obtained using a simple t-test. It is worth noting that this methodology assumes that the expected excess return every month is equal to the average excess return calculated over the short window: $\tau = t - 23, t - 22, \dots, t$.

5.10. Conclusion

This chapter introduces and comprehensively describes the methodologies used in asset pricing to test linear and non-linear models: the time-series tests, GRS and the cross-sectional tests based on the Fama and MacBeth methodology.

The simplest methodology to test an asset pricing model requires the researcher to conduct several time-series regressions to determine the risk factors which have a significant coefficient. The CAPM states that only beta should be significant. However, it is empirically well known that variables such as E/P, leverage, the B/M ratio, size (market capitalization), amongst others, will affect returns significantly. The time series approach requires the application of a quadratic test of the pricing errors to test the hypothesis that the alphas, jointly considered, are not significantly different from zero (the GRS test).

Cross-sectional tests, and in particular the Fama and MacBeth (1973) methodology, try to estimate whether certain factors are significantly priced and can explain the difference in returns across assets. The methodology is known as a two-step methodology which requires in the first step a rolling time series regression to estimate the sensitivity of the asset returns to the risk factors (factor loadings), and in the second step a monthly cross-sectional regression to estimate the monthly market premium. The estimated market premium is then simply an average of the monthly estimated market premia.

However, the assumption that beta is constant over time has been widely criticized, and the time-varying nature of beta is accepted by researchers as a more realistic

assumption, especially where beta is a function of the business cycle. This time-varying nature of beta lies at the heart of conditional models, in which the asset returns are a function of their exposure to conditional betas. Therefore, if an asset were particularly exposed to risk when investors require a high risk premium – for example, in a recession – that asset would be expected conditionally to yield higher returns.

The second part of the chapter introduces conditional models and describes how to incorporate time variation of the parameters into the models using DCC multivariate GARCH and by introducing switching regimes. The major problem associated with the conditional models branch of asset pricing research is how to derive the time-varying factor loadings. In general, the assumption is made that beta follows a certain statistical process as in the case of the GARCH or the Markov Switching (MS), or that beta is indeed a function of some economic conditioning variables that can describe the business cycle and anticipate future market returns.

This thesis presents some methodological innovations. The Four-Moment CAPM is derived so that the risk premia equal the market excess return, applying DCC to the risk factor sensitivities, and estimating risk premia separately in different regimes. The introduction of switching regimes represents an important innovation compared to the simple separation in upmarkets and downmarkets according to observed market returns. The regimes are determined by the data as suggested by the stochastic latent process of an unobservable variable, that is, the state of the market excess return in the case of this thesis. The Markov-switching allows the researcher to back out the probability of being in each regime, and the risk premia are then estimated using a panel regression. The t-test is then applied to the time series of risk premia to test whether the factors are rewarded by the market.

Finally, the method used for the test of the CAPM and the Four-Moment CAPM on individual assets was presented. Specifically, while the factor loadings are obtained through rolling regressions in the first step, the average excess returns of individual stocks calculated over the window used to estimate the factor loadings are regressed at cross section over the conditional factor loadings to estimate the risk premia in the second step. The monthly conditional risk premia are then treated as time series. Specifically, the time series average of the risk premia is the risk premium, and its significance is tested using a t-test as in the Fama and MacBeth methodology.

Chapter 6

Data Description and Summary Statistics

6.0. Introduction

The objective of this chapter is to describe the asset data employed in this thesis, and in particular the descriptive statistics for each variable studied. Specifically, given that the main topic of this thesis is the higher-moment CAPM, the focus is on the average returns, variance, skewness and kurtosis of the distribution of returns of the assets, and in the measures of systematic covariance, coskewness and cokurtosis. Systematic covariance, coskewness and cokurtosis are obtained using the extension to the fourth moment (kurtosis) definition given by Kraus and Litzenberger (1973) for the higher moments, that is, the comoments standardized by the variance, skewness and kurtosis of the market portfolio.

In this thesis the assets used to test the CAPM and the Four-Moment CAPM are portfolios of stocks as well as individual stocks. The portfolios are the Fama and French portfolios of US stocks sorted on market capitalization, the book-to-market ratio, and double sorted on market capitalization and the book-to-market ratio. The data for the individual stocks are obtained from the CRSP database and include the AMEX, NYSE and NASDAQ indexes.

6.1. Portfolios and summary statistics

6.1.1. Descriptive statistics for the 10 ME portfolios

In this subsection the summary statistics for the ten portfolios of stocks sorted on market capitalization are presented and discussed. All of the portfolios discussed in this chapter are downloaded from the website of Kenneth French. All of the stocks

belonging to NYSE, AMEX and NASDAQ are ranked on market capitalization as at the end of June of year t and are then allocated into ten deciles according to the NYSE breakpoints (not all of the exchanges are used in order to avoid small stocks dominating in the period following the introduction of Nasdaq in 1973). The choice of June each year is to ensure that the accounting variables are known to investors in advance of the returns they are used to explain. Therefore the accounting data at the end of the fiscal year $t-1$ are matched with the returns for July of year t to June of year $t+1$. The portfolios are then reformed yearly in July. Therefore, every year the deciles portfolios can contain different stocks according to stock migration from one characteristic to the others. Small-capitalized firms might grow in size and move from one decile to another. Table 6.1 shows the descriptive statistics for ten portfolios of stocks sorted on market capitalization. Panel A shows the statistics for the full sample 1926-2011. Panel B shows the statistics for the sample 1980-2011. From Panel A, it can be noted that the average excess return decreases monotonically with market capitalization. Small stocks yield a much higher average return than large stocks (1.15% for the smallest decile versus 0.57% for the largest decile). Small stocks have larger standard deviations, skewness and kurtosis than large stocks, meaning that they are more prone to extreme outcomes (riskier), although more positively skewed. The normality of returns is strongly rejected for all of the portfolios, which confirms that one of the assumptions of the standard CAPM is violated. It is worth noting that not only do returns decrease with market capitalization but also that beta, coskewness and cokurtosis decrease monotonically with market capitalization.

In other words, small stocks are characterized by a higher beta, higher coskewness and higher cokurtosis than large stocks. Therefore, the addition of the higher moment of the distribution of returns can help explain why small stocks yield higher average returns,

i.e. the presence of a cokurtosis premium – a risk premium that investors demand for the higher probability of extreme negative outcomes.

An examination of Figure 6.1, which shows the relationship between returns and standardized comoments, reveals a strong positive relationship between returns and, respectively, beta and cokurtosis. It may be appreciated that this can justify partly the interest in the higher-moment CAPM. However, the figure also shows a positive relationship between returns and coskewness, which contradicts theoretical expectations (returns should decrease with coskewness, as investors like positive skewness and should be willing to forego some returns in exchange for positive skewness).

The results are quite different when looking at the subsample 1980-2011 reported in Panel B of Table 6.1. The small premium almost disappears (the mid deciles yield the highest average returns), and the returns are not very dispersed. The standard deviation is still negatively correlated with market capitalization, but there is no longer a clear pattern for skewness or kurtosis. Normality of returns is strongly rejected again. The dispersion of betas is very low, and the same can be said for coskewness and cokurtosis.

From Figure 6.2, which shows the relationship between returns and standardized comoments, it is noted that the positive relationship tends to flatten out with respect to the vertical axis, meaning that the linear relationship becomes less strong.

Table 6.1 – Descriptive statistics for the 10 ME portfolios

Panel A: The Fama and French 10 Size Portfolios 1926-2011

	Means	Standard Deviations	Skewness	Excess Kurtosis	Jarque- Bera	Beta	Coskewness	Cokurtosis
Market	0.62	5.45	0.16	7.53	0.00	1.00	1.00	1.00
S1	1.15	10.23	3.70	37.00	0.00	1.44	7.07	1.76
S2	0.97	8.93	2.26	22.15	0.00	1.39	4.32	1.51
S3	0.97	8.17	1.93	20.36	0.00	1.34	4.15	1.50
S4	0.92	7.56	1.55	15.84	0.00	1.25	3.61	1.38
S5	0.88	7.25	1.14	13.06	0.00	1.24	3.03	1.36
S6	0.88	6.92	1.02	12.11	0.00	1.21	3.04	1.33
S7	0.83	6.55	0.80	11.00	0.00	1.16	2.46	1.23
S8	0.77	6.21	0.74	10.79	0.00	1.11	2.27	1.19
S9	0.71	5.92	0.56	10.41	0.00	1.07	1.90	1.14
S10	0.57	5.13	0.09	6.42	0.00	0.93	0.77	0.91

Panel B: The Fama and French 10 Size Portfolios 1980-2011

	Means	Standard Deviations	Skewness	Excess Kurtosis	Jarque- Bera	Beta	Coskewness	Cokurtosis
Market	0.57	4.61	-0.80	2.47	0.00	1.00	1.00	1.00
S1	0.65	6.15	-0.32	3.21	0.00	1.03	1.38	1.13
S2	0.69	6.33	-0.44	2.70	0.00	1.15	1.37	1.21
S3	0.75	5.96	-0.78	2.39	0.00	1.14	1.33	1.19
S4	0.70	5.79	-0.78	2.53	0.00	1.12	1.28	1.18
S5	0.76	5.68	-0.78	2.42	0.00	1.13	1.25	1.17
S6	0.74	5.23	-0.80	2.51	0.00	1.07	1.16	1.10
S7	0.76	5.18	-0.79	3.02	0.00	1.07	1.14	1.12
S8	0.71	5.13	-0.72	2.34	0.00	1.07	1.10	1.08
S9	0.68	4.76	-0.73	2.47	0.00	1.01	0.99	1.01
S10	0.54	4.44	-0.54	1.68	0.00	0.94	0.86	0.90

Panel A and Panel B show the descriptive statistics for portfolios sorted on market capitalization from July 1926 to December 2011 and from January 1980 to December 2011, respectively. S1 through S10 show the ten deciles (from the smallest to the largest) in terms of market capitalization. The means are the average excess returns over the risk-free rate of a Treasury Bill. Jarque-Bera reports the probability under the null hypothesis of the normality of returns. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1973) as $c\hat{\sigma}v(r_i, r_m)/c\hat{\sigma}v(r_m, r_m)$, $c\hat{\sigma}v(r_i, r_m^2)/c\hat{\sigma}v(r_m, r_m^2)$, and $c\hat{\sigma}v(r_i, r_m^3)/c\hat{\sigma}v(r_m, r_m^3)$ respectively.

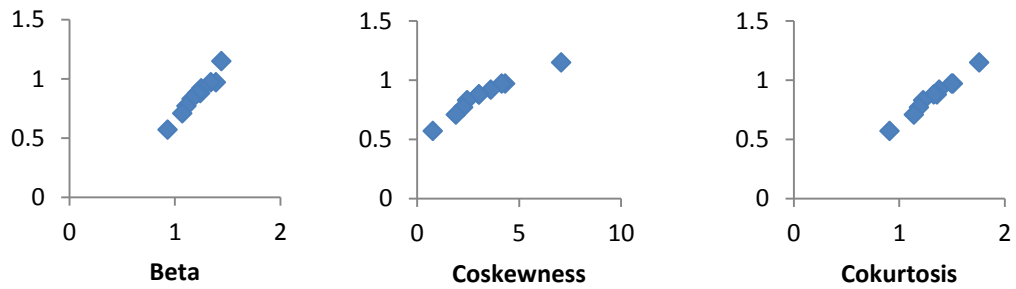


Figure 6.1 – Returns versus comoments for the 10 ME portfolios 1926-2011

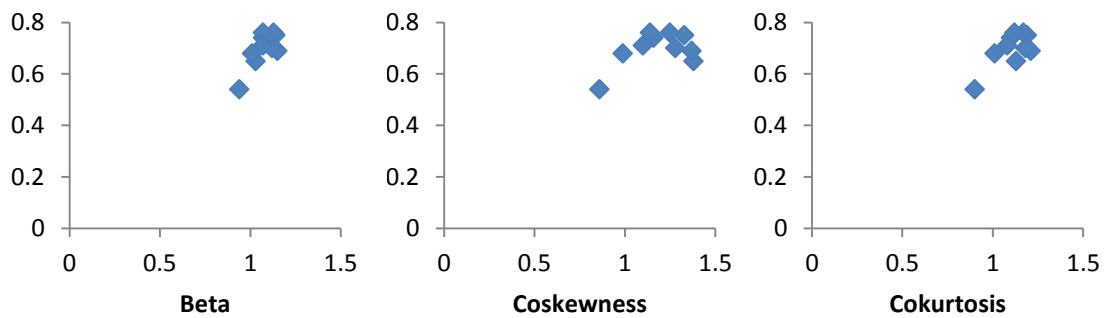


Figure 6.2 – Returns versus comoments for the 10 ME portfolios 1980-2011

6.1.2. Descriptive statistics for the 10 BM portfolios

Here, the summary statistics related to the ten portfolios of stocks sorted on the book-to-market ratio are presented and discussed. All of the stocks belong to the NYSE, AMEX and NASDAQ indexes and are ranked on the book-to-market (BM) ratio at the end of June of year t , obtained as book equity BE at the end of the last fiscal year divided by market equity (number of shares times price) at the end of December of year $t-1$, and then allocated to ten deciles. Portfolios are reformed yearly in July as in the case of ME portfolios.

Table 6.2 shows the descriptive statistics for ten portfolios of stocks sorted on book-to-market ratios. Panel A shows the results for the full sample 1926-2011. Returns increase monotonically with the BM ratio from 0.56% for the lowest decile to 1.06% for the highest decile. There is also a clear pattern in the other statistical measures: standard

deviation increases with the BM ratio, and the same can be noted for skewness and excess kurtosis. In particular, the higher average returns can be explained by larger beta and larger cokurtosis for high BM portfolios, although these portfolios also have the highest coskewness. Therefore, looking at the descriptive statistics for the whole sample, there seems to be some rationale for the use of a higher-moment CAPM, i.e. cokurtosis might add some explanatory power to the cross-section of returns of BM sorted portfolios – a result also valid for the ME portfolios over the sample 1926-2011. The result is shown intuitively with the charts that show the relationship between returns and standardized comoments in Figure 6.3.

Panel B shows the results for the subsample 1980-2011 and indeed a different scenario. The BM premium is still present, although it is smaller in comparison with the full sample, and is particularly clustered on the last two deciles, showing a much larger return than the other deciles (0.80 and 0.90, respectively). However, there is no longer a clear pattern in terms of the other statistical measures. All of the portfolios have a negative skewness, in contrast to the results for the full sample. The differences in betas, coskewness and cokurtosis are no longer much dispersed, questioning the use of a higher-moment CAPM to explain the value premium.

The situation can be readily appreciated with reference to Figure 6.4 which describes the relationship between returns and standardized comoments. Specifically, the Figure shows that the positive relationship between beta and returns, and between beta and cokurtosis, has disappeared, leading to a rejection of the CAPM and of the higher-moment CAPM as explanations of the cross-section of returns.

Table 6.2 – Descriptive statistics for the 10 BM portfolios

Panel A: The Fama and French 10 BM Portfolios 1926-2011								
	Means	Standard Deviations	Skewness	Excess Kurtosis	Jarque- Bera	Beta	Coskewness	Cokurtosis
Market	0.62	5.45	0.16	7.53	0.00	1.00	1.00	1.00
H1	0.56	5.76	-0.03	4.98	0.00	1.00	0.47	0.95
H2	0.64	5.52	-0.09	5.00	0.00	0.97	0.32	0.91
H3	0.66	5.36	-0.22	4.77	0.00	0.94	0.04	0.86
H4	0.64	6.08	1.24	15.88	0.00	1.06	3.18	1.22
H5	0.70	5.67	0.84	12.32	0.00	0.98	2.29	1.09
H6	0.74	6.21	0.94	16.24	0.00	1.07	2.60	1.24
H7	0.75	6.68	1.83	20.51	0.00	1.12	4.56	1.38
H8	0.91	7.00	2.13	24.44	0.00	1.16	5.13	1.46
H9	0.97	7.60	1.33	14.44	0.00	1.24	3.91	1.41
H10	1.06	9.41	2.39	24.25	0.00	1.45	6.49	1.81

Panel B: The Fama and French 10 BM Portfolios 1980-2011								
	Means	Standard Deviations	Skewness	Excess Kurtosis	Jarque- Bera	Beta	Coskewness	Cokurtosis
Market	0.57	4.61	-0.80	2.47	0.00	1.00	1.00	1.00
H1	0.50	5.31	-0.40	1.16	0.00	1.07	0.92	1.01
H2	0.62	4.78	-0.68	2.38	0.00	0.99	0.96	1.00
H3	0.65	4.76	-0.74	2.86	0.00	0.97	0.99	0.98
H4	0.70	4.95	-0.73	2.76	0.00	0.99	1.01	1.03
H5	0.62	4.68	-0.69	3.01	0.00	0.92	0.96	0.96
H6	0.63	4.62	-0.74	3.01	0.00	0.93	0.94	0.97
H7	0.69	4.45	-0.46	1.34	0.00	0.84	0.75	0.81
H8	0.62	4.53	-1.06	4.31	0.00	0.84	0.92	0.90
H9	0.80	4.65	-0.78	2.13	0.00	0.87	0.93	0.89
H10	0.90	5.80	-0.25	6.29	0.00	1.01	1.13	1.09

Panel A and Panel B show the descriptive statistics for portfolios sorted on the book-to-market ratio from July 1926 to December 2011 and from January 1980 to December 2011, respectively. H1 through H10 show the ten deciles (from the highest to the lowest) in terms of the book-to-market ratio. Means are the average excess returns over the risk-free rate of a Treasury Bill. Jarque-Bera reports the probability under the null hypothesis of the normality of returns. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1973) as $c\hat{\partial}v(r_i, r_m)/c\hat{\partial}v(r_m, r_m)$, $c\hat{\partial}v(r_i, r_m^2)/c\hat{\partial}v(r_m, r_m^2)$, and $c\hat{\partial}v(r_i, r_m^3)/c\hat{\partial}v(r_m, r_m^3)$ respectively.

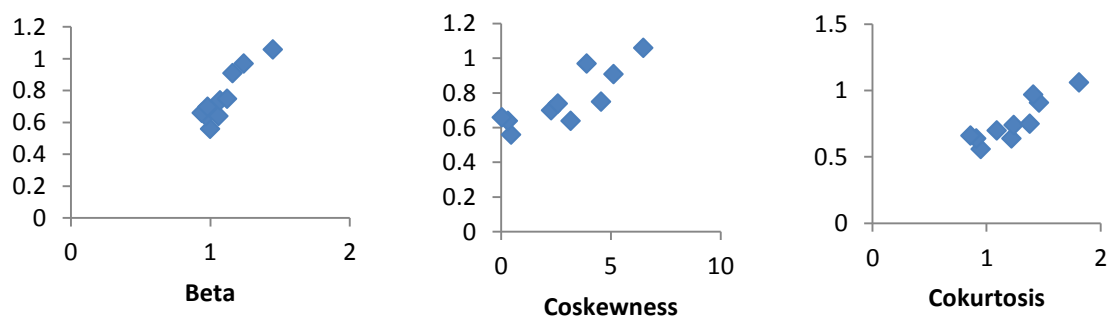


Figure 6.3 – Returns versus comoments of the 10 BM portfolios 1926-2011

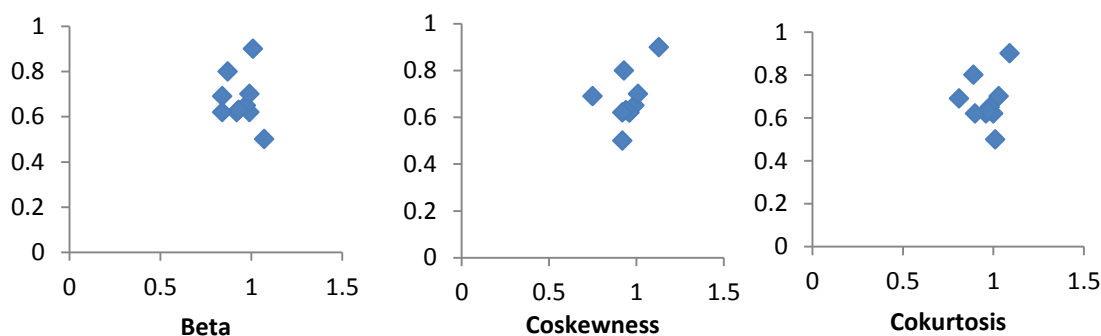


Figure 6.4 – Returns versus comoments of the 10 BM portfolios 1980-2011

6.1.3. Descriptive statistics for the 25 ME/BM portfolios

In this section the summary statistics related to the 25 portfolios of stocks sorted on market capitalization and the book-to-market ratio are presented and discussed. The portfolios are obtained by the intersection of 5 portfolios formed on size (ME, market equity) and 5 portfolios formed on the book-to-market ratio (BM, book equity-to-market equity). The size breakpoints are the market equity quintiles at the end of June of year t , the BM breakpoints are book-to-market quintiles obtained as book equity as at the fiscal year end $t-1$ divided by market equity, resulting in December of year $t-1$. Portfolios are held from July of year t to June of year $t+1$, and are reformed yearly in July.

Table 6.3 shows the descriptive statistics for 25 portfolios double sorted on market capitalization and book-to-market ratios. Panel A shows the results for the full sample 1926-2011. The average excess return increases monotonically with the BM ratios ranging from 0.45% for the smallest stocks with low book-to-market ratios to 1.39% for the smallest stocks with high book-to-market ratios. The pattern is the same for the first four quintiles, and it is only contradicted by the fifth quintile due to the poor return of the largest portfolio of stocks with high book-to-market ratios (-0.25%). In the first quintile, the standard deviation decreases monotonically with BM ratios, whereas kurtosis increases monotonically with BM ratios in all of the quintiles. Interestingly, the measures of standardized risk tend to increase monotonically with the BM ratios as shown in Figure 6.5. In other words, value portfolios have a higher beta, a higher coskewness and a higher cokurtosis. Therefore, for the 25 portfolios a higher-moment CAPM also seems to be able to partly explain the cross-section of returns for the full sample, though some outliers such as the large value portfolio and the small growth portfolio worsen the performance of the CAPM.

Panel B of Table 6.3 shows the descriptive statistics for the 25 ME/BM portfolios for the subsample 1980-2011. The evidence of a value premium is even stronger for this subsample, with returns increasing monotonically with BM ratios for all of the quintiles. Figure 6.6 shows that standard deviations are negatively correlated with market capitalization, whereas it is very interesting to note that value portfolios are characterized by negative skewness and larger kurtosis than the other portfolios. Nevertheless, beta, coskewness and cokurtosis show a similar pattern as they decrease monotonically with the BM ratio. Therefore, the returns of the 25 ME/BM portfolios do not appear to be explained by a higher-moment CAPM. However, there is some hint that idiosyncratic skewness and kurtosis might partly explain the cross-section of returns, questioning the effects or the real possibility of diversification as systematic and

not idiosyncratic risk should matter. In particular, whereas there is no relationship between returns and the standardized comoments, there is a clear positive relationship between returns and idiosyncratic excess kurtosis, and a clear negative relationship between returns and idiosyncratic skewness as it is shown clearly in Figure 6.7.

6.2. Individual stocks

The data for individual stocks refer to monthly returns of all common stocks listed in the NYSE, AMEX, and NASDAQ (only from January 1972) exchanges from January 1926 to December 2010, obtained from the Centre for Research in Security Prices (CRSP). The empirical tests are conducted on individual stocks for the sample 1930-2010 and for the subsample 1980-2010. In the cross-sectional regressions used in the empirical tests, only stocks with 24 months of returns are included, since beta is calculated using short windows of 24 months.

Table 6.4 reports the summary statistics on the cross section of all stocks on December 1940/50/60/70/80/90 and 2000/10.

Some considerations can be made. The number of stocks in the cross section has increased over the years from less than 1,000 stocks until the 1950 to reach a peak of 9,681 in December 2000. The number of stocks appears to be related to the process of growth in the industrialization but also to time of economic boom from the 1980 to 2000. The second observation is that the standard deviation of the returns across stocks has increased since the 80's. In other words, there seem to be two different regimes in terms of volatility: before the 80's and after the 1980. Finally, the normality of the distribution of returns (joint hypothesis that skewness is equal to zero and kurtosis is 3) at cross-section is always rejected.

Table 6.3 – Descriptive statistics for the 25 ME/BM portfolios

Panel A: The Fama and French 25 Size/BM Portfolios 1926-2011

	B1	B2	B3	B4	B5	Avg	B1	B2	B3	B4	B5	Avg
	Means						Standard Deviations					
S1	0.45	0.81	1.02	1.16	1.39	0.97	12.25	10.58	9.23	8.65	9.59	10.06
S2	0.58	0.94	1.03	1.07	1.20	0.96	7.99	7.89	7.34	7.61	8.76	7.92
S3	0.67	0.87	0.97	0.99	1.13	0.93	7.65	6.60	6.75	6.82	8.63	7.29
S4	0.68	0.74	0.84	0.94	1.04	0.85	6.24	6.29	6.41	7.02	8.99	6.99
S5	0.59	0.60	0.65	0.68	-0.25	0.45	5.49	5.25	5.75	6.91	13.24	7.33
Avg	0.59	0.79	0.90	0.97	0.90		7.92	7.32	7.10	7.40	9.84	
	Skewness						Excess Kurtosis					
S1	2.74	4.45	1.82	2.79	3.12	2.98	28.16	57.84	15.76	30.83	30.61	32.64
S2	0.35	1.92	2.12	1.74	1.80	1.59	5.03	21.43	22.57	18.37	17.73	17.03
S3	1.03	0.30	1.06	1.22	1.94	1.11	10.63	6.66	14.54	13.29	19.73	12.97
S4	-0.22	0.85	0.99	1.86	2.07	1.11	3.52	12.33	14.76	20.72	22.07	14.68
S5	-0.01	-0.08	0.86	1.90	-4.83	-0.43	5.32	5.14	14.59	23.70	36.88	17.13
Avg	0.78	1.49	1.37	1.90	0.82		10.53	20.68	16.44	21.38	25.40	
	Jarque-Bera						Beta					
S1	0.00	0.00	0.00	0.00	0.00	0.00	1.63	1.46	1.38	1.30	1.39	1.43
S2	0.00	0.00	0.00	0.00	0.00	0.00	1.24	1.27	1.18	1.22	1.37	1.26
S3	0.00	0.00	0.00	0.00	0.00	0.00	1.27	1.13	1.14	1.13	1.38	1.21
S4	0.00	0.00	0.00	0.00	0.00	0.00	1.07	1.10	1.10	1.17	1.44	1.18
S5	0.00	0.00	0.00	0.00	0.00	0.00	0.97	0.92	0.98	1.13	1.23	1.05
Avg	0.00	0.00	0.00	0.00	0.00		1.24	1.18	1.16	1.19	1.36	
	Coskewness						Cokurtosis					
S1	3.86	5.51	4.76	4.76	5.43	4.86	1.64	1.69	1.58	1.51	1.60	1.60
S2	-0.39	3.55	4.06	3.93	4.75	3.18	0.99	1.40	1.36	1.42	1.60	1.35
S3	2.68	1.07	2.97	3.00	5.86	3.12	1.32	1.09	1.31	1.26	1.72	1.34
S4	-0.29	2.50	2.40	4.54	6.55	3.14	0.93	1.20	1.21	1.39	1.84	1.31
S5	0.54	0.36	2.41	4.95	-0.20	1.61	0.92	0.87	1.13	1.45	1.24	1.12
Avg	1.28	2.60	3.32	4.24	4.48		1.16	1.25	1.32	1.41	1.60	

Panel B: The Fama and French 25 Size/BM Portfolios 1980-2011

	B1	B2	B3	B4	B5	Avg	B1	B2	B3	B4	B5	Avg
	Means						Standard Deviations					
S1	0.00	0.79	0.88	1.00	1.07	0.75	8.06	6.79	5.66	5.30	5.76	6.31
S2	0.41	0.76	0.96	0.93	0.93	0.80	7.27	5.86	5.18	5.11	5.89	5.86
S3	0.54	0.81	0.83	0.84	1.10	0.82	6.75	5.43	4.91	4.87	5.22	5.44
S4	0.73	0.72	0.71	0.81	0.83	0.76	6.10	5.24	5.20	4.71	5.26	5.30
S5	0.58	0.66	0.55	0.56	0.67	0.60	4.84	4.71	4.64	4.50	5.07	4.75
Avg	0.45	0.75	0.79	0.83	0.92		6.60	5.61	5.12	4.90	5.44	
	Skewness						Excess Kurtosis					
S1	0.06	0.01	-0.46	-0.70	-0.87	-0.39	2.99	4.44	3.49	3.94	3.40	3.65
S2	-0.39	-0.82	-0.96	-1.16	-1.06	-0.88	1.75	3.05	3.63	3.82	3.56	3.16
S3	-0.50	-0.78	-0.83	-0.68	-1.01	-0.76	1.58	3.21	2.88	2.71	3.77	2.83
S4	-0.31	-0.86	-0.86	-0.70	-0.80	-0.71	1.93	3.81	3.67	2.23	3.15	2.96
S5	-0.42	-0.59	-0.53	-0.58	-0.54	-0.53	1.37	2.01	2.42	1.92	1.65	1.87
Avg	-0.31	-0.61	-0.73	-0.76	-0.86		1.92	3.30	3.22	2.92	3.11	
	Jarque-Bera						Beta					
S1	0.00	0.00	0.00	0.00	0.00	0.00	1.37	1.15	1.00	0.91	0.98	1.08
S2	0.00	0.00	0.00	0.00	0.00	0.00	1.35	1.11	0.97	0.93	1.04	1.08
S3	0.00	0.00	0.00	0.00	0.00	0.00	1.29	1.07	0.94	0.90	0.93	1.03
S4	0.00	0.00	0.00	0.00	0.00	0.00	1.21	1.06	1.01	0.89	0.95	1.02
S5	0.00	0.00	0.00	0.00	0.00	0.00	0.99	0.94	0.90	0.81	0.86	0.90
Avg	0.00	0.00	0.00	0.00	0.00		1.24	1.07	0.96	0.89	0.95	
	Coskewness						Cokurtosis					
S1	1.59	1.40	1.25	1.23	1.40	1.37	1.39	1.22	1.09	1.04	1.13	1.17
S2	1.41	1.30	1.19	1.23	1.32	1.29	1.35	1.19	1.07	1.06	1.13	1.16
S3	1.35	1.18	1.07	0.97	1.15	1.14	1.28	1.15	1.00	0.96	1.04	1.09
S4	1.14	1.14	1.14	0.90	1.04	1.07	1.16	1.14	1.09	0.90	1.00	1.06
S5	0.86	0.92	0.87	0.75	0.88	0.86	0.94	0.94	0.93	0.80	0.88	0.90
Avg	1.27	1.19	1.10	1.02	1.16		1.22	1.13	1.04	0.95	1.04	

Panel A and Panel B show the descriptive statistics for portfolios double sorted on market capitalization and the book-to-market ratio from July 1926 to December 2011 and from January 1980 to December 2011, respectively. S1 through S5 show the five quintiles (from the smallest to the largest) in terms of market capitalization. B1 through B5 show the five quintiles (from the highest to the lowest) in terms of book-to-market. The means are the average excess returns over the risk-free rate of a Treasury Bill. Jarque-Bera reports the probability under the null hypothesis of the normality of returns. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1973) as $c\hat{v}(r_i, r_m)/c\hat{v}(r_m, r_m)$, $c\hat{v}(r_i, r_m^2)/c\hat{v}(r_m, r_m^2)$, and $c\hat{v}(r_i, r_m^3)/c\hat{v}(r_m, r_m^3)$ respectively.

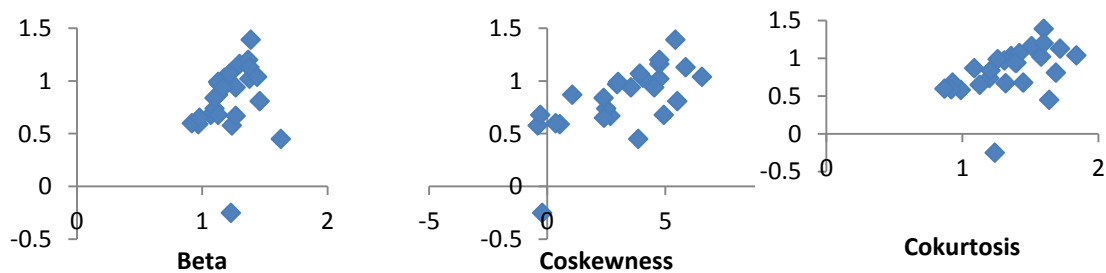


Figure 6.5 – Returns versus comoments of the 25 ME/BM portfolios 1926-2011

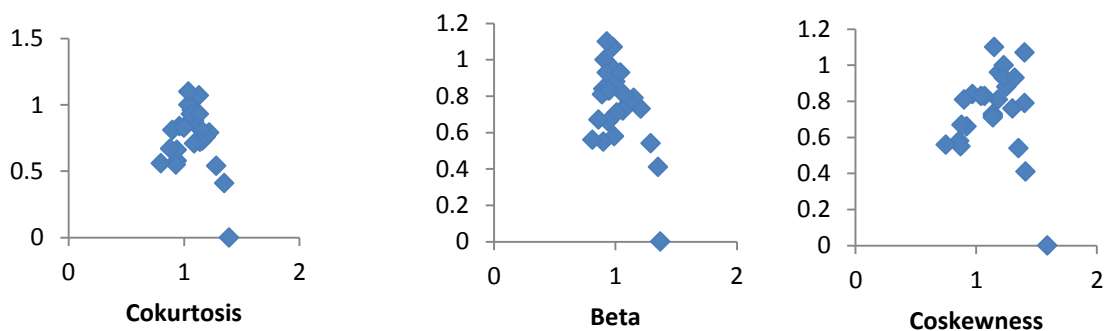


Figure 6.6 – Returns versus comoments of the 25 ME/BM portfolios 1980-2011

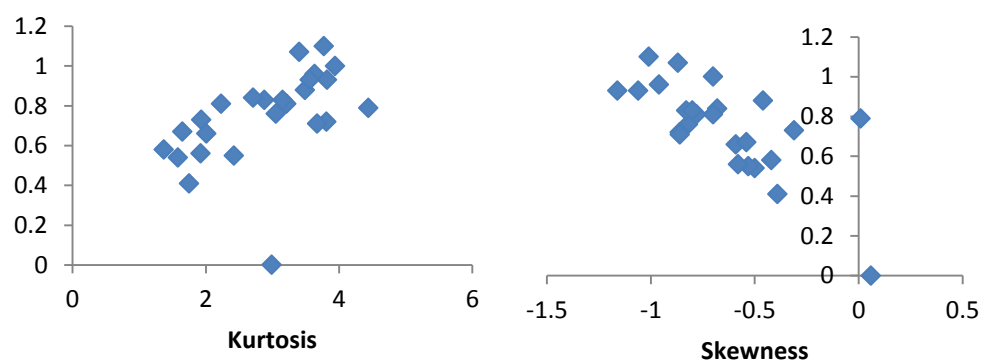


Figure 6.7 – Returns versus idiosyncratic higher moments of the 25 ME/BM portfolios 1980-2011

Table 6.4 Summary statistics on the cross section of all stocks on December 1940/50/60/70/80/90 and 2000/10

December Year	1940	1950	1960	1970	1980	1990	2000	2010
Number of stocks	852	986	1,159	2,242	4,733	7,507	9,681	7,820
Mean Excess Return	0.54%	2.00%	0.39%	-1.55%	1.63%	-0.50%	0.62%	1.84%
t-statistic	7.17*	50.02*	7.03*	-36.00*	40.72*	-15.34*	16.11*	42.85*
Standard deviation	0.022	0.013	0.019	0.020	0.028	0.028	0.038	0.038
Skewness	1.14	0.28	-7.10	-0.70	1.18	0.47	1.34	0.71
Kurtosis (excess)	9.46	0.65	146.79	1.06	3.04	20.09	9.65	18.77

This Table reports the summary statistics for the cross-sectional regression of all stocks on December of 1950/50/60/70/80/90 and 2000/10: number of stocks, mean excess return, t-statistic, standard deviation, skewness and excess kurtosis.

6.3. Conclusion

This Chapter has presented the summary statistics of the data used in this thesis. Specifically, the first part deals with portfolios of stocks sorted on market capitalization and/or book-to-market. The main statistics show that portfolios returns are positively correlated with standardized measures of covariance, coskewness and cokurtosis for the period 1926-2011. Small stocks and stocks with high book-to-market ratio tend to yield higher returns but to have also higher standard deviation, coskewness and cokurtosis. This evidence appears to favour a higher-moment CAPM as a premium for the higher moment might reconcile the observed returns with the systematic risk accounted for by beta, coskewness and cokurtosis. However, the positive relation between returns and systematic risk disappears over the period 1980-2011, especially for portfolios double sorted on ME and BM. Interestingly, there is a positive relationship between average returns and idiosyncratic kurtosis and a negative relationship between average returns and idiosyncratic skewness over the period 1980-2011 which might suggest that diversification is not fully obtained by investors.

The second part of the chapter has shown the descriptive statistics of individual stocks in US from 1930 to 2010. Results show that returns have become more volatile from 1980, that returns are not normally distributed across stocks and that the number of stocks has increased over time with a boom in the period surrounding 2000.

In the next chapter the main empirical tests of the CAPM will be conducted on the portfolios of stocks sorted on ME and/or BM, that is the Gibbon, Ross and Shanken (1989) time series test, the Black, Jensen and Scholes (1972) cross-sectional test and the Fama and MacBeth (1973) two-pass methodology. In the second part of the next chapter, the Four-Moment CAPM which incorporates coskewness and cokurtosis will be tested on the same portfolios.

Chapter 7

The Capital Asset Pricing Model and the Cross-Section of Equity Returns

7.0. Introduction

This and the following chapter discuss the empirical results of the models and methodologies applied in this thesis and outlined in Chapter 5. Specifically, the first section of this chapter deals with the time-series and cross-sectional test of the unconditional CAPM. The traditional CAPM is applied to the Fama and French portfolios (10 size portfolios, 10 book-to-market portfolios, and 25 portfolios of stocks double sorted on size and the book-to-market ratio) in order to demonstrate the extent of the problems encountered by the CAPM when confronted with portfolios sorted on market capitalization and book-to-market ratios – the size and book-to-market anomalies. The CAPM is first applied to the period from July 1926 to July 2011 in order to test whether it works quite well over the long term, as highlighted by Ang and Chen (2002), but has several problems in the later period (1980-2011).

Recall that the CAPM can be written as:

$$E(R_i - R_f) = \alpha + [E(R_m - R_f)]\beta_{im} \quad \forall i = 1, \dots, N \quad (7.1)$$

with the expected market risk premium $\lambda_\beta = [E(R_m - R_f)]$, it becomes:

$$E(R_i - R_f) = \alpha + \lambda_\beta \beta_{im} \quad (7.2)$$

and the hypotheses to test are that the intercept is zero, and the risk premium is positive and equal to the expected excess return on the market portfolio:

$$\left\{ \begin{array}{l} \alpha = 0 \\ \lambda_\beta > 0 \\ \lambda_\beta = \overline{(R_m - R_f)} \end{array} \right. \quad (7.3)$$

This chapter reports the results of the standard tests of the CAPM. First, a simple cross-sectional regression with full sample constant betas is applied using average excess returns as the dependent variable (Black, Jensen, and Scholes, 1972, henceforth BJS). Next, the Fama and MacBeth (1973) methodology is introduced first with rolling regression betas and then with dynamic conditional correlations betas obtained through a Multivariate GARCH. In terms of time-series test of the CAPM, the GRS (Gibbons, Ross, and Shanken, 1989) test is performed for each set of portfolios for the overall sample and for the more recent period 1980-2011. The hypothesis to be tested is that all the intercepts are jointly insignificant, that is, in mathematical terms:

$$H_0: \alpha_i = 0, \quad \forall i = 1, \dots, N$$

Moreover, following Pettengill et al. (1995), the CAPM is tested separately in an up and a downmarket to test whether there is a segmented relationship between beta and returns. The sample is split into up- and downmarket according to the positive or negative sign of the monthly market excess return, and the risk premium is estimated separately in the two subsamples. The objective of the first section of this chapter is one hand to provide an overview of the main empirical failures of the unconditional CAPM in order to underpin the empirical study which follows in which extensions of the CAPM are investigated and on the other hand to show the results of the traditional tests of the CAPM using more recent data.

In summary, in the first section of this chapter the results of the following empirical tests are reported:

- The simple cross-sectional regression, BJS test;
- The Fama and MacBeth cross-sectional methodology;
- The times series test, GRS;
- The Fama and MacBeth cross-sectional methodology with DCC betas;

- The Pettengill et al. conditional test.

The second section of the chapter deals with the important extension of the CAPM which incorporates higher moments of the distribution of returns of the market portfolio. The CAPM of Lintner (1964) and Sharpe (1965) is derived under the assumption of normally distributed returns¹⁵ or quadratic utility functions with investors interested in only the first two moments of the distribution of returns of the market portfolio. Under this assumption investors make their investment choice on the basis of a mean-variance trade-off. However, stylized facts (see for example Taylor, 2005) show that asset returns are not normally distributed. Typically, they are asymmetric and leptokurtic, that is, their distribution is characterized by thick tails with extreme outcomes which are more likely than is predicted by the normal distribution (excess of kurtosis), and with large negative returns more likely than large positive returns (asymmetry or skewness). The evidence that returns exhibit non-normal distributions with skewness and excess kurtosis, and the observation that investors dislike stocks with extreme outcomes, together form the basis for the introduction of a multi-moment CAPM.

Certain studies (Kraus and Litzenberger, 1973 and Fang and Lai, 1997, among others) have argued that the introduction of an additional premium for cokurtosis and coskewness might explain some asset pricing puzzles. While skewness measures the asymmetry of the distribution of returns, kurtosis measures the sensitivity to extreme returns or the thickness of the tails of the distribution of returns. Investors have a preference for higher skewness and lower kurtosis, as they naturally prefer a higher probability for returns above the average and they dislike extreme outcomes (extreme losses). Therefore, if an asset contributes by reducing the skewness or increasing the

¹⁵ In general elliptical distributed returns, of which normally distributed returns are a particular case, are compatible with the CAPM.

kurtosis of the market portfolio, its risk would be underestimated by beta alone. Hence, if small stocks and high book-to-market stocks have a coskewness and cokurtosis premium, these premia might explain the small and value puzzles. That is, the returns required for the higher moments might explain why small and high book-to-market stocks yield on average higher returns. However, it might be that small and high book-to-market stocks add to negative skewness and large kurtosis in the market portfolio when investors are more risk-averse, in a recession, and therefore their conditional coskewness and cokurtosis might explain the anomalies of the CAPM. This is the rationale for a conditional Four-Moment CAPM.

In this section of the chapter a higher-moment extension of the CAPM, beyond mean and variance analysis, is implemented and tested. In particular, it is shown that a multi-moment CAPM, using conditional betas obtained with a DCC GARCH approach, is not capable of explaining the returns of a cross section of portfolios sorted on size or book-to-market ratios over the subsample 1980-2011. The model does not perform particularly well when compared with the more parsimonious CAPM. However, the introduction of DCC-based loadings improves both models (the CAPM and Four-Moment CAPM) considerably when compared to rolling regression betas, especially when the models are tested on ME-sorted portfolios. In particular, this chapter shows the empirical results of the Four-Moment CAPM tested on the well-known size and book-to-market sorted portfolios – the Fama and French portfolios: 10 ME portfolios, 10 BM portfolios, and 25 ME/BM portfolios.

The study presents a number of innovations: (i) the higher moment CAPM is derived such that the sum of the risk premium for all factors (beta, systematic coskewness and systematic cokurtosis) equals the market excess return; (ii) non-standardized coskewness is used, as the market portfolio skewness might be close to zero; (iii) the conditional coskewness and cokurtosis are estimated as counterparts of the conditional

covariance using a DCC GARCH approach; and (iv) non-standardized coskewness is used so that the estimate of the coefficient associated with skewness should be negative and independent of the market skewness.

Recall briefly that the model to be tested is as that given in Equation 7.4:

$$\begin{aligned} E[(R_i) - R_f] &= \lambda_\beta \frac{E(r_i \cdot r_m)}{E(r_m^2)} + \lambda_s E(r_i \cdot r_m^2) + \lambda_k \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} \\ &= \lambda_\beta \beta_i + \lambda_s s_i + \lambda_k k_i \end{aligned} \quad (7.4)$$

and the hypotheses tested are:

$$\left\{ \begin{array}{l} \lambda_\beta > 0 \\ \lambda_s < 0 \\ \lambda_k > 0 \\ \lambda_\beta + \lambda_s s_m + \lambda_k > 0 \\ \lambda_\beta + \lambda_s s_m + \lambda_k = \overline{(R_m - R_f)} \end{array} \right. \quad (7.5)$$

The risk premium is expected to be positive and approximately equal to the average historical market excess return. The premia for systematic covariance and systematic cokurtosis are expected to be positive, whereas the premium for the systematic coskewness is expected to be negative as investors have a preference for positive skewness. The Four-Moment CAPM is tested unconditionally using rolling regression betas and then using DCC betas. In Chapter 8, time-varying risk premia will be introduced to more closely approximate reality. The empirical results report both the multi-moment CAPM and the traditional CAPM to allow a comparison between the two models.

7.1. The BJS and GRS Test

The CAPM states that the difference in returns across assets depends exclusively on market betas. The easiest way to conduct a preliminary test of the CAPM is by means of the BJS procedure, which is to regress the average excess returns of individual assets

(stocks or portfolios) on full sample betas. In other words, first, for each portfolio beta is estimated through a full-sample time series regression, and second, a cross-sectional regression of the average excess returns on the estimated betas is conducted. This methodology is applied to the groups of portfolios under examination, even acknowledging the well known limitations of this methodology; it is reported in this thesis for the historical relevance and ease of intuition.

All tests conducted will refer to a significance level of 5%.

7.1.1. Results based on 10 ME portfolios

Table 7.1 reports the regression results based on the BJS approach. As Panel A shows, when the full sample (1926-2011) is used, the average monthly excess returns of ten portfolios sorted on market capitalization is well explained by the market betas. The slope is positive and highly significant. The R-squared is also very large (95%), suggesting a strong relation between systematic risk and expected returns. This is depicted in the left hand side graph in Figure 7.1 and in Panel A of Table 7.2. The linearity seems very pronounced. Panel A shows that excess returns increase monotonically with the market beta. The smallest portfolios have both higher returns (1.15%) and higher betas (1.44), while the largest portfolios have lower returns and lower systematic risks. Despite this, however, the evidence contradicts the CAPM because the intercept is not only significant but also negative. In some respects, the relationship between average returns and betas seems too pronounced, that is, the slope is too steep, which is one possible reason why the intercept is negative.¹⁶ Indeed, the risk premium gives a cumulative yearly excess return of 12%, which appears excessive. The results for the sub-sample period (1980-2011) given in Panel B of Table 7.1 are weaker. The risk premium is still positive and significant, but it is of smaller magnitude.

¹⁶ One other possibility is the absence of sufficient observations with excess returns and betas near zero.

The R-squared is substantially reduced to 56%. Finally, the intercept is negative but insignificant. This might suggest acceptance of the CAPM since the intercept is not significantly different from zero. However, the right hand side of Figure 7.1 shows that both returns and betas cluster in a limited area compared with the full sample. As can be seen from Panel B of Table 7.2, beta ranges from 0.94 to a maximum of 1.15.

It should also be noted that the market betas have declined over time, especially for small stock portfolios (from 1.44 to 1.03). Another point of note concerns the dispersion of returns. Excess returns are not at all dispersed (the monthly standard deviation of returns is 0.16 for the full sample but only 0.0006 for the more recent period), meaning that a market capitalization or size strategy does not represent a significant anomaly any longer, another widely recognized observation in the asset pricing literature (Schwert, 2003; Cochrane 2001).

One important conclusion from these preliminary results is that the CAPM should not be tested on portfolios that do not show dispersion in average returns or betas, as in this case. Without dispersion there is little to be tested. One possible explanation, however, is the difference between conditional and unconditional testing. Small stocks tend to lose more in a downmarket and to gain more in an upmarket. This might not be captured by unconditional models using beta alone. Finally, the BJS approach is well known for its limitations with respect to producing biased standard errors (Cochrane, 2001), and thus, the significant t-statistics may simply be due to under-estimated standard errors.

The time-series test using the GRS procedure does not support the CAPM. The rejection of the null hypothesis that all alphas are jointly insignificant depends once more on the sample used. For the overall period the GRS statistic is 1.14, considerably lower than the theoretical or critical value $F_{10,1021}$ of 1.83. Therefore, the full sample GRS test

results support the CAPM. However, for the last 30-year period (1980-2011), the GRS statistic is 1.91 with an $F_{10,368}$ of 1.83. The CAPM is therefore rejected.

7.1.2. Results based on 10 BM portfolios

Table 7.1, Panel C and D, shows the results of the test on the CAPM using ten book-to-market portfolios. As Panel C in Table 7.1 shows, when the full sample (1926-2011) is used, the average monthly excess returns of ten portfolios sorted on book to market are well explained by the market betas with an estimated risk premium of 0.96%. Moreover the R-squared is as high as 81%. Indeed, looking at the full sample, no value anomaly appears to be present as the SML shown in Figure 7.2 has a positive slope, although slightly declining in the highest-beta portfolio. However, the intercept is negative and significant and this leads to a rejection of the CAPM's assumption that the alpha should be insignificantly different from zero. This result is dramatically altered in the 1980-2011 period, shown in Panel D of Table 7.1. The simple cross-sectional regression of the average returns over the market betas has a negative adjusted R-squared and a negative estimated market premium with a positive but insignificant intercept. Therefore, the CAPM is rejected firmly for this second period as neither is the estimated market premium positive nor the intercept is insignificantly different from zero.

As can be seen from Panel C of Table 7.2, the high book-to-market portfolios have a higher return and a much higher beta than the low book-to-market portfolios for the full sample. The high beta is the result of a higher beta in the first half of the sample. Indeed, examining Panel D of Table 7.2, it becomes clear that the betas of the high book-to-market portfolios have declined significantly over time. Although the value premium is still present, especially in the 9th and 10th decile, the market beta does not explain the returns any longer – there is indeed very low dispersion in the market betas across portfolios and this is the main reason why the CAPM fails. The result is due

mainly to the decline in beta for the high book-to-market portfolios (consisting of value stocks).

In terms of the time-series test of the CAPM, the GRS statistic is 1.41 for the full sample and 0.82 for the second period, leading to a non rejection of the null hypothesis that the alphas are jointly insignificantly different from zero in both cases. However, whereas for the full sample the slope of the SML is positive and the model cannot be rejected, for the second period the hypothesis that the alphas are jointly zero cannot be rejected, but the slope of the SML, as shown in Figure 7.2, is negative, and therefore the model cannot be accepted.

7.1.3. Results based on 25 ME/BM portfolios

Finally, the CAPM is tested on the 25 portfolios double sorted on size and book-to-market, which arguably represents a greater challenge for the CAPM. The simple cross-sectional regression of the average returns of the portfolios on the market betas leads to a positive but insignificant relationship between beta and returns for the full sample (Panel E of Table 7.1), with an insignificant intercept. However, the R-squared is only 1% showing a very poor linear relationship between returns and beta. The cross-sectional regression for the subsample (Panel F of Table 7.1) shows a significant negative relationship between beta and returns, the opposite sign to that expected in theory, with a significant large intercept (1.62%) and an R-squared of 28%. The results of the cross-sectional test are therefore negative for both samples and lead to a rejection of the CAPM.

With regard to the time series test, the GRS statistic for the full sample is 3.77, leading to a rejection of the null hypothesis of zero pricing errors. The GRS test statistic for the subsample 1980-2011 is of 5.12 which is larger than the theoretical F-statistic leading

again to a firm rejection of the null hypothesis, a result caused mainly by the large pricing errors of the small-growth and small-value portfolios.

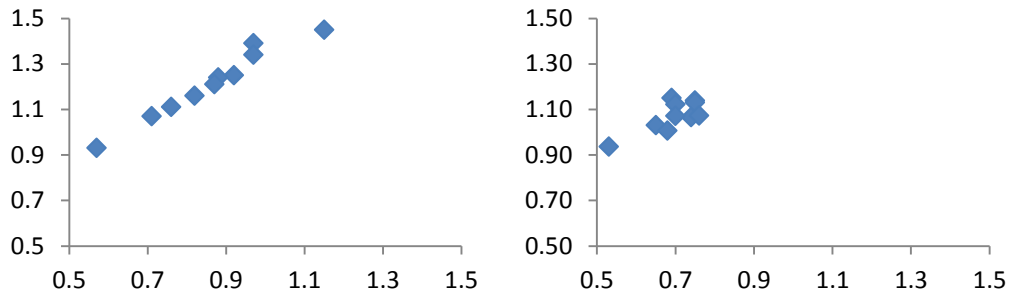
Table 7.1: The BJS test for the CAPM using 10 ME, 10 BM, and 25 ME/BM portfolios over the period 1926-2011 and 1980-2011

10 ME portfolios			
Sample Period	α	λ_β	R^2
Panel A: 1926-2011	-0.0035 (-3.96)*	0.0100 (13.66)*	0.95
Panel B: 1980-2011	-0.0011 (-0.48)	0.0075 (3.41)*	0.54
10 BM portfolios			
Sample Period	α	λ_β	R^2
Panel C: 1926-2011	-0.0029 (-1.74)*	0.0095 (6.36)*	0.81
Panel D: 1980-2011	0.0091 (1.96)	-0.0025 (-0.51)	-0.09
25 ME/BM portfolios			
Sample Period	α	λ_β	R^2
Panel E: 1926-2011	0.0030 (0.64)	0.0043 (1.11)	0.01
Panel F: 1980-2011	0.0162 (5.92)*	-0.0086 (-3.24)*	0.28

The Table reports the intercept, slope and R-squared of the cross sectional regression of the average returns of, respectively, 10 ME-sorted portfolios (Panel A and B), 10 BM-sorted portfolios (Panel C and D), and 25 ME/BM-sorted portfolios (Panel E and F) on their full-sample beta for the sample period 1926-2011 and the sub-sample 1980-2011. T-statistics are given in parentheses. The asterisk shows significance at the 5% level.

The results refer to the model:

$$\overline{R}_i - R_f = \alpha + \lambda_\beta \beta_{im}$$



(1926-2011)

(1980-2011)

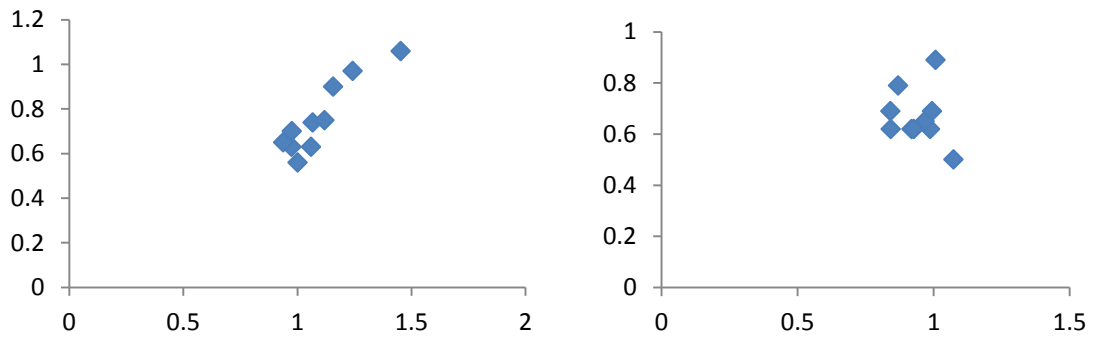
The Figure shows a scatter diagram of the full-sample beta against average excess returns for 10 ME-sorted portfolios against their full-sample beta for the periods 1926-2011 and 1980-2011. Excess returns are on the vertical axis and betas are on the horizontal axis.

Figure 7.1 SML for 10 ME portfolios

Table 7.2: Average excess returns and full-time beta for the ten ME and the ten BM portfolios for the period 1926-2011 and 1980-2011

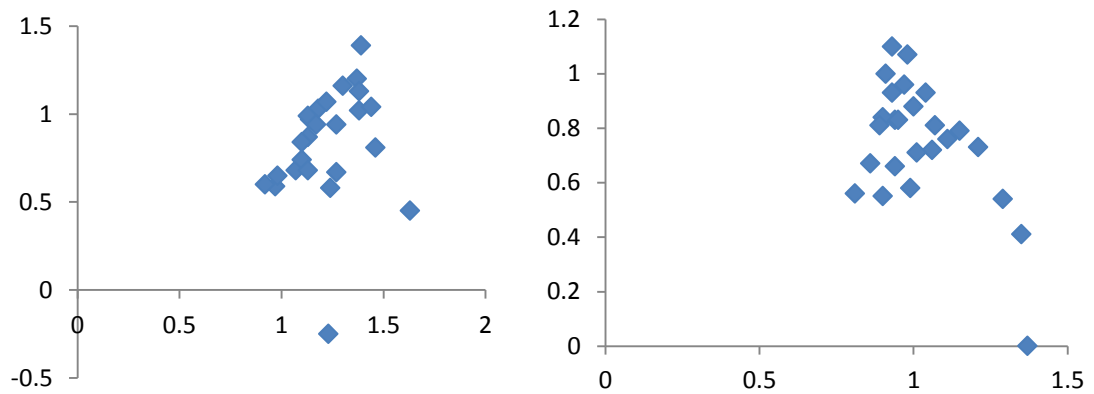
ME portfolios										
Deciles	Small	2	3	4	5	6	7	8	9	Big
Panel A: (1926-2011)										
Excess Returns (%)	1.15	0.97	0.97	0.92	0.88	0.87	0.82	0.76	0.71	0.57
Beta	1.44	1.39	1.34	1.25	1.24	1.21	1.16	1.11	1.07	0.93
Panel B: (1980-2011)										
Excess Returns (%)	0.65	0.69	0.75	0.7	0.75	0.74	0.76	0.7	0.68	0.53
Beta	1.03	1.15	1.14	1.12	1.13	1.07	1.07	1.07	1.01	0.94
BM portfolios										
Deciles	Low	2	3	4	5	6	7	8	9	High
Panel C: (1926-2011)										
Excess Returns (%)	0.56	0.63	0.65	0.63	0.7	0.74	0.75	0.9	0.97	1.06
Beta	1.00	0.97	0.94	1.06	0.98	1.07	1.12	1.16	1.24	1.45
Panel D: (1980-2011)										
Excess Returns (%)	0.5	0.62	0.65	0.69	0.62	0.62	0.69	0.62	0.79	0.89
Beta	1.07	0.99	0.97	0.99	0.92	0.93	0.84	0.84	0.87	1.01

The Table shows the average excess return and full-sample beta for 10 ME-sorted portfolios (Panel A and B), and for 10 BM-sorted portfolios (Panel C and D).



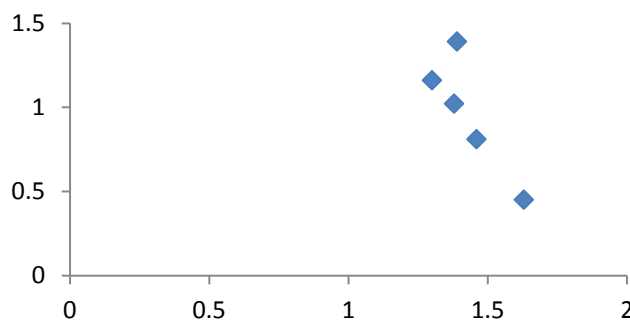
The Figure shows a scatter diagram of full-sample beta against average excess returns for 10 BM-sorted portfolios against their full-sample beta for the periods 1926-2011 and 1980-2011. Excess returns are on the vertical axis and betas are on the horizontal axis.

Figure 7.2 SML for 10 BM portfolios



The Figure shows a scatter diagram of full-sample beta against average excess returns for 25 ME/BM-sorted portfolios against their full-sample beta for the periods 1926-2011 and 1980-2011. Excess returns are on the vertical axis and betas are on the horizontal axis.

Figure 7.3 SML for 25 ME/BM portfolios



The Figure shows a scatter diagram of full-sample beta against average excess returns for the 5 of the 25 ME/BM-sorted portfolios in the smallest quintile for market capitalization against their full-sample beta for the periods 1926-2011. Excess returns are on the vertical axis and betas are on the horizontal axis.

Figure 7.4. SML for small size quintile portfolios 1926-2011

From the plot of the excess returns against the market betas in Figure 7.3, there is no evidence of a clear and positive SML. The outlier with the high beta and very low excess returns is the small-growth portfolio in the first size quintile – the main reason underlying the historically poor performance of small-growth stocks.

Figure 7.4 demonstrates well that when the quintile of small size is considered, the excess returns decrease with beta; thereby contradicting markedly the CAPM as the SML is inverted. The excess returns conditional on market capitalization are not positively related to the betas, but to the book-to-market ratio.

It is interesting to observe the alphas and their significance for each size quintile over the last 30 years, in order to understand how the positive alphas are related not to higher betas but to the book-to-market characteristic, as shown in Table 7.3. Most of the portfolios with alphas different from zero are in the small and medium size deciles – 8 portfolios out of 15 are in the first three quintiles versus 1 out of 10 in the last 2 quintiles. In almost all of the quintiles beta decreases with size and the returns increase with the book-to-market ratio. The anomaly is mainly due to the positive alphas of the small value portfolios and the low returns and high betas of the small growth portfolios. Growth portfolios tend to have negative alphas, especially in the smaller size quintiles, whereas alpha becomes positive and increases with the value characteristic (high book-to-market).

7.2. The Fama and MacBeth (1973) methodology

A single cross-sectional regression provides a good basis for understanding the problems of the CAPM, but suffers the shortcomings of a constant beta versus a time-varying beta, as well as the cross-sectional correlation problem of returns across portfolios. The methodology discussed in this section should address these problems.

The Fama and MacBeth (1973) methodology requires an estimation of the market betas

from time series rolling regressions for 60 months in the first step, and an estimation of the market premium in the second step by running monthly cross sectional regressions of the excess returns on the estimated betas, as in Equation 7.6:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1} \lambda_t \quad i = 1, 2, \dots, N \quad \text{for each } t=60, \dots, T \quad (7.6)$$

This widely employed methodology is applied to the three groups of portfolios under observation. The subscript $\hat{\beta}_{i,t-1}$ indicates that beta is time varying and is estimated with one lag.

7.2.1. Results based on the 10 ME portfolios

The results for the intercept and the estimated market premium, together with the historical average risk premium, are presented in Table 7.4, Panels A and B, for the ten ME portfolios when the Fama and MacBeth procedure is applied. The results reported in Panel A suggest that the market premium (the average slope of the cross sectional regressions) is 1.41% and is significant at the 5% level, whereas the alpha is insignificant and negative though quite large in magnitude, at -0.70%. The estimated market premium is much larger than the historical average market excess return for the full sample (1.41% against 0.62%). Therefore, the CAPM seems to hold relatively well for portfolios sorted on market capitalization for the full sample.

The CAPM is tested on the same portfolios over the most recent period 1980-2011 in Panel B. The estimated market premium is 0.77% but is insignificant, and the intercept is not significantly different from zero. Overall, from the results on Table 7.4, it can be argued that the CAPM explains the cross-section of equity returns when portfolios are sorted on market capitalization reasonably well, especially in the long run.

Table 7.3 Excess returns, beta and alpha of the 25 ME/BM portfolios divided into 5 quintiles for the period 1980-2011

	B1	B2	B3	B4	B5	Avg
Excess Return						
S1	0.01	0.79	0.87	0.99	1.07	0.75
S2	0.40	0.76	0.95	0.93	0.92	0.79
S3	0.54	0.80	0.83	0.84	1.1	0.82
S4	0.72	0.71	0.71	0.80	0.82	0.72
S5	0.58	0.65	0.55	0.55	0.67	0.60
Avg	0.45	0.74	0.78	0.82	0.92	0.74
Beta						
S1	1.37	1.15	1	0.91	0.98	1.08
S2	1.35	1.11	0.97	0.93	1.04	1.08
S3	1.29	1.07	0.94	0.9	0.93	1.03
S4	1.21	1.06	1.01	0.89	0.95	1.02
S5	0.99	0.94	0.9	0.81	0.86	0.9
Avg	1.24	1.07	0.96	0.89	0.95	1.02
Alpha						
S1	-0.77 (2.99)*	0.14 (0.65)	0.31 (1.82)	0.48 (2.87)*	0.51 (2.79)*	0.13
S2	-0.35 (-1.85)	0.13 (0.89)	0.4 (3.02)*	0.4 (2.83)*	0.33 (1.89)	0.18
S3	-0.18 (-1.13)	0.20 (1.70)	0.29 (2.50)*	0.33 (2.51)*	0.57 (3.73)*	0.24
S4	0.04 (0.33)	0.11 (1.15)	0.14 (1.17)	0.30 (2.54)*	0.28 (1.90)	0.17
S5	0.02 (0.26)	0.12 (1.27)	0.04 (0.37)	0.10 (0.77)	0.18 (1.14)	0.09
Avg	-0.19	0.14	0.24	0.32	0.37	0.16

This Table shows the full-sample beta, the excess return, the intercept (alpha) of the CAPM, and the t-statistic for the 25 ME/BM-sorted portfolios when divided into 5 quintiles in terms of market capitalization. S1 through S5 show the five quintiles (from the smallest to the largest) in terms of market capitalization. B1 through B5 show the five quintiles (from the highest to the lowest) in terms of the book-to-market ratio.

7.2.2. Results based on the 10 BM portfolios

The CAPM is next tested on ten book-to-market portfolios, the results of which are reported in Table 7.4, in Panels C and D. For the full sample (Panel C) the market premium has the correct positive sign (0.47%) but is not significant at the 5% level, whereas the intercept is not significantly different from zero, as expected theoretically.

However, the result is dramatically different for the later period. As shown in Panel D, the alpha is significant at the 5% level, whereas the market premium has the incorrect (negative) sign though is insignificant, reflecting the inability of the CAPM to price the high book-to-market portfolio over this 30-year span. The positive and large intercept shows that most of the cross-sectional variation of returns is not captured by systematic risk (beta). Therefore, the CAPM does not hold when BM portfolios are introduced, especially over the last three decades. These results are in line with the findings of the previous literature starting with Fama and French (1992), and confirm that a simple time-variation in beta, as the one obtained by rolling regression, cannot rescue the CAPM.

7.2.3. Results based on 25 ME/BM portfolios

Finally, the CAPM is tested on the 25 portfolios, double sorted on size and the book-to-market ratio, arguably the greatest challenge for the CAPM as empirical studies have shown that beta cannot explain the size and book-to-market anomalies. For the full sample, as reported in Panel E of Table 7.4, the intercept is found to be positive and significant, with a small positive but insignificant market premium. The result therefore strongly contradicts the theory. The model deteriorates when the CAPM is tested on the same portfolios over the later period of 1980-2011, as shown in Panel F. The alpha is positive, larger and even more significant than before, contradicting the theory of the CAPM, whereas the market risk premium has a negative sign though is insignificant.

This result confirms the result obtained by Fama and French (1992) that when portfolios are formed using the book-to-market characteristic there is no longer a positive relationship between beta and returns. The decline in beta, especially for the high book-to-market portfolios, makes it impossible for the CAPM to price the 25 portfolios. The results therefore confirm that the CAPM works quite well over the long horizon but that it has some problems in more recent decades (as documented by Fama and French, 1992), especially when the BM characteristic is used to sort portfolios.

7.3. The CAPM with DCC betas

The first attempt to save the single-factor model, or at least to make it more realistic, requires the use of time-varying betas where the time variation is obtained in a more sophisticated way than in the simple rolling regressions. As betas are predictive in their nature (meaning that investors should invest on the basis of the ex-ante expectations), one possible solution is to adopt conditional betas obtained through dynamic conditional correlations and multivariate GARCH. In other words, the betas obtained through this methodology, which allows the estimation of volatilities, covariances and correlations, might help to obtain more accurate betas than the simple rolling regression betas. Beta is obtained as the estimated conditional covariance divided by the estimated conditional volatility where the correlations between asset returns and market returns change over time.

The Dynamic Conditional Correlation (DCC) approach is used to estimate the time-varying betas, and the Fama and MacBeth methodology is applied to estimate the market premium, that is, every month the excess returns are regressed at cross section on the DCC betas to estimate the market premium. Finally, the average of the estimated market premia is the estimated market premium and the significance is tested with a t-test in the usual way. The results of the CAPM for the three groups of portfolios under examination are reported in Table 7.5.

Table 7.4 The CAPM for the 10 ME, 10 BM, and 25 ME/BM portfolios for the period 1926-2011 and 1980-2011

10 ME portfolios			
Sample Period	α	λ_β	$\overline{(r_m - r_f)}$
Panel A: 1926-2011	-0.0070 (-1.64)	0.0141 (2.97)*	0.0062
Panel B: 1980-2011	-0.0015 (-0.29)	0.0077 (1.34)	0.0056

10 BM portfolios		
Sample Period	α	λ_β
Panel C: 1926-2011	0.0024 (0.89)	0.0047 (1.41)
Panel D: 1980-2011	0.0103 (2.71)*	-0.0044 (1.08)

25 ME/BM portfolios		
Sample Period	α	λ_β
Panel E: 1926-2011	0.0060 (2.50)*	0.0025 (0.85)
Panel F: 1980-2011	0.0114 (3.44)*	-0.0043 (-1.11)

The Table reports the intercept and slope of the Fama and MacBeth cross sectional regression of the monthly returns of, respectively, 10 ME-sorted portfolios (Panel A and B), 10 BM-sorted portfolios (Panel C and D), and 25 ME/BM-sorted portfolios (Panel E and F) on their rolling regression beta for the full sample period 1926-2011 and the sub-sample 1980-2011. T-statistics are given in parentheses. The asterisk shows significance at the 5% level. Results are obtained from the following regression:

$$\overline{R_i - R_f} = \alpha + \lambda_\beta \beta_{im}$$

Table 7.5 The CAPM for 10 ME, 10 BM and 25 ME/BM portfolios for the period 1926-2011 and 1980-2011

Portfolios	α	λ_β
Panel A: 1926-2011		
10 ME portfolios	0.0006 (0.18)	0.0065 (1.91)
10 BM portfolios	0.0016 (0.69)	0.0049 (1.77)
25 ME/BM portfolios	0.0118 (5.07)*	-0.0031 (-1.25)
Panel B: 1980-2011		
10 ME portfolios	-0.0021 (-0.40)	0.0083 (1.62)
10 BM portfolios	0.0084 (2.24)*	-0.0026 (-0.64)
25 ME/BM portfolios	0.0107 (3.31)*	-0.0033 (-0.91)

The Table reports the intercept and slope of the Fama and MacBeth cross sectional regression of the monthly returns of, respectively, 10 ME-, 10 BM-, and 25 ME/BM-sorted portfolios on their DCC betas for the full sample period 1926-2011 and the sub-sample 1980-2011. T-statistics are given in parentheses. The asterisk shows significance at the 5% level. Results are obtained from the following regressions:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1}\lambda_t$$

7.3.1. Results of the CAPM with DCC betas for the 10 ME portfolios

The estimated market premium for the ten size portfolios (0.65%) is now rather different from the simple rolling regressions beta (1.14%) for the full sample (Panel A), and is very close to the average historical market premium (0.62%), and moreover it is almost significant, whereas the intercept is not significantly different from zero. The improvement obtained with DCC betas is marked. For the subsample (Panel B), the estimation of the CAPM leads to a positive and insignificant risk premium (0.83%) and to a negligible negative intercept which is insignificant. Therefore, although being statistically rejected at 5% level, the CAPM appears to hold reasonably well in economic terms for the ten size portfolios.

7.3.2. Results of the CAPM with DCC betas for the 10 BM portfolios

The model also improves over the full sample for the ten BM portfolios, with an estimated risk premium of 0.49% which is almost significant at 5% level, and with the intercept insignificant and very close to zero. However, for the ten book-to-market portfolios over the subsample (Panel B) the risk premium is estimated to give the incorrect negative sign and moreover the intercept is positive and significant. Therefore, for the BM portfolios, the CAPM with conditional betas seems to hold only for the full sample, but not for the last 30 years. In other words, the DCC betas improve the performance of the model for the ten ME portfolios for the full sample and for the subsample, whereas they improve the performance of the CAPM only for the full sample when the ten BM portfolios are used.

7.3.3. Results of the CAPM with DCC betas for the 25 ME/BM portfolios

Finally, for the 25 size/BM portfolios the usual negative slope of the SML is observed both for the full sample and for the subsample, although it is insignificant. The model does not price the 25 portfolios as can be noted from the large positive and significant intercept (1.18% and 1.07% in the full sample and subsample, respectively), meaning that the CAPM cannot explain why some of the ME/BM-sorted portfolios earn higher returns than others.

Hence, for the 25 portfolios, the problem of a negative market premium already obtained for the ten BM portfolios is further exacerbated, with a very significant positive unexplained alpha and an insignificant but negative risk premium. Therefore, one can conclude that whilst the DCC betas do not improve the estimates for the later 30 year period, they do improve the estimates for the ten ME portfolios over the full sample.

7.4. The CAPM in up and downmarkets: the dual test of Pettengill et al. (1995)

7.4.1. Dual tests of the CAPM with rolling regression betas

Confronted with the failure of the unconditional CAPM, the introduction of the conditional CAPM is suggested on the assumption that time-varying factor loadings and risk premia might better explain the cross section of average returns. Pettengill et al. (1995) introduce a dual test of the CAPM in up and downmarkets, depending on the positive or negative sign of the excess market return. In other words, the risk premium is estimated separately in up and downmarkets and the hypotheses that the risk premium is, respectively, positive and negative are tested.

The results and the estimation of the up and downmarket with rolling regression betas are reported in Table 7.6. For the size portfolios, the results show a substantially larger market premium in an upmarket than in a downmarket in absolute value, and with the opposite sign (positive in an upmarket and negative in a downmarket). A symmetric risk premium is obtained for the CAPM when the book-to-market dimension is introduced. Consistent with Pettengill et al., the beta premium is significant and positive in an upmarket and significant and negative in a downmarket. For the 25 portfolios, the risk premium is larger in absolute value in a downmarket than in an upmarket: 2.90% in an up market and -3.70% in a down market, both values significant. In summary, there is a positive relationship between beta and returns in an upmarket and a negative relationship in a downmarket.

Table 7.6 The dual test of Pettengill et al. for the CAPM using ten ME portfolios, ten BM portfolios and 25 ME/BM portfolios for the period 1926-2011 and 1980-2011

Portfolios	Up	Down
Panel A: 1926-2011		
λ_β 10 ME portfolios	0.0436 (6.43)*	-0.0305 (-5.55)*
λ_β 10 BM portfolios	0.0302 (6.42)*	-0.0335 (-8.95)*
λ_β 25 ME/BM portfolios	0.0290 (7.01)*	-0.0372 (-11.17)*
Panel B: 1980-2011		
λ_β 10 ME portfolios	0.0352 (5.09)*	-0.0339 (-3.69)*
λ_β 10 BM portfolios	0.0159 (3.03)*	-0.0355 (-6.14)*
λ_β 25 ME/BM portfolios	0.0207 (4.16)*	-0.0424 (-8.45)*

The Table reports the slope of the Fama and MacBeth cross sectional regression of the monthly returns of, respectively, 10 ME-, 10 BM-, and 25 ME/BM-sorted portfolios on their rolling regression betas for the full sample period 1926-2011 and the sub-sample 1980-2011 split into two samples: up when the excess market return is positive and down when the excess market return is negative. T-statistics are given in parentheses. The asterisk shows significance at the 5% level.

The dual test is replicated for the three groups of portfolios for the period 1980-2011, and the results are shown in Panel B. The results show that the risk premium has declined in the up regime: from 4.30% to 3.50% for the 10 ME portfolios, from 3.0% to 1.60% for the 10 BM portfolios, and from 2.90% to 2.0% for the 25 ME/BM portfolios. While a symmetric risk premium is obtained for the 10 ME portfolios, the risk premium is much larger in absolute value in a downmarket than in an upmarket when the book-to-market characteristic is introduced. The magnitude of the risk premium and the stronger negative relationship between risk and return in a downmarket in comparison to the positive risk premium and the positive relationship between risk and return in an upmarket may be the main reasons why the CAPM fails unconditionally. Although the risk-return relationship is positive in an upmarket and the historical frequency of upmarkets is higher than that for downmarkets, the magnitude of the relationship between risk and return in an upmarket does not seem to be strong enough to obtain a positive and significant unconditional average risk premium.

7.4.2. Dual tests of the CAPM with DCC betas

The dual test of the CAPM can be replicated using the DCC betas instead of the rolling regression betas. This is done and the results for the three groups of portfolios are reported in Table 7.7, which shows the estimated risk premium λ_β with t-statistics in parentheses. The CAPM shows a symmetric behaviour in the risk premium, as suggested by Pettengill et al. The results in Panel A show that the estimated risk premia are fairly symmetric over the full sample (3% in absolute value) for the ten size portfolios and for the ten BM portfolios. However, the risk-return relationship is asymmetric for the 25 ME/BM portfolios.

Panel B shows that for the ten size portfolios with DCC over the more recent period, the risk premia are still symmetric in the CAPM, although they are slightly larger in up than in downmarkets. For the ten BM portfolios the results confirm that the risk premium in negative markets is larger in absolute terms than in positive markets (-4.20% against 2.30%). Overall, the dual test of the CAPM confirms that beta is positively related to returns in an upmarket and negatively related to returns in a downmarket, i.e. riskier stocks earn more in upmarkets, but lose more in downmarkets, which is consistent with the concept of risk.

This dual test is however criticized as investors do not know ex-ante whether the market will be bullish or bearish, as the test uses realized returns as opposed to expected returns and, moreover, as up and downmarkets are characterized, respectively, by a prolonged positive and negative trend which can easily contain some movements in the opposite direction.

7.5. Tests of the CAPM and Four-Moment CAPM with rolling regression betas

This section of the chapter introduces the empirical results and tests of an important extension of the CAPM, specifically the Four-Moment CAPM. The CAPM and the Four-Moment CAPM are tested on the ten size portfolios for the overall sample 1926-2011 using rolling regression betas obtained from 60-month rolling time series regressions. This is indeed the most frequently used method to estimate conditional or time-varying covariance and its higher moments counterparts. However, for the multi-moment CAPM, the systematic covariance, coskewness, and cokurtosis cannot be estimated as the coefficients of a multiple time-series regression, but need to be estimated as the covariance of the returns with the de-meaned market excess returns, de-meaned market excess returns squared, and de-meaned market excess returns to the power of three. In other words, for the multi-moment CAPM we cannot use the first step of the methodology of Fama and MacBeth (1973) that is employed for the single factor CAPM. Table 7.8 reports the estimated parameters and their t-statistics for both models. Alternatively, the three comoments can be estimated conducting a simple regression of the asset returns on the market portfolio excess returns, the market portfolio excess returns squared and the market portfolio excess returns to the power of three separately as opposed to a multiple regression.

7.5.1. Results for 10 ME portfolios

The results of Panel A of Table 7.8 suggest that according to the CAPM, the market premium (the average slope of the cross sectional regressions) is 1.42% and is significant (with a t-statistic of 2.97), whereas the alpha is insignificant and negative, though gives a fairly sizeable -0.70%. The only significant factor for the Four-Moment CAPM is the beta premium of 0.89% which is much lower than the single-factor CAPM beta premium. However, the sign of the coskewness and cokurtosis premia are incorrect and the estimated total market premium is positive (0.43%) but insignificant. Overall, it

seems that the additional higher moments are not needed to explain the cross-section of equity returns and indeed that beta is useful in explaining the cross section of returns of stocks sorted on market capitalization in the long period, although the estimated risk premium seems to be overemphasized.

The Four-Moment CAPM is also tested on the same portfolios over the later period of 1980-2011, which represents the most problematic period of time for the simple CAPM. The results are reported in Panel B. The worsening in the performance of the CAPM appears clear. None of the parameters in the subsample estimation are significant, with an estimated market premium of 0.78% for the single CAPM. The intercept is insignificant in both models. The Four-Moment CAPM appears to perform worse than the CAPM, with a negative and insignificant total risk premium and with no significant additional factors. Specifically, the premium for beta is positive (0.62%) but insignificant (t-statistic of 1.15), whereas coskewness and cokurtosis have the theoretically incorrect positive and negative signs, respectively.

What explanation can be made for the poor performance of the CAPM and Four-Moment CAPM for this sample? Historically, over the last 30 years market betas have declined, especially for small stock portfolios. Furthermore, excess returns are not very dispersed, that is, the market capitalization or size strategy does not represent an anomaly any longer (see, for example, Schwert, 2003), suggesting that the CAPM should not be tested on portfolios that show no dispersion of average returns. Therefore, the poor performance of the models on the ten ME portfolios in the unconditional test does not necessarily mean that the models have to be rejected, but that the model should be tested on a set of assets which shows greater dispersion in returns. In other words, the CAPM is doomed by the low dispersion of returns in the portfolios under investigation.

7.5.2. Results for the 10 BM portfolios

The Four-Moment CAPM is tested on the ten book-to-market portfolios for the full sample and for the latter period and results are reported in Table 7.9. For the full sample (Panel A) the simple CAPM again seems to perform better than the Four-Moment model as it gives a positive (0.47%) although insignificant risk premium (t-statistic of 1.41), whereas the beta premium alone has the correct positive sign (0.87%) and is significant (t-statistic of 2.49) in the Four-Moment CAPM. Cokurtosis appears to be priced, but it has a negative sign (-2.7%), opposite to that expected from theory. However, the results change significantly for the later subsample period as shown in Panel B.

Although the beta premium and the coskewness factor have the correct signs (positive and negative, respectively), they are not significant and the estimated market premium required by investors for the total systematic risk is large and negative, which is inconsistent with the theory according to which investors require a positive market premium to hold risky assets. The CAPM does not perform any better, with a large alpha which is significant at the 5% level and an incorrect negative sign for the market premium, reflecting the inability of the CAPM to price the high book-to-market portfolio over this 30-year period. Although the value premium is still present, especially in the 9th and 10th deciles, unlike the size premium, the market beta does not explain the returns any longer, a result due largely to the decline in beta for high book-to-market portfolios.

Table 7.7 The dual test of Pettengill for the CAPM using time-varying betas with dynamic correlations for the period 1926-2011 and 1980-2011 on 10 ME portfolios, 10 BM portfolios and 25 ME/BM portfolios

Portfolios	Up	Down
Panel A: 1926-2011		
λ_β 10 ME portfolios	0.0291 (6.51)*	-0.0274 (-5.74)*
λ_β 10 BM portfolios	0.0322 (8.96)*	-0.0360 (-10.14)*
λ_β 25 ME/BM portfolios	0.0236 (7.42)*	-0.0434 (-13.07)*
Panel B: 1980-2011		
λ_β 10 ME portfolios	0.0289 (4.44)*	-0.0230 (-2.95)*
λ_β 10 BM portfolios	0.0229 (4.51)*	-0.0418 (-7.11)*
λ_β 25 ME/BM portfolios	0.0212 (4.73)*	-0.0408 (-8.65)*

The Table reports the slope of the Fama and MacBeth cross sectional regression of the monthly returns of, respectively, 10 ME-, 10 BM-, and 25 ME/BM-sorted portfolios on their DCC betas for the full sample period 1926-2011 and the sub-sample 1980-2011 split into two samples: up when the excess market return is positive and down when the excess market return is negative. T-statistics are given in parentheses. The asterisk shows significance at the 5% level.

Table 7.8 Test of the CAPM and Four-Moment CAPM on ten ME portfolios using rolling regression betas for the period 1926-2011 and 1980-2011

Panel A: 1926-2011					
MODELS	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	-0.0017 (-0.51)	0.0089 (2.29)*	24.16 (0.30)	-0.0048 (-0.31)	0.0044 (0.37)
CAPM	-0.00701 (-1.64)	0.0142 (2.97)*			
Panel B: 1980-2011					
	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	-0.0001 (0.00)	0.0062 (1.15)	55.813 (0.40)	-0.0072 (-0.46)	-0.0032 (-0.24)
CAPM	-0.0016 (-0.29)	0.0078 (1.34)			

This Table reports the results of the monthly cross-sectional regressions of the ten ME portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM. Panel A shows the coefficients of the regression for the overall sample 1926-2011. Panel B shows the coefficients for the latter period 1980-2011. The coefficients are reported for the intercept alpha, the conditional covariance, the conditional coskewness, the conditional cokurtosis, and for the estimated overall premium for the Four-Moment CAPM. The t-statistics are reported in brackets and the significant coefficients at 5% level of significance are indicated with an asterisk.

Results are obtained from the following models:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1} \lambda_{\beta t} \quad \text{CAPM}$$

$$R_{i,t} - R_{f,t} = \lambda_\beta \beta_{i,t-1} + \lambda_s (s_{i,t-1} - s_{m,t-1} \beta_{i,t-1}) + \lambda_k (k_{i,t-1} - \beta_{i,t-1}) \quad \text{Four-Moment CAPM}$$

The introduction of the higher moments of the distribution of returns does not provide any evidence of ‘rescuing’ the model and does not offer a rational explanation for the empirical anomalies. In particular, it is striking that investors should require a negative premium for systematic cokurtosis, in plain contrast with the theory.

Finally, the Four-Moment CAPM is tested on the 25 portfolios, double-sorted on size and the book-to-market ratio but the results are very similar to the 10 BM portfolios and the signs of the risk factors are again inconsistent with expectations; therefore these results are not reported in this thesis.

The simple cross-sectional regression of the average returns for the portfolios on the market betas leads to a negative relationship between beta and returns, inconsistent with theoretical expectations. On the other hand, a problem with the Four-Moment CAPM might be that the volatility of the last 30 years makes it more challenging to estimate the comoments for which there is always a problem of error-in-variables as for the market beta.

Therefore, up to this point in the thesis, the inclusion of the higher moments into the CAPM does not seem to have any positive effect on the performance of the model. Indeed the results do not support either the findings of Kraus and Litzenberger (1973) of a positive beta and negative coskewness together with an insignificant intercept, or the findings of Fang and Lai (1997) which support a Four-Moment CAPM. Indeed, most of the time, the model results seem to deteriorate with the introduction of the higher moments. It is worth pointing out, however, that the portfolios used in this thesis are sorted according to different criteria from those used by Kraus and Litzenberger, and Fang and Lai¹⁷, and therefore the diversity of the results raises some doubts on the robustness of the higher-moment CAPM and does not allow the comparison of the

¹⁷ Kraus and Litzenberger use portfolios of stocks sorted on beta and coskewness, whereas Fang and Lai use portfolios of stocks sorted on beta, coskewness and cokurtosis.

results with those of other authors who conducted the empirical tests on different portfolios. However, the findings seem to confirm that there is little evidence that the higher moments are priced and that cokurtosis is unexpectedly negatively rewarded for BM portfolios, a result confirmed recently by Heaney et al. (2012). In contrast to Dittmar (2002), the results of this thesis are not in favour of the Four-moment CAPM, though again Dittmar uses industry portfolios.

7.6. Tests of the CAPM and Four-Moment CAPM with DCC betas

The first attempt to improve the multi-moment CAPM involves the use of DCC time-varying betas. As already undertaken for the CAPM in this thesis, the approach is to adopt conditional betas obtained through dynamic conditional correlations and a multivariate GARCH. The market betas of the assets, stocks in the case of this thesis, may change with the business cycle and with conditioning information (see for example Ferson and Harvey, 1999, and Cochrane, 2001). Since the development of the GARCH model, this technique has become popular for estimating the conditional volatilities of equity returns.

The DCC is a multivariate development of the GARCH which enables the researcher to obtain dynamic correlations and covariances, and therefore conditional betas. As the Four-Moment CAPM is obtained as a linear function of bivariate covariances, the methodology is particularly suitable for the time-varying parameters of the model. Coskewness and cokurtosis can in fact be construed as counterparts of the covariance.

The DCC GARCH allows the estimation of the covariance of the portfolio returns with the market portfolio (covariance), the market portfolio de-meaned squared (coskewness) and the market portfolio to the power of three (cokurtosis), but also the covariances of the market portfolio returns with the market portfolio (variance of the market portfolio), with the market portfolio squared (skewness), and with the market portfolio to the

power of three (kurtosis). The DCC estimates the time-varying betas, and then the second step (monthly cross-sectional regressions) of the Fama and MacBeth (1973) methodology is applied to estimate the market premium.

7.6.1. Results for the 10 ME portfolios

The test for the ten size portfolios gives the results reported in Table 7.10. The DCC betas improve both models considerably. For the full sample (Panel A) the risk premium in the CAPM becomes almost significantly positive with an average risk premium of 0.65% per month and an insignificant intercept. Interestingly, the Four-Moment CAPM performs well over the whole sample as all of the factors have the correct sign. Covariance and cokurtosis have a positive coefficient as theoretically expected, whereas coskewness is negative as expected, yet the coefficients in each case are insignificant. However, the estimated market premium is very large compared to the historical market premium and significant. The intercept is not significant in either model. Therefore, the full sample results give some credit to Kraus and Litzenberger (1973) and Fang and Lai (1997) for acknowledging the importance of coskewness and cokurtosis in explaining the cross-section of returns.

In the later period as shown in Panel B, the CAPM estimates a positive though insignificant risk premium of 0.84%. As for the Four-Moment CAPM, all of the signs of the factors are consistent with theory, with a positive but insignificant total risk premium, though none of the model's factors are significant.

Table 7.9: Test of the CAPM and Four-Moment CAPM on ten BM portfolios using rolling regression betas for the period 1926-2011 and 1980-2011

Panel A: 1926-2011					
MODELS	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	-0.0018 (-0.59)	0.0087 (2.489)*	102.589 (1.30)	-0.0270 (-2.52)*	-0.0162 (-2.15)*
CAPM	0.0025 (0.89)	0.0047 (1.41)			
Panel B: 1980-2011					
	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	0.0023 (0.54)	0.0035 (0.79)	-135.516 (-1.01)	-0.0386 (-2.22)*	-0.0207 (-1.95)
CAPM	0.0133 (2.71)*	-0.0045 (-1.08)			

This Table reports the results of the monthly cross-sectional regressions of the ten BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM. Panel A shows the coefficients of the regression for the overall sample 1926-2011, Panel B shows the coefficients for the latter period 1980-2011. The coefficients are reported for the intercept alpha, the conditional covariance, the conditional coskewness, the conditional cokurtosis, and for the estimated overall premium for the Four-Moment CAPM. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk.

Results are obtained from the following models:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1} \lambda_{\beta t} \quad \text{CAPM}$$

$$R_{i,t} - R_{f,t} = \lambda_\beta \beta_{i,t-1} + \lambda_s (s_{i,t-1} - s_{m,t-1} \beta_{i,t-1}) + \lambda_k (k_{i,t-1} - \beta_{i,t-1}) \quad \text{Four-Moment CAPM}$$

Both models appear to work for the ME portfolios when the DCC betas are applied, especially for the one-factor CAPM. For the simple CAPM, alpha is insignificant and the market premium is 0.83% though is not significant at the 5% level. For the Four-Moment CAPM, the signs of the factors are consistent with expectations but are insignificant at the 5% level, and the intercept is quite large (0.59%) though insignificant.

7.6.2. Results for the 10 BM portfolios

When the models are tested over the BM portfolios, as shown in Table 7.11 for the full sample, the beta premium is found to be significantly positive (0.58%) in the Four-Moment CAPM. Co-skewness and co-kurtosis have the incorrect signs (positive and negative, respectively), and the overall market premium is 0.50%, which is comparable

with the historical market premium of 0.62%. Moreover, the intercept is insignificant as expected from theory.

In the simple CAPM, the market premium is estimated to be 0.50% and it is almost significant at the 5% level. Therefore, once again, the traditional CAPM with DCC betas appears to outperform the Four-Moment CAPM and to offer a good approximation of reality for the ten BM portfolios over the long run.

For the later period, as shown in Panel B, none of the factors are significant, confirming the inability of both the CAPM and of the multi-moment CAPM to price the book-to-market portfolios. However, the intercept becomes insignificant in the multi-moment CAPM that also shows a positive overall risk premium, but given that three factors are adopted and that there are only 10 portfolios the results are most likely spurious.

Finally, both models perform very poorly when tested on the 25 ME/BM portfolios, with the usual negative slope for the SML; therefore these results are not reported in this thesis.

What explanation can be provided for this poor performance? Five explanations are offered here: (i) there is a clear problem with portfolios of stocks double-sorted on size and book-to-market that might require the researcher to apply the models to individual assets, i.e. portfolio formation might smooth out the important cross-sectional behaviours of assets; (ii) the tests are always conducted on realized returns whereas the theory is in itself based on expected returns; (iii) the model is misspecified and other risk factors should be included, i.e. multifactor models; (iv) the market portfolio proxy used in the test is inefficient; and (v) a behavioural finance explanation exists whereby some psychological traits might prevent investors from rationally measuring the risk-expected-return relationship. In particular, it is worth considering the undershooting or overshooting effect according to which investors might underestimate the risk in bullish markets, and overestimate the risk in bearish markets.

Table 7.10 Test of the CAPM and Four-Moment CAPM on ten ME portfolios using DCC betas for the period 1926-2011 and 1980-2011
Panel A: 1926-2011

MODELS	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	0.0004 (0.14)	0.0063 (1.82)	-23.100 (-1.11)	0.0050 (1.55)	0.0119 (2.33)*
CAPM	0.0006 (0.18)	0.0065 (1.91)			

Panel B: 1980-2011					
MODELS	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	0.0059 (1.28)	0.0009 (0.19)	-25.362 (-0.91)	0.0011 (0.30)	0.0017 (0.28)
CAPM	-0.0021 (-0.40)	0.0084 (1.62)			

This Table reports the results of the monthly cross-sectional regressions of the ten ME portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM. Panel A shows the coefficients of the regression for the overall sample 1926-2011, Panel B shows the coefficients for the latter period 1980-2011. The coefficients are reported for the intercept alpha, the conditional covariance, the conditional coskewness, the conditional cokurtosis, and for the estimated overall premium for the Four-Moment CAPM. The t-statistics are reported in brackets and the significant coefficients at the 5% level are indicated with an asterisk.

Results are obtained from the following models:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1} \lambda_{\beta t} \quad \text{CAPM}$$

$$R_{i,t} - R_{f,t} = \lambda_\beta \beta_{i,t-1} + \lambda_s (s_{i,t-1} - s_{m,t-1} \beta_{i,t-1}) + \lambda_k (k_{i,t-1} - \beta_{i,t-1}) \quad \text{Four-Moment CAPM}$$

Table 7.11 Test of the CAPM and Four-Moment CAPM on ten BM portfolios using DCC betas for the period 1926-2011 and 1980-2011
Panel A: 1926-2011

MODELS	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	0.0001 (0.04)	0.0058 (2.04)*	50.681 (1.96)*	-0.0011 (-1.17)	0.0050 (1.66)
CAPM	0.0016 (0.69)	0.0049 (1.77)			

Panel B: 1980-2011					
MODELS	α	λ_β	λ_s	λ_k	$\lambda_\beta + \lambda_s s_m + \lambda_k$
Four-Moment CAPM	0.0030 (0.78)	0.0018 (0.44)	86.760 (1.88)	-0.0011 (-0.62)	0.0011 (0.25)
CAPM	0.0084 (2.24)*	-0.0027 (-0.64)			

This Table reports the results of the monthly cross-sectional regressions of the ten BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM. Panel A shows the coefficients of the regression for the overall sample 1926-2011, Panel B shows the coefficients for the latter period 1980-2011. The coefficients are reported for the intercept alpha, the conditional covariance, the conditional coskewness, the conditional cokurtosis, and for the estimated overall premium for the Four-Moment CAPM. The t-statistics are reported in brackets and the significant coefficients at the 5% level are indicated with an asterisk.

Results are obtained from the same models as in Table 7.10.

7.7. Conclusion

This chapter has provided a set of simple empirical tests of the CAPM. The main objective has been to implement standard methodologies to test the model, and to highlight its limitations in explaining the cross-section of equity returns when portfolios are sorted using size and the book-to-market ratio, especially over the last three decades. The literature (the work of Fama and French, for instance) points out that most of the problems of the CAPM occur from 1960-1970 onwards. Indeed, the simple tests applied in this chapter confirm that the traditional model works quite well over the whole sample 1926-2011, apart from the case of portfolios double sorted on size and book-to-market. However, the traditional model suffers serious shortcomings over the period 1980-2011. The results show that the size and book-to-market anomalies still affect the cross-section of asset returns.

In this chapter, market betas are obtained using rolling regressions and dynamic conditional correlations. Although these advancements provide a better proxy of reality than constant market betas, they do not allow the model to be rescued. In the second part of the chapter, an extension of the CAPM is introduced in the search for a rational explanation of the failures of the traditional model. In particular, the model is extended to the third and fourth moment of the distribution of returns of the market portfolio.

Confronted with the failure of the CAPM, and based on the empirical evidence that returns show skewness and excess kurtosis, researchers have proposed extending the model to include the third and the fourth moment of the distribution of returns: the Four-Moment CAPM. The main idea is that the risk of an asset depends not only on how its returns covary with the aggregate market portfolio, but also on how they covary with the volatility and skewness of the market portfolio.

Only if the returns are normally distributed or investors have a quadratic utility function (as assumed by the CAPM) do investors make their investment choice following the

mean-variance trade-off rule. Stylized facts show that returns are not normally distributed and that they exhibit asymmetry and an excess of kurtosis, that is, large negative returns are more likely than large positive returns and also that extreme outcomes are more likely than that predicted by the CAPM (thick distribution tails). Investors are indeed concerned about extreme movements in returns that appear to be more frequent than forecasted by the normal distribution. Assets that instead tend to have positive returns when market volatility increases are considered safer and therefore investors might be willing to sacrifice some returns to hold them. If returns are not normally distributed, the risk might be under or over-estimated when the asset's beta is considered alone. If small stocks and high book-to-market stocks contribute to the negative skewness and the larger kurtosis of the market portfolio, then the coskewness and cokurtosis premia might rationalize the size and book-to-market anomalies.

The second part of the chapter presents the results of a Four-Moment CAPM when tested on 10 ME, 10 BM, and 25 ME/BM portfolios for US stocks over the period 1926-2011, and the more recent subsample period of 1980-2011. In general, the Four-Moment CAPM does not improve the explanation of the cross-section of average returns when compared with the more parsimonious one-factor CAPM. However, the results show that the introduction of DCC time-varying betas can markedly improve both models, especially when the portfolios are sorted on size. In sum, both models perform better over a longer horizon, whereas they fail to explain returns over the last 30 years, though DCC betas appear to significantly outperform rolling regression betas. The poor results of the Four-Moment CAPM are not conclusive given the limited number of portfolios used in this study, and given the fact that only one country's stock market is tested, albeit the most important and efficient in the financial world. The performance of the models appears to be linked to the particular portfolios tested and

their formation criteria. There is some evidence, however, that coskewness and cokurtosis are priced in ME sorted portfolios.

Further research might extend the Four-Moment CAPM for individual assets and employ a larger number of ME sorted portfolios. However, one problem that remains is that the portfolios used to test the model may in themselves be misleading. In the real world, there is little reason why investors should hold portfolios with such a large proportion of small-growth stocks, as implied by sorting portfolios on the book-to-market ratio, and therefore it may be advisable to test asset pricing models on individual assets – this further step is undertaken in Chapter 8 - or at least on less artificial portfolios which are not too extreme. Moreover, portfolio formation might lead to a smoothing out of the cross-sectional behaviour of the assets, as, for example, beta is particularly sensitive to extreme results which might be diluted in a portfolio. Another important point to note is that the Four-Moment CAPM might work conditionally, with coskewness and cokurtosis premia that are time-varying over the business cycle. This important possibility is investigated in Chapter 8 with the introduction of switching regimes and time-varying risk premia.

Chapter 8

Conditional CAPM and Four Moment CAPM with time-varying risk premia

8.0. Introduction

In the previous chapter, the CAPM was modified to account for the time-varying nature of beta using dynamic conditional correlations (DCC) as opposed to a constant beta or simple rolling regression betas and the traditional single factor model was augmented to incorporate higher moments (coskeweness and cokurtosis) in a Four-Moment CAPM. The results showed that neither the introduction of higher moments nor the introduction of loadings obtained from a DCC model help to explain the cross section on the returns of portfolios of stocks sorted on ME or BM.

In this chapter a further feature is added to the preceding conditional models. This extension allows for time-varying risk premia. Thus, the betas are no longer the only element that is assumed to follow a dynamic process. The risk premia are now assumed to change over the business cycle. Specifically, one of the main assumptions made here is the presence of two different risk premia: one associated with bullish markets and another with bearish markets. Indeed, if the sensitivity to risk of stocks that yield higher average returns were to increase at a time when the risk premia required by investors were to increase as well, the conditional change of both factor loadings and risk premia might explain the difference in returns among different assets in the cross-section of returns. For instance, suppose small stocks have a higher beta when the risk premium required by the market also increases then the explanation for the higher returns of small stocks might lie in the time variation in both beta and the risk premium.

The problem is therefore how to model the time variation in beta and the risk premium. Whereas for the factor loadings the problem is solved by introducing time varying betas obtained with a Multivariate GARCH DCC, the problem remains how to model the time variation in the risk premia. In particular, the choice is made to have different risk premia for each market regime.

The basic idea draws from the model of Pettengill et al. (1995). The analysis of this thesis therefore commences by using this model as a baseline. In their model, the definition of bull and bear markets is simply assumed to strictly follow the sign of the market return, that is, when the market return is positive the market is deemed to be bullish, and vice versa. Thus, the initial definition of market regimes adopted in this chapter will be the simple identification of Pettengill et al. (1995). Given that the regime is unobserved and that it cannot reasonably be assumed to be perfectly correlated with the sign of the market return, the restrictive definition of Pettengill et al. (1995) is relaxed by adopting a Markov switching regimes approach. In this approach, the regime is unobservable but is assumed to follow some probabilistic process, which can possibly include the market model.

Whereas the betas are assumed to vary over time following a DCC GARCH process, the risk premia are estimated using panel data in order to overcome the obstacle of having to estimate two sets of risk premia having only one set of factor loadings each time. Specifically, the set of factor loadings is obtained with a DCC GARCH model, whereas two sets of risk premia need to be estimated each time: one for the bullish regime and one for the bearish regime (each with its own probability). This task would not be feasible with a simple cross-sectional regression approach, and that is why panel data are introduced in order to increase the dimension of the equations from which the unknown variables are estimated.

Recall that the two models to be tested are the CAPM and the Four-Moment CAPM. The main steps in the estimation procedure are as follows (as detailed in chapter 5). First, given the probability that the market is in a bull regime, p_t , returns (for asset i at time t) are given by the model:

$$R_{it} - R_{ft} = \gamma_0 + \gamma_{12}p_t\beta_{it} + \gamma_2\beta_{it} + \varepsilon_{it} \quad (8.1)$$

where $\gamma_{12} = \gamma_1 - \gamma_2$ is the difference between the risk premium in a bull market and the risk premium in a bear market. Once $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are obtained, a time series test is undertaken on the mean of $\Gamma_t = p_t\hat{\gamma}_1 + (1 - p_t)\hat{\gamma}_2$ as a t-test with HAC (Heteroscedasticity and Autocorrelation Consistent) standard errors corrected for autocorrelation.

The unconditional CAPM is upheld if the two risk premia are not significantly different. In this case the conditional CAPM in Equation 8.1 reduces to the unconditional CAPM with only one risk premium. Otherwise there are two different risk premia and the researcher thus needs to understand the sign and magnitude of these risk premia and what factors drive them.

For the Four-Moment CAPM the model can be stated as in Equation 8.2:

$$R_{it} - R_{ft} = \Gamma_{\beta t}\beta_{it} + \Gamma_{st}(s_{it} - s_m\beta_{it}) + \Gamma_{kt}(k_{it} - \beta_{it}) + \varepsilon_{it} \quad (8.2)$$

where

$$\Gamma_{\beta t} = p_t\lambda_{\beta 1} + (1 - p_t)\lambda_{\beta 2},$$

$$\Gamma_{st} = p_t\lambda_{s1} + (1 - p_t)\lambda_{s2},$$

$$\Gamma_{kt} = p_t\lambda_{k1} + (1 - p_t)\lambda_{k2},$$

are respectively, the covariance, coskewness, and cokurtosis premia.

As in the case of the simple conditional CAPM in Equation 8.1, the operational model for estimation is given by replacing $\Gamma_{\beta t}$, Γ_{st} , and Γ_{kt} in Equation 8.2 with:

$$\Gamma_{\beta t} = p_t \lambda_{\beta 12} + \lambda_{\beta 2},$$

$$\Gamma_{s t} = p_t \lambda_{s 12} + \lambda_{s 2},$$

$$\Gamma_{k t} = p_t \lambda_{k 12} + \lambda_{k 2},$$

where $\lambda_{\beta 12} = \lambda_{\beta 1} - \lambda_{\beta 2}$, $\lambda_{s 12} = \lambda_{s 1} - \lambda_{s 2}$, and $\lambda_{k 12} = \lambda_{k 1} - \lambda_{k 2}$ are the difference between the beta, coskewness and cokurtosis premium in a bull and bear market.

As before, p_t is the probability of a bull market at time t (so $(1 - p_t)$ is the probability of a bear market). These probabilities are obtained from a Markov switching process as follows. First, assume a simple non-linear model for the market excess return:

$$R_{Mt} - R_{ft} = \mu_{Mi} + \sigma_{Mi} \varepsilon_t \quad (8.3)$$

The coefficients, μ_{Mi} , and σ_{Mi} , $i = 1, 2$ take one of two values, depending on the regime, and ε_t is a random disturbance assumed to be normally distributed. The filtered probabilities are estimated using the Expected Maximization algorithm of Hamilton (1989). With these probabilities, panel regressions are used to obtain estimates of the risk premia for each state. The regressions give estimates of $\Gamma_{\beta t}$ for the CAPM, in addition to $\Gamma_{s t}$ and $\Gamma_{k t}$ for the Four-Moment CAPM. These can be treated as time series of conditional risk premia. The t-test can therefore be applied to test for the null hypotheses that these premia are not different from zero. The expected signs of the conditional risk premia are the same as those for the standard (unconditional) models.

The first hypothesis to be tested for the CAPM is that the risk premiums in bullish and bearish regime are not significantly different:

$$H_0: \gamma_1 - \gamma_2 = 0$$

$$H_A: \gamma_1 - \gamma_2 \neq 0$$

This is a test of unconditional CAPM. Failing to reject the null hypothesis is consistent with the unconditional CAPM. The alternative can be taken as evidence in favour of a conditional CAPM. However, a full test of the unconditional CAPM requires a further step, namely that $\gamma_1 = \gamma_2 > 0$. Specifically, the risk premium should be positive to

uphold the CAPM, meaning that investors require a positive risk premium to hold risky assets.

Similarly, the conditional CAPM requires a further hypothesis, namely that the weighted risk premia are positive. Specifically, the following hypothesis on the conditional risk premium is tested:

$$H_0: E(\Gamma_t) = 0$$

$$H_A: E(\Gamma_t) > 0$$

where $\Gamma_t = p_t \hat{\gamma}_1 + (1 - p_t) \hat{\gamma}_2$. The conditional CAPM is upheld if the null hypothesis is rejected.

For the Four-Moment CAPM, the hypotheses are somewhat more complicated as there are three factors. The first hypothesis for the Four-Moment CAPM regards the unconditional test of the model, that is, whether the risk premia for the comoments are insignificantly different:

$$H_0: \begin{cases} \lambda_{\beta 1} - \lambda_{\beta 2} = 0 \\ \lambda_{s1} - \lambda_{s2} = 0 \\ \lambda_{k1} - \lambda_{k2} = 0 \end{cases}$$

against the alternative hypothesis that at least one of the set of risk premia is different from zero:

$$H_A: \begin{cases} \lambda_{\beta 1} - \lambda_{\beta 2} \neq 0 \\ \lambda_{s1} - \lambda_{s2} \neq 0 \\ \lambda_{k1} - \lambda_{k2} \neq 0 \end{cases}$$

The second hypothesis is related to the sign of the three comoments. The premium for standardized covariance and cokurtosis are expected to be positive, whereas coskewness should have a negative premium, as investors have a preference for positive skewness and an aversion for variance and kurtosis

$$H_0: \begin{cases} E(\Gamma_{\beta t}) = 0 \\ E(\Gamma_{st}) = 0 \\ E(\Gamma_{kt}) = 0 \end{cases}$$

Against

$$H_A: \begin{cases} E(\Gamma_{\beta t}) > 0 \\ E(\Gamma_{st}) < 0 \\ E(\Gamma_{kt}) > 0 \end{cases}$$

where $\Gamma_{\beta t} = p_t \lambda_{\beta 1} + (1 - p_t) \lambda_{\beta 2}$, $\Gamma_{st} = p_t \lambda_{s1} + (1 - p_t) \lambda_{s2}$, and $\Gamma_{kt} = p_t \lambda_{k1} + (1 - p_t) \lambda_{k2}$. The rejection of the null would confirm the conditional CAPM.

These conditional versions of the models are estimated on the 25 ME/BM portfolios for the subsample as they present the greatest challenge for the CAPM, and as they give a larger size of observations from which the models can be tested. Indeed, the empirical literature from Fama and French (1993) show that the CAPM is vulnerable to the size and value anomalies especially in the period following the 70s. The subsample choice is justified for two main reasons: (i) two regimes can describe the market over the last 30 years, but a more complicated Markov switching process would be required to cover the full sample (3 or 4 regimes), given also the high volatility changes; and (ii) this period of time is the most problematic for the CAPM.

The introduction of higher-moment CAPM with sensitivity obtained with a DCC GARCH and time-varying risk premia according to a Markov Switching regime represents a novel approach in asset pricing. In particular, instead of splitting the sample into up and down markets on the basis of the probability of each regime, the panel data allows the researcher to use all information weighted by that probability.

The last section of this chapter introduces the empirical tests of the CAPM, the Four-Moment CAPM, and some alternative models using individual US assets over the period 1930-2010 and 1980-2010. Individual assets show a larger dispersion in betas and this might be one reason why the estimates of the risk premia are more precise. In particular a short-window regression methodology is employed, in which the factor loadings are determined through short-window time-series, and the expected returns (calculated as the average return for each window) are regressed at cross section over

the factor loadings to estimate the risk premia. The choice of a short window regression is inspired by the work of Lewellen and Nagel (2006). Whilst acknowledging that better choices can be made to estimate the beta of the individual assets, the main purpose of this thesis is to see whether tests of the CAPM and four-moment CAPM produces different results for individual assets as opposed to portfolios.

8.1. Dual tests of the Four-Moment CAPM with rolling regression betas

This section starts with the replication of the study of Pettengill et al. It is therefore assumed that the probability of a bull market is unity when the market return is positive, and zero when it is negative.

Pettengill et al. (1995) introduce a dual test of the CAPM in up and downmarkets, depending on the positive or negative sign of the market return, arguing that there is a positive relationship between beta and realized returns when the market portfolio excess return is positive, and a negative relationship when the market portfolio excess return is negative.

Recall the main proposition of the CAPM:

$$E(R_i) - R_f = \alpha + [E(R_m) - R_f]\beta_{im} \quad \forall i = 1, \dots, N \quad (8.4)$$

Thus, if the expected market portfolio excess return, $E(R_m) - R_f$, is greater than 0 then stocks with a higher beta should yield higher returns, whereas if the market portfolio excess return, $E(R_m) - R_f$, is less than 0 then the stocks with a higher beta should yield a lower or even negative return. Pettengill et al. use realized returns as proxy for the expected returns (a common practice in asset pricing).

Arguably, riskier stocks, with a higher beta, lower coskewness and higher cokurtosis, should yield a higher return in a bull market and a lower return in a bear market. Therefore, it is expected that the beta premium and the cokurtosis premium are positive in a bull market and negative in a bear market, whereas the coskewness premium should

be positive in a bear market (as safer stocks lose less in a bear market) and negative in a bull market (as safer stocks earn less in a bull market). Finally, the overall risk premium should be positive in a bull market and negative in a bear market. In summary, following the logic of Pettengill et al., the hypotheses in a bull market are:

$$\left\{ \begin{array}{l} \lambda_{\beta} > 0 \\ \lambda_s < 0 \\ \lambda_k > 0 \\ \lambda_{\beta} + \lambda_s s_m + \lambda_k > 0 \end{array} \right.$$

and in a bear market:

$$\left\{ \begin{array}{l} \lambda_{\beta} < 0 \\ \lambda_s > 0 \\ \lambda_k < 0 \\ \lambda_{\beta} + \lambda_s s_m + \lambda_k < 0 \end{array} \right.$$

The results and the estimation of the bullish and bearish market with rolling regression betas for 10 ME-sorted portfolios are reported in Table 8.1.

8.1.1. Results for the 10 ME portfolios

The results of the Pettengill test on ten ME portfolios reported in Table 8.1 show a positive and significant risk premium for the CAPM in a bull market and a negative and significant risk premium in a bear market, as expected theoretically. The risk premium in a bull market is substantially larger in magnitude than the risk premium in a bear market, whereas the estimated premium is almost symmetric for the Four-Moment CAPM (2.60% versus -2.81%) across the two states, and is lower than in the case of the single CAPM.

In the case of the four-CAPM, the premium for beta is positive in a bull market and negative in a bear market, as expected. Coskewness is positive as expected in the bear market, though is insignificant, whereas it is positive and insignificant in a bull market where the coefficient is, as expected, negative and significant. Cokurtosis is insignificant and negative in both bull and bear markets, whereas it is expected to be

positive in a bull market and negative in a bear market. However, the simple CAPM is preferable to the multi-moment CAPM as none of the additional factors are either significant or have the theoretically expected sign, i.e. the higher comoments do not add anything to the explanation of the cross section of returns.

For the subsample, reported in Panel B of Table 8.1, the risk premium appears to be symmetric for the CAPM, with, respectively, a positive and significant risk premium in a bull market and a negative and significant risk premium in a bear market.

In the case of the Four-Moment CAPM, only beta is significant, whereas the other comoments are insignificantly priced. The beta premium is positive and significant in a bull market, whereas coskewness is positive, contrary to the expectations though insignificant, and cokurtosis is negative (inconsistent with theory) and is insignificant. Furthermore, the risk premium is larger in magnitude in a bear market than in a bull market. From the results for the subsample, the CAPM appears preferable to the Four-Moment CAPM as it is more parsimonious and the additional factors of the Four-Moment CAPM are not significant in the explanation of the returns of the ten ME portfolios.

8.1.2. Results for the 10 BM portfolios

The results for the ten BM portfolios are reported in Panel C of Table 8.1. Panel C shows that for the CAPM, the risk premium in a bull market is of the same magnitude as the risk premium in a bear market, and that the risk premium is positive and significant in a bull market and negative and significant in a bear market.

In the case of the Four-Moment CAPM, the beta premium is positive and significant in a bull market, though a little larger than in the case of the CAPM, and the beta premium is negative and significant in a bear market, though almost 0.60% larger than in the case of the CAPM. Coskewness has a positive risk premium in both a bull and a bear market

and is insignificant, whereas the expectation was for a negative premium in a bull market and a positive premium in a bear market. Interestingly, portfolios with higher cokurtosis seem to perform better in a bear market and much worse in a bull market. Cokurtosis has a negative and significant coefficient in a bull market against theoretical expectations, and a positive and insignificant coefficient in a bear market. The overall risk premium is negative in both market regimes and is insignificant. In summary, the additional factors do not appear to be relevant and therefore the CAPM is still preferred to the Four-Moment CAPM.

The results of the Pettengill et al. test for the CAPM and for the Four-Moment CAPM for the 25 ME/BM-sorted portfolios are very similar to those of the 10 BM portfolios and are therefore not reported in this thesis.

In summary, unsurprisingly and consistent with Pettengill et al. (1995), the beta premium is significant and strongly positive in a bull market and negative in a bear market, but in general the coskewness and cokurtosis factors do not add explanatory power to the model when tested on the ME or BM portfolios. There is some evidence that coskewness is positively rewarded in a bear market, but cokurtosis displays a theoretically incorrect sign as it is negative in a bull (it should be positive) and positive in a bear market (it should be negative). The results confirm that beta does not fully capture the risk for the 25 ME/BM portfolios, and that the inclusion of the two higher moments does not lead to better model performance.

Table 8.1: The Dual test of Pettengill for the CAPM and Four-Moment CAPM on Ten ME and Ten BM portfolios for the period 1926-2011 and 1980-2011

10 ME portfolios Dual Test				
	Four-Moment CAPM		CAPM	
Panel A: 1926-2011				
	Bull	Bear	Bull	Bear
λ_β	0.0371 (7.076)*	-0.0334 (-6.72)*	0.0437 (6.43)*	-0.0306 (-5.55)*
λ_s	31.3094 (0.31)	13.4385 (0.10)		
λ_k	-0.0079 (-0.42)	-0.0003 (-0.02)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0260 (1.48)	-0.0281 (-2.00)*		
Panel B: 1980-2011				
λ_β	0.0367 (5.58)*	-0.0402 (-5.21)*	0.0352 (5.09)*	-0.0339 (-3.69)*
λ_s	105.0600 (0.65)	-19.0418 (-0.08)		
λ_k	-0.0071 (-0.34)	-0.0074 (-0.31)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0210 (1.22)	-0.0400 (-2.07)*		
10 BM portfolios Dual Test				
	Four-Moment CAPM		CAPM	
Panel C: 1926-2011				
	Bull	Bear	Bull	Bear
λ_β	0.0327 (6.79)*	-0.0275 (-6.52)*	0.0303 (6.42)*	-0.0336 (-8.95)*
λ_s	71.0388 (0.70)	149.9164 (1.18)		
λ_k	-0.0548 (-3.79)*	0.0148 (0.96)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	-0.0195 (-1.81)	-0.0115 (-1.16)		
Panel D: 1980-2011				
λ_β	0.0205 (3.40)*	-0.0224 (-4.09)*	0.0160 (3.03)*	-0.0356 (-6.14)*
λ_s	-144.5077 (-0.88)	-121.8500 (-0.53)		
λ_k	-0.0767 (-3.58)*	0.0194 (0.68)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	-0.0392 (-2.77)*	0.0075 (0.48)		

This Table reports the results of the monthly cross-sectional regressions of, respectively, the ten ME portfolios and the ten BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM. The results for the period 1926-2011 are reported in Panel A and C, and the results for the period 1980-2011 are reported in Panel B and D, for the two regimes: bull when the monthly excess market return is positive and bear when the monthly excess market return is negative. The coefficients are reported for the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. The t-statistics are reported in brackets and the significant coefficients at the 5% level are indicated with an asterisk.

8.2. Dual tests of the Four-Moment CAPM with DCC betas

The poor results of the multi-moment CAPM may in theory be due to the use of rolling regression betas as this approach to capturing the time-variation of beta is a very simple and rough tool. It is possible that the use of a more sophisticated methodology to derive the time-variation of beta can lead to more significant results. Therefore the same dual test of Pettengill for the CAPM and for the Four-Moment CAPM is conducted using the DCC methodology for time-varying betas.

8.2.1. Results for the 10 ME portfolios

The results of the dual test of Pettengill for the ten ME portfolios are reported in Table 8.2. For the full sample, the introduction of DCC betas leads to a considerable improvement in the Four-Moment CAPM, where all of the factors have the expected sign. Specifically, in a bull market, covariance and cokurtosis are positive and significant, whereas coskewness is negative but insignificant. In a bear market, covariance and cokurtosis are both negative as expected, but only covariance is statistically significant whereas coskewness is positive, as expected, but insignificant. Stocks with higher covariance and cokurtosis tend to outperform in a bull market and to underperform in a bear market. These results confirm both the expectations and the results obtained for other countries by Gelagedera and Maharaj (2004) and Teplova and Shutova (2011). Stocks with higher coskewness have lower returns in bull markets as investors give up some returns in exchange for stocks whose returns are positive when market volatility increases. The overall risk premium is positive and significant in a bull market and negative and significant in a bear market with a larger magnitude than the single CAPM risk premia, and with a positive risk premium which is larger in absolute value than the negative risk premium. The CAPM shows a symmetric behaviour in the risk premium, as determined by Pettengill et al. (1995). This difference is mainly due to the negative sign of cokurtosis in a bull market.

Panel B of Table 8.2 shows that for the ten size portfolios with DCC betas estimated over the subsample period, the risk premia are still symmetric in the CAPM, although the risk premium is slightly larger in bull markets. Interestingly, all of the factors of the Four-Moment CAPM have the expected sign in bull markets and bear markets, as is the case for the full sample. The negative risk premium for beta in bear markets is however estimated to be slightly larger than in bull markets according to the Four-Moment CAPM, meaning that stocks tend to lose more in negative markets than they gain in absolute terms in positive markets. The overall risk premium is positive and significant in a bull market and negative and significant in a bear market, with a larger magnitude in absolute value terms in a bear market, suggesting either that realized returns have been poor in the subsample or that risk aversion is larger in bear markets, and therefore a negative risk premium is larger in absolute value than a positive risk premium.

8.2.2. Results for 10 BM portfolios

The dual test of Pettengill conducted on the ten BM portfolios shown in Panels C and D for the CAPM and for the Four-Moment CAPM in Table 8.2 shows that the simple CAPM is preferable to the Four-Moment CAPM for these portfolios as no additional factors in the Four-Moment CAPM are significant. The Four-Moment CAPM shows that the estimated risk premia are symmetric over the full sample (3% in absolute value). Beta is positively rewarded in bull markets and is symmetrically negatively rewarded in bear markets. Coskewness is positively rewarded in both bull and bear markets, but is insignificant. Cokurtosis is negatively rewarded in both bull and bear markets, whereas it is expected to be positive in a bull market and negative in a bear market, though the coefficients are not significant.

For the subsample 1980-2011, shown in Panel D of Table 8.2, the results confirm that the risk premium in negative markets is larger than that in positive markets for the

CAPM (-4.20% against 2.30%, respectively). Panel D shows that for the Four-Moment CAPM, only coskewness has a significant positive effect on the returns (158.9 with a t-statistic of 2.10) in a bear market, apart from beta. Specifically, the beta premium is positive and significant in a bull market and negative and significant in a bear market, with a larger magnitude in the bear market. Coskewness has a positive sign in a bear market, as expected, and is significant. In a bull market coskewness is instead positive and insignificant. Cokurtosis is insignificantly different from zero in both a bull and a bear market. The overall risk premium is positive in a bull market and negative in a bear market with a larger absolute value in a bear market.

For the 25 portfolios, results show that the risk premium is larger in a bear market than in a bull market, and again no additional factors to beta appear to be significant; therefore the results are not reported in this thesis.

Therefore, it can be concluded that the introduction of time-varying factor loadings with a DCC Multivariate GARCH model and of the higher comoments (coskewness and cokurtosis) do not solve the problem of the CAPM in explaining the returns of portfolios of stocks double sorted on ME and BM. One further solution is to introduce time-varying risk premia in the next section.

Table 8.2: The Dual test of Pettengill for the CAPM and Four-Moment CAPM on ten ME portfolios and ten BM portfolios for the period 1926-2011 and 1980-2011 with time-varying betas with dynamic correlations

10 ME portfolios dual test with DCC betas				
	Four-Moment CAPM		CAPM	
Panel A: 1926-2011				
	Bull	Bear	Bull	Bear
λ_β	0.0289 (6.10)*	-0.0277 (-6.24)*	0.0291 (6.51)*	-0.0274 (-5.74)*
λ_s	-39.0910 (-1.36)	0.9240 (0.03)		
λ_k	0.0105 (2.43)*	-0.0033 (-0.69)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0402 (5.56)*	-0.0307 (-5.08)*		
Panel B: 1980-2011				
λ_β	0.0206 (3.64)*	-0.0293 (-4.26)*	0.02900 (4.44)*	-0.0231 (-2.95)*
λ_s	-43.0890 (-1.18)	1.6999 (0.04)		
λ_k	0.0023 (0.53)	-0.0009 (-0.15)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0226 (3.09)*	-0.0302 (-3.08)*		
10 BM portfolios dual test with DCC betas				
	Four-Moment CAPM		CAPM	
Panel C: 1926-2011				
	Bull	Bear	Bull	Bear
λ_β	0.0322 (8.88)*	-0.0340 (-9.13)*	0.0322 (8.96)*	-0.0360 (-10.14)*
λ_s	50.1525 (1.48)	51.4751 (1.30)		
λ_k	-0.0008 (-0.59)	-0.0015 (-1.29)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0315 (8.13)*	-0.0350 (-8.85)*		
Panel D: 1980-2011				
λ_β	0.0241 (4.89)*	-0.0322 (-5.36)*	0.0230 (4.51)*	-0.0418 (-7.11)*
λ_s	39.5623 (0.68)	158.8167 (2.10)*		
λ_k	-0.0015 (-0.56)	-0.0004 (-0.25)		
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0224 (3.81)*	-0.0313 (-4.76)*		

This Table reports the results of the monthly cross-sectional regressions of, respectively, the ten ME portfolios and the ten BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM. The results for the period 1926-2011 are reported in Panels A and C and the results for the period 1980-2011 are reported in Panels B and D for the two samples: bull when the monthly excess market return is positive, and bear when the monthly excess market return is negative. The coefficients are reported for the conditional covariance, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. The t-statistics are reported in brackets and the significant coefficients at the 5% level are indicated with an asterisk.

8.3. Markov switching regimes

One limitation of the dual test of Pettengill et al. (1995), in which two different risk premia are estimated conditional on the sign of the excess market return, is that investors do not know ex-ante whether the market will be bullish or bearish and that, moreover, a bullish/bearish market might contain some negative/positive market excess returns but is fundamentally characterised by an overall positive/negative trend. Therefore, some negative/positive periods should actually be accounted for in the bullish/bearish estimation.

Table 8.3: Markov switching parameters for the market model¹⁸

Parameters	Coeff.	T-Stat.
μ_1	0.0155	6.74
μ_2	0.0	-
p_{12}	0.0423	1.72
p_{21}	0.0716	1.72
σ_1	0.0312	14.63
σ_2	0.0613	15.31

This Table shows the parameters of the Markov switching process for the market model. The parameters reported are the two means, the transition probabilities, and the two standard deviations.

Switching regimes are introduced where two regimes (bull and bear market) are obtained from a Markov switching process with a probability which depends on the realization of an unobservable variable, the state or regime itself, which is random but assumed to be determined by the realization of the market return. First, a simple non-linear model for the market excess return is assumed:

$$R_{Mt} - R_{ft} = \mu_{Mi} + \sigma_{Mi}\varepsilon_t \quad (8.5)$$

The coefficients, μ_{Mi} , and σ_{Mi} , $i = 1,2$ take one of two values, depending on the regime, and ε_t is a random disturbance assumed to be normally distributed. The filtered probabilities are estimated using the Expected Maximization algorithm of Hamilton

¹⁸ For a detailed explanation of the parameters please see Chapter 5 which discusses the methodology.

(1989) and are reported in Figure 8.1. The two regimes are estimated with a different mean and different standard deviation.

When applying the switching regimes methodology, the mean in the bearish market is found to be not significantly different from zero and therefore the Markov switching process is repeated, forcing the mean in a bear market to be 0. More explicitly, the bear market average return is found to be 0.45% but with a t-statistic of 0.84, insignificant at the 5% level (with a p-value 0.39). Therefore, the switching regime is repeated, imposing the average return in a bear market to be zero.

From Table 8.3, it can be noted that the first regime, the more likely of the two, is the bullish regime characterized by a positive average return of 1.55% and a standard deviation of 3.12%, whereas the bearish market is characterized by low returns (on average 0%) and a high standard deviation of 6.13%. The first regime is the bullish regime with an average return of 1.55% is typical of the 1980s, and is especially very prolonged in the decade of the 1990s, and after the dotcom crisis in the early years of the new millennium with the burst of the high-tech bubble.

The second regime, which is the bear regime, is typical of the period following the early 1980s, the end of the 1980s, the high volatility period of the late 1990s, the early years of the new millennium, and the financial crisis of 2007. Both regimes are persistent with very small transition probabilities (only 7.16% transition probability from a bearish regime to a bullish regime, and only 4.23% transition probability from a bullish regime to a bearish regime, which shows that regimes are very sticky, i.e. once in a regime it is difficult to switch to a different state).

It is noted that the two regimes are identified mainly by their different volatilities, so that they are more precisely: i) a bullish regime with a positive return and a low volatility; and ii) a high volatility regime with low average returns (rather than the latter being a real bearish regime characterized by negative returns). The two regimes (bullish

and bearish) might then be defined more appropriately as a bullish quiet regime and a high risk/volatility regime that might lead to either high positive or high negative returns, respectively. Interestingly, it is worth recalling that the risk premium for the market portfolio is a function of both risk aversion and volatility. Therefore, the two regimes (different in terms of volatility and returns) should capture the variation in risk aversion and hence should reveal the time-varying risk premium. Thus, risk aversion is not directly modelled in this thesis, but is indirectly revealed by the time-varying risk premium which depends on the regime.

Figure 8.1 shows that the market is mainly in a bullish regime with an average of 62.7% of the time, and in a bearish regime only 37.3% of the time. This difference is important as the different risk premia demanded in a bull and a bear market might provide a positive weighted average risk premium. In particular, it might be that the conditional risk premia and the time-varying factor sensitivities are correlated in such a way as to explain the main anomalies of the CAPM.

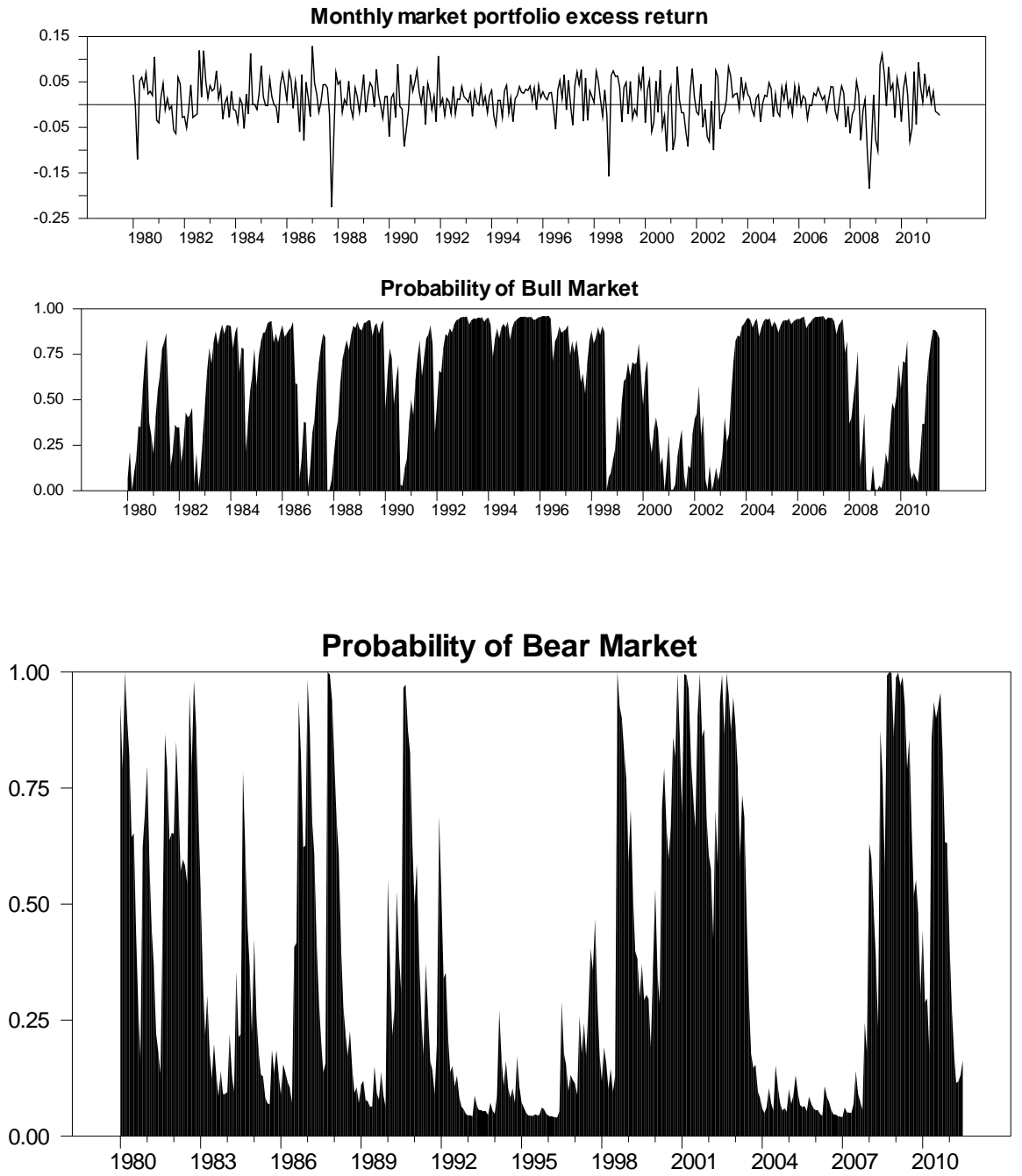


Figure 8.1 Filtered probabilities of the bull and bear regime for the period 1980-2011

8.4. Individual-fixed effects panel for the CAPM and Four-Moment CAPM for the 25ME/BM portfolios over the period 1980-2011

In this section the approach discussed in chapter 5 and recalled in equation 8.1 and 8.2, i.e. panel data, is presented with its empirical results for the 25 portfolios double sorted on ME and BM for the period 1980-2011. The period 1980-2011 is chosen as a multi-stage Markov switching model is required for long periods of time and the last 30 years avoid bias due to the high inflation of the 1970s; moreover these 30 years are the most problematic empirically for the CAPM.

The introduction of Markov switching regimes allows the researcher not only to obtain the parameters but also the probabilities of the regimes. However, one main problem from an econometric perspective is that there are two sets of risk premia to estimate but only one set of betas or factor loadings. In order to overcome this estimation problem, panel data are introduced to augment the dimension of the equations from which the risk premia can be estimated.

The first methodology that is applied to test the models here involves individual fixed-effects panel data in which the intercepts are allowed to vary across individual assets (the 25 ME/BM-sorted portfolios), but such intercepts are kept constant over time. The intercepts will then capture an individual effect that affects the portfolios but does not change over time. The effect of introducing this method is that the intercept is removed and therefore only the two risk premia are obtained, as reported in Table 8.4. Interest then focuses on three items: (i) the coefficients of the risk premia; (ii) the results of the tests of the unconditional CAPM and unconditional Four-Moment CAPM; and (iii) the weighted average risk premium.

The conditional models are obtained first using a DCC GARCH approach covering the full sample (as shown in Table 8.4) and then using a DCC GARCH approach starting

from 1980 (as shown in Table 8.5). In other words, the factor loadings are estimated with a Multivariate DCC GARCH approach for the period 1926-1980 for the estimation process, and then only the subsample 1980-2011 in order to test the robustness of the results.

The results in Table 8.4 show that for the CAPM the bear risk premium is estimated to be negative (-0.75%) and significant (t-statistic of -2.09), whereas the risk premium in a bull market is significant and positive with a coefficient of 1.18% per month. Therefore, the conditional signs of the risk premia are consistent with theory. However, the unconditional CAPM is strongly rejected as the difference between the two risk premia is 1.94%, statistically significant with a t-statistic of 11.80.

When the time series of the risk premium is tested with a t-test methodology with standard errors corrected for autocorrelation (using four lags), the results show that the risk premium becomes significantly positive and is estimated as 0.46% per month. These results are affected by the usual limitation of a low dispersed risk premium reflected in a low standard error and a high t-statistic. The results therefore appear to support a conditional CAPM in which beta seems to be priced, especially in bullish markets. This finding is extremely interesting as it appears to pave the way for a rational explanation of the cross-section of returns for the most challenging portfolios in the asset pricing literature. This result is, however, the effect of the varying intercepts across portfolios that most likely capture the alphas. In this context, the cross-section of the 25 double sorted portfolios can be explained by the conditional CAPM with time-varying betas and time-varying risk premia.

Table 8.4 Test of the conditional CAPM and conditional Four-Moment CAPM using Individual-Fixed effects panel data for the 25 ME/BM portfolios for the period 1980-2011

	Four-Moment CAPM		CAPM	
	Bull	Bear	Bull	Bear
λ_β	0.0163 (4.49)*	-0.0097 (-2.67)*	0.0118 (3.43)*	-0.0075 (-2.09)*
λ_s	11.6488 (0.40)	-8.3265 (-1.38)		
λ_k	0.0069 (4.65)*	0.0054 (6.37)*		
$\lambda_{\beta 1} - \lambda_{\beta 2}$		0.0250 (12.62)*		0.0194 (11.80)*
$\lambda_{s 1} - \lambda_{s 2}$		51.9712 (2.25)*		
$\lambda_{k 1} - \lambda_{k 2}$		0.0006 (0.32)		
$\Gamma_{\beta t}$		0.0066 (7.66)*		0.0046 (7.13)*
Γ_{st}		4.2051 (6.31)*		
Γ_{kt}		0.0064 (127.24)*		
Γ_t		0.0132 (14.20)*		

This Table reports the results of the panel data regressions with individual-fixed effects of the 25 ME/BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM over the period 1980-2011 using a DCC GARCH model starting from 1926 for the two regimes: bull and bear. The coefficients are reported for the conditional beta, the conditional coskewness, the conditional cokurtosis, the tests of differences in the three risk premia, and for the weighted average beta, coskewness, cokurtosis premium, and the overall risk premium. The t-statistics are reported in brackets and the significant coefficients at the 5% level are indicated with an asterisk. Results are obtained as follows:

$$R_{it} - R_{ft} = \gamma_0 + \gamma_{12} p_t \beta_{it} + \gamma_2 \beta_{it} + \varepsilon_{it} \quad \text{CAPM}$$

$$R_{it} - R_{ft} = \Gamma_{\beta t} \beta_{it} + \Gamma_{st} (s_{it} - s_m \beta_{it}) + \Gamma_{kt} (k_{it} - \beta_{it}) + \varepsilon_{it} \quad \text{Four-moment CAPM}$$

The same individual-fixed effects panel data methodology is applied for the Four-Moment CAPM. The results are reported in Table 8.5 and show that; i) in a bull market the beta premium is positive (1.63%) and significant as expected and ii) cokurtosis is positive (0.69%) and significant as expected, whereas iii) coskewness is positive as opposed to the expected negative sign and is insignificant. In theory assets with higher risk, that is – higher exposure to covariance and cokurtosis – are positively rewarded by the market, whereas assets with higher coskewness, being safer, tend to earn less in a bull market. However, the results show that assets more exposed to cokurtosis are

rewarded in both markets. The explanation of this apparent anomaly is that portfolios with high book-to-market ratios tend also to have higher cokurtosis and hence the value premium is partly reflected in this cokurtosis premium. In other words, high book-to-market ratio stocks tend to do better than low book-to-market ratio stocks in both bull and bear markets, and they also have higher cokurtosis.

The coskewness premium is consistently insignificant at the 5% level, and therefore does not appear to add anything to explanations of the cross-section of returns. Nevertheless, the signs of the coefficients associated to coskewness are opposite to theoretical expectations.

In a bear market, the beta premium is estimated as negative (-0.97%) and is significant, whereas coskewness is negative (-8.32), as opposed to the expected positive sign, and is statistically significant. Cokurtosis is positive and significant, whereas it was expected to be negative in a bear market. Furthermore, the unconditional CAPM is rejected as the risk premia are significantly different in the two regimes, apart from the case of the cokurtosis premium.

Finally, the t-test is applied to the time series of the risk premia. Beta premium and cokurtosis have the positive sign expected from theory, and they are a statistically significant 0.66% and 0.64% monthly, respectively. The coskewness premium is unexpectedly positive and significant. The total risk premium is positive and significant, but it appears to be excessively large at 1.32% monthly compared to the historical market premium.

In summary, the results show that beta and cokurtosis (but not coskewness) can better explain the cross section of returns. However, whereas beta has the sign expected from theory that is positive in a bull market and negative in a bear market, cokurtosis is significantly positively rewarded in both bull and bear markets. The instability of the

coefficients of higher moments is also well documented in the findings of Lambert and Hubner (2010) and Heaney et al. (2012).

The fixed effects methodology is then repeated including only the later period of 1980-2011 in the DCC GARCH model to estimate the time-varying betas, the results of which are reported in Table 8.5. In the case of the CAPM, the results show that the risk premium is only significant in the bear market and not in the bull market. Moreover, the risk premium in the bear market is -1.78% and of a much larger magnitude in absolute value than the risk premium in the bull market. The results suggest therefore that the CAPM works in a bear market but not in a bull market. The explanation can partly be attributed to the high correlation between assets in downmarkets. However, it appears that riskier stocks lose more in a bear market whereas in a bull market all assets tend to yield positive returns independently of their beta, suggesting a 'boom and bust' effect. Thus, the null hypothesis that an unconditional CAPM holds is firmly rejected as the difference between the risk premia is positive (2.08%) and significant. Moreover the weighted average risk premium is significantly negative. Therefore, the conditional CAPM is not supported by the data as the risk premium is negative and highly significant, a result which contrasts with theory. Investors hold risky assets for a positive premium and not for a negative premium.

The fixed effect individual approach is applied to the Four-Moment CAPM and the results are reported in Table 8.5. In a bull market, the beta premium is insignificantly different from zero, whereas in a bear market it is significantly negative. Coskewness is positive and significant in a bull market (incorrect sign) and negative and significant in a bear market (incorrect sign). However, cokurtosis is significantly positive in a bull market and significantly negative in a bear market, a result that is consistent with

theoretical expectations. Thus the unconditional Four-Moment CAPM is firmly rejected in favour of a conditional model as all of the risk premia pairs are significantly different.

Table 8.5: Test of the conditional CAPM and conditional Four-Moment CAPM using Individual-Fixed effects panel data for the 25 ME/BM portfolios for the period 1980-2011 with DCC betas calculated over the period 1980-2011

	Four-Moment CAPM		CAPM	
	Bull	Bear	Bull	Bear
λ_β	0.0011 (0.32)	-0.0722 (-16.04)*	0.0030 (0.85)	-0.0178 (-4.63)*
λ_s	396.0209 (7.88)*	-40.0984 (-6.27)*		
λ_k	0.0040 (21.11)*	-0.0688 (-21.24)*		
$\lambda_{\beta 1} - \lambda_{\beta 2}$	0.0734 (25.23)*		0.0208 (12.08)*	
$\lambda_{s 1} - \lambda_{s 2}$	436.1194 (8.49)*			
$\lambda_{k 1} - \lambda_{k 2}$	0.0729 (21.24)*			
$\Gamma_{\beta t}$	-0.0262 (-10.71)*		-0.0047 (-6.84)*	
Γ_{st}	233.5041 (16.06)*			
Γ_{kt}	-0.0231 (-9.51)*			
Γ_t	-0.0632 (-15.57)*			

This Table reports the results of the panel data regressions with individual-fixed effects for the 25 ME/BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM over the period 1980-2011 using a DCC GARCH model starting from 1980 for the two regimes: bull and bear. The coefficients are reported for the conditional beta, the conditional coskewness, the conditional cokurtosis, the tests of differences in the three risk premia, and for the weighted average beta, coskewness, cokurtosis premium, and the overall risk premium. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk. Results are obtained as follows:

$$R_{it} - R_{ft} = \gamma_0 + \gamma_{12} p_t \beta_{it} + \gamma_2 \beta_{it} + \varepsilon_{it} \quad \text{CAPM}$$

$$R_{it} - R_{ft} = \Gamma_{\beta t} \beta_{it} + \Gamma_{st} (s_{it} - s_m \beta_{it}) + \Gamma_{kt} (k_{it} - \beta_{it}) + \varepsilon_{it} \quad \text{Four-moment CAPM}$$

All of the coefficients for the weighted average risk premia have the opposite sign to that which was expected in theory, showing that the Four-Moment conditional CAPM is not supported, and that it can only be partly rescued by allowing the DCC beta process to start from 1926 in the sample data. More specifically, beta and cokurtosis have negative and significant premia, whereas coskewness has a positive and significant premium (contrary to theoretical expectations). Finally, the total risk premium is

significant and negative, a result that markedly contradicts theory as the risk premium should be positive. The conclusion is that only the conditional CAPM with time-varying betas obtained with the DCC GARCH approach tested on the full sample can explain the returns of the 25 double sorted portfolios.

8.5. Random individual effects panel data for the CAPM and Four-Moment CAPM for the 25 ME/BM portfolios over the period 1980-2011

In this section a random effects panel model is introduced. In this case, as in the case of the individual fixed-effects panel model, the intercepts are allowed to vary across individual assets (stocks or portfolios), but not over time. However, the intercepts have a common component and a random component.

For the CAPM, the Hausman Test, which tests whether the random effects are uncorrelated with the explanatory variables, is rejected strongly with a low probability of 0.04, suggesting that the random effects model is not appropriate. The results show that the random effects model is not supported and that the individual fixed-effects model is preferable, leading to a positive market premium of 0.46% per month for the period 1980-2011.

The random individual effect technique is applied for the Four-Moment CAPM and the results are reported in Table 8.6. The intercept is positive and strongly significant, showing that a large part of the variability of portfolio returns is not captured adequately by the model. In a bear market, both covariance and cokurtosis have a significant coefficient which is positive for the cokurtosis premium, as expected, and negative for the covariance premium in contrast to theoretical expectations. Coskewness is negative but insignificant. Overall, the Four-Moment CAPM does not describe well the cross-section of portfolio returns in a bearish market. In a bull market, the beta premium is a positive at 1.14% and significant and thus coskewness is positive in contrast to

theoretical expectations. The cokurtosis has a positive and significant premium of 0.63% per month.

The Four-Moment CAPM therefore seems to describe reasonably well the expected returns in bullish markets as all the factors except coskewness have the signs expected from theory. In particular, beta and cokurtosis appear to be the most important factors in explaining the cross section of returns in a bull market.

In addition, the Hausman test with 6 degrees of freedom is not rejected, with probability of 0.22, and thus the model is appropriately described by a random effects panel data. The unconditional version of the model is rejected firmly as only the cokurtosis premia are not significantly different from each other whereas both beta and cokurtosis are significantly differently rewarded in the two regimes. The results show that in this case all of the coefficients of the weighted average risk premia are positive and significant. Covariance and cokurtosis have the positive sign expected from theory, whereas coskewness is positive, contrary to theoretical expectations. Furthermore, the total risk premium is 0.85% per month and is significant, which is a sensible result given that the historical risk premium was 0.56% over the period 1980-2011.

For the Four-Moment CAPM, therefore, using individual random effects may be appropriate and leads to a better performance of the Four-Moment CAPM in comparison with the conditional CAPM. In summary, the results show that the conditional CAPM is better explained by an individual fixed-effects panel model, yielding a positive beta premium (0.46% monthly), whereas the conditional Four-Moment CAPM is better described by a random-effects panel model in which time-varying beta and cokurtosis are the main explanatory factors for returns.

Table 8.6: Test of the conditional CAPM and conditional Four-Moment CAPM using random effects panel data for the 25 ME/BM portfolios for the period 1980-2011

	Four-Moment CAPM		CAPM	
	Bull	Bear	Bull	Bear
α		0.0070 (2.54)*		0.0083 (2.97)*
λ_β	0.0114 (4.15)*	-0.0136 (-4.90)*	0.0063 (2.41)*	-0.0131 (-4.71)*
λ_s	41.8571 (2.05)*	-10.1141 (-1.73)		
λ_k	0.0063 (4.55)*	0.0057 (6.93)*		
$\lambda_{\beta 1} - \lambda_{\beta 2}$		0.0251 (12.63)*		0.0196 (11.89)*
$\lambda_{s 1} - \lambda_{s 2}$		51.9713 (2.26)*		
$\lambda_{k 1} - \lambda_{k 2}$		0.0006 (0.32)		
$\Gamma_{\beta t}$		0.0020 (2.46)*		-0.0009 (-1.39)
Γ_{st}		22.49 (12.98)*		
Γ_{kt}		0.0061 (305.92)*		
Γ_t		0.0085 (9.82)*		

This Table reports the results of the panel data regressions with random effects for the 25 ME/BM portfolios on the three factors of the Four-Moment CAPM and on the single factor of the CAPM over the period 1980-2011 using a DCC GARCH model starting from 1926 for the two regimes: bull and bear. The coefficients are reported for the conditional beta, the conditional coskewness, the conditional cokurtosis, for the tests of differences in the three risk premia, and for the weighted average beta, coskewness, cokurtosis premium, and the overall risk premium. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk.

8.6. Empirical tests of the CAPM and Four-Moment CAPM on individual assets

In previous chapters, the CAPM has been extended by incorporating higher comoments and introducing time-varying factor loadings using a Multivariate GARCH model with DCC. In the first part of this chapter, time-varying risk premia, which are dependent on market regimes are introduced. With a few exceptions, these extensions have been unable to explain the anomalies of the CAPM – size and value premium – especially for the last 30 years of stock returns.

However, so far in this thesis the two models (the CAPM and the Four-Moment CAPM) have been tested on portfolios of stocks sorted on market capitalization and/or the book-

to-market ratio. In the remainder of this chapter, these two models are tested on individual US stocks over the periods 1930-2010 and 1980-2010.

Authors such as Ang et al. (2008) and Kim (1995) have criticized the use of portfolios to estimate the market premium, arguing that the spread in betas is too small when portfolios are formed, leading to very large standard errors in the estimation of the risk premium. Kim argues that when portfolios are formed, the behaviour of individual stocks is smoothed out, losing important information for the estimation of the risk premium. Accordingly, in this section, individual assets are used to test the CAPM and the higher moment CAPM using an extension of the short-window regression methodology proposed by Lewellen and Nagel (2006). The section then proceeds with the approach of Kraus and Litzenberger (1973) and an extension of the Fama and French three factor model in sections 8.7. and 8.8, respectively.

Lewellen and Nagel's approach is simple, involving two main steps. First, short-window (24 months) time series regressions of monthly individual asset excess returns over the market excess return are used to estimate conditional betas in the CAPM, and rolling short windows (24 months) are used to estimate conditional betas, coskewness, and cokurtosis in the Four-Moment CAPM. The average excess returns of the individual stocks over the short windows are assumed to be the expected excess returns of the individual stocks.

In the second step, the average excess return of individual stocks, calculated over the window used to estimate the factor loadings, are regressed at cross section against the conditional factor loadings (calculated over the same time window as in the first step) to estimate the risk premia. The monthly conditional risk premia are then treated as time series. Specifically, the time series average of the risk premia is the risk premium, and its significance is tested using a t-test as in the Fama and MacBeth methodology. The

models are conditional in so far as the factor loadings are obtained using rolling short windows of 24 months. The use of time-varying betas with dynamic correlations (DCC) would be too cumbersome for individual assets. Another problem is that the number of individual assets in the monthly cross sectional regressions varies over time. The results of the empirical test conducted over the period 1930-2010 are reported in Table 8.7 and the results for the period 1980-2010 are reported in Table 8.8.

8.6.1. Results of the test of the conditional CAPM on individual assets

The results of the test of the conditional CAPM given in Table 8.7 show a different scenario when individual assets are used as opposed to portfolios. Indeed, the market premium is found to be significant and positive. For the full sample, the intercept is significant, but very close to zero (0.07%), but the estimated market premium is positive and significant (0.67% per month, which is equivalent to 8.34% per year). Inconsistent with theory, the intercept is significant and positive, though it is quite small at 0.07% per month. The positive intercept is a finding that is common in the literature and might be related to the use of an inappropriate proxy for the risk-free rate (the one month T-Bill rate). As is well known in econometrics, the positive intercept could also be due to not having enough observations around zero.

For the subsample 1980-2010, the intercept is still significant and positive (0.07%). Interestingly, the beta premium has a positive and significant coefficient (0.59% per month), equal to a compound rate of return of 7.31% per year. Moreover, the risk premium appears to have declined over the second period in comparison to the first. Therefore, when using individual assets the CAPM seems to hold quite well as the risk premium is sensible despite the positive alpha.

8.6.2. Results of the test of the conditional Four-Moment CAPM on individual assets

For the Four-Moment CAPM, as shown in Table 8.7 for the full sample, the intercept is significant and positive, although very close to zero (0.06%). The beta premium is significant and positive, as expected from theory, and of a magnitude of 0.67% per month. Coskewness is negatively priced, as expected theoretically, whereas cokurtosis is not significantly rewarded. This finding is different from the result obtained for individual assets by Heaney et al. (2012) who instead find coskewness and cokurtosis to be significant, but not beta. The overall premium is estimated at 0.73% per month, equal to 9.12% per year, larger than the 8.34% estimated for the simple CAPM.

For the subsample, as shown in Table 8.8, and unlike the single factor CAPM, the intercept is insignificantly different from zero (at 0.04% per month), the beta premium is positive (at 0.59%) and is significant as expected theoretically, which is very close to the risk premium of the CAPM. Coskewness has a negative and significant premium, as expected by the theory, and so investors are willing to sacrifice some return in exchange for positive skewness. The only insignificant factor is cokurtosis, suggesting that cokurtosis is not priced by the market and that, unexpectedly, investors do not seem to award a premium to this risk factor.

The overall risk premium is 0.56% per month, equal to 6.93% per year, slightly lower than in the case of the CAPM and again smaller than the risk premium estimated over the full sample. Thus, the risk premium seems to have declined over time. However, the results seem to show that beta and coskewness are priced factors of risk in the cross-section of returns of individual stocks, whereas cokurtosis is not.

Table 8.7: Test of the 4-CAPM and CAPM using short-windows regressions on individual assets (1930-2010)

	Four-Moment CAPM	CAPM
α	0.0006 (3.33)*	0.0007 (3.80)*
λ_β	0.0067 (23.88)*	0.0067 (22.14)*
λ_s	-73.16 (14.27)*	
λ_k	-0.0004 (-0.45)	
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0073 (19.43)*	

This Table reports the results of the monthly cross-sectional regression of a stock's average returns (24 months) on the three factors of the Four-Moment CAPM and on the single factor of the CAPM over the period 1930-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk.

Results are obtained from the following models:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1} \lambda_{\beta t} \quad \text{CAPM}$$

$$R_{i,t} - R_{f,t} = \lambda_\beta \beta_{i,t-1} + \lambda_s (s_{i,t-1} - s_{m,t-1} \beta_{i,t-1}) + \lambda_k (k_{i,t-1} - \beta_{i,t-1}) \quad \text{Four-Moment CAPM}$$

Table 8.8: Test of the 4-CAPM and CAPM using short-windows regressions on individual assets (1980-2010)

	Four-Moment CAPM	CAPM
α	0.0004 (1.32)	0.0007 (4.03)*
λ_β	0.0059 (19.10)*	0.0059 (18.98)*
λ_s	-32.79 (-3.15)*	
λ_k	0.0007 (0.72)	
$\lambda_\beta + \lambda_s s_m + \lambda_k$	0.0056 (10.61)*	

This Table reports the results of the monthly cross-sectional regression of a stock's average returns (24 months) on the three factors of the Four-Moment CAPM and on the single factor of the CAPM over the period 1980-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk.

Results are obtained from the following models:

$$R_{i,t} - R_{f,t} = \alpha_t + \hat{\beta}_{i,t-1} \lambda_{\beta t} \quad \text{CAPM}$$

$$R_{i,t} - R_{f,t} = \lambda_\beta \beta_{i,t-1} + \lambda_s (s_{i,t-1} - s_{m,t-1} \beta_{i,t-1}) + \lambda_k (k_{i,t-1} - \beta_{i,t-1}) \quad \text{Four-Moment CAPM}$$

8.7. Results of the test of the conditional Four-Moment CAPM and three-moment CAPM of Kraus and Litzenberger on individual assets

In this section results are reported for the tests conducted on the individual assets for the three-moment CAPM of Kraus and Litzenberger (1973), with systematic covariance and systematic coskewness, and for the Four-Moment CAPM including also systematic cokurtosis. For comparison, results are also reported for the three-moment CAPM obtained by excluding systematic cokurtosis from the Four-Moment CAPM model used in this thesis (3-CAPM adjusted). The results for the full sample are reported on Table 8.9, and the results for the subsample 1980-2011 are reported in Table 8.10.

The approach adopted here is the same as in the previous section (i.e. using the rolling window approach of Lewellen and Nagel), except that the coskewness is computed differently. Specifically, the model estimated here is

$$R_i - R_f = \lambda_\beta \frac{E(r_i \cdot r_m)}{E(r_m^2)} + \lambda_s \frac{E(r_i \cdot r_m^2)}{E(r_m^3)} + \lambda_k \frac{E(r_i \cdot r_m^3)}{E(r_m^4)} + \varepsilon_i \quad (8.6)$$

This is in line with the derivation of the 3-moment CAPM of Kraus and Litzenberger, that is adding a standardized measure of cokurtosis. This model will be referred as unadjusted Four-Moment CAPM.

With the term adjusted model the Four-Moment CAPM derived in this thesis and introduced in Chapter 5 will be identified:

$$R_i - R_f = \lambda_\beta \beta_i + \lambda_s (s_i - s_m \beta_i) + \lambda_k (k_i - \beta_i) + \varepsilon_i \quad (8.7)$$

With the term adjusted three-moment CAPM the following model will be identified:

$$R_i - R_f = \lambda_\beta \beta_i + \lambda_s (s_i - s_m \beta_i) + \varepsilon_i \quad (8.8)$$

Table 8.9: Test of the unadjusted 4-CAPM, unadjusted 3-CAPM and adjusted 3-CAPM using short-windows regressions on individual assets (1930-2010)

	Four-Moment CAPM unadjusted	3-CAPM unadjusted	3-CAPM adjusted
α	0.0006 (3.33)*	0.0006 (2.91)*	0.0006 (2.91)*
λ_β	0.0062 (11.78)*	0.0061 (17.10)*	0.0068 (23.14)*
λ_s	0.0010 (3.41)*	0.0008 (5.63)*	-70.4126 (-17.08)*
λ_k	-0.0004 (-0.90)		
$\lambda_\beta + \lambda_s + \lambda_k$	0.0067 (23.88)*	0.0068 (23.14)*	0.0076 (25.84)*

This Table reports the results of the monthly cross-sectional regression of a stock's average returns (24 months) on the three factors of the Four-Moment CAPM and on the factors of the unadjusted and adjusted 3-CAPM over the period 1930-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1973).

Table 8.10: Test of the unadjusted 4-CAPM, unadjusted 3-CAPM and adjusted 3-CAPM using short-windows regressions on individual assets (1980-2010)

	Individual Assets		
	Four-Moment CAPM unadjusted	3-CAPM unadjusted	3-CAPM adjusted
α	0.0004 (2.77)*	0.0004 (2.48)*	0.0004 (2.48)*
λ_β	0.0063 (15.48)*	0.0064 (22.01)*	0.0059 (19.19)*
λ_s	-0.0011 (-3.15)*	-0.0004 (-3.36)*	-35.57 (-4.38)*
λ_k	0.0007 (1.45)		
$\lambda_\beta + \lambda_s + \lambda_k$	0.0052 (8.42)*	0.0059 (19.18)*	0.0055 (14.45)*

This Table reports the results of the monthly cross-sectional regression of a stock's average returns (24 months) on the three factors of the Four-Moment CAPM and on the factors of the unadjusted and adjusted 3-CAPM over the period 1980-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk. Beta, coskewness and cokurtosis are obtained as in Kraus and Litzenberger (1973).

8.7.1. Results for the conditional unadjusted Four-Moment CAPM

The conditional unadjusted Four-Moment CAPM is tested on individual assets.

For the full sample, as shown in Table 8.9, the intercept is significant and positive, although very small at 0.06% per month. The beta premium is positive and significant, as expected theoretically, whereas coskewness is significant, but with a positive sign at 0.10%, whereas the market portfolio skewness is positive for the full sample, therefore creating an expectation for a negative coskewness premium. Cokurtosis is not significantly priced, and again the overall risk premium is positive and significant, and is estimated to be of a magnitude of 0.67% per month against 0.73% in the case of the Four-Moment CAPM.

For the subsample, as shown in Table 8.10, the intercept is still significant and positive (at 0.04%), whereas it is insignificant in the adjusted model. The beta premium is positive and significant as expected, whereas coskewness has a negative and significant premium. The market portfolio skewness is negative over the subsample and therefore commands a theoretical positive coskewness premium. Cokurtosis is not priced significantly, while the overall risk premium is positive and significant at 0.52% per month, smaller than the 0.56% estimated for the adjusted Four-Moment CAPM.

8.7.2. Results for the conditional unadjusted three-moment of Kraus and Litzenberger (1973) and the adjusted three-moment CAPM

As the results in Table 8.9 and Table 8.10 show that cokurtosis is not priced in the cross section of individual assets, cokurtosis is dropped from the model and the three-moment CAPM is tested. Two models are tested: the three-moment model of Kraus and Litzenberger (1973), and the three-moment model obtained from the Four-Moment CAPM model used in this thesis by restricting the coefficient of cokurtosis to zero. For the full sample, for the unadjusted 3-moment CAPM, as shown in Table 8.9, the

intercept is significantly positive for the KS model, the beta premium is positive and significant, as expected, and the coskewness premium is positive and significant. The overall risk premium is 0.68% per month and it is significant. The results differ somewhat for the adjusted three-moment model. The intercept is positive and significant, and the beta premium is positive and significant, though larger than in the adjusted model. However, coskewness has a negative coefficient, as expected, and it is significant. The total risk premium is 0.76% per month, which is significant and equal to 9.51% annually compounded, and bigger than the risk premium estimated using the adjusted models and by the CAPM and the unadjusted Four-Moment CAPM.

For the subsample, as shown in Table 8.10 for the adjusted three-moment CAPM, the intercept is positive and significant. Beta is significantly positively rewarded, whereas coskewness has a negative premium, and the market portfolio skewness is negative. The overall premium is positive and significant, at 0.59% monthly. For the adjusted model, the intercept is positive and significant. Beta is positively and significantly rewarded by the market, whereas the coskewness premium has the negative and significant sign expected from theory. The total risk premium is 0.55% and is significant. Further, the risk premium is estimated to have declined significantly over time. The results show that the adjusted three-moment CAPM is the preferred model, although the intercept is still positive and significant, as the coskewness coefficient is negative as expected from theory.

8.8. Four-Moment CAPM augmented with Fama and French factors

In this final section, the results of the test on individual assets are reported for a conditional Four-Moment CAPM to which SMB and HML, the two factors of the Fama and French model (1993), are added. SMB represents the return of a portfolio of small-capitalized stocks minus the return on a portfolio of large-capitalized stocks. HML

represents the returns of a portfolio of high book-to-market stocks minus the return on a portfolio with low book-to-market stocks.

The three-factor model of Fama and French is widely used in empirical studies of asset pricing, therefore it would be interesting to augment the conditional Four-Moment CAPM with the SMB and HML factors in order to see whether the FF factors remain relevant in asset pricing even after correcting for additional moment sensitivities.

The results are reported in Table 8.11 for the full sample and in Table 8.12 for the subsample. On the right hand side of the Tables empirical results for the three-factor model of Fama and French are reported.

The augmentation which is similar to Smith (2006) and Engle and Bali (2010) is given by:

$$R_i - R_f = \lambda_\beta \beta_i + \lambda_s (s_i - s_m \beta_i) + \lambda_k (k_i - \beta_i) + \lambda_{smb} \frac{\text{cov}(r_i, \text{smb})}{\text{cov}(\text{smb}, \text{smb})} + \lambda_{hml} \frac{\text{cov}(r_i, \text{hml})}{\text{cov}(\text{hml}, \text{hml})} + \varepsilon_i \quad (8.9)$$

Therefore, the standardized covariances between returns of the stocks with the SMB and HML factors are obtained from univariate regressions.

8.8.1. Results for the Four-Moment CAPM augmented with SMB and HML

The Four-Moment CAPM augmented with SMB and HML is tested on individual assets for the full sample and for the period 1980-2010.

For the full sample, as shown in Table 8.11, the intercept is significant and positive, showing that either the model cannot capture fully the cross-sectional variation of returns, or that the TBill rate is not an adequate proxy for the risk-free rate. The beta premium is positive and significant, coskewness has a negative and significant premium, as expected, whereas cokurtosis is positive and significant as expected. SMB is not made insignificant by the Four-Moment CAPM, showing that SMB retains some

explanatory additional power; so that coskewness and cokurtosis do not explain the size effect. SMB is positive and significant, showing that stocks that have more exposure to SMB yield higher returns over the full sample. HML is significantly negatively priced (-0.04% per month), although very close to zero.

For the subsample, shown in Table 8.12, the intercept becomes insignificant. The beta premium is still positive and significant, although smaller than in the case for the full sample. Coskewness has a significant negative sign, as expected in theory, and cokurtosis is positively and significantly priced, as expected. SMB retains explanatory power for the subsample with an estimated premium of 0.28% per month, reflecting the good returns yielded by small stocks over the last 30 years. HML retains a negative sign, but is no longer significant. Surprisingly, the SMB premium seems to remain, whereas the HML premium is not there anymore. The findings are different from the results obtained by Heaney et al. (2012) who find that the higher moments are encompassed by SMB and HML. Indeed, HML does not appear to be significantly priced.

8.8.2. Results of the test of the three-factor model of Fama and French (1993) using short-window regressions

Finally, for completeness the three-factor model of Fama and French is tested on individual assets.

For the full sample, as reported in Table 8.11, the intercept is significant and positive, thereby contradicting the expectations for the intercept to be zero. Beta is significantly and positively priced at 0.53% per month, SMB has a positive significant premium, showing that stocks with small market capitalization tend to yield higher returns over the full sample which are not fully explained by the difference in beta. HML has a negative and significant coefficient, although it is very small at -0.05% per month.

For the subsample 1980-2010, reported in Table 8.12, the intercept is positive but insignificant. Beta is positively and significantly rewarded by the market, and SMB has a positive and significant coefficient of 0.26% per month. The results confirm that small stocks yielded higher returns than large stocks over the last 30 years. Finally, HML has a negative coefficient, but is insignificant.

Therefore, it appears that the best description of the cross-section of returns for individual stocks is obtained when the Four-Moment CAPM is augmented with SMB, as all of the factors are significant and have the theoretically expected sign. More specifically, beta and cokurtosis are positively rewarded, coskewness is negatively rewarded and SMB is positively rewarded.

Table 8.11: Test of the adjusted Four-Moment CAPM augmented with SMB and HML and the CAPM augmented with SMB and HML using short-windows regressions on individual assets (1930-2010)

	Individual Assets	
	Four-Moment CAPM adjusted + FF	3-Factor F&F
	1930-2010	
α	0.0007 (3.77)*	0.0009 (4.65)*
λ_β	0.0058 (22.83)*	0.0053 (20.00)*
λ_s	-49.4709 (-11.70)*	
λ_k	0.0014 (3.60)*	
<i>smb</i>	0.0016 (8.29)*	0.0016 (8.19)*
<i>hml</i>	-0.0004 (-2.97)*	-0.0005 (-3.87)*

This Table reports the results of the monthly cross-sectional regression of a stock's average returns (24 months) on the three factors of the Four-Moment CAPM and on the three factors of the F&F model over the period 1930-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, SMB and HML. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk.

Table 8.12: Test of the adjusted Four-Moment CAPM augmented with SMB and HML and the CAPM augmented with SMB and HML using short-windows regressions on individual assets (1980-2010)

	Individual Assets	
	Four-Moment CAPM adjusted + FF	3-Factor F&F
	1980-2010	
α	0.0002 (1.17)	0.0003 (1.69)
λ_β	0.0043 (14.78)*	0.0041 (15.28)*
λ_s	-10.32 (-2.08)*	
λ_k	0.0018 (3.38)*	
<i>smb</i>	0.0028 (13.60)*	0.0026 (14.73)*
<i>hml</i>	-0.0003 (-1.55)	-0.0003 (-1.57)

This Table reports the results of the monthly cross-sectional regression of a stock's average returns (24 months) on the three factors of the Four-Moment CAPM and on the three factors of the F&F model over the period 1980-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, SMB and HML. The t-statistics are reported in brackets and the significant coefficients at 5% level are indicated with an asterisk.

8.9. Conclusion

This chapter investigates the potential explanatory power of a conditional CAPM and conditional Four-Moment CAPM, respectively, using conditional time-varying risk sensitivities and conditional time-varying risk premia. The sensitivities (or factor loadings) have been estimated using a DCC GARCH. The conditional risk premia have been estimated by first identifying the probability of a bull and a bear regime according to a Markov Switching process and then estimating the conditional risk premia using a panel regression. This approach is in itself a novelty in asset pricing.

In the first part of this chapter, the main findings are: (i) the unconditional models are always rejected in favour of a conditional time-varying risk premium; and (ii) the conditional models tend to work better when risk sensitivities are estimated using the full sample 1926-2011. In particular, the conditional CAPM estimates a positive risk premium of 0.46% per month, equal to 5.52% annually. This result appears to provide some evidence that both time-varying betas and time-varying risk premia are required to obtain significant results in asset pricing. Moreover, the individual fixed-effects method

appears to be preferable to a random-effects method given the overall failure of the Hausman test, but also given that the 25 double sorted portfolios span the whole market and therefore a fixed-effects approach should be more appropriate, whereas in the presence of randomly selected portfolios or assets the random-effects approach is more justified theoretically.

In the second part of the chapter, a two-pass methodology with a rolling short-window is applied to individual assets. The results of the empirical tests on the conditional CAPM and conditional Four-Moment CAPM, together with the tests on some alternative models, show that when individual assets are used the risk premium is positive and significant as expected from the theory underpinning the CAPM, and that also coskewness is significant and has the expected negative sign. In particular, the best results are obtained when the Four-Moment CAPM is augmented to include the *Small Minus Big* factor of FF, whereby all of the factors are significant and have the expected sign, with the sensitivity to the factor SMB exerting an important positive effect on the cross-section of stock returns. This suggests that the Four-Moment CAPM can indeed improve the performance of the standard CAPM, but it appears that there is room for improvement. In particular, the SMB sensitivity significantly adds to the explanation of the cross section of expected returns.

The use of individual assets in empirical asset pricing tests allows for a larger spread in systematic measures of risk such as beta. Researchers are therefore able to obtain more precise estimates of the risk premia than in the case of portfolios of stocks. Furthermore, the use of a moving average to proxy for the expected returns seems to improve the performance of the asset pricing models considered in this chapter. Although not perfect, this proxy appears to be better than realized returns. Indeed, the main reason why the conditional models seem to fail is the use of realized returns as proxies for

expected returns. The use of conditional models highlights even more the need for a better proxy of expected return in asset pricing to find meaningful results. Nevertheless, it is worth noting that even in the presence of good estimates of expected returns, the book-to-market anomaly remains unresolved. Indeed, either the expected returns for high book-to-market portfolios are larger than those of low book-to-market portfolios, which means that investors expect actually lower returns for small growth stocks despite their higher beta – a statement that is quite difficult to support or, more likely, the realized returns fall short of the real expectations due more probably to investor overconfidence. However, this would entail venturing inside the boundaries of behavioural finance, which is beyond the scope of this thesis. Still, that would indeed be an interesting venue for future research.

Chapter 9

Conclusions and Suggestions for Future Research

9.0. Introduction

After more than forty years of theoretical and empirical research into the CAPM, contemporary research is still struggling with several anomalies. Many systematic empirical observations cannot be explained by the CAPM. More importantly, the predictive ability of the model has proven to be extremely weak. There are portfolios of assets for which the relationship between beta and return is weak or even negative, and there are security characteristics such as dividend yield, price-earnings, the book-to-market ratio, and market capitalization that can significantly better explain the differences in returns across such portfolios than beta.

This thesis begins with the traditional CAPM model and attempts to address some of the questions raised by its empirical failures. In particular, the objective of this thesis is to consider possible extensions of the CAPM and to investigate whether such extensions can offer a rational explanation for the size and book-to-market anomalies for the cross-section of US equity returns.

9.1. Research Questions and Objectives

This thesis focuses on four main additions to the traditional model: (i) the use of time-varying factor loadings obtained from multivariate GARCH and dynamic conditional correlations; (ii) the introduction of higher moments of returns in addition to mean and variance, that is, using a Four-Moment CAPM which incorporates coskewness and cokurtosis; (iii) the use of time-varying risk premia, which are assumed to change according to the regime of the market and with regimes defined by a Markov-Switching

process; and (iv) the use of individual assets to test the conditional CAPM and conditional Four-Moment CAPM, as opposed to portfolios of stocks sorted on a particular characteristic.

In particular, the main objectives of this thesis are:

Objective 1: To evaluate the performance of the CAPM, both unconditionally and conditionally

This task was accomplished in Chapter 7 with: (i) the test that the intercepts in the time series of portfolio returns are not significantly different from zero, by means of the Gibbons, Ross, and Shanken (1989) test; and (ii) the Fama and MacBeth (1973) two-pass methodology to estimate the premium associated with the risk factors, and whether the models can explain the cross-section of average returns. The conditional version with time-varying factor loadings was obtained using a Multivariate GARCH with dynamic conditional correlations as opposed to the simple rolling regression method.

Specifically, in Chapter 7, the time-series Gibbons, Ross, and Shanken test of the CAPM and the Fama and MacBeth cross-sectional test of the CAPM were conducted for portfolios of stocks sorted on market capitalization and/or the book-to-market ratio, and the results showed compelling evidence that, especially for the subsample 1980-2011, the model is not supported by the empirical findings. The premium associated with beta is not positive and significant as expected from theory, and the alphas are not jointly equal to zero – this is particularly true for portfolios sorted on the book-to-market ratio for which the positive relationship between beta-risk and returns does not seem to exist.

The first amendment to the traditional CAPM model was carried out by introducing time-varying betas obtained through Multivariate GARCH and dynamic conditional correlations. In other words, the first attempt to improve the model rested on the

introduction of conditional betas in which both covariances of the individual returns with the market portfolio returns and variance of the market portfolio returns are time-varying. The use of the GARCH is supported by the literature as this methodology allows the researcher to capture not only heteroskedasticity or time-varying volatility but also clusters in volatility that are empirically observed (period of large volatility and other periods of relative tranquillity).

The results of the tests on the portfolios of stocks showed that this methodology improves the performance of the CAPM, especially for stocks sorted on market capitalization, but it does not explain the cross-section of US stocks returns or explain the book-to-market anomaly. While the CAPM is derived assuming that investors use a mean-variance criterion for their investment decision and that higher moments of the distribution of the market portfolio returns are irrelevant, empirical evidence suggests that investors are averse to large losses (extreme outcomes) and that returns are not normally distributed. Therefore, skewness and kurtosis should not be neglected. Skewness is a measure of the asymmetry of returns whereas kurtosis is a measure of the extreme movements/outcomes and of the peakedness of the distribution of returns. A distribution with large kurtosis means that extreme outcomes are more likely than predicted by the normal distribution. Investors fear large losses and therefore they dislike kurtosis, whereas they have a preference for positive skewness, as it means that large positive returns are more likely than large negative returns. These considerations lead to the second objective of this thesis:

Objective 2: To test an extended version of the CAPM, which includes systematic skewness and kurtosis

This task requires the derivation of a model which includes coskewness and cokurtosis, and the two-pass methodology of Fama and MacBeth, to test the significance of the

higher moments for US equity portfolio returns. The major contribution of this thesis here has been to assume time-varying sensitivity in the higher moments of the distribution of returns obtained with a Multivariate GARCH with dynamic conditional correlation.

The derivation of the Four-Moment CAPM used in this thesis presents some significant innovations: (i) the higher moment CAPM is derived such that the sum of the risk premium for all factors (beta, systematic coskewness and systematic cokurtosis) equals the market excess return; (ii) non-standardized coskewness is employed, as the market portfolio skewness might approach zero; (iii) the conditional coskewness and cokurtosis are estimated as counterparts of the conditional covariance using DCC GARCH; and (iv) non-standardized coskewness is used so that the estimated or expected coefficient associated with coskewness should be negative and independent of the sign of market skewness.

The introduction of the Four-Moment CAPM with time-varying betas does not rescue the CAPM. In most of the empirical findings, coskewness and cokurtosis do not have explanatory power additional to beta. It must be underlined that in this thesis the findings do not confirm the results of Kraus and Litzenberger who find that coskewness improves the explanation of the cross section of returns considerably, nor the findings of Fang and Lai who find a negative premium associated with coskewness and a positive premium associated with cokurtosis, but in this thesis the portfolios under investigation were those sorted on the two characteristics that have proved more problematic for the CAPM: size and book-to-market.

Confronted with findings that do not support the extensions so far introduced, the next step was to introduce time-varying risk premia. This was undertaken in Chapter 8 to address objective number 3:

Objective 3: To introduce time variation in systematic risks (covariance, coskewness, cokurtosis)

The assumption of a constant required rate of return may be too strong. Risk premia should depend on uncertainty (measured usually as volatility) and on risk aversion. Therefore, it is reasonable to assume that risk aversion is time-varying. Specifically, investors are expected to be more optimistic when financial markets and economic news are positive, and hence less risk averse, and more pessimistic when financial markets and economic news are negative and hence more risk averse at such times, when incidentally volatility is also expected to be greater given the uncertainty.

The analysis of the performance of the models in different regimes, determined by a latent stochastic process, has the appealing feature that the parameters of the models are not derived from a set of conditioning information whose choice might be arbitrary.

In particular, the assumption was made in this thesis that there are two regimes, each with a probability that is returned by a Markov Switching process, and it was assumed that there are two different sets of risk premia in each regime. Whereas the factor loadings were still conditional and determined through a Multivariate GARCH, the risk premia were estimated in a panel data regression, and the average risk premia were calculated as the average of the time series of the weighted average of the two risk premia where the weights were represented by the probability of being in each regime. The main objective of the research for this element was to investigate whether the addition of time-varying factor loadings and time-varying risk premia could explain the cross-section of US average returns.

In this thesis, time-varying risk aversion was not modelled directly. Instead time-varying risk premia were introduced under the assumption that investors require a different risk premium for each risk factor according to a particular regime or their

belief of being in a particular regime. The regimes were simply classified as bull and bear regimes determined by a Markov Switching process in which the regime is the unobservable variable, but whose probability is inferred from a market model. This has the great benefit of avoiding exogenous identification of the regime (for instance, using the sign of the monthly market return).

However, the problem in the estimation of the risk premia is that in a monthly cross-section there are two risk premia to estimate for each factor and only one beta for each stock. The problem has been overcome with the use of panel data techniques.

In particular, the findings reported in Chapter 8 showed that the unconditional models are always rejected in favour of a conditional time-varying risk. In particular, the conditional CAPM estimates a positive risk premium of 0.46% per month, equal to 5.52% annually, when the individual-fixed effect is used. However, the results do not appear robust over time, i.e. they depend on the sample used to estimate the conditional betas in the multivariate GARCH.

In the last section of Chapter 8, the common practice of forming portfolios of stocks and testing asset pricing models on them instead of on individual stocks was questioned to address objective number 4:

Objective 4: To estimate a conditional version of the CAPM and Four-Moment CAPM using individual stocks as test assets

The common practice in testing asset pricing models is to build portfolios of stocks and then investigate the return-beta relationship in cross-sectional regressions. Ang, Liu, and Schwarz (2008) suggest that individual stocks lead to more efficient tests of whether the factors are priced. The common practice of forming portfolios of stocks has been motivated by the attempt to reduce the estimation errors in the betas, as forming

portfolios reduces the idiosyncratic risk. However, Ang et al. argue that forming portfolios causes a lower dispersion in estimated betas and a loss of information that results in higher standard errors in the estimates of the premia, and indeed that the CAPM should be valid for individual assets too.

In the case of individual assets there is more dispersion in betas and therefore more information for the cross-sectional estimation of the risk premium, hence a more precise risk premium. The use of individual assets is more in line with the assumption of a single period investment made by the CAPM, whereas testing asset pricing models on portfolios is more in line with the testing of different investment strategies. Furthermore, portfolio formation might lead to a smoothing out of the cross-sectional behaviours of the assets, for instance beta is particularly sensitive to extreme results which might be diluted in a portfolio (Kim, 1995). Therefore, in this thesis the CAPM and the Four-Moment CAPM were tested on individual US stocks to draw a comparison of the results of the tests involving such individual assets and the results obtained with tests conducted on portfolios.

Specifically, in the second part of Chapter 8, a two-pass methodology with a rolling short-window was applied to individual assets. The results of the empirical tests on the conditional CAPM and conditional Four-Moment CAPM, together with the tests of some alternative models, showed that when individual assets are used, the risk premium is positive and significant as expected from the theory underpinning the CAPM, and that coskewness is significant and has the expected negative sign. In particular, the best results were obtained when the Four-Moment CAPM was augmented to include the SMB factor of FF. In that case all of the factors are significant and have the expected sign, with SMB producing an important positive effect on the cross-section of stock

returns. Thus, the Four-Moment can indeed improve the performance of the CAPM, but it cannot explain the SMB factor.

The previous discussion summarised this thesis in terms of matching the analysis of this thesis with the objectives set out in the introductory chapter. It is also interesting to see how this thesis has responded to the research questions posed in the first chapter.

The first two research questions are dealt with in chapter 7. Specifically, after testing the CAPM on the portfolios sorted on market capitalization and book-to-market, the following questions can be addressed:

RQ1: Is a higher-moment CAPM that incorporates systematic skewness and kurtosis (Four-Moment CAPM) capable of explaining the cross-section of US average returns?

RQ2: Can a conditional CAPM or conditional Four-Moment CAPM with time-varying betas explain the cross-section of US asset returns?

The first question may be addressed in the negative. The unconditional Four-Moment CAPM is rejected as coskewness and cokurtosis are not found to have additional explanatory power for the cross-section of returns of portfolios of stocks sorted on market capitalization and book-to-market.

As to the second question, the conditional CAPM finds marginal support. The introduction of a time-varying beta with DCC improves the performance of the CAPM, especially for portfolios of stocks sorted on market capitalization. But the model is still rejected for the portfolios of stocks sorted on the book-to-market ratio. The conditional Four-Moment CAPM is also rejected since the DCC time-varying factor loadings are not enough to make coskewness and cokurtosis significant.

In the first part of Chapter 8, an attempt to answer the third research question is made.

RQ3: How do the CAPM and the Four-Moment CAPM perform under different regimes for the US equities market?

The introduction of the conditional CAPM and the conditional Four-Moment CAPM with time-varying risk premia (changing according to the regimes) leads to interesting results as the hypothesis of time-varying risk premia is never rejected and the conditional CAPM, when tested with individual-fixed effect panel data, produces a positive beta premium for the 25 portfolios of stocks double-sorted on market capitalization and the book-to-market ratio. This suggests that beta is positively rewarded by the market, but that other risk factors are probably left out by the model.

This is a very interesting result and represents something novel in the literature as beta appears to be clearly priced when both time-varying beta and time-varying risk premia are accounted for using this innovative methodology. It must be emphasized that the use of panel data as opposed to simple cross-sectional regressions and splitting the sample in two subsamples defined as up and down markets allows the establishment of a weighted average risk premium that can be considered as “unconditional”.

The final empirical part deals with individual assets, and a short-window regression methodology and provides an answer to the final research question.

RQ4: What is the performance of the CAPM and of the Four-Moment CAPM when the models are tested on individual assets (stocks) instead of portfolios of stocks?

Interestingly, the results support the CAPM for individual stocks over the last 30 years. Even more interestingly, the Four-Moment CAPM seems to work especially when the SMB factor is added to the model. All of the factors have the expected sign: beta demands a positive premium, coskewness a negative premium and cokurtosis a positive premium. Interestingly, SMB retains significance and has a positive risk premium. Small stocks tend to earn higher returns even after accounting for the comoments.

Therefore, it seems that small stocks retain some further risk, such as a liquidity risk. However, the factor HML has no relevance when individual stocks are used in the tests.

These results are extremely interesting and appear to confirm the results of Ang et al. (2008) and Avramov and Chordia (2005) that a conditional version of asset pricing models on individual assets sheds light upon positive results that appear to confirm a rational explanation of the cross-section of returns. Probably the most interesting result is that, together with the higher moments, SMB appears to be priced, and is sensibly related to a liquidity premium, whereas HML does not appear to be priced.

9.2. Limitations of the research

As with any study, physical, financial and time constraints cause the present study to be subject to several limitations. Although the work undertaken in this thesis has introduced many extensions, some of which appear to be promising and may well improve the predictive power of the (extended) CAPM, some improvements may well be warranted. There are six limitations which can be identified here.

First, the thesis only uses data from the US stock market. Although this is the most developed financial market, the results may not be generalizable to other markets, especially in the developing and emerging countries. It would be interesting to see if the conditional Four-Moment CAPM would also hold in other markets, and whether the various prices of risk are significant, and if so, whether they are of similar magnitude.

Second, the Four-Moment CAPM requires the estimation of the higher comoments. This is the usual error in variable problem that affects the standard CAPM beta. While Shanken's correction can be applied in the standard case, there are no known corrections for the error in variable relating to coskewness and cokurtosis. The two step procedure, therefore, may produce biased standard errors.

Third, the time-varying risk premia obtained in this study are based on the assumption that only two regimes exist. The assumption of a bull and a bear regime represents a simplistic approximation of reality. In particular, when using data that span a long period of time, it might not be appropriate to describe the stochastic process generating the observed data by two states. More regimes may be appropriate. For example, bull and bear states can be augmented by a 'stable' state.

Fourth, a further limitation stems from the fact that estimated regimes are obtained from a simple model for the market return. This model includes an intercept but no other exogenous variables. This may not reflect the stochastic process generating market excess returns. For example, market returns might be influenced by certain macroeconomic variables such as credit spread, interest rates, and gross domestic product.

Fifth, although the liquidity CAPM has been discussed in the literature review, it has not been tested in this thesis, though it is indeed planned as future research. There is evidence that the liquidity premium might explain part of the size anomaly and the book-to-market anomaly. In particular, future research will employ the conditional four-moment CAPM including a liquidity premium obtained as in Amihud (2002) or as trading turnover. The trading turnover sensitivity might also reveal investors' sentiment and therefore might be used as a behavioural indicator. As a matter of fact, stocks more sensitive to turnover should yield a lower return in a downmarket as the required return should be higher given the liquidity risk and should demand a lower risk premium in an upmarket. The asymmetric liquidity risk premium might indeed explain part of the asset pricing puzzle. It is also interesting to apply the liquidity CAPM to the individual assets. Therefore, a future avenue of research will be the conditional liquidity four-moment CAPM.

A decision was made to avoid testing empirically the liquidity CAPM in this thesis as this requires assuming risk factors additional to market portfolio risk. The rationale for this decision is that coskewness and cokurtosis premia are justified by the extension of the CAPM in the case of a non normal distribution, and preference for positive skewness without exploring additional risk factors.

Finally, a major limitation that is common with most existing studies is the use of realized rather than expected returns. The CAPM and any extension of it is stated in terms of expected returns. Although the average realized return has been used in this study as a proxy to expected return, it is well known that average returns at times bear no relation to expected returns. Nevertheless, this is a major limitation in most of the existing empirical work in asset pricing in the existing literature.

9.3. Implications and suggestions for future research

The use of time-varying betas and time varying risk premia is necessary to obtain a positive beta risk premium. Indeed the use of DCC GARCH together with a Markov Switching process and panel data is capable of rescuing the CAPM and finding some support for the four-moment CAPM.

However, the estimation of the higher moments is exposed to the error-in-variables problem and therefore it might be preferable to use some high frequency (daily or weekly) data.

On the other hand, the use of a simple methodology such as short window rolling regressions appears to confirm that both the CAPM and the four-moment CAPM work well for individual assets, showing evidence that there is room for a rational explanation of the cross-section of returns.

Given that the four-moment CAPM is particularly useful in the case of a lack of normally distributed returns, the models of this thesis might be implemented by hedge funds for the purposes of performance evaluation. Moreover, investors should consider the reward for cokurtosis and liquidity risk when estimating the returns on their assets.

A problem remains for portfolios double-sorted on size and book-to-market, but it might be that introducing a third regime will some light be shed on this issue.

Future research will include some mimicking portfolios such as book-to-market, size, momentum and reversal anomalies, and the implementation of the four-moment CAPM as undertaken in this thesis.

One of the main conclusions of this thesis is that a range of extensions do not appear to solve the well-established problems of the traditional unconditional CAPM when portfolios of stocks double-sorted on market capitalization and the book-to-market ratio are employed as test assets. The conditional versions of the CAPM and the Four-Moment CAPM do not improve significantly on their unconditional counterparts.

This section identifies possible further directions for research in this field. First of all, the most immediate extension tackles the limitations of focusing only on data for the US market, and involves applying the same models and methodologies developed in this thesis on stocks belonging to other markets, such as the UK and European exchanges, in order to produce more generalized and robust results.

Secondly, as one of the problems of the CAPM appears to be related to the formation of portfolios on an ad hoc basis, such as portfolios of stocks sorted on book-to-market and market capitalization, a more realistic and practical approach would involve testing the

extensions derived in this thesis on investment fund returns, consistent with the work of Tan (1991) who examines funds with different investment styles.

Thirdly, the results of this thesis show that more sophisticated techniques for the estimation of beta do not lead to a solution for the empirical problems of the CAPM. Therefore, a possible way forward requires modelling risk premia in order to better estimate expected returns. The empirical tests of the existing literature are always conducted on realized returns, whereas the theory of asset pricing is based on expected returns. Therefore, an interesting direction for future research entails the estimate of expected returns, using analysts' forecasts and reports, and then testing the CAPM using expected returns.

A final potential direction for future research enters the realms of behavioural finance, and might focus on the psychological traits that prevent investors from rationally measuring the risk-expected-return relationship. Evidently, investors appear to underestimate risk in bullish markets and overestimate risk in bearish markets. The conditional asset pricing models developed in this thesis might incorporate a behavioural component. For instance, whereas beta, coskewness and cokurtosis premia might explain the rational part of the required return, a proxy for investors's sentiment might explain why risk (and therefore required return) is underestimated or overestimated. This proxy might be represented by the sensitivity of a stock's trading volume to the aggregate trading volume, which should command a negative premium in a bull market and a positive premium in a bear market. This might help to explain how expected returns are obtained by investors. In particular, the turnover or volume of trading might reveal the expectations of investors and help to arrive at fairer expectations. The development of models using consumer confidence or investor confidence might offer indirect progress in this direction. Arguably, stock turnover

should reveal the expectations of investors. Large purchasing volume should be associated with an optimistic market, lower required return or lower risk aversion, whereas large volumes on the selling side or very thin markets should be typical of a market dominated by pessimism, uncertainty with high risk aversion, and very low expectations for the stock price growth.

Another proxy to consider is the sensitivity of the stock's returns to the distance between the stock's fundamentals (the price-to-earnings ratio of the stock) and the industry's fundamentals (the price-to-earnings ratio of the industry). *Ceteris paribus*, investors using fundamental analysis might deem stocks whose fundamentals are far below the fundamentals of the industry less risky, regardless of the stock's beta.

References

- Acharya, V., and Pedersen, L. (2005) Asset pricing with liquidity risk. *Journal of Financial Economics*. 77, pp. 375-410.
- Adesi, G. (1985) Arbitrage equilibrium with skewed asset returns. *Journal of Financial and Quantitative Analysis*. 20, pp. 299-313.
- Adrian, T., and Franzoni, F. (2002) Learning about beta: an explanation for the value premium. *Working Paper*. Massachusetts Institute of Technology.
- Adrian, T., and Franzoni, F. (2005) Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM. *Working Paper*, Federal Reserve Bank.
- Adrian, T., and Rosenberg, J. (2008) Stock returns and volatility: pricing the short-run and long-run components of market risk. *Journal of Finance*. 63, pp. 2997-3030.
- Agarwal, V., Bakshi, G., and Huij, J. (2008) Higher-moment equity risk and the cross-section of hedge fund returns. *Working Paper*. Robert H. Smith School of Business, University of Maryland.
- Akbas, F., Boehmer, E., Genc, E., and Petkova, R. (2010) The time-varying liquidity risk of value and growth stocks. *Edhec-Risk Institute*.
- Amihud, Y. (2002) Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*. 5, pp. 31-56.
- Amihud, Y., and Mendelson, H. (1986) Asset pricing and the bid-ask spread. *Journal of Financial Economics*. 17, pp. 223-249.
- Amihud, Y., and Mendelson, H. (1986) Liquidity and stock returns. *Financial Analysts Journal*. 42, pp. 43-48.
- Andersen, T., Bollerslev, T., Diebold, F., and Labys, P. (2003) Modeling and forecasting realized volatility. *Econometrica*. 71, pp. 529-626.
- Ang, A., and Bekaert, G. (2004) How regimes affect asset allocation? *Financial Analysts Journal*. 60, pp. 86-99.
- Ang, A., and Chen, J. (2007) CAPM over the long run: 1926-2001. *Journal of Empirical Finance*. 14, pp. 1-40.
- Ang, A., Hodrick, R., Xing, Y., and Zhang, X. (2006) The cross-section of volatility and expected returns. *Journal of Finance*. 61, pp. 259-299.
- Ang, A., Liu, J., and Schwarz, K. (2008) Using stocks or portfolios in tests of factor

models. *Working Paper*. Columbia University.

Anthonisz, S., and Putnins, T. (2013) Asymmetric liquidity risks and asset pricing. *Working Paper*. University of Technology Sydney.

Arditti, F. (1967) Risk and the required return on equity. *Journal of Finance*. 22, pp. 19-36.

Arditti, F. and Levy, H. (1972) Distribution moments and equilibrium: A comment. *Journal of Financial and Quantitative Analysis*. 7, pp. 1429-1433.

Arditti, F. and Levy, H. (1975) Portfolio efficiency analysis in three-moments: the multiperiod case. *Journal of Finance*. 30, pp. 797-809.

Aretz, K., Bertram, S., and Pope, P. (2007) Macroeconomic risks and characteristic-based factor models. *Working Paper*. Manchester Business School.

Athayde, G. and Flores, R. (1997) A CAPM with higher moments: Theory and econometrics. *Ensaio Economicos*. 317, Rio de Janeiro.

Athayde, G. and Flores, R. (1999) Introducing higher moments in the CAPM: Some basic ideas. *Working Paper*. Fundacao Vargas.

Athayde, G. and Flores, R. (2004) Do higher moments really matter in portfolio choice? *Working Paper*. Fundacao Vargas.

Avramov, C., Chordia, T., and Goyal, A. (2005) Liquidity and autocorrelations in individual stock returns. *Journal of Finance*. 61, pp. 2365-2394.

Baker, M., and Wurgler, J. (2006) Investor sentiment and the cross-section of stock returns. *Journal of Finance*. 4, pp. 1645-1680.

Ball, R. (1978) Anomalies in relationships between securities' yields and yield-surrogates. *Journal of Financial Economics*. 6, pp. 103-126.

Banz, R. (1981) The relationship between return and market value of common stocks. *Journal of Financial Economics*. 9, pp. 3-18.

Barberis, N., and Shleifer, A. (2003) Style investing *Journal of Financial Economics*. 68, pp. 161-199.

Basu, S. (1983) The relationship between earnings yield, market value, and return for NYSE common stocks: Further evidence. *Journal of Financial Economics*. 12, pp. 129-156.

Basu, S. (1977) Investment performance of common stocks in relation to their price-earnings ratios: A test of the Efficient Market Hypothesis. *Journal of Finance*. 32, pp.

663-682.

Bauer, R., Cosemans, M., and Schotman, P. (2010) Conditional asset pricing and stock market anomalies in Europe. *European Financial Management*. 16, pp. 165-190.

Bawa, V., and Lindenberg, E. (1977) Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics*. 5, pp. 189-200.

Berkman, H., Jacobsen, B., and Lee, J. (2011) Rare disaster risk and the equity premium puzzle. *Working Paper*. University of Auckland.

Bhandari, L. (1988) Debt/equity ratio and expected common stock returns: Empirical evidence. *Journal of Finance*. 43, pp. 507-528.

Black, F. (1972) Capital market equilibrium with restricted borrowing. *Journal of Business*. 45, pp. 444-454.

Black, F. (1993) Beta and return. *Journal of Portfolio Management*. 20, pp. 8-18.

Black, F., Jensen, M., and Scholes, M. (1972) The Capital Asset Pricing Model: Some empirical tests. *Studies in the Theory of Capital Markets*. Ed. New York: Praeger Publishers.

Blume, M. (1970) Portfolio theory: A step toward its practical application. *Journal of Business*. 43, pp. 152-173.

Bodurtha, J., and Nelson, M. (1991) Testing the CAPM with time-varying risks and returns. *Journal of Finance*. 46, pp. 1485-1505.

Bollerslev, T., Engle, R., and Wooldridge, J. (1988) A Capital Asset Pricing Model with time varying covariances. *Journal of Political Economy*. 96, pp. 116-131.

Bossaerts, P., and Green, R. (1989) A general equilibrium model of changing risk premia: Theory and tests. *Review of Financial Studies*. 2, pp. 467-494.

Brav, A., Lehavy, R., and Michaely, R. (2005) Using expectations to test asset pricing models. *Working Paper*. Duke University.

Brennan, M., Chordia, T., and Subrahmanyam, A. (1998), Alternative factor specifications, security characteristics and the cross-section of expected stock returns, *Journal of Financial Economics*. 49, pp. 345-373.

Brooks, C. (2008) *Introductory Econometrics for Finance*. Cambridge University Press.

Campbell, J. (1991) A variance decomposition for stock returns. *Economic Journal*. 101, pp. 157-179.

- Campbell, J., and Shiller, R. (1988) The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies*. 1, pp. 195-227.
- Campbell, J., and Vuolteenaho, T. (2004) Bad beta, good beta. *American Economic Review*. 94, pp. 1249-1275.
- Campbell, J., Polk, C., and Vuolteenaho, T. (2008) Growth or glamour? Fundamentals and systematic risk in stock returns. *Working Paper*. Harvard University and NBER.
- Carhart, M. (1997) On persistence in mutual fund performance. *Journal of Finance*. 52(1), pp. 57-82.
- Chan, K., and Nai-fu Chen (1988) An unconditional asset-pricing test and the role of firm size as an instrumental variable for risk. *Journal of Finance*. 43, pp. 309-325.
- Chan, K., Nai-fu Chen, and Hsieh, D. (1985) An exploratory investigation of the firm size effect. *Journal of Financial Economics*. 14, pp. 451-471.
- Chan, L., and Lakonishok, J. (1993) Are the reports of beta's death premature? *Journal of Portfolio Management*. 19, pp. 51-62.
- Chan, L., Jegadeesh, N., and Lakonishok, J. (1995) Evaluating the performance of value versus glamour stocks: The impact of selection bias. *Journal of Financial Economics*. 38, pp. 269-296.
- Chen, L., Novy-Marx, R., and Zhang, L. (2011) An alternative three-factor model. *Working Paper*. Washington University.
- Chen, L., Petkova, R., Zhang, L. (2008) The expected value premium. *Journal of Financial Economics*. 87, pp. 269-280.
- Chen, Nai-fu, Roll, R., and Ross, S. (1986) Economic forces and the stock market. *Journal of Business*. 59, pp. 383-403.
- Chordia, T., Subrahmanyam, A., and Anshuman, V. (2001) Trading activity and expected stock returns. *Journal of Financial Economics*. 59, pp. 3-32.
- Christie-David, R. and Chaudhry, M. (2001) Coskewness and cokurtosis in futures markets. *Journal of Empirical Finance*. 8, pp. 55-81.
- Chunachinda, P., Danpani, K., Hamid, S. and Prakash, A. (1997) Portfolio selection and skewness: Evidence from international stocks markets. *Journal of Banking and Finance*. 21, pp. 143-167.
- Chung, P., Johnson H., and Schill, M. (2006) Asset pricing when returns are nonnormal: Fama-French factors versus higher-order systematic co-moments. *Journal of Business*. 79, pp. 923-940.

- Cochrane, J. (2001) *Asset Pricing*. Princeton University Press.
- Cochrane, J. (2011) Presidential Address: Discount rates. *Journal of Finance*. 4, pp. 1047-1108.
- Cochrane, J. Asset pricing program review: Liquidity, trading and asset prices
- Conrad, J., Dittmar, R. and Ghysels, E. (2008) Ex ante skewness and expected stock returns. *Journal of Finance*. 68, pp. 85-124.
- Daniel, K., Hirshleifer, D., and Subrahmanyam, A. (2001) Overconfidence, arbitrage, and equilibrium asset pricing. *Journal of Finance*. 3, pp. 921-959.
- Datar, V., Naik, N., and Radcliffe, R. (1998) Liquidity and stock returns: An alternative test. *Journal of Financial Markets*. 1, pp. 203-219.
- Davis, J. (1994) The cross-section of realized stock returns: The pre-Compustat evidence. *Journal of Finance*. 49, pp. 1579-1593.
- DeBondt, W. and Thaler, R. (1985) Does the stock market overreact? *Journal of Finance*. 40, pp. 793-805.
- DeBondt, W. and Thaler, R. (1987) Further evidence on investor overreaction and stock market seasonality. *Journal of Finance*. 42, pp. 557-581.
- Dempsey, M. (2013) The Capital Asset Pricing Model (CAPM): The history of a failed revolutionary idea in Finance? *Abacus*. 49, pp. 7-23.
- Diacogiannis, G. (1994) Three-parameter asset pricing. *Managerial and Decision Economics*. 15, pp. 149-158.
- Diacogiannis, G. (1999) A three-dimensional risk-return relationship based upon the inefficiency of a portfolio: Derivation and implications. *European Journal of Finance*. 5, pp. 225-235.
- Dittmar, R. (2002) Nonlinear pricing kernels, kurtosis preference and cross-section of equity returns. *Journal of Finance*. 57, pp. 369-403.
- Doan, M., Lin, C., and Chng, M. Higher moments and asymmetric behaviour of market risk: Evidence from Australia. *Working Paper*. University of Melbourne.
- Doan, P., Lin C., and Zurbruegg R. (2010) Pricing assets with higher moments: Evidence from the Australian and US stock markets. *Int. Fin. Markets, Inst. and Money*. 20, pp.51-67.
- Douglas, G. (1969) Risk in the equity markets: An empirical appraisal of market

efficiency. *Yale Economic Essays*. 9, pp. 3-45.

Elton, E. (1999) Expected return, realized return and asset pricing tests. Presidential Address. *Journal of Finance*. 54, pp. 1199-1220.

Engle, R. (2002) Dynamic conditional correlation - A simple class of multivariate GARCH models. *Journal of Business and Economic Statistics*. 20, pp. 339-350.

Engle, R., and Bali, T. (2008) Resurrecting the conditional CAPM with dynamic conditional correlations. *Working Paper*. New York University Stern School of Business.

Estrada, J. (2002) Systematic risk in emerging markets: The D-CAPM. *Emerging Markets Review*. 3, pp. 365-379.

Fama, E. (1965) The behaviour of stock market prices. *Journal of Business*. 38, pp. 34-105.

Fama, E. and French, K. (1992) The cross-section of expected stock returns. *Journal of Finance*. 47, pp. 427-465.

Fama, E. and French, K. (1993) Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*. 33, pp. 3-56.

Fama, E. and French, K. (1995) Size and book-to-market factors in earnings and returns. *Journal of Finance*. 50, pp. 131-155.

Fama, E. and French, K. (1996) Multifactor explanations of asset pricing anomalies. *Journal of Finance*. 51, pp. 55-84.

Fama, E., and French, K. (1988) Dividend yields and expected stock returns. *Journal of Financial Economics*. 22, pp. 3-25.

Fama, E., and French, K. (1996) The CAPM is wanted, dead or alive. *Journal of Finance*. 51, pp. 1947-1958.

Fama, E., and French, K. (1998) Value versus growth: The international evidence. *Journal of Finance*. 53, pp. 1975-1999.

Fama, E., and French, K. (2004) The Capital Asset Pricing Model: Theory and evidence. *Working Paper*. Tuck Business School.

Fama, E., and MacBeth, J. (1973) Risk, return and equilibrium: Some empirical tests. *Journal of Political Economy*. 81, pp. 607-636.

Fang, H. and Lai, T. (1997) Co-kurtosis and capital asset pricing. *The Financial Review*.

32, pp. 293-307.

Ferson, W., and Harvey, C. (1991) The variation of economic risk premiums. *Journal of Political Economy*. 99, pp. 385-415.

Ferson, W., and Harvey, C. (1998) Fundamental determinants of national equity market returns: A perspective on conditional asset pricing. *Journal of Banking and Finance*. 21, pp. 1625-1665.

Ferson, W., and Harvey, C. (1999) Conditioning variables and the cross section of stock returns. *Journal of Finance*. 54, pp. 1325-1360.

Ferson, W., Kandel, S., and Stambaugh, R. (1987) Tests of asset pricing with time-varying expected risk premiums and market betas. *Journal of Finance*. 42, pp. 201-220.

Fletcher, J. and Kihanda, J. (2005) An examination of alternative CAPM-based models in UK stock returns. *Journal of Banking and Finance*. 29, pp. 2995-3014.

Franzoni, F. (2002) Where is beta going? *Working Paper*. University of Lugano.

Freeman, M., and Guermat, C. (2006) The conditional relationship between beta and returns: a reassessment. *Journal of Business Finance and Accounting*. 33, pp. 1213-1239.

French, K., Schwert, W., and Stambaugh, R. (1987) Expected stock returns and volatility. *Journal of Financial Economics*. 19, pp. 3-29.

Friend, I. and Westerfield, R. (1980) Co-skewness and capital asset pricing. *Journal of Finance*. 35, pp. 897-914.

Friend, I., and Blume, M. (1970) Measurement of portfolio performance under uncertainty. *American Economic Review*. 60, pp. 561-575.

Galagedera, D., and Brooks, R. (2007) Is co-skewness a better measure of risk in the downside than downside beta? Evidence in emerging market data. *Journal of Multinational Financial Management*. 17, pp. 214-230.

Galagedera, D., and Maharaj, E. (2004) Wavelet timescales and conditional relationship between higher-order systematic co-moments and portfolio returns: evidence in Australian data. *Working Paper*. Monash University.

Gibbons, M. (1982) Multivariate tests of financial models: A new approach. *Journal of Financial Economics*. 10, pp. 3-27.

Gibbons, M., Ross, S. and Shanken, J. (1989) A test of the efficiency of a given portfolio. *Econometrica*. 57, pp. 1121-1152.

Glosten, L., Jagannathan, R., and Runkle, D. (1993) On the relation between the

- expected value and the volatility of the nominal excess returns on stocks. *Journal of Finance*. 48, pp. 1779–1801.
- Guermat, C., Bulkley, G., Freeman, M., and Harris R. (2004) Testing the CAPM: A simple alternative to Fama and MacBeth (1973). *Working Paper*. University of Exeter.
- Guidolin, M. (2009) Regime shifts in empirical pricing kernels: A mixture CAPM. *Working Paper*. Manchester Business School.
- Guidolin, M., and Timmermann, A. (2002) Economic implications of bull and bear regimes in UK stock returns. *The Economic Journal*. 115, pp. 111-143.
- Guidolin, M., and Timmermann, A. (2006) Asset allocation under multivariate regime switching. *Working Paper*. Federal Reserve Bank of St. Louis.
- Guo, H., and Jiang, X. (2010) Aggregate distress risk is priced with a positive premium. *Working Paper*. University of Cincinnati.
- Hahn, J., and Lee, H. (2005) Yield spreads as alternative risk factors for size and book-to-market. *Journal of Financial and Quantitative Analysis*. 41, pp. 245-269.
- Hamilton, J. (1994) *Time Series Analysis*. Princeton
- Harlow, V., and Rao, R. (1989) Asset pricing in a generalized mean-lower partial moment framework. *Journal of Financial and Quantitative Analysis*. 24, pp. 285-311.
- Harvey, C. (1989) Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics*. 24, pp. 289-317.
- Harvey, C. and Siddique, A. (2000) Conditional skewness in asset pricing tests. *Journal of Finance*. 55, pp. 1263-1295.
- Harvey, C. and Siddique, A. (2000) Time-varying conditional skewness and the market risk premium. *Research in Banking and Finance*. 1, pp. 25-58.
- Hasan, Z., and Kamil, A. (2014) Contribution of co-skewness and co-kurtosis of the higher moment CAPM for finding the technical efficiency. *Economics Research International*.
- Haugen, R. (1995) *The New Finance: The Case against Efficient Markets*. Englewood Cliffs, New Jersey: Prentice Hall
- Heaney, R., Lan, Y., and Treepongkaruna, S. (2012) A test of co-skewness and co-kurtosis: the relevance of size, book to market and momentum in asset pricing. *Working Paper*. University of Western Australia.
- Heaton, J., and Lucas, D. (2000) Portfolio choice and asset prices: The importance of

entrepreneurial risk. *Journal of Finance*. 55, pp. 1163-1198.

Hogan, W., and Warren, J. (1974) Toward the development of an equilibrium capital-market model based on semivariance. *The Journal of Financial and Quantitative Analysis*. 9, pp. 1-11.

Huang, C., and Litzenberger, R. (1988) *Foundations for Financial Economics*. Prentice Hall.

Hung, C. (2008) Return predictability of higher-moment CAPM market models. *Working Paper*. Durham University.

Hung, D., Shackleton, M., and Xu, X. (2003) CAPM, higher co-moment and factor models of UK stock returns. *Working Paper*. Lancaster University.

Hwang, S. and Satchell, S. (1999) Modelling emerging market risk premia using higher moments. *International Journal of Finance and Economics*. 4, pp. 271-296.

Iqbal, J., Brooks, R., and Galagedera, D. (2008) Testing conditional asset pricing model: an emerging market perspective. *Working Paper*. Monash University, Australia.

Jagannathan, R., and Wang, Z. (1996) The conditional CAPM and the cross-section of expected returns. *Journal of Finance*. 51, pp. 3-54.

Javid, A., and Ahmad, E. (2008) Test of Multi-moment Capital Asset Pricing Model: Evidence from Karachi Stock Exchange. *Working Paper*. Pakistan Institute of Development Economics.

Jean, W. (1971) The extension of portfolio analysis to three or more parameters. *Journal of Financial and Quantitative Analysis*. 6, pp. 505-515.

Jean, W. (1972) Distribution moments and equilibrium: reply. *Journal of Financial and Quantitative Analysis*. 7, pp. 1435-1437.

Jean, W. (1973) More on multidimensional portfolio analysis. *Journal of Financial and Quantitative Analysis*. 8, pp. 475-49.

Jegadeesh, N. (1992) Does market risk really explain the size effect? *Journal of Financial and Quantitative Analysis*. 27, pp. 337-351.

Jegadeesh, N., and Titman, S. (1993) Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*. 48, pp. 65-91.

Jondeau, E. and Rockinger, M. (2003) Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements. *Journal of Economic Dynamic and Control*. 27, pp. 1699-1737.

Jondeau, E., Rockinger, M., (2003). Testing for differences in the tails of stock market

returns. *Journal of Empirical Finance*. 10, pp. 559-581.

Jostova, G., and A. Philipov, A. (2005) Bayesian analysis of stochastic betas. *Journal of Financial and Quantitative Analysis*. 40, pp. 747-778.

Jurczenko, E. and Maillet, B. (2001) The three-moment CAPM: Theoretical foundations and an asset pricing models comparison in an unified framework. *Working Paper*. University of Paris.

Jurczenko, E. and Maillet, B. (2006) *Multi-moment Asset Allocation and Pricing Models*. John Wiley.

Kahneman, D., and Tversky, A. (1979) Prospect theory: An analysis of decision under risk. *Econometrica*. 47, pp. 263-291.

Kandel, S., and Stambaugh, R. (1995) Portfolio inefficiency and the cross-section of expected stock returns. *Journal of Finance*. 50, pp. 157-184.

Kapadia, N. (2006) The next Microsoft? Skewness, idiosyncratic volatility, and expected returns. *Working Paper*. Kenan-Flagler Business School.

Keim, D. (1983) Size-related anomalies and stock return seasonality: Further empirical evidence. *Journal of Financial Economics*. 12, pp. 13-32.

Keim, D., and Stambaugh, R. (1986) Predicting returns in stock and bond markets. *Journal of Financial Economics*. 17, pp. 357-390.

Kim, C., Piger, J., and Startz, R. (2005) Estimation of Markov regime-switching regression models with endogenous switching. *Working Paper*. Federal Reserve Bank of St. Louis.

Kim, D. (1995) The errors in the variables problem in the cross-section of expected stock returns. *Journal of Finance*. 50, pp. 1605-1634.

Koch, S., and Westheide, C. (2013) The conditional relation between Fama-French betas and return. *Schmalenbach Business Review*. 65, pp. 334-358.

Kostakis, A., Muhammad, K., and Siganos A. (2012) Higher co-moments and asset pricing on London Stock Exchange. *Journal of Banking and Finance*. 36, pp. 913-922.

Kothary, S., Shanken, J., and Sloan, R. (1995) Another look at the cross-section of expected stock returns. *Journal of Finance*. 50, pp. 185-224.

Kraus, A. and Litzenberger, R. (1976) Skewness preference and the valuation of risk assets. *Journal of Finance*. 31, pp. 1085-1100.

Kraus, A., and Litzenberger, R. (1983) On the distributional conditions for a

- consumption-oriented three moment CAPM. *Journal of Finance*. 38, pp. 1381-1391.
- Kumar, A. (2005) Institutional Skewness Preferences and the idiosyncratic skewness premium. *Working Paper*. University of Notre Dame.
- Lakonishok, J., Schleifer, A., and Vishny, R. (1994) Contrarian investment, extrapolation, and risk. *Journal of Finance*. 49, pp. 1541-1578.
- Lambert, M., and Hubner, G. (2013) Comoment risk and stock returns. *Journal of Empirical Finance*. 23, pp. 191-205.
- Lettau, M., and Ludvigson, S. (2001) Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy*. 109, pp. 1238-1287.
- Levy, H. (1969) A utility function depending on the first three moments. *Journal of Finance*. 24, pp. 715-719.
- Lewellen, J. (1999) The time-series relations among expected return, risk, and book-to-market. *Journal of Financial Economics*. 54, pp. 5-43.
- Lewellen, J., and Nagel, S. (2006) The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics*. 82, pp. 289-314.
- Lewellen, J., Nagel, S., and Shanken, J. (2005) A sceptical appraisal of asset-pricing tests. *Journal of Financial Economics*. 96, pp. 175-194.
- Liew, J. and Vassalou, M. (2000) Can book-to-market, size, and momentum be risk factors that predict economic growth? *Journal of Financial Economics*. 57, pp. 221-245.
- Lim, K. (1989) A new test of the three-moment capital asset pricing model. *Journal of Financial and Quantitative Analysis*. 24, pp. 205-216.
- Limkriangrai, M., Durand, R., and Watson, I. (2008) Is liquidity the missing link? *Accounting and Finance*. 48, pp. 829-845.
- Lintner, J. (1965) The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*. 47, pp. 13-37.
- Liu, W. (2006) A liquidity-augmented capital asset pricing model. *Journal of Financial Economics*. 82, pp. 631-671.
- Lo, A. and MacKinlay, A. C. (1990) Data-snooping biases in tests of financial asset pricing models. *Review of Financial Studies*. 3, pp. 431-468.
- Lo, A., and Wang, J. (2006) Trading volume: Implications of an intertemporal capital asset pricing model. *Journal of Finance*. 6, pp. 2805-2839.
- Markowitz, H. (1959) *Portfolio Selection: Efficient Diversification of Investments*.

Wiley, New York

Mayers, D. (1972) Nonmarketable assets in capital market equilibrium. *Studies in the Theory of Capital Markets*. Praeger Publishers

Merton, R. (1973) An intertemporal capital asset pricing model. *Econometrica*. 41, pp. 867-887.

Morana, C. (2009) Realized betas and the cross-section section of expected returns. *Applied Financial Economics*. 19, pp. 1371 - 1381.

Moreno, D., Rodriguez, R., (2009). The value of coskewness in mutual fund performance evaluation. *Journal of Banking and Finance*. 33, pp. 1664-1676.

Mumpower, J. and McClelland, G. (2002) Measurement error, skewness, and risk analysis: coping with the long tail of the distribution. *Risk Analysis*. 22, pp. 277-290.

Nelson, D. (1991) Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*. 59, pp. 347-370.

Officer, R. (1972) The distribution of stock returns. *Journal of the American Statistical Association*. 67, pp. 807-812.

Pastor, L., and Stambaugh, R. (2003) Liquidity risk and expected stock returns. *Journal of Political Economy*. 11, pp. 642-685.

Pedersen, C., and Hwang, S. (2004) Asymmetric risk measures when modelling emerging market equities: evidence for regional and timing effects. *Emerging Markets Review*. 5, pp. 109-128.

Pedersen, C., and Hwang, S. (2007) Does downside beta matter in asset pricing? *Applied Financial Economics*. 17, pp. 961-978.

Peiró, A. (1999) Skewness in financial returns. *Journal of Banking and Finance*. 23, pp. 847-862.

Petkova, R. and Zhang, L. (2005) Is value riskier than growth? *Journal of Financial Economics*. 78, pp. 187-202.

Pettengill, G., Sundaram, S., and Mathur, I. (1995) The conditional relation between beta and returns. *The Journal of Financial and Quantitative Analysis*. 30, pp. 101-116.

Post, T., and Van Vliet, P. (2004) Conditional downside risk and the CAPM. *Working Paper*. Erasmus Research Institute of Management.

Prakash, A., Chang, C., and Pactwa, T. (2003) Selecting a portfolio with skewness: Recent evidence from US, European, and Latin American equity markets. *Journal of*

Banking and Finance. 27, pp. 1375-1390.

Price, K., and Nantell, T. (1982) Variance and lower partial moment measures of systematic risk: some analytical and empirical results. *Journal of Finance*. 37, pp. 843-855.

Racine, M. (1998) Asset valuation and coskewness in Canada. *Working Paper*. Wilfrid Laurier University.

Ranaldo, A. and Favre, L. (2003) How to price hedge funds: from two-to-four-moment CAPM. *UBS Global Asset Management & UBS Wealth Management*, October 2003.

Ranaldo, A. and Favre, L. (2005) Hedge fund performance and higher-moment market models. *Swiss National Bank*, Winter 2005.

Reinganum, M. (1981) A new empirical perspective of the CAPM. *Journal of Financial and Quantitative Analysis*. 16, pp. 439-460.

Reinganum, M. (1981) Misspecification of capital asset pricing: Anomalies based on earnings' yields and market values. *Journal of Financial Economics*. 9, pp. 19-46.

Reinganum, M. (1990) Market microstructure and asset pricing: An empirical investigation of NYSE and NASDAQ securities. *Journal of Financial Economics*. 28, pp. 127-147.

Roll, R. (1977) A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory. *Journal of Financial Economics*. 4, pp. 129-176.

Roll, R. (1981) A possible explanation of the small firm effect. *Journal of Finance*. 36, pp. 879-888.

Roll, R. (1983) Was ist das? The turn-of-the-year effect and the return premia of small firms. *Journal of Portfolio Management*. 9, pp. 18-28.

Roll, R., and Ross, S. (1994) On the cross-sectional relation between expected returns and betas. *Journal of Finance*. 49, pp. 101-121.

Rosenberg, B., Reid, K., and Lanstein, R. (1985) Persuasive evidence of market inefficiency. *Journal of Portfolio Management*. 11, pp. 9-17.

Ross, S. (1976) The arbitrage theory of capital asset pricing. *Journal of Economic Theory*. 13, pp. 341-360.

Rubinstein, M. (1973) The fundamental theorem of parameter preference and security valuation. *Journal of Financial and Quantitative Analysis*. 8, pp. 61-69.

Samuelson, P. (1970) The fundamental approximation theorem of portfolio analysis in

terms of mean, variances, and higher moments. *Review of Economic Studies*. 37, pp. 537-542.

Schwert, G. (1983) Size and stock returns, and other empirical regularities. *Journal of Financial Economics*. 12, pp. 3-12.

Scott, R. and Horvath, P. (1980) On the direction of preference for moments of higher order than the variance. *Journal of Finance*. 35, pp. 915-919.

Sears, S. and Wei, J. (1985) Asset pricing, higher moments, and the market risk premium: A note. *Journal of Finance*. 40, pp. 1251-1253.

Shanken, J. (1990) Intertemporal asset pricing: An empirical investigation. *Journal of Econometrics*. 45, pp. 99-120.

Shanken, J., and Winstein, M. (2006) Economic forces and the stock market revisited. *Journal of Empirical Finance*. 13, pp. 129-144.

Sharpe, W. (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*. 19, pp. 425-442.

Simkowitz, M. and Beedles, W. (1978) Diversification in three-moment world. *Journal of Financial and Quantitative Analysis*. 13, pp. 927-941.

Singleton, J. and Wingender, J. (1986) Skewness persistence in common stock returns. *Journal of Financial and Quantitative Analysis*. 21, pp. 335-341.

Smith, D. (2006) Conditional coskewness and asset pricing. *Journal of Empirical Finance*. 14, pp. 91-119.

Stambaugh, R. (1982) On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis. *Journal of Financial Economics*. 10, pp. 237-268.

Stattman, D. (1980) Book values and stock returns. *The Chicago MBA: A Journal of Selected Papers*. 4, pp. 25-45.

Sun, Q. and Yan, Y. (2003) Skewness persistence with optimal portfolio selection. *Journal of Banking & Finance*. 27, pp. 1111-1121.

Tan, K. (1991) Risk return and the three-moment capital asset pricing model: Another look. *Journal of Banking and Finance*. 15, pp. 449-460.

Taylor, S. (2005) *Asset Price Dynamics, Volatility and Prediction*. Princeton

Teplova, T., and Shutova, E. (2011). A higher moment downside framework for conditional and unconditional CAPM in the Russian Stock Market. *Eurasian Economic Review*. 1, pp. 157-177.

- Timmermann, A. (1993) How learning in financial markets generates excess volatility and predictability in stock prices. *The Quarterly Journal of Economics*. 108, pp. 1135-1145.
- Tinic, S., and West, R. (1984) Risk and return: January vs. the rest of the year. *Journal of Financial Economics*. 13, pp. 561-574.
- Tinic, S., and West, R. (1986) Risk, return, and equilibrium: A revisit. *The Journal of Political Economy*. 94, pp. 126-147.
- Vassalou, M. (2003) News related to future GDP growth as a risk factor in equity returns. *Journal of Financial Economics*. 68, pp. 47-73.
- Vassalou, M., and Xing, Y. (2004) Default risk and equity returns. *Journal of Finance*. 59, pp. 831-868.
- Vassalou, M., Chen, J., and Zhou, L. (2005) The interrelation of liquidity risk, default risk, and equity returns. *Working Paper*. Columbia University.
- Wang, K. (2003) Asset pricing with conditioning information: A new test. *Journal of Finance*. 58, pp. 161-196.
- Young, B., Christoffersen, P., and Jacobs, K. (2010) Market skewness risk and the cross-section of stock returns. *Working Paper*. McGill University.
- Zhang, C. (2010) On the returns to small-growth stocks. *Working Paper*. Hong Kong University of Science and Technology.
- Zhang, L. (2005) The value premium. *Journal of Finance*. 60, pp. 67-103.