Identification and U-control of a state-space system with time-delay

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Summary:

This article presents a state-space model with time-delay to map the relationship between known input-output data for discrete systems. For the given input-output data, a model identification algorithm combining parameter estimation and state estimation is proposed in line with the causality constraints. Consequently, this article proposes a least squares parameter estimation algorithm, and analyses its convergence for the studied systems to prove that the parameter estimation errors converge to zero under the persistent excitation conditions. In control system design, the U-model based control is introduced to provide a unilateral platform to improve the design efficiency and generality. A simulation portfolio from modelling to control is provided with computational experiments to validate the derived results.

KEYWORDS:

adaptive filtering, parameter estimation, recursive identification, U-model based controller design.

1 | INTRODUCTION

Various conventional algorithms have been used first for estimating the parameters of control systems[1-4] and some identification algorithms have been developed for linear systems[5,6] and nonlinear systems[7-11] but some of them assume that input-output data are available at every sampling instant and most of control algorithms assume that the parameters of the considered systems are known.[12,13] The law of dynamic motion of a system can be described by differential/difference equations. These equations can be transformed into a set of the first-order differential/difference equations, which are called the continuous-time/discrete-time state-space models. The state space models are convenient for controller design, system modelling and identification, signal processing and filtering,[14] for example, the pole placement, the observer design,[15] and Kalman filter. Time-delay systems are very common in practical situations in nature like, for instance, transmission problems, communications, population growth models and other control systems.[16]

Many control processes (e.g., three-tank water tank, the continuous reactor and the distillation tower) can be best modelled by systems involving time-delay in the state.[17] Effects of time-delay on the stability and performance of control systems have drawn attention of many investigators in different engineering disciplines, including structural systems, chemical processes, remotely controlled undersea and aerospace robots and structures, and manufacture processes. Sanz et al. studied the observation and stabilization of linear time-varying systems with time-varying measurement delay,[18] Javier and Zheng proposed the unknown input functional observability of descriptor systems with neutral and distributed delay effects,[19] Chen et al. presented the variational Bayesian approach for ARX systems with missing observations and varying time-delays.[20] The recursive or iterative search schemes can be used for finding the solutions of linear and nonlinear matrix equations[21-24] and of deriving new system identification algorithms.[25-31] The convergence analysis of identification algorithms is important to provide assurance/effectiveness for the applications in control system operation[32] and the other related areas. Earlier convergence analysis assumed that the input and output signals of the system under consideration had finite nonzero power, and the noise was an independent and uniformly distributed random sequence with finite fourth-order moments. Such ideal assumptions are difficult to meet in practice. The convergence rate of the standard recursive least squares parameter estimation is obtained by assuming that the process noise has finite second order and higher order moments. Since then, most of the convergent results of the least squares algorithm have made so-called "weak" assumptions. Zhang et al. proposed the recursive parameter estimation methods and convergence analysis for bilinear systems.[33-35]

Since a foundation study, the U-model based control, the U-control in short, has received certain range of attention, even not greatly. Regarding the U-control approach, it is different from those model based design approaches in essence of separating system performance from plant models and also different from data driven design approaches as using the plant models to determine the controller output by solving the dynamic inverse of the plant models. The merit U-control claimed is that the control design on this platform is no long classify linear/nonlinear, polynomial/state space model structures, secondly U-control provides great simplicity/generality using linear control system design principle to nonlinear control systems, especially in specifying the system transient response and steady state performance in a systematic formulation.

For the U-control progression, polynomial including ration (total nonlinear) model based design has been predominantly studied with pole placement,[36] general predictive control, and Neural control,[37] the U-Smith predictor enhanced control with input delay. A comprehensive review of U-control has been reported. It has noted that the critical challenge in U-control is the uncertainty in dynamic inversion. This article expands the U-control to a class of linear state space models with state delay. Except a recent work using U-control for dealing nonlinear systems with input delay, the U-control has not been introduced for dealing systems with state delay.

Regarding the related research, it should be noted that the stochastic gradient (SG) identification algorithm has a small amount of calculation, but low estimation accuracy, and slow convergence speed.[38] This article focuses on the modeling and control of state-space systems with time-delays. The key idea is to integrate the Kalman filter and the recursive least squares algorithm to directly identify the parameters of the systems, which has high accuracy. Compared with the previous work in Reference 39, the proposed algorithm gives the joint state and parameter estimation to improve the convergence speed, and gives the convergence analyzes to ensure the stability of the algorithm.

The main contributions of this article are listed as follows.

- The convergence of the least squares algorithm is proved better than that of the stochastic gradient algorithm.
- In order to reduce computation and storage costs, a joint state and parameter estimation is proposed for state-space systems with time-delays and its convergence of the proposed algorithm is proved under the weak persistent excitation condition.
- U-control expansion to state space model with state delay, which considers the problem by dynamic inversion, different from the predictor based and the other popular approaches.
- Provides a simulation portfolio with model identification and U-control system design, which could be an integrated package for potential users with their ad hoc applications.

For the rest of this article, Section 2 derives the Kalman filter to estimate the states of the time-delay system. Section 3 proves the convergence of the parameter estimates obtained from the proposed recursive least squares algorithm. Section 4 gives the U-model control. Section 5 presents the case studies. Numerical validation of the analytical results is derived in Sections 2–3. Finally, Section 6 gives some concluding remarks.

2 | AUGMENTED STATE ESTIMATION ALGORITHM

Consider the following state-space system with time-delay,

$$\boldsymbol{x}(t+1) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{A}_d\boldsymbol{x}(t-1) + \boldsymbol{b}_1u(t) + \boldsymbol{w}(t), \tag{1}$$

$$y(t) = \boldsymbol{c}_1\boldsymbol{x}(t) + v(t), \tag{2}$$

$$A_1 := \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ a_n & a_{n-1} & a_{n-2} & \dots & a_1 \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad b_1 := \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \in \mathbb{R}^n,$$

$$A_d := \begin{bmatrix} a_{d1} \\ a_{d2} \\ \vdots \\ a_{dn} \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad a_{dj} \in \mathbb{R}^{1\times n}, \ j = 1, 2, \dots, n,$$

$$c_1 := [1, 0, \dots, 0] \in \mathbb{R}^{1\times n},$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the system input, $y(t) \in \mathbb{R}$ is the system output, $v(t) \in \mathbb{R}$ is a random noise with zero mean, $w(t) \in \mathbb{R}^n$ is a noise vector with zero mean, $A_1 \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $b_1 \in \mathbb{R}^n$ and $c_1 \in \mathbb{R}^{1\times n}$ are the system parameter matrices/vectors.

Let I be an identity matrix of appropriate sizes, $v(t)$ is the observation noise which is assumed to be zero mean Gaussian white noise with covariance σ^2.

Remark 1. If $A_d = 0$, then Equations (1)–(2) have no time-delay term $A_d x(t-1)$ and thus reduce to the standard state-space model. Because the delay term $A_d x(t-1)$ exists, the standard Kalman filtering method for the standard state-space model with known parameters cannot be applied to the time-delay state-space system in (1)–(2). This is the difficulty of the state estimation of the time-delay state-space system with unknown parameters. This motivates us to present new parameter and state estimation algorithm for time-delay state-space systems.

Remark 2. This article is to develop the Kalman filter and recursive algorithm for combined parameter and state estimation based on the given measurement data $\{u(t), y(t)\}$, taking causality constraints into consideration. The identifiability of a system depends on its controllability and observability. Therefore, it is very important whether the state-space models with time-delays are controllable and observable. We assume that the state-space model with time-delay is a minimal implementation. The controllability and observability can be achieved under a mild condition.

Remark 3. In real industry, not all states can be measured by sensors. In the face of this problem, one approach is to consider the corresponding input-output representation by eliminating the available state vector.[40-43] But such an approach cannot solve the identification and state estimation of the system under consideration. In this article, a recursive state estimation algorithm is presented to update the state estimation by constructing a state observer.

Define an expanded state vector and some matrix/vectors:

$$X(t) := \begin{bmatrix} x(t) \\ x(t-1) \end{bmatrix} \in \mathbb{R}^{2n},$$

$$A := \begin{bmatrix} A_1 & A_d \\ I & 0 \end{bmatrix} \in \mathbb{R}^{(2n)\times(2n)}, \quad b := \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \in \mathbb{R}^{2n},$$

$$C := [c_1, 0] \in \mathbb{R}^{1\times(2n)}.$$

Equations (1) and (2) can be equivalently rewritten as

$$X(t+1) = \begin{bmatrix} A_1 & A_d \\ I & 0 \end{bmatrix} X(t) + \begin{bmatrix} b_1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} w(t) \\ 0 \end{bmatrix}, \tag{3}$$

$$\begin{aligned} y(t) &= [c_1, 0]X(t) + v(t) \\ &= CX(t) + v(t). \end{aligned} \tag{4}$$

The Kalman state estimation algorithm of estimating the state vector $X(t)$ in (3)–(4) can be expressed as

$$\hat{X}(t+1) = A\hat{X}(t) + bu(t) + L(t)[y(t) - C\hat{X}(t)], \tag{5}$$

$$L(t) = AP(t)C^{\mathrm{T}}[\sigma^2 + CP(t)C^{\mathrm{T}}]^{-1}, \tag{6}$$

$$P(t+1) = AP(t)A^{\mathrm{T}} - AP(t)C^{\mathrm{T}}[\sigma^2 + CP(t)C^{\mathrm{T}}]^{-1}CP(t)A^{\mathrm{T}}. \tag{7}$$

About the state estimation algorithm of the system in (1)–(2), we have the following theorem.

Theorem 1. *If the parameter matrices/vector A_1, A_d and b_1 are known, then we have the following state estimation algorithm to generate the estimate $\hat{x}(t)$ of the state vector $x(t)$ in (1) and (2):*

$$\begin{aligned}\hat{x}(t+1) = {} & A_1\hat{x}(t) + A_d\hat{x}(t-1) + A_dL_2(t-1)[y(t-1) - c_1\hat{x}(t-1)] \\ & + b_1u(t) + L_1(t)[y(t) - c_1\hat{x}(t)],\end{aligned} \tag{8}$$

$$L_1(t) = [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}\eta^{-1}(t), \tag{9}$$

$$L_2(t) = P_1(t)c_1^{\mathrm{T}}\eta^{-1}(t), \tag{10}$$

$$\begin{aligned}P_1(t+1) = {} & [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]A_1^{\mathrm{T}} + [A_1P_{12}(t) + A_dP_2(t)]A_d^{\mathrm{T}} \\ & - [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}[c_1P_1(t)A_1^{\mathrm{T}} + c_1P_{12}(t)A_d^{\mathrm{T}}]\eta^{-1}(t),\end{aligned} \tag{11}$$

$$P_{12}(t+1) = A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) - [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}c_1P_1(t)\eta^{-1}(t), \tag{12}$$

$$P_2(t+1) = P_1(t) - P_1(t)c_1^{\mathrm{T}}c_1P_1(t)\eta^{-1}(t). \tag{13}$$

The proof is given in the appendix.

When the parameter matrices/vector A_1, A_d, and b_1 are unknown, then we use the estimated parameter vector $\hat{\theta}(t) := [\hat{A}_1(t), \hat{A}_d(t), \hat{b}_1(t)]^{\mathrm{T}}$ to construct the estimates $\hat{A}_1(t)$, $\hat{A}_d(t)$, and $\hat{b}_1(t)$ of A_1, A_d, and b_1, and use the estimates to compute the estimate $\hat{x}(t)$ of the state vector $x(t)$:

$$\begin{aligned}\hat{x}(t+1) = {} & \hat{A}_1(t)\hat{x}(t) + \hat{A}_d(t)\hat{x}(t-1) + \hat{A}_d(t)L_2(t-1)[y(t-1) - c_1\hat{x}(t-1)] \\ & + \hat{b}_1(t)u(t) + L_1(t)[y(t) - c_1\hat{x}(t)],\end{aligned} \tag{14}$$

$$L_1(t) = [\hat{A}_1(t)P_1(t) + \hat{A}_d(t)P_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}\eta^{-1}(t), \tag{15}$$

$$L_2(t) = P_1(t)c_1^{\mathrm{T}}\eta^{-1}(t), \tag{16}$$

$$\begin{aligned}P_1(t+1) = {} & [\hat{A}_1(t)P_1(t) + \hat{A}_d(t)P_{12}^{\mathrm{T}}(t)]\hat{A}_1^{\mathrm{T}}(t) + [\hat{A}_1(t)P_{12}(t) + \hat{A}_d(t)P_2(t)]\hat{A}_d^{\mathrm{T}}(t) \\ & - [\hat{A}_1(t)P_1(t) + \hat{A}_d(t)P_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}[c_1P_1(t)\hat{A}_1^{\mathrm{T}}(t) + c_1P_{12}(t)\hat{A}_d^{\mathrm{T}}(t)]\eta^{-1}(t),\end{aligned} \tag{17}$$

$$P_{12}(t+1) = \hat{A}_1(t)P_1(t) + \hat{A}_d(t)P_{12}^{\mathrm{T}}(t) - [\hat{A}_1(t)P_1(t) + \hat{A}_d(t)P_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}c_1P_1(t)\eta^{-1}(t), \tag{18}$$

$$P_2(t+1) = P_1(t) - P_1(t)c_1^{\mathrm{T}}c_1P_1(t)\eta^{-1}(t), \tag{19}$$

$$\hat{A}_1(t) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ \hat{a}_n(t) & \hat{a}_{n-1}(t) & \hat{a}_{n-2}(t) & \dots & \hat{a}_1(t) \end{bmatrix}, \quad \hat{A}_d(t) = \begin{bmatrix} \hat{a}_{d1}(t) \\ \hat{a}_{d2}(t) \\ \vdots \\ \hat{a}_{dn}(t) \end{bmatrix}, \tag{20}$$

$$\hat{\boldsymbol{b}}_1(t) = [\hat{b}_1(t), \hat{b}_2(t), \dots, \hat{b}_n(t)]^{\mathrm{T}}. \tag{21}$$

The estimated parameters are used to compute the system states, the proposed algorithm has low computational cost and high accuracy.

3 | RECURSIVE PARAMETER ESTIMATION ALGORITHM

In order to derive the identification model of the time-delay system in (1) and (2), here ignores the noise term $\boldsymbol{w}(t)$. From (1), we have

$$x_i(t+1) = x_{i+1}(t) + \boldsymbol{a}_{di}\boldsymbol{x}(t-1) + b_i u(t), \quad i = 1, 2, \dots, n-1, \tag{22}$$

$$x_n(t+1) = a_n x_1(t) + a_{n-1}x_2(t) + \cdots + a_1 x_n(t) + \boldsymbol{a}_{dn}\boldsymbol{x}(t-1) + b_n u(t). \tag{23}$$

Let $\boldsymbol{a} := [a_n, a_{n-1}, \dots, a_1]^{\mathrm{T}} \in \mathbb{R}^n$. Using the property of the shift operator z and multiplying (22) by z^{-i} and (23) by z^{-n} give

$$x_i(t-i+1) = x_{i+1}(t-i) + \boldsymbol{a}_{di}\boldsymbol{x}(t-i-1) + b_i u(t-i), \quad i = 1, 2, \dots, n-1,$$
$$x_n(t-n+1) = \boldsymbol{a}\boldsymbol{x}(t-n) + \boldsymbol{a}_{dn}\boldsymbol{x}(t-n-1) + b_n u(t-n).$$

Adding all expressions gives

$$\begin{aligned}
x_1(t) &= \boldsymbol{a}\boldsymbol{x}(t-n) + \boldsymbol{a}_{d1}\boldsymbol{x}(t-2) + \boldsymbol{a}_{d2}\boldsymbol{x}(t-3) + \cdots + \boldsymbol{a}_{dn-1}\boldsymbol{x}(t-n) + \boldsymbol{a}_{dn}\boldsymbol{x}(t-n-1) \\
&\quad + b_1 u(t-1) + b_2 u(t-2) + \cdots + b_n u(t-n) \\
&= \boldsymbol{a}\boldsymbol{x}(t-n) + \sum_{i=1}^{n} \boldsymbol{a}_{di}\boldsymbol{x}(t-i-1) + \sum_{j=1}^{n} b_j u(t-j).
\end{aligned} \tag{24}$$

Define the parameter vector $\boldsymbol{\theta}'$ and the information vector $\boldsymbol{\phi}(t)$ as

$$\boldsymbol{\theta}' := [\boldsymbol{a}_{d1}, \dots, \boldsymbol{a} + \boldsymbol{a}_{dn-1}, \boldsymbol{a}_{dn}, \boldsymbol{b}_1^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n^2+n}, \tag{25}$$

$$\boldsymbol{\phi}(t) := [\boldsymbol{x}^{\mathrm{T}}(t-2), \boldsymbol{x}^{\mathrm{T}}(t-3), \dots, \boldsymbol{x}^{\mathrm{T}}(t-n-1), u(t-1), \dots, u(t-n)]^{\mathrm{T}} \in \mathbb{R}^{n^2+n}. \tag{26}$$

The identification model of the system is

$$y(t) = \boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{\theta}' + v(t). \tag{27}$$

Equation (27) is the parameter identification model of the systems. The proposed algorithms in this article are based on this identification model. Many identification methods are derived based on the identification models of the systems. Since $v(t)$ is a white noise, forming a quadratic cost function,[44-48]

$$J_1(\boldsymbol{\theta}') = \frac{1}{2}\sum_{j=1}^{t}[y(j) - \boldsymbol{\phi}^{\mathrm{T}}(j)\boldsymbol{\theta}']^2,$$

and minimizing $J_1(\boldsymbol{\theta}')$ lead to the following recursive least squares algorithm of estimating $\boldsymbol{\theta}'$:

$$\hat{\boldsymbol{\theta}}'(t) = \hat{\boldsymbol{\theta}}'(t-1) + \boldsymbol{L}_3(t)[y(t) - \boldsymbol{\phi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}'(t-1)], \tag{28}$$

$$\boldsymbol{L}_3(t) = \boldsymbol{P}_3(t-1)\boldsymbol{\phi}(t)[1 + \boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{P}_3(t-1)\boldsymbol{\phi}(t)]^{-1}, \tag{29}$$

$$\boldsymbol{P}_3^{-1}(t) = \boldsymbol{P}_3^{-1}(t-1) + \boldsymbol{\phi}(t)\boldsymbol{\phi}^{\mathrm{T}}(t). \tag{30}$$

Because the information vector $\boldsymbol{\phi}(t)$ contains the unknown variables $\boldsymbol{x}(t-i)$, the above algorithm cannot be implemented. The solution is to replace the unknown $\boldsymbol{x}(t-i)$ in $\boldsymbol{\phi}(t)$ with its estimate $\hat{\boldsymbol{x}}(t-i)$, we have the state estimate-based recursive least squares parameter estimation algorithm for time-delay state-space systems:

$$\hat{\boldsymbol{\theta}}'(t) = \hat{\boldsymbol{\theta}}'(t-1) + \boldsymbol{L}_3(t)[y(t) - \hat{\boldsymbol{\phi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}'(t-1)], \tag{31}$$

$$\boldsymbol{L}_3(t) = \boldsymbol{P}_3(t-1)\hat{\boldsymbol{\phi}}(t)[1 + \hat{\boldsymbol{\phi}}^{\mathrm{T}}(t)\boldsymbol{P}_3(t-1)\hat{\boldsymbol{\phi}}(t)]^{-1}, \tag{32}$$

$$\boldsymbol{P}_3^{-1}(t) = \boldsymbol{P}_3^{-1}(t-1) + \hat{\boldsymbol{\phi}}(t)\hat{\boldsymbol{\phi}}^{\mathrm{T}}(t), \tag{33}$$

$$\hat{\boldsymbol{\phi}}(t) = [\hat{\boldsymbol{x}}^{\mathrm{T}}(t-2), \hat{\boldsymbol{x}}^{\mathrm{T}}(t-3), \dots, \hat{\boldsymbol{x}}^{\mathrm{T}}(t-n-1), u(t-1), \dots, u(t-n)]^{\mathrm{T}}, \tag{34}$$

$$\hat{\boldsymbol{\theta}}'(t) = [\hat{\boldsymbol{a}}_{d1}(t), \dots, \hat{\boldsymbol{a}}(t) + \hat{\boldsymbol{a}}_{dn-1}(t), \hat{\boldsymbol{a}}_{dn}(t), \hat{\boldsymbol{b}}_1^{\mathrm{T}}(t)]^{\mathrm{T}}. \tag{35}$$

The initial value $\hat{\boldsymbol{\theta}}'(0)$ is taken as a zero matrix of appropriate sizes, $y(i)$, $\hat{\boldsymbol{\phi}}(i)$, $\hat{\boldsymbol{x}}(i)$, $v(i)$, $u(i)$, and $y(i)$ are zero for $i \leq 0$. The proposed state and parameters estimation algorithm for time-delay state-space systems in this article can combine other parameter estimation algorithms[49-56] to study the identification problem of other linear and nonlinear stochastic systems with colored noises[57-63] and can be used to estimate the parameters of other linear, bilinear, and nonlinear systems and can be applied to fields[64-68] such as chemical process control systems.

The steps of computing the state estimation vector in (14)–(21) and the parameter estimation vector $\hat{\boldsymbol{\theta}}'(t)$ in (31)–(35) are listed in the following.

1. Let $t = 1$, set the initial values $\hat{\boldsymbol{\theta}}'(0) = \mathbf{1}_{n^2+n}/p_0$, $\boldsymbol{P}_3(0) = p_0\boldsymbol{I}_{n^2+n}$, $\boldsymbol{P}_1(1) = \boldsymbol{I}_n$, $\boldsymbol{P}_2(1) = \boldsymbol{I}_n$, $p_0 = 10^6$, $\hat{\boldsymbol{x}}(1) = \mathbf{0}$.
2. Collect the input-output data $u(t)$ and $y(t)$, form $\hat{\boldsymbol{\phi}}(t)$ by (34).
3. Compute the gain vector $\boldsymbol{L}_3(t)$ by (32) and the covariance matrix $\boldsymbol{P}_3(t)$ by (33).
4. Update the parameter estimation vector $\hat{\boldsymbol{\theta}}'(t)$ by (31).
5. Read $\hat{a}_i(t)$, $\hat{\boldsymbol{a}}_{dj}(t)$ and $\hat{b}_i(t)$ from $\hat{\boldsymbol{\theta}}'(t)$ according to the definition of $\hat{\boldsymbol{\theta}}'(t)$.
6. Form $\hat{\boldsymbol{A}}_1(t)$, $\hat{\boldsymbol{A}}_d(t)$ and $\hat{\boldsymbol{b}}_1(t)$ by (20) and (21).
7. Compute the gain vector $\boldsymbol{L}_3(t)$ by (32) and the covariance matrix $\boldsymbol{P}_3(t)$ by (33).
8. Compute the state estimation vector $\hat{\boldsymbol{x}}(t+1)$ by (14).
9. Increase t by 1 and go to step 2, continue the recursive calculation.

Remark 4. The novelty of the article is to present a new parameter estimate-based state estimation algorithm in (14)–(21) and the state estimate-based parameter estimation algorithm in (31)–(35), that is a joint state and parameter estimation algorithm for time-delay state-space systems with unknown parameters. Although the least squares is basic for linear systems, the parameter estimation algorithm in this article is based on the state estimates because the states in the information vector are unknown and they are replaced with the estimated states in our algorithm. Also, we have analyzed the convergence of the proposed algorithm.

Remark 5. The recent work used the dynamic regressor extension and mixing (DREM) technique to establish the parameter convergence of continuous-time linear regression without the usual requirement of the regressor persistency of excitation. Also, the DREM technique was also applied to nonlinear regressions with "partially" monotonic parameter dependence.[69] Here we study the convergence of the state estimate-based parameter estimation algorithm under the persistent excitation condition. The condition can be improved by means of the DREM technique.

Theorem 2. *For the system in (1)–(2) and the recursive least squares algorithm, suppose that* $\{v(t)\}$ *is a white noise sequence with zero mean,* $\mathrm{E}[v(t)] = 0, \mathrm{E}[v(t)v(i)] = 0, t \neq i$, $\mathrm{E}[v^2(t)] = \sigma_v^2(t) \leq \sigma_v^2 < \infty$, *and that there exist constant* $0 < \alpha_1 \leq \beta_1 < \infty$ *and* $t_0 > 0$ *for* $t \geqslant t_0$, *the persistent excitation condition hold:*[70]

$$\alpha_1 \boldsymbol{I} \leq \frac{1}{t}\sum_{i=1}^{t}\boldsymbol{\phi}(i)\boldsymbol{\phi}^{\mathrm{T}}(i) \leq \beta_1 \boldsymbol{I}, \quad \text{a.s.}$$

assume that $\mathrm{E}[\|\hat{\boldsymbol{\theta}}'(0) - \boldsymbol{\theta}'\|^2] = \delta_1 < \infty$, $\hat{\boldsymbol{\theta}}'(0)$ *and* $v(t)$ *are uncorrelated. Then the parameter estimates* $\hat{\boldsymbol{\theta}}'(t)$ *given by the recursive least squares algorithm converge uniformly to the true parameters* $\boldsymbol{\theta}'$ *at the speed of* $(1/\sqrt{t})$,

$$\mathrm{E}[\|\hat{\theta}'(t)-\theta'\|^2] \leq \frac{2\lambda_{\max}^2[P_3^{-1}(0)]\delta_1}{\alpha_1^2 t^2} + \frac{2n\sigma_v^2}{\alpha_1 t} = \frac{2\delta_1}{\alpha_1^2 p_1^2 t^2} + \frac{2n\sigma_v^2}{\alpha_1 t} =: f_1(t), t \geqslant t_0,$$

or $\lim_{t\to\infty} \hat{\theta}'(t) = \theta'$, m.s., *where* $\lambda_{\max}[X]$ *is the maximum eigenvalue of matrix* X.

The proof can be done in a similar to the way in Reference 70.

4 | U-MODEL CONTROL

Consider a class of general state-space (including state delayed) representation for single-input single-output (SISO) nonlinear discrete time systems

$$x(k+1) = F(x(k), x(k-d), u(k)), \tag{36}$$

$$y(k) = h(x(k)), \tag{37}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}$ is the control input, $d > 0$ is an integer denoting state delay, $y(k) \in \mathbb{R}$ is the system output respectively. $F \in \mathbb{R}^n$ is a smooth vector function to describe the model dynamics and $h \in \mathbb{R}$ is a smooth function to map the state and input to the output. Throughout the study, assume the system relative degree r equals to the system order n and has stable zero dynamics (that is the system model has stable inverse), the full state vector x is available for measurement.

The state space model of (36)–(37) can be converted into a multilayer U model as

$$\begin{aligned} x_1(k+1) &= \sum_{j=0}^{M_1} \lambda_{1j}(k) f_{1j}(x_2(k)), \\ &\vdots \\ x_{n-1}(k+1) &= \sum_{j=0}^{M_{n-1}} \lambda_{n-1j}(k) f_{n-1j}(x_n(k)), \\ x_n(k+1) &= \sum_{j=0}^{M_n} \lambda_{nj}(k) f_{nj}(u(k)), \\ y(k) &= h(x(k)), \end{aligned}$$

where $f_{ij}(x_i(k))$ is a smooth function of the ith state.

Back stepping root resolving routing to determine control input $u(k)$ from the U-state space model. The algorithm is listed below.

1. For a given desired trajectory $y_m(k)$, assign $y(k+1) = y_m(k)$.
2. Solve $x_1(k+1) = h^{-1}(y_m(k))$.
3. In back step order, solve

$$\begin{aligned} x_2(k+1) &\in x_1(k+1) - \sum_{j=0}^{M_1} \lambda_{1j}(k) f_{1j}(x_2(k)) = 0, \\ &\vdots \\ x_n(k+1) &\in x_{n-1}(k+1) - \sum_{j=0}^{M_1} \lambda_{n-1j}(k) f_{n-1j}(x_{n-1}(k)) = 0. \end{aligned}$$

4. To determine the control input $u(k)$, solve the last line of the U-state space model by

$$u(k) \in x_n(k+1) - \sum_{j=0}^{M_n} \lambda_{nj}(k) f_{nj}(u(k)) = 0.$$

With reference to Figure 5, the design procedure is outlined below:

1. Establish a stable linear feedback control system structured in Figure 5, assign G for the closed loop system transfer function.
2. Specify G as a linear system with damping ratio, undamped natural frequency, and steady state error and/or the other performance indices (such as poles and zeros, and frequency response).
3. Let the plant model be a constant unit or the virtual pant $G_{ip} = G_p^{-1}G_p = 1 : u \to y$ have been achieved. To determine a linear invariant controller G_{c1} by taking inverse of the closed loop transfer function G using $G_{c1} = \frac{G}{1-G} = (1-G)^{-1}G$. Accordingly, the desired system output is equivalently determined by the output y_m of the controller G_{c1}.
4. Convert plant model G_p into G_p (U-model).
5. To achieve $G_{ip} = G_p^{-1}G_p = 1 : u \to y$ to guarantee the desired output $y_m(t)$, determine the controller output $u(t)$ by solving an equation $y_m(t) - G_p$(U-model)$= 0$, that is, $u(t) \in y_m(t) - G_p$(U-model)$= 0$.
6. Locate/connect the blocks with reference to Figure 5.

5 | CASE STUDIES

Case study 1. The model identification

Consider the following state-space system with time-delay:

$$x(t+1) = \begin{bmatrix} 0 & 1 \\ -0.25 & -0.95 \end{bmatrix} x(t) + \begin{bmatrix} 0.35 & -0.38 \\ 0.18 & -0.05 \end{bmatrix} x(t-1) + \begin{bmatrix} 1.00 \\ 0.97 \end{bmatrix} u(t),$$

$$y(t) = [1, 0]x(t) + v(t).$$

In simulation, the initial value is randomly generated and should be distributed in the unit circle to ensure the stability of the system.[70] The input $\{u(t)\}$ is taken as an uncorrelated persistent excitation signal sequence with zero mean and unit variance, and $\{v(t)\}$ as a white noise sequence with zero mean and variances $\sigma^2 = 0.10^2$ and $\sigma^2 = 0.50^2$. Applying the proposed RLS algorithm to estimate the parameters of this example system, the parameter estimates and their estimation errors are shown in Tables 1–3 and Figures 1–4.

From Tables 1–3 and Figures 1–4, we can draw the following conclusions.

- The convergence of the least squares algorithm is better than that of the stochastic gradient algorithm.
- The proposed algorithm is effective for estimating the parameters of the state-space model. With the data length increasing, the parameter estimation errors become smaller and converge to zero.
- A low noise variance leads to higher accuracy of parameter estimates. As the data length t increases, the parameter estimates approach their true values.
- It is clear that the proposed state observer can generate accurate state estimates because the state estimates are close to their true values as t increases. The predicted outputs match well with the actual outputs.

TABLE 1 The RLS parameter estimates and errors with $\sigma^2 = 0.50^2$

k	100	200	500	1000	2000	3000
$a_2 = -0.25$	−0.33525	−0.28095	−0.25048	−0.24896	−0.25789	−0.25759
$a_1 = -0.95$	−0.98993	−0.91664	−0.91787	−0.92302	−0.95503	−0.95990
$a_{21} = 0.18$	0.17029	0.16224	0.16971	0.17816	0.18697	0.18745
$a_{22} = -0.05$	−0.10814	−0.04455	−0.02085	−0.02857	−0.05779	−0.05399
$b_2 = 0.97$	1.01024	0.94950	0.98804	0.96431	0.97949	0.97858
$a_{11} = 0.35$	0.33398	0.34289	0.34241	0.35291	0.35606	0.34878
$a_{12} = -0.38$	−0.34817	−0.33635	−0.35828	−0.35574	−0.36590	−0.36977
$b_1 = 1.00$	0.75219	0.94818	0.95514	0.99924	0.98418	0.98300
δ (%)	12.19427	4.04025	3.17803	1.95805	1.25696	1.21002

TABLE 2 The RLS parameter estimates and errors with $\sigma^2 = 0.10^2$

k	100	200	500	1000	2000	3000
$a_2 = -0.25$	−0.26399	−0.25426	−0.24912	−0.24401	−0.25495	−0.25148
$a_1 = -0.95$	−0.96078	−0.94307	−0.94232	−0.93860	−0.95422	−0.95184
$a_{21} = 0.18$	0.17273	0.17103	0.17933	0.18115	0.18146	0.18047
$a_{22} = -0.05$	−0.05890	−0.04254	−0.04315	−0.04081	−0.05417	−0.05022
$b_2 = 0.97$	0.72608	0.82876	0.93568	0.94028	0.96826	0.97125
$a_{11} = 0.35$	0.33903	0.34153	0.35004	0.35390	0.35013	0.34874
$a_{12} = -0.38$	−0.38443	−0.38131	−0.37458	−0.37553	−0.37624	−0.37910
$b_1 = 1.00$	0.84029	0.98227	0.96844	1.00587	0.98853	0.98538
δ (%)	12.84049	6.29245	2.11585	1.53185	0.64056	0.66005

TABLE 3 The SG parameter estimates and errors with $\sigma^2 = 0.10^2$

k	100	200	500	1000	2000	3000
$a_2 = -0.25$	0.90469	0.42713	−0.10452	−0.19756	−0.21675	−0.24012
$a_1 = -0.95$	−0.30568	−0.35767	−0.45342	−0.64689	−0.89432	−0.93467
$a_{21} = 0.18$	0.19274	0.19183	0.17453	0.16455	0.15176	0.15756
$a_{22} = -0.05$	−0.01935	−0.01478	−0.03982	−0.04378	−0.05017	−0.05867
$b_2 = 0.97$	0.58348	0.63886	0.72358	0.80438	0.90646	0.94965
$a_{11} = 0.35$	0.24783	0.24765	0.26434	0.28356	0.31073	0.31564
$a_{12} = -0.38$	−0.30453	−0.31907	−0.32970	−0.32776	−0.34235	−0.36875
$b_1 = 1.00$	0.80989	0.84217	0.85841	0.90527	0.91013	0.91788
δ (%)	62.22479	48.02295	32.33475	20.88767	11.85797	6.86405

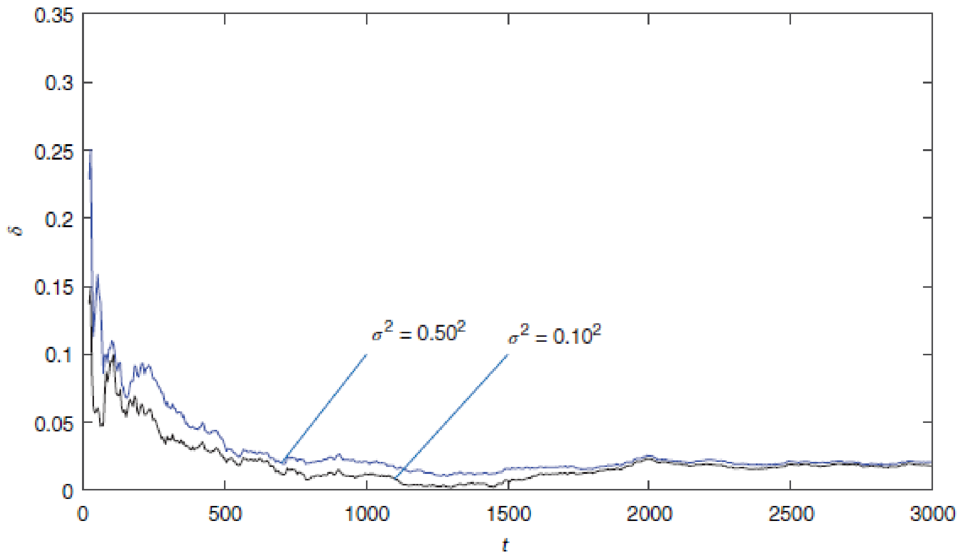

FIGURE 1 The parameter estimation error δ versus t

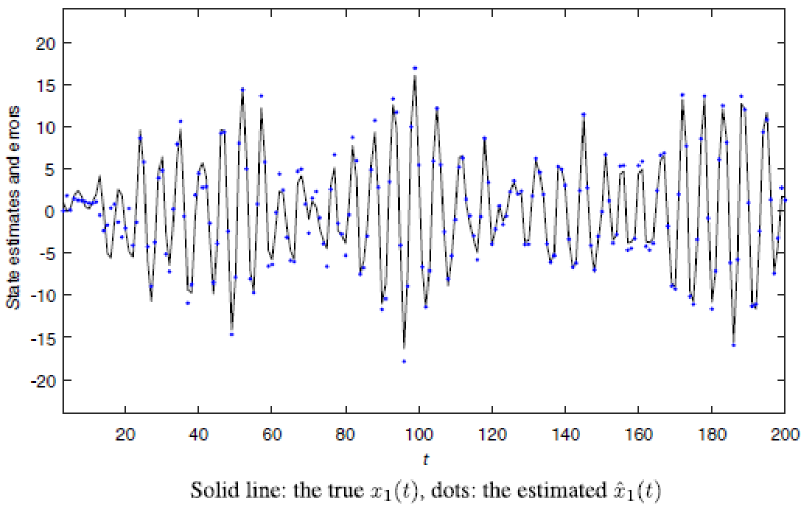

FIGURE 2 The estimate of the state $x_1(t)$

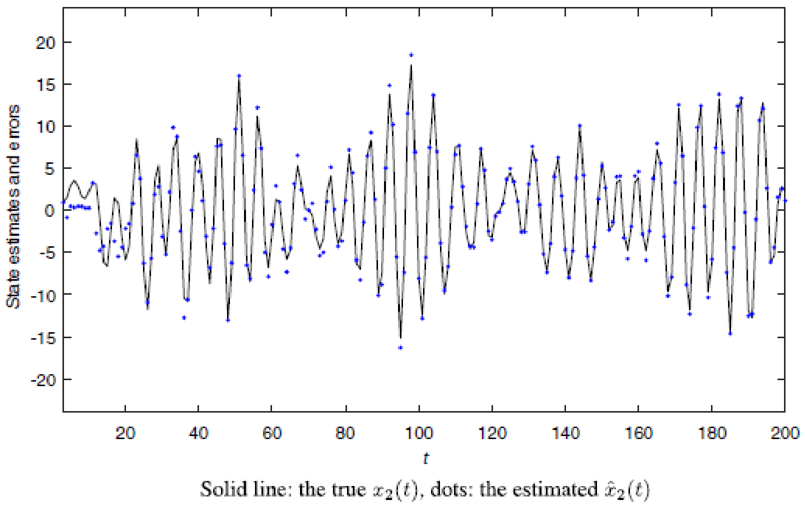

FIGURE 3 The estimate of the state $x_2(t)$

Case study 2. U-control system design

For showing off the efficiency of the U-control system design directly, take up the plant model

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.01 & -0.22 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.19 & -0.08 \\ 0.16 & -0.12 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

$$y(t) = [1,\ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

and convert it into U-state space model below

$$\begin{aligned} x_1(k+1) &= \lambda_{10}(k) + \lambda_{11}(k)x_2(k), \\ x_2(k+1) &= \lambda_{20}(k) + \lambda_{21}(k)u(k), \\ y(k) &= x_1(k), \end{aligned}$$

where

$$\lambda_{10}(k) = 0.21x_1(k-1) - 0.01x_2(k-1),$$
$$\lambda_{11}(k) = 1,$$
$$\lambda_{20}(k) = -0.01x_1(k) - 0.22x_2(k) + 0.16x_1(k-1) + 0.12x_2(k-1),$$
$$\lambda_{21}(k) = 1.$$

With reference to the general U-control system design routine, make the following step-by-step design:

1. Establish a stable linear feedback control system structured in Figure 5, assign the closed loop system transfer function with $\frac{Y(z)}{W(z)} = G(z) = \frac{0.45z^{-1}}{1-0.6z^{-1}+0.05z^{-2}}$, where z is the Z transform operator, the two poles are $p_1 = 0.1$, $p_2 = 0.5$, and no steady state error to a step input.
2. To determine a linear invariant controller G_{c1} by taking inverse of the closed loop transfer function G gives $G_{c1} = \frac{G}{1-G} = (1-G)^{-1}G = \frac{0.45z^{-1}}{1-1.05z^{-1}+0.05z^{-2}}$. Accordingly, the desired system output is equivalently determined by the output y_m of the invariant controller G_{c1}.
3. To achieve $G_{ip} = G_p^{-1}G_p = 1 : u \rightarrow y$ to guarantee the desired output $y_m(t)$, determine the controller output $u(t)$ by solving an equation $y_m(t) - G_p$(U-model)= 0, that is, $u(t) \in y_m(t) - G_p$(U-model)= 0. In this case, the back stepping routine is used in the root solving.
4. The established control system is consistent with structure in Figure 5.

Figures 5 and 6 show the simulated responses, which confirm the specified performance and design efficiency.

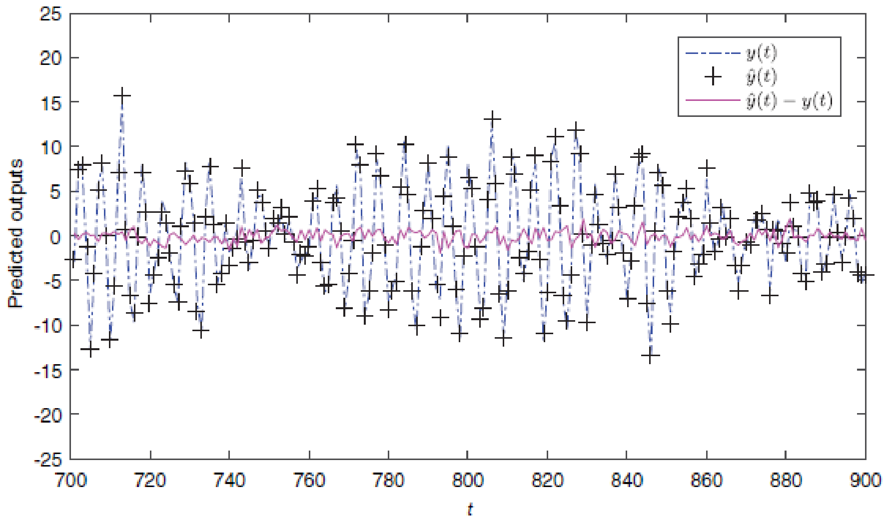

FIGURE 4 The actual output $y(t)$ and the estimated output $\hat{y}(t)$ versus t

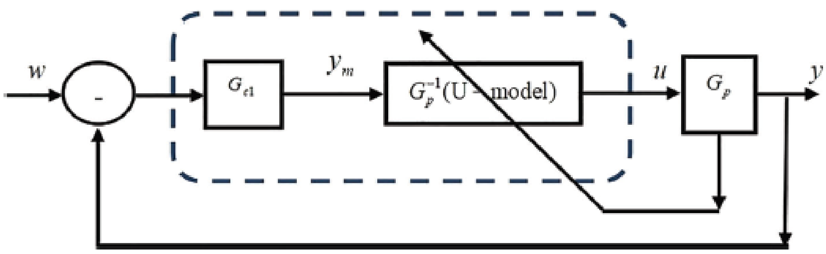

FIGURE 5 Plant output

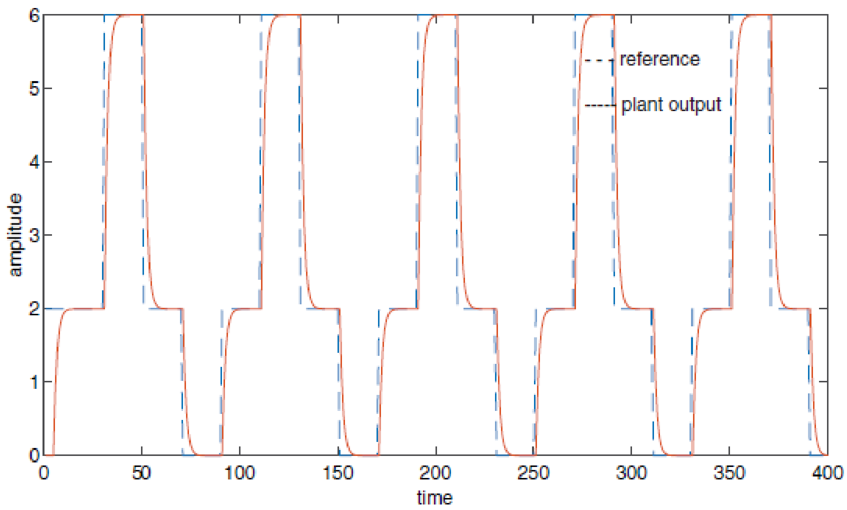

FIGURE 6 Control input

6 | CONCLUSIONS

This article has taken up a category of state-space models with state time delay as the research background, and accordingly developed the solution for the model identification (parameter and state estimation in specific) and control system design. The unknown states of the system are obtained under the framework of the state observer. The unknown parameters are estimated by the least squares from sampled data. The numerical example shows that the parameter estimates converge to their true values and the state observer based on the estimated parameters makes the estimated state curves match the actual state curves. The proposed state and parameters estimation for dual-rate state-space systems with time-delays in this article can combine other estimation algorithms[71-76] to study the parameter identification of other linear and nonlinear stochastic systems with colored noises[77-82] and can be applied to other literatures such as information processing and transportation communication systems.[83-90] The U-control has been adopted from the authors' recent research, but this article has presented a comprehensive back stepping routine for dynamic inversion for the U-state space models in such delayed control system design, which is different from those predominant approaches in this field. Hopefully, this can stimulate a new research/application direction in the future. The modeling-control studies have been explored and demonstrated that the proposed algorithms/procedures are effective and efficient in design and implementation.[91]

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DATA AVAILABILITY STATEMENT

All data generated or analyzed during this study are included in this article.

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APPENDIX. PROOF OF THEOREM

The proof of Theorem 1. Let

$$L(t) := \begin{bmatrix} L_1(t) \\ L_2(t) \end{bmatrix} \in \mathbb{R}^{2n},$$

$$P(t) := \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \in \mathbb{R}^{(2n)\times(2n)},$$

$$\hat{X}(t) := \begin{bmatrix} \hat{x}(t) \\ \hat{x}_1(t-1) \end{bmatrix} \in \mathbb{R}^{2n}.$$

From Equation (5), we have

$$\begin{aligned} \begin{bmatrix} \hat{x}(t+1) \\ \hat{x}_1(t) \end{bmatrix} &= \begin{bmatrix} A_1 & A_d \\ I & \mathbf{0} \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{x}_1(t-1) \end{bmatrix} + \begin{bmatrix} b_1 \\ \mathbf{0} \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} L_1(t) \\ L_2(t) \end{bmatrix} \left\{ y(t) - [c_1, \mathbf{0}] \begin{bmatrix} \hat{x}(t) \\ \hat{x}_1(t-1) \end{bmatrix} \right\} \\ &= \begin{bmatrix} A_1\hat{x}(t) + A_d\hat{x}_1(t-1) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ \mathbf{0} \end{bmatrix} u(t) + \begin{bmatrix} L_1(t) \\ L_2(t) \end{bmatrix} [y(t) - c_1\hat{x}(t)]. \end{aligned} \tag{A1}$$

Then, we have

$$\hat{x}(t+1) = A_1\hat{x}(t) + A_d\hat{x}_1(t-1) + b_1u(t) + L_1(t)[y(t) - c_1\hat{x}(t)], \tag{A2}$$

$$\hat{x}_1(t) = \hat{x}(t) + L_2(t)[y(t) - c_1\hat{x}(t)]. \tag{A3}$$

Equation (6) gives

$$\begin{aligned} \begin{bmatrix} L_1(t) \\ L_2(t) \end{bmatrix} &= \begin{bmatrix} A_1 & A_d \\ I & \mathbf{0} \end{bmatrix} \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \begin{bmatrix} c_1^{\mathrm{T}} \\ \mathbf{0} \end{bmatrix} \left\{ \sigma^2 + [c_1, \mathbf{0}] \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \begin{bmatrix} c_1^{\mathrm{T}} \\ \mathbf{0} \end{bmatrix} \right\}^{-1} \\ &= \begin{bmatrix} A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) & A_1P_{12}(t) + A_dP_2(t) \\ P_1(t) & P_{12}(t) \end{bmatrix} \begin{bmatrix} c_1^{\mathrm{T}} \\ \mathbf{0} \end{bmatrix} [\sigma^2 + c_1P_1(t)c_1^{\mathrm{T}}]^{-1} \\ &:= \begin{bmatrix} [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}} \\ P_1(t)c_1^{\mathrm{T}} \end{bmatrix} \eta^{-1}(t), \end{aligned}$$

where $\eta(t) := \sigma^2 + c_1P_1(t)c_1^{\mathrm{T}}$. Thus, we have

$$L_1(t) = [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}\eta^{-1}(t), \tag{A4}$$

$$L_2(t) = P_1(t)c_1^{\mathrm{T}}\eta^{-1}(t). \tag{A5}$$

Similarly, from Equation (7), we have

$$\begin{aligned} \begin{bmatrix} P_1(t+1) & P_{12}(t+1) \\ P_{12}^{\mathrm{T}}(t+1) & P_2(t+1) \end{bmatrix} &= \begin{bmatrix} A_1 & A_d \\ I & \mathbf{0} \end{bmatrix} \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \begin{bmatrix} A_1 & A_d \\ I & \mathbf{0}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ &\quad - \begin{bmatrix} A_1 & A_d \\ I & \mathbf{0} \end{bmatrix} \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \begin{bmatrix} c_1^{\mathrm{T}} \\ \mathbf{0} \end{bmatrix} \left\{ \sigma^2 + [c_1, \mathbf{0}] \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \begin{bmatrix} c_1^{\mathrm{T}} \\ \mathbf{0} \end{bmatrix} \right\}^{-1} \end{aligned}$$

$$
\begin{aligned}
&\times [c_1, \mathbf{0}] \begin{bmatrix} P_1(t) & P_{12}(t) \\ P_{12}^{\mathrm{T}}(t) & P_2(t) \end{bmatrix} \begin{bmatrix} A_1 & A_d \\ I & \mathbf{0}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\
&= \begin{bmatrix} A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) & A_1P_{12}(t) + A_dP_2(t) \\ P_1(t) & P_{12}(t) \end{bmatrix} \begin{bmatrix} A_1^{\mathrm{T}} & I \\ A_d^{\mathrm{T}} & \mathbf{0} \end{bmatrix} \\
&- \begin{bmatrix} A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) & A_1P_{12}(t) + A_dP_2(t) \\ P_1(t) & P_{12}(t) \end{bmatrix} \begin{bmatrix} c_1^{\mathrm{T}} \\ \mathbf{0} \end{bmatrix} [c_1P_1(t), c_1P_{12}(t)] \begin{bmatrix} A_1^{\mathrm{T}} & I \\ A_d^{\mathrm{T}} & \mathbf{0} \end{bmatrix} \eta^{-1}(t) \\
&= \begin{bmatrix} [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]A_1^{\mathrm{T}} + [A_1P_{12}(t) + A_dP_2(t)]A_d^{\mathrm{T}} & A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) \\ P_1(t)A_1^{\mathrm{T}} + P_{12}(t)A_d^{\mathrm{T}} & P_1(t) \end{bmatrix} \\
&- \begin{bmatrix} [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}} \\ P_1(t)c_1^{\mathrm{T}} \end{bmatrix} [c_1P_1(t)A_1^{\mathrm{T}} + c_1P_{12}(t)A_d^{\mathrm{T}}, c_1P_1(t)]\eta^{-1}(t) \\
&= \begin{bmatrix} [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]A_1^{\mathrm{T}} + [A_1P_{12}(t) + A_dP_2(t)]A_d^{\mathrm{T}} & A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) \\ P_1(t)A_1^{\mathrm{T}} + P_{12}(t)A_d^{\mathrm{T}} & P_1(t) \end{bmatrix} \\
&- \begin{bmatrix} [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}[c_1P_1(t)A_1^{\mathrm{T}} + c_1P_{12}(t)A_d^{\mathrm{T}}] & [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}c_1P_1(t) \\ P_1(t)c_1^{\mathrm{T}}[c_1P_1(t)A_1^{\mathrm{T}} + c_1P_{12}(t)A_d^{\mathrm{T}}] & P_1(t)c_1^{\mathrm{T}}c_1P_1(t) \end{bmatrix} \eta^{-1}(t).
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
P_1(t+1) &= [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]A_1^{\mathrm{T}} + [A_1P_{12}(t) + A_dP_2(t)]A_d^{\mathrm{T}} \\
&\quad - [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}[c_1P_1(t)A_1^{\mathrm{T}} + c_1P_{12}(t)A_d^{\mathrm{T}}]\eta^{-1}(t),
\end{aligned} \tag{A6}
$$

$$
P_{12}(t+1) = A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t) - [A_1P_1(t) + A_dP_{12}^{\mathrm{T}}(t)]c_1^{\mathrm{T}}c_1P_1(t)\eta^{-1}(t), \tag{A7}
$$

$$
P_2(t+1) = P_1(t) - P_1(t)c_1^{\mathrm{T}}c_1P_1(t)\eta^{-1}(t). \tag{A8}
$$

Equations (A2)–(A8) form the state estimation algorithm in (8)–(13) for the time-delay state-space systems in (1) and (2): The proof is completed.