Dynamic Analysis of Footbridge to Eurocode
(Case Study on Leventis Footbridge, Aba Road, Port Harcourt)

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ABSTRACT

This research project presents a dynamic structural analysis of footbridges as per Eurocode (case study on Leventis Footbridge, Aba Road, Port Harcourt). The footbridge was simulated using a commercial finite element software; Midas FEA. The crowd-footbridge is modelled as a structural oscillator to which some external load is applied. Therefore, the crowd is taken as an imposed load rather than as a dynamical system. The eigenvalue analysis was carried out to obtain the natural frequency ω_n, which is obtained as the eigenvalue of the characteristic equation and the corresponding mode shape is obtained as the eigenvector of the characteristic equation. The values are compared to Eurocode specification for comfort criteria on footbridges. Time history analysis was also carried out. This method was used to construct time histories of variables, such as displacement and acceleration, by calculating the response at the end of a succession of very small time steps. The peak deck acceleration is obtained and compared to Eurocode specification for comfort criteria on footbridges. From the time history graph, it was observed that the maximum acceleration of the footbridge is $0.56m/s^2$ which is greater than the value stated in the Eurocode standard specification of $0.5m/s^2$ for maximum comfort. This implies that the footbridge acceleration obtained is not satisfactory and that some measures need to be taken in order to remediate this effect and control the vibrations. Some measures to reduce vibration are recommended.

KEYWORDS: Acceleration, Dynamics, Footbridge, Natural frequency, Vibration.

INTRODUCTION

Footbridges (or pedestrian bridges) are structures whose primary purpose is to carry pedestrians over a physical obstacle. They are not designed for vehicular traffic, but for pedestrians, cyclists and even animals. A popular case is the London Millennium Bridge which was opened in 2000 and experienced synchronous lateral excitation exerted by crowded pedestrians

(Dallard et al., 2001). This has triggered investigations on crowd-structure interaction on footbridges. The footbridge was closed soon after opening due to lateral swaying experienced as uncomfortable by pedestrians (Setra, 2006). The excessive vibration induced by the crowd-structure interaction phenomenon, also called Synchronous Lateral Excitation (SLE) by pedestrians who walk on crowded footbridges, has attracted increasing attention in the past two decades. While designing footbridges, human-induced loads became an important parameter to consider, since the dynamic effect of pedestrian load can cause uncomfortable and

excessive vibrations due to low vibration frequency.

The phenomenon of the vulnerability of footbridges to vibration also occurs due to resonant vibration effect of the structure with fundamental frequencies close to the frequency of the pedestrian walking load. In this case, the dynamic load will have a great effect on the functionality of the bridge as human beings are very sensitive to vibration levels (Bachmann, 2002).

To remedy such vibration issues due to dynamic loads induced by pedestrians on modern footbridges, it is therefore pertinent that structures are designed according to recommendations of standards, which requires an analysis of super-structures in the two limit states: Ultimate Limit State (ULS) and Serviceability Limit State (SLS). Few researchers have investigated dynamic analysis on footbridges around the world.

Pantak et al. (2012) reported on the application of EN 1990/AI vibration serviceability limit state requirements for steel footbridges. According to the report, brief characteristics of vibration comfort criteria requirements for footbridges according to EN 1990/AI and their comparison with recommendations of different authors and guidelines were presented.

Fiammetta et al. (2007) proposed a mathematical model and a computational approach to study the complex multi-physical nonlinear coupled system that results from the interaction between a moving platform and the moving pedestrians. The mathematical model accounts for several nonlinear features of the problem. It also considers the effects of discontinuities in the crowd flow, such as obstructions, traffic jams and stop-and-go phenomena. They recommended that microscopic or mesoscopic models of the pedestrian flow could be included in the framework of crowd-structure interaction.

Figueiredo et al. (2008) developed a realistic load model to incorporate the effects induced by people walking on the dynamical response of footbridges. The method is based on the motion equations, including coefficients of rate of pedestrian lateral force, pedestrian density and the rate of synchronized pedestrians to large vibration amplitude. The proposed prediction model was validated by field measured data of two bridges that suffer from lateral vibration.

Ingolfsson and Georgakis (2011) studied the phenomenon of excessive pedestrian-induced lateral vibrations and conducted full-scale measurements of some selected bridges under crowd loading. Self-excited pedestrian force was determined *via* an extensive experimental campaign and a stochastic load model has been proposed for modelling of the frequency-and-amplitude-dependent pedestrian-induced lateral forces of footbridges.

According to Flaga (2002), the fundamental parameters governing the intensity of vibration perception by humans are: vibration amplitude, frequency characteristics of vibrations, direction of the vibration, vibrations impact time (exposure time), repeatability of the vibration and human activity.

Shum-Ichi and Toshitsugu (2006) investigated the lateral vibration of footbridges by synchronous walking. A dynamic model was then proposed to evaluate the pedestrians' synchronized dynamic forces. According to the available literature, results revealed that cable-supported bridges have both structural rationality and elegant features and are very attractive.

Prior to static analysis, dynamic structural analysis is equally required to determine the effect of dynamic loads on the structure's dynamic response and the influence on users. To satisfy serviceability requirement, while carrying out dynamic analysis of footbridges, it should be verified that the proposed design provides a sufficiently high level of comfort. In order to ensure user comfort and safety and prevent outright structural failure, many footbridges today are equipped with tuned mass dampers (also known as harmonic absorbers).

RESEARCH SIGNIFICANCE

Our study's aim is to carry out dynamic analysis on the Leventis footbridge subject to human-induced dynamic loads and compared to the comfort criteria recommended by Eurocode. This aim would be

achieved using the following objectives:

➢ To carry out free vibration analysis (eigenvalue analysis) on the footbridge to obtain the natural angular frequency, ω_n.

➢ To carry out time history analysis on the footbridge to obtain variables such as: deck acceleration, displacement and velocity using forward integration in the time domain.

➢ To compare variables, such as natural angular frequency and deck acceleration of the footbridge, with those recommended in Eurocode.

THEORETICAL BASIS

Eigenvalue Analysis

Eigenvalue analysis was employed to undertake free vibration analysis of the bridge. Conventionally, vibration in a structure is analyzed as a 3-D damped dynamical system. In this analysis, damping and excitation are not considered.

$$M\ddot{u}(t) + Ku(t) = 0 \qquad (1)$$

where K = Stiffness matrix of the structure;

M = Mass matrix of the structure;
u (t) = Displacement vector of the structure;
$\ddot{U}(t)$ = Acceleration vector.

If the displacement vector u is presumed to be a linear combination of mode-shape vectors, characterized by the mode shape matrix $\emptyset$ and the combination factors for the selected modes are defined by a vector of time-functions, Y(t), the displacement vector becomes:

$$u = \emptyset Y(t) \qquad (2)$$

Substituting Equation (2) into Equation (1) yields:

$$M\emptyset\ddot{Y} + K\emptyset Y = 0$$

The combination factors $y_m(t)$ are assumed as harmonic functions in time defined as:

$$y_m(t) : \cos(\omega_n t + \beta_m)$$

Due to Midas FEA, the time function Y(t) is defined as:

$$Y(t) = \{y_1(t) \dots y_m(t) \dots y_n(t)\}^T$$

where n is the total number of freedom degrees in the system.

For implementation in Midas FEA, the second derivative $\ddot{y}_m(t)$ with time of the harmonic function can be written as inverse multiplication with a constant factor $\lambda_m = \omega_n{}^2$ of the original function, $y_m(t)$.

$$\ddot{y}_m(t) = -\lambda_m y_m(t) \qquad (3)$$

Consequently, this assumption can be used to transform Equation (3) into:

$$(-M\emptyset\lambda + K\emptyset)Y = 0 \qquad (4)$$

where the matrices λ and $\emptyset$ are assembled as:

$$\lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_m & \\ & & \lambda_n \end{bmatrix}$$

$$\lambda_m = \omega_n{}^2 \qquad (5)$$

$$\emptyset = [\emptyset_1 \dots \emptyset_m \dots \emptyset_n] \qquad (6)$$

Equation (4) is valid for every harmonic function, which implies that every $y_m(t)$ can be transformed into:

$$K\emptyset_m - \lambda_m M\emptyset_m = 0 \qquad (7)$$

Equation (7) represents an eigenvalue problem which must satisfy the condition of Equation (8) and from this, the natural angular frequency is obtained as the eigenvalue of the characteristic Equation in equation (8).

$$|K - \lambda_m M| = 0 \qquad (8)$$

The velocities of the structure are expressed in terms of natural frequencies, f_m (cycle/time).

$$\omega_n = 2\pi f_m \qquad (9)$$

Time History Analysis

Time history analysis was performed using the direct integration method. In this approach, the total analysis time range is sub-divided into a number of finite steps and numerical integration of the dynamic equilibrium equation is performed at each time step (Midas FEA, 1989).

This method is particularly suitable for describing a dynamic system experiencing nonlinearity of stiffness of damping. Several methods are used to perform numerical integration, but MIDAS uses the average acceleration method of the Newmark-β method in which the acceleration $\ddot{U}(t)$ in the time range $t_i < t < t_{i+1}$ is assumed constant at the average of $\ddot{U}_i$ and $\ddot{U}_{i+1}$ in Equation (10).

$$\ddot{U}_{(t)} = \frac{\ddot{U}_i + \ddot{U}_{i+1}}{2} = \text{Constant} \qquad (10)$$

Consequently, the velocity and displacement at $t = t_{i+1}$ are expressed as:

$$\dot{U}_{i+1} = \dot{U}_i + \frac{\ddot{U}_i + \ddot{U}_{i+1}}{2}\Delta t \qquad (11)$$

$$U_{i+1} = U_i + \dot{U}_i \Delta t + \frac{\ddot{U}_i + \ddot{U}_{i+1}}{4}\Delta t^2 \qquad (12)$$

Let's use the integration variables of the Newmark-β method to characterize Equations (11) and (12).

$$\dot{U}_{i+1} = \dot{U}_i + (1-\gamma)\Delta t \ddot{U}_i + \gamma \Delta t \ddot{U}_{i+1} \qquad (13)$$

$$U_{i+1} = U_i + \Delta t \dot{U}_i + \left(\frac{1}{2}-\beta\right)\Delta t^2 \ddot{U}_i + \beta \Delta t^2 \ddot{U}_{i+1} \qquad (14)$$

where $\beta = 0.25$, $\gamma = 0.5$.

Rearranging Equation (14) gives the acceleration at the end of a successive time-step:

$$\ddot{U}_{i+1} = \frac{1}{\beta \Delta t^2}\left\{U_{i+1} - U_i - \Delta t \dot{U}_i - \left(\frac{1}{2}-\beta\right)\Delta t^2 \ddot{U}_i\right\} \qquad (15)$$

Equation (15) is substituted into Equation (13) and gives the following expression for the velocity at the end of the time-step:

$$\dot{U}_{i+1} = \frac{\gamma}{\beta \Delta t}U_{i+1} - \frac{\gamma}{\beta \Delta t}U_i + \left(1-\frac{\gamma}{\beta}\right)\dot{U}_i + \left(1-\frac{\gamma}{2\beta}\right)\Delta t \ddot{U}_i \qquad (16)$$

Equations (15) and (16) are substituted back into the dynamic equation of motion and rearranged for the displacement response U_{i+1} at the end of the increment as follows:

$$\left(\frac{1}{\beta \Delta t^2}M + \frac{\gamma}{\beta \Delta t}C + K\right)U_{i+1} = F + M\begin{bmatrix}\frac{1}{\beta \Delta t^2}U_i + \frac{1}{\beta \Delta t}\dot{U}_i + \\ \left(\frac{1}{2\beta}-1\right)\ddot{U}_i\end{bmatrix} +$$

$$C\left[\frac{\gamma}{\beta \Delta t}U_i + \left(\frac{\gamma}{\beta}-1\right)\dot{U}_i + \left(\frac{\gamma}{2\beta}-1\right)\Delta t \ddot{U}_i\right] \qquad (17)$$

NUMERICAL SIMULATION

Description of the Footbridge (Structural Model)

Detailed geometrical information on the structural model of the footbridge is presented here for proper discussion. The footbridge investigated in this research project is the Leventis footbridge, Aba Road, Port Harcourt. The bridge is supported by five piers (columns) at 7m interval each and is currently used for pedestrian crossing. The footbridge is made of reinforced concrete all through and spanning 33m lengthwise. The pier is 5.625m high with 0.6m diameter. The deck is supported by a main girder along the longitudinal direction of the beam. The beam is also made of reinforced concrete and is 1m wide, 0.6m deep and 2m long. The bridge width is 2m with a 0.15m thick concrete slab.

Figure (1): Leventis footbridge, Aba Road, Port Harcourt

Computational Model

The footbridge was simulated using a commercial finite element software, Midas FEA. The entire footbridge was discretized into 56775 elements and 69407 nodes. A full-scale model of the bridge was carried out. The piers were modelled using 3D 8-noded parametric solid element. The deck was modelled using 3D solid element defined by the elastic failure criterion. The embedded reinforcements in the deck were modelled using embedded reinforcement concept and defined using the von Mises failure criterion. The base of the piers was restrained in the translational and rotational degrees of freedom.
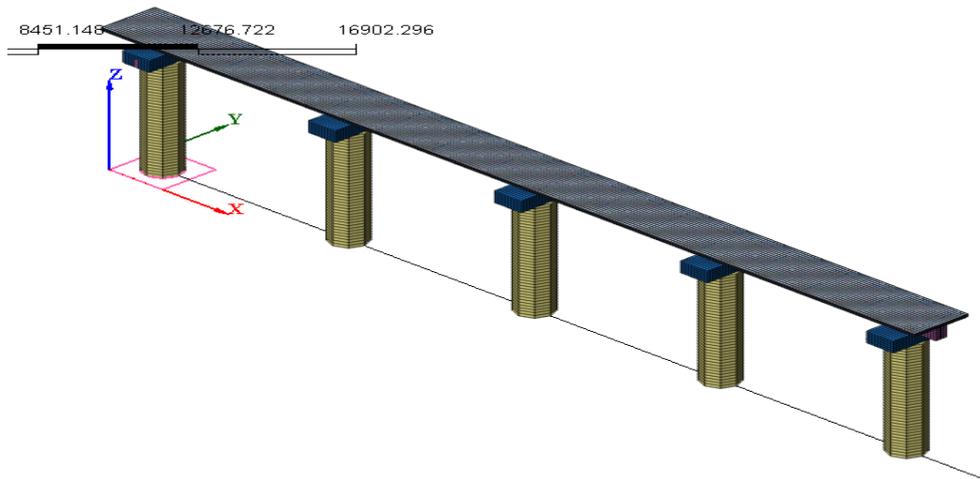
Figure (2): Model of Leventis footbridge, Aba Road, Port Harcourt

Modelling of the Dynamic Actions Induced by Walking Pedestrians

Various models were examined, but the model proposed by Figueiredo et al. (2008) was adopted in this paper. Load mode 1 is described as follows: in this model, the dynamic forces that represent the walking loads were estimated using Equation (13). It is applied on the basis that only one resonant load harmonic was applied on the footbridge's highest modal amplitude point at the midspan. It is assumed that the excitation frequency is equal to the footbridge fundamental frequency.

$$F(t) = P\alpha_i \cos(2\pi i f_s t) \qquad (18)$$

where:

P: individual weight taken as 700-800N [Ellis and Ji (1994)].

α_i: dynamic coefficient for the ith harmonic force component.

i: step frequency harmonic multiple.

f_s: step frequency.

t: time in seconds.

ANALYSIS OF RESULTS

Natural Frequencies and Mode Vibrations

The footbridge natural frequencies were determined with the aid of finite element method simulations. Figure 3 presents the natural frequencies and mode shapes for the footbridge when it freely vibrates in a particular mode, moving up and down with a certain configuration or mode shape.

Fig. 4 illustrates the mode shapes corresponding to the first five natural frequencies of the pedestrian footbridge with a span equal to 33m. It could be clearly observed that the maximum displacement occurs in mode shape 4 at 0.097m and at 4.7%.

History Analysis Results

The present analysis proceeds with the evaluation of the footbridge performance in terms of vibration serviceability effect due to human activities. In this case, dynamic forces are induced by pedestrian walking.

The footbridge peak acceleration was determined based on the developed finite element model (FEM) as revealed by the time history analysis graph shown in Fig. 5.
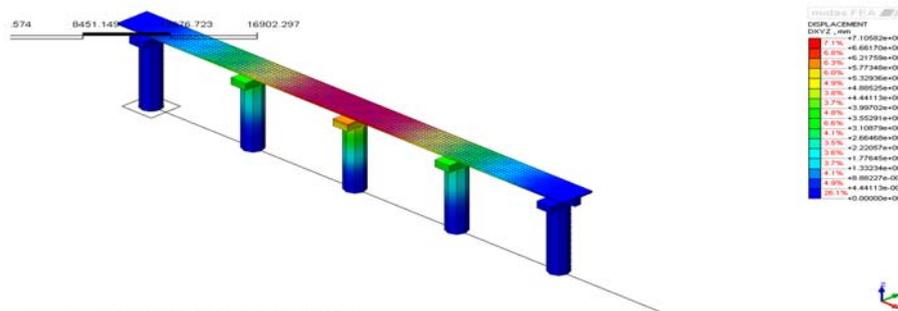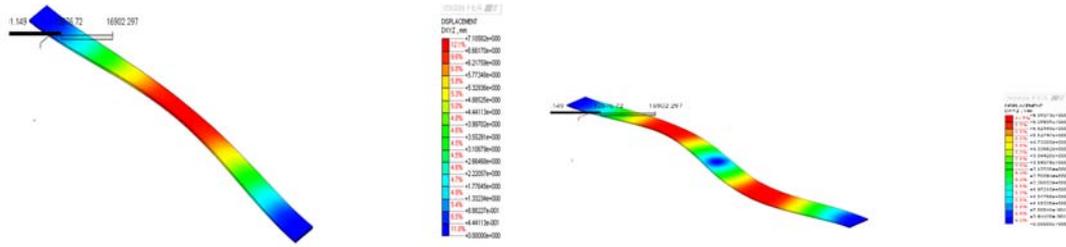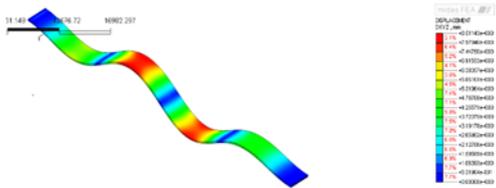

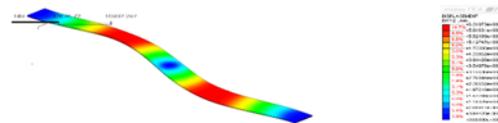**Figure (3): Displacement contour of the footbridge**

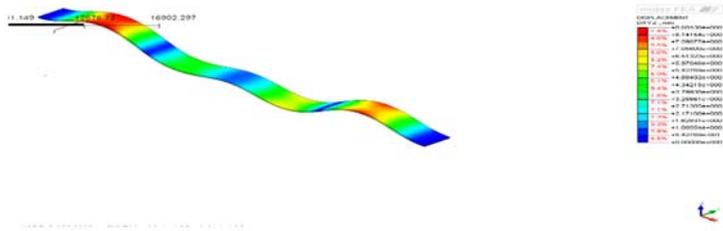Mode shape associated with the 1st natural frequency: f_{01} = 9.14 Hz

Mode shape associated with the 1st natural frequency: f_{02} = 16.0 Hz

Mode shape associated with the 1st natural frequency: f_{03} = 25.5 Hz

Mode shape associated with the 1st natural frequency: f_{04} = 25.7 Hz

Mode shape associated with the 1st natural frequency: f_{05} = 30.2 Hz
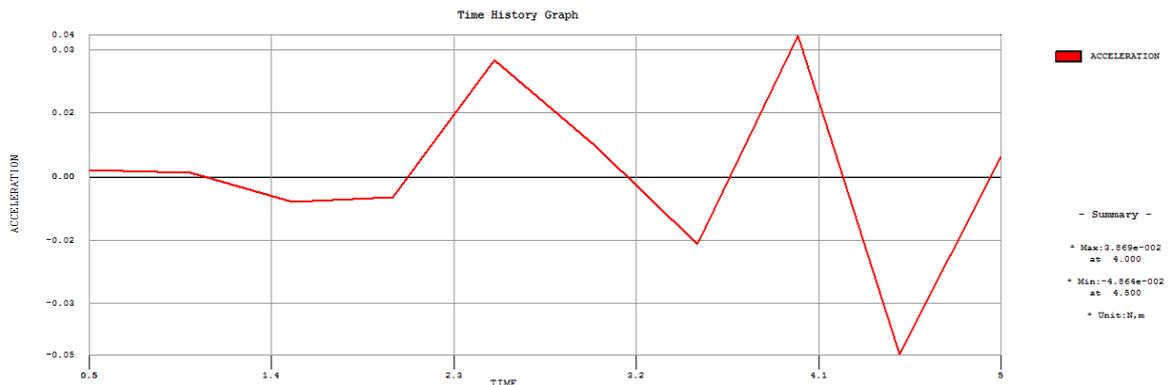
Figure (4): Footbridge vibration modes

Figure (5): Time history graph

From the time history graph, it is shown that the maximum acceleration of the footbridge is $0.56 m/s^2$, which is greater than the Eurocode standard specification value of $0.5 m/s^2$ for maximum comfort (i.e., vertical vibration). This therefore implies that the footbridge acceleration as analyzed is not satisfactory and so some measures need to be taken in order to remediate this effect and control the vibrations. There are different ways to control vibration on a footbridge deck. Some standard methods could be modifying the stiffnes and changing the structural mass (i.e., changing some member sizes), thereby changing the entire structural layout. Another standard method may be done by introducing dampers into the structure.

CONCLUSIONS

Pedestrian bridges are very often lively structures prone to vibrations, necessitating the vibration serviceability assessment in the design stage. The Eurocode enables the designer to check the vibration serviceability of the footbridge based on the prediction of the maximum acceleration levels. In this research project, the Leventis Footbridge, Aba Road, Port Harcourt has been studied. The finite element model of the 33m span footbridge was developed to simulate the physical and dynamic behaviour of the structure and to predict the response under human-induced loading.

In the design stage, uncertainty with regard to the predicted dynamic properties of the footbridge is inevitable. The footbridge results were obtained through a simple modelling scheme using Midas FEA. Two distinct analyses were carried out; namely, eigenvalue analysis which reveals the natural frequencies and the vibration shapes and the time history analysis which reveals the peak (maximum) acceleration of the structure with respect to time.

From the time history graph, it is shown that the maximum acceleration of the footbridge is $0.56 m/s^2$, which is greater than the Eurocode standard specification value of $0.5 m/s^2$ for maximum comfort. This, therefore, implies that the footbridge acceleration as analyzed is not satisfactory and that some measures need to be taken in order to remediate this effect and control the vibrations. There are different ways to control vibration on a footbridge deck. Some standard methods could be modifying the stiffness and changing the structural mass (i.e., changing some member sizes), thereby changing the entire structural layout. Another standard method may be done by introducing dampers into the structure.

Having studied the Leventis footbridge, Aba Road, Port Harcourt, results obtained from eigenvalue and time history analysis show that the peak acceleration exceeds the Eurocode specification value for maximum comfort. It is therefore pertinent to recommend ways to avoid/reduce the effect of vibration, *viz*:

❖ Modifying stiffness by changing structural arrangement or elements.
❖ Modifying the mass of the structure by introducing an extra mass.
❖ Modifying the damping properties of the structure by using advanced techniques (dampers, Time Division Multiplexing (TDM),… etc).

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