

Neural Adaptive Global Stability Control for Robot Manipulators with Time-varying Output Constraints

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Abstract

In this paper, a novel adaptive control scheme is proposed based on radial basis function neural network (RBFNN). The considered system is deduced by the structure of RBFNN with non-zero time-varying parameter that installed in the fore-end and terminal of RBFNN. With this structure and the Taylor expansion of any smooth continuous nonlinear function, a universal approximation of RBFNN is addressed according to the analysis of the character of continuous homogenous function and the Euler's theorem. The approximation accuracies can be adjusted on-line by the non-zero time-varying parameter in the device with the degree of continuous homogenous function, which expand the semi-globally stability to global stability over conventional neural controller design approaches. Based on the theory analysis of barrier Lyapunov function, the violation of time-varying constraints can be subjugated without wrecked. Finally, simulation results are carried out to verify the effectiveness by the design methods.

Keywords: Radial basis function neural network (RBFNN), Robot manipulator, Adaptive control, Time-varying constraints, Global uniformly ultimately bounded(GUUB).

I. INTRODUCTION

In recent years, many researchers and scholars are incline to the design of intelligent control to cope with different regions or fields with sophisticated and system identification in engineering environments, and hence artificial intelligence control has become an indispensable significant technique [1]–[3]. For example, in [4], an intelligence shared control system with brain computer interface was developed to achieve accurate object for robot manipulator system, in which the user's mind is regarded as a commander. Parameters identifications are also be populated for frequently usage in various engineering such as hovercraft system [5], robot system [6], [7], vehicle engines [8], wiener system [9], etc. However, parameters identifications only be suited to some class of known structured model systems in [5], [6], [8], [9]. Although some novel adaptive parameter estimation control schemes were constructed for the unknown robotic system in [7], the assumption of persistent excitation condition is necessary to be known beforehand. In many real applications, unknown nonlinearities and unstructured uncertainties in complex dynamical systems are inevitable existed, neural network space (NNs) and fuzzy logic systems (FLS) have been received much more attention since they are two powerful artificial intelligent instruments to deal with uncertainties or unknown model in nonlinear complex dynamic systems [10]–[17]. In literatures [12], [18]–[22], to investigate different nonlinear dynamical systems with unknown nonlinearities terms or unstructured models, several excellent NN controllers were developed. For the uncertain nonlinear robot manipulators systems, extreme learning machine (ELM) was studied to oppose the uncertain terms or unknown model such that some pre-given performances can be satisfied in [11], [14]. In [23], in order to guaranteed the good tracking performance of closed-loop systems, NNs combining with wave variable was employed to form a teleoperation controller. For multiagent systems with uncertain nonlinearities, distributed adaptive NNs controls [24], [25], multiple NN with supervisory control [22], consensus control [26], distributed output feedback control [27] and quantized control [20], [28] with asymmetric actuator backlash were proposed.

Recently, robot or telerobot systems recently have a significant of control design in multifarious intricate environments [23], [29], [30], and consequently the NN adaptive controls design for these kinds of systems have been become a hot interesting topic. In [29], RBFNNs was utilized to counteract the effect of uncertainties of Baxter robot. A robust output feedback control based on self-recurrent wavelet NNs with observer dynamical surface was put forward to the flexible joint electrically driven robot systems in [31]. For the flexible link manipulators in civil and military applications, an adaptive cerebellar model articulation control with NNs was designed in [32]. A robust NN output feedback controller with observer was implemented for the motion of robot manipulators in [33]. In [34]–[36], robust adaptive NN controls were brought forward for continuous and discrete-time robot systems respectively. For a class of robot arm systems with higher nonlinear structure, a NN algorithm control was presented to obtain good control performance in [37]. Nonetheless, in these literatures, it is important to point out that the constraint control problem was not considered in the design of the NN controllers.

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More outstanding works about barrier function were engendered from the pioneer works in [38], [39], where an originality idea was proposed to prevent the constraint violation for the output of closed-loop systems. For uncertain systems with states bounded constraints, an adaptive NN control strategy was developed by uniting novel integral barrier Lyapunov functions (BLFs) in [40]. The full-state constraints for a kind of stochastic nonlinear systems, and all the states under the function of controller can be ensured in their constraint bounds without being destroyed in [41]. An adaptive NN tracking controller combining with RBF was designed for a kind of marine surface vessels with unknown parameters and full state constraints in [42]. Howbeit the state constraints in these mention literatures only focused on the invariability of compact sets, and the result is that these approaches will become invalid when the constraints are defined in some variable domain sets. In [43], the full states were constrained in time-varying regions, and an adaptive controller was addressed for a class of nonlinear strict-feedback systems. It is necessary should be pointed out that the constraints were mainly researched on the states of closed-loop systems in [40]–[43]. The objects of these studies were generally some specific nonlinear systems, and the outputs constraint of uncertain or unstructured robot systems were not be considered. The outputs time-varying constraint of unstructured model of robotic manipulators were studied in [44] where an adaptive control design scheme was proposed to guarantee good tracking performance without outputs violation. In [45], two NNs were employed to design controller for the tracking problem of the output time-varying constrained in a sort of pure-feedback nonlinear systems.

In these above discussed works, the control approaches only be guaranteed to semi-globally uniformly ultimately bounded (SGUUB), which are ascribed to the property of approximation on a given certain compact domain. Consequently, the global stability control design is a challenge for work by using NNs. In [46], [47], global stability NNs controls based on backstepping design method were devised and satisfied for some given certain tracking performances, but the method can be only applied to the strict feedback systems and uncertain hypersonic flight vehicle respectively. A novel global uniformly ultimately bounded (GUUB) switching NN control was designed for bimanual robots systems with given tailored transient performances in advance in [48], it should be noted that the approximation accuracies of RBFNN can not be adjusted on-line automatically.

With these mentioned analyses, although the aforementioned abundant research works about NNs have significant guidance for the designing of intelligent controls, it is necessary to exploit other unified novelty methods to meet more performance requirements in application real world. Nevertheless, we attempt to break the limitation of conventional NN control methods, and construct updated laws according to the approximation accuracies of RBFNN. In this work, the control procedure of RBFNNs with non-zero adjusted parameter be divided two folds, the states of closed-loop system will be arrived at the sliding surface with limited time by using adaptive laws when they go out of the approximation domain, and then the adaptive RBFNNs controller be employed to satisfy the output of unstructured robot manipulators with outputs time-varying constraints. The major advantage of the proposed design method is that the on-line training burdens of RBFNNs can be eased greatly. Meanwhile, the high approximation accuracies of RBFNNs can be guaranteed owing to the relationship designed between adjusted laws of non-zero parameter and approximation accuracies.

In Section 2, descriptions of dynamic robot manipulator and some assumptions are given, and the structure of RBFNN with non-zero parameter and limiter is designed, which induced a new and original universal approximation with integer degree. Tracking adaptive control integrating with RBFNNs is presented in Section 3, and simulation example is carried out to validate controller in Section 4. The conclusion is addressed finally.

II. SYSTEM DESCRIPTIONS AND PRELIMINARIES

A. Descriptions and Assumptions of the Robot Manipulators

The dynamic system can be described as the following equation with n -link robot manipulators

$$\begin{cases} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u(t) - J^T(q)\tilde{F}(t) \\ y = q \end{cases} \quad (1)$$

where, $q \in R^{n \times 1}$ is the joint angular position, $\dot{q} \in R^{n \times 1}$ and $\ddot{q} \in R^n$ denote the velocity and acceleration, respectively; $M(q) \in R^{n \times n}$ expresses the Inertia matrix; $C(q, \dot{q}) \in R^{n \times n}$ is the centripetal and coriolis, and $G(q) \in R^{n \times 1}$ denotes the gravity; $u(t) \in R^{n \times 1}$ indicates the input torque; $J(q)$ denotes reversible Jacobian matrix, and the constrained force $\tilde{F}(t)$ with bounded, that is to say, there exists a given positive constant \tilde{f} and satisfies $\|\tilde{F}(t)\| \leq \tilde{f}$ for $t > 0$; $y = q$ denotes the output of the robot manipulators.

Assumption 1: [7] The Inertia matrix $M(q)$ is positive definite and its inverse matrix $M^{-1}(q)$ exists.

If we denote $q = x_1 = [x_{11}, x_{12}, \dots, x_{1n}]^T$, $\dot{q} = \dot{x}_1 = [x_{21}, x_{22}, \dots, x_{2n}]^T$, then the robot manipulator system (1) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = M^{-1}(x_1)[u(t) - J^T(q)\tilde{F}(t) - C(x_1, x_2)x_2 - G(x_1)] \\ y = x_1 \end{cases} \quad (2)$$

In this paper, the output position of (1) be assumed to be constrained on a time-varying compact set with the following form:

$$\underline{h}_s(t) \leq q \leq \bar{h}_s(t), \forall t \geq 0 \quad (3)$$

where $\underline{h}_s(t) = [\underline{h}_{s_1}, \underline{h}_{s_2}, \dots, \underline{h}_{s_n}]^T$, $\bar{h}_s(t) = [\bar{h}_{s_1}(t), \bar{h}_{s_2}(t), \dots, \bar{h}_{s_n}(t)]^T$ with $\bar{h}_{s_i}(t) > \underline{h}_{s_i}(t)$, $\forall t \in R_+, i = 1, \dots, n$.

Assumption 2: [40] There exist a series of constants \underline{H}_{sij} and \bar{H}_{sij} such that $|\bar{h}_{s_i}^{(j)}| \leq \bar{H}_{sij}$ and $|\underline{h}_{s_i}^{(j)}| \geq \underline{H}_{sij}$ ($j = 0, 1, 2, \dots, n$).

Assumption 3: The two function vectors $\bar{X}_0(t) < \bar{h}_s$ and $\underline{X}_0(t) > \underline{h}_s$ all hold for $\forall t > 0$, and there exist some known positive constants X_i with satisfying $|y_{di}| \leq X_i$ and $\underline{X}_0(t) \leq y_d(t) \leq \bar{X}_0(t)$.

Defining

$$\begin{cases} h_{a_i}(t) = y_{di}(t) - \underline{h}_{s_i}(t) \\ h_{b_i}(t) = \bar{h}_{s_i}(t) - y_{di}(t) \end{cases} \quad (4)$$

With the condition of Assumption 2 and 3, the result is that there exist some positive constants \underline{h}_{b_i} , \bar{h}_{b_i} , \underline{h}_{a_i} and \bar{h}_{a_i} , which can make sure the following inequalities are satisfied

$$\begin{cases} \underline{h}_{a_i}(t) \leq h_{a_i}(t) \leq \bar{h}_{a_i}(t) \\ \underline{h}_{b_i}(t) \leq h_{b_i}(t) \leq \bar{h}_{b_i}(t) \end{cases} \quad (5)$$

We introduce the ideal reference of the manipulated object as $y_d = [y_{d1}, y_{d2}, \dots, y_{dn}]^T$, and the tracking error be defined as

$$e = x_1 - y_d, z = \dot{q} - \beta \quad (6)$$

where $e = [e_1, e_2, \dots, e_n]^T$ is the position tracking error vector, $z = [z_1, z_2, \dots, z_n]^T$ represents the velocity tracking error robot manipulator in joint space, $\dot{q} = [\dot{q}_1, \dot{q}_2, \dots, \dot{q}_n]^T$, and $\beta = [\beta_1, \beta_2, \dots, \beta_n]^T$ stands for a virtual control vector which will be designed in the following procedure.

Introduce the following coordinates that will be used in the control design

$$\begin{cases} \zeta_a = [\frac{e_1}{h_{a_1}}, \frac{e_2}{h_{a_2}}, \dots, \frac{e_n}{h_{a_n}}]^T \\ \zeta_b = [\frac{e_1}{h_{b_1}}, \frac{e_2}{h_{b_2}}, \dots, \frac{e_n}{h_{b_n}}]^T \\ \zeta_{a_i} = \frac{e_i}{h_{a_i}}, \zeta_{b_i} = \frac{e_i}{h_{b_i}} \\ \zeta_i = p_i(e_i)\zeta_{b_i} + (1 - p_i(e_i))\zeta_{a_i} \end{cases} \quad (7)$$

$$\text{where } p_i(e_i) = \begin{cases} 1, e_i > 0 \\ 0, e_i \leq 0 \end{cases}.$$

Assumption 4: The unknown function vector is defined as $\varphi(X) = [\varphi_1(X), \dots, \varphi_n(X)]^T$, in which, the unknown model function $\varphi_i(X)$ is assumed as continuous and bounded with satisfies $|\varphi_i(X)| \leq \bar{\varphi}_i$, and $\bar{\varphi}_i$ is a known bounded positive constant.

With these Assumptions, there exist some known information about this unknown model in system such as continuous and up-bounded, which is necessary to design the control in the process of analysis the model.

The follows two Lemmas are introduced:

Lemma 1: [44] For any given $|\zeta_i| < 1$, and any integer $m > 0$, then $\log(\frac{1}{1-\zeta_i^{2m}}) < \frac{\zeta_i^{2m}}{1-\zeta_i^{2m}}$ holds.

Lemma 2: [44] If $-h_{a_i}(t) < e_{1i}(t) < h_{b_i}(t)$, then $|\zeta_i(t)| < 1$ can be obtained.

B. Descriptions of NNs

For the unknown nonlinearities in robot manipulator system (1), the RBFNN is fitted with non-zero parameter ρ and limiter as displayed in Fig.1

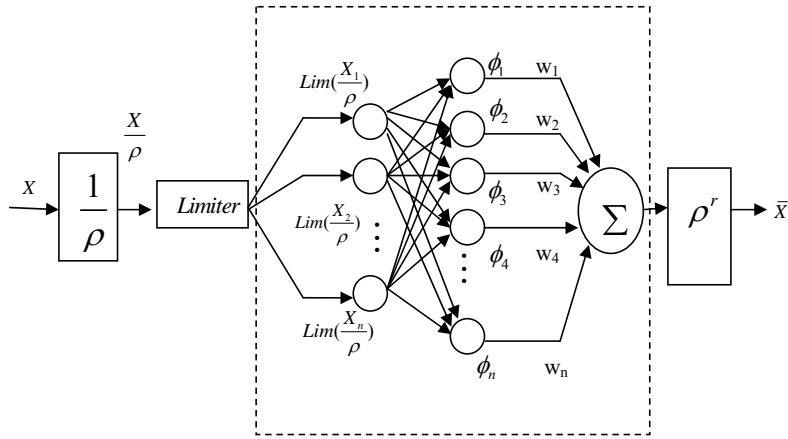


Fig. 1. The structure of RBFNN with the non-zero parameter ρ and the limiter

Remark 1: The zoomer $\frac{1}{\rho}$ is added in the input of RBFNN, then $\frac{X}{\rho}$ goes to the limiter. The working mechanism of the zoomer and limiter is that the input vector X can be limited at the approximation domain field of RBFNN, then other zoomer ρ^r (r denotes the degree of unknown continuous homogeneous function that is a known integer) is loaded at the terminal of the RBFNN, which can force the input states to converge to the origins.

Based on the structure of RBFNN with the parameter ρ and the limiter, the final output in Fig. 1 can be obtained by

$$\bar{X} = \rho^r W^T \phi\left(\frac{X}{\rho}\right) \quad (8)$$

where $X = [X_1, X_2, \dots, X_n]^T$, $W = [W_1, W_2, \dots, W_n]^T$ is the weight of the RBFNN, r denotes a known integer, $\phi\left(\frac{X}{\rho}\right) = [\phi_1\left(\frac{X}{\rho}\right), \phi_2\left(\frac{X}{\rho}\right), \dots, \phi_n\left(\frac{X}{\rho}\right)]^T$ is the radial basis function vector. Generally speaking, the radial basis function be chosen as Gaussian function form

$$\phi_k\left(\frac{X}{\rho}\right) = \exp\left[-\frac{\|\lim\left(\frac{X}{\rho}\right) - c_k\|^2}{2b_k^2}\right], (k = 1, 2, \dots, n) \quad (9)$$

where $\|\ast\|$ denotes 2-norm, $c_k = [c_{k1}, c_{k2}, \dots, c_{kn}]^T$ represents neural center, and b_k is the width of Gaussian function.

The working principle of the RBFNN in Fig.1 is that the input vector be entered into the device with a non-zero time-varying parameter ρ , and be went to the following limiter, the input vector X can be limited to a certain domain and then go through another device with ρ^r , hence the final output of the Fig.1 can be used to approximate any given unknown continuous homogeneous function with r degree (a known integer) as

$$\psi(\theta X) = \theta^r \psi(X) \quad (10)$$

where θ is any given known real constant. On a certain compact set, RBFNN (8) can be employed to approximate any given uncertain continuous homogeneous function with any degree of accuracy (In Fig.1, the degree is r).

Remark 2: For any unknown smooth nonlinear function $\varphi(X, t)$ at $X = 0$, by using the Taylor expansion of $\varphi(X, t)$, the following result can be obtained:

$$\varphi(X, t) = \sum_{i=1}^s \varphi_{k_i}(X) + o(X, t) \quad (11)$$

where $\sum_{i=1}^s \varphi_{k_i}(X)$ is the sum of s items, the remainder be denoted as $o(X, t)$.

With this property of Taylor expansion (11), we know that the degree k_i of the function $\varphi_{k_i}(X)$ can be obtained as $1, 2, 3, \dots, s$ (s is a positive integer).

Based on the above analysis, every unknown item $\varphi_{k_i}(X)$ in (11) that can be compensated by RBFNN as shown in Fig.1.

Now, the following Lemma 3 is proposed, which play an important role in the adaptive RBFNNs control design procedure.

Lemma 3: Consider the uncertain continuous nonlinear homogeneous function $\psi(X) \in R^n$ with known degree r , there exist RBFNN that be denoted as $\bar{F}(X)$ with approximation accuracy ε satisfying $\sup_{X \in U_0} |\psi(X) - \bar{F}(X)| \leq \varepsilon$. On the compact set

$U_0 = \{X \mid \|X\| \leq |\rho|\alpha\}$, then the following property of approximation is true

$$\sup_{X \in U_0} |\psi(X) - \bar{F}\left(\frac{X}{\rho}\right)| \leq |\rho|^r \varepsilon \quad (12)$$

Proof: Because we know the degree r of the uncertain homogeneous function $\psi(X)$, so $\psi(X) - \rho^r \psi(\frac{X}{\rho}) = 0$ is obtained. On the domain $U_0 = \{X \mid \|X\| \leq |\rho|\alpha\}$, then the following inequality holds:

$$\begin{aligned} |\psi(X) - \rho^r \bar{F}(\frac{X}{\rho})| &= |\psi(X) - \rho^r \psi(\frac{X}{\rho}) + \rho^r [\psi(\frac{X}{\rho}) - \bar{F}(\frac{X}{\rho})]| \\ &\leq |\psi(X) - \rho^r \psi(\frac{X}{\rho})| + |\rho|^r |\psi(\frac{X}{\rho}) - \bar{F}(\frac{X}{\rho})| \\ &= |\rho|^r |\psi(\frac{X}{\rho}) - \bar{F}(\frac{X}{\rho})| \leq |\rho|^r \varepsilon \end{aligned} \quad (13)$$

The proof of Lemma 3 is completed.

In this paper, let \hat{W} and $\hat{\varepsilon}$ represent the estimation value of W and ε , respectively, the errors be defined as $\tilde{W} = \hat{W} - W$ and $\tilde{\varepsilon} = \hat{\varepsilon} - \varepsilon$. The adaptive laws \hat{W} and $\hat{\varepsilon}$ will be proposed in the design procedure of control.

Because the dynamical model of robot manipulators is unknown, some RBFNNs need to be employed to compensate the unstructured model. We know the information that the higher degree be assumed as v , and hence the unstructured model can be represented as linear combinatorial function. Along these analyses, a novel RBFNNs adaptive control be designed such that the tracking global stability performance in Fig.2 can be achieved.

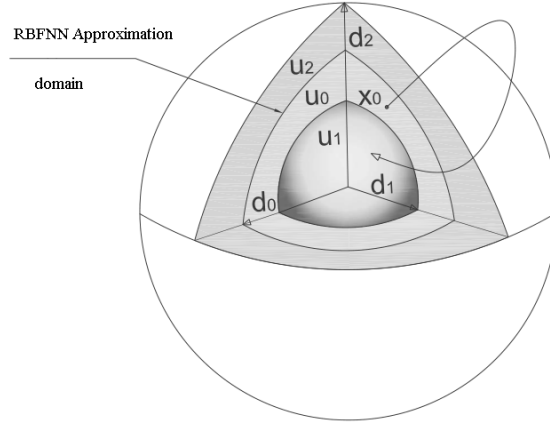


Fig. 2. Global stability performance

From Lemma 3, it is known that the RBFNN in Fig.1 be utilized to approximate the unstructured robot manipulators on the approximation domain U_0 in Fig.2. If the tracking signals run out of the domain U_0 , the sliding model surface $s = 0$ can be designed that pull the tracking signals back to the domain U_0 . Then RBFNN adaptive control is worked such that the signals are ensured to enter the compact U_1 , which make all signals to be GUUB guaranteed.

III. GLOBAL ADAPTIVE RBFNN CONTROL DESIGN

In this section, adaptive RBFNN control will be devised, we choose the following Lyapunov function:

$$V_1 = \sum_{i=1}^n \left(\frac{p_i}{2} \log \frac{h_{b_i}^2}{h_{b_i}^2 - e_i^2} + \frac{1-p_i}{2} \log \frac{h_{a_i}^2}{h_{a_i}^2 - e_i^2} \right) \quad (14)$$

Then, the result of \dot{V}_1 is can be got as

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^n \left[\frac{p_i \zeta_{b_i}}{h_{b_i}(1-\zeta_{b_i}^2)} + \frac{(1-p_i)\zeta_{a_i}}{h_{a_i}(1-\zeta_{a_i}^2)} \right] \dot{e}_i \\ &\quad - \sum_{i=1}^n \left[\frac{p_i \zeta_{b_i}}{h_{b_i}(1-\zeta_{b_i}^2)} \frac{\dot{h}_{b_i}}{h_{b_i}} + \frac{(1-p_i)\zeta_{a_i}}{h_{a_i}(1-\zeta_{a_i}^2)} \frac{\dot{h}_{a_i}}{h_{a_i}} \right] e_i \\ &= \sum_{i=1}^n \frac{\zeta_i^2}{(1-\zeta_i^2)} \dot{e}_i - \sum_{i=1}^{n_o} \left[\frac{p_i \zeta_{b_i}}{h_{b_i}(1-\zeta_{b_i}^2)} \frac{\dot{h}_{b_i}}{h_{b_i}} + \frac{(1-p_i)\zeta_{a_i}}{h_{a_i}(1-\zeta_{a_i}^2)} \frac{\dot{h}_{a_i}}{h_{a_i}} \right] e_i \end{aligned} \quad (15)$$

Let $R = [\frac{\zeta_1^2}{1-\zeta_1^2}, \frac{\zeta_2^2}{1-\zeta_2^2}, \dots, \frac{\zeta_n^2}{1-\zeta_n^2}]^T$, then it yields

$$\dot{V}_1 = R^T \dot{e} - \sum_{i=1}^n [\frac{p_i \zeta_{b_i}}{h_{b_i}(1-\zeta_{b_i}^2)} \frac{\dot{h}_{b_i}}{h_{b_i}} + \frac{(1-p_i)\zeta_{a_i}}{h_{a_i}(1-\zeta_{a_i}^2)} \frac{\dot{h}_{a_i}}{h_{a_i}}] e_i \quad (16)$$

In terms of the output of system (2) and the given ideal reference y_d , we define the differential of tracking error as

$$\dot{e} = \dot{x}_1 - \dot{y}_d = \begin{bmatrix} \dot{x}_{11} - \dot{y}_{d1} \\ \dot{x}_{12} - \dot{y}_{d2} \\ \vdots \\ \dot{x}_{1n} - \dot{y}_{dn} \end{bmatrix} = \begin{bmatrix} z_{21} + \beta_{11} - \dot{y}_{d1} \\ z_{22} + \beta_{12} - \dot{y}_{d2} \\ \vdots \\ z_{2n} + \beta_{1n} - \dot{y}_{dn} \end{bmatrix} \quad (17)$$

Let $\beta = [\beta_{11}, \beta_{12}, \dots, \beta_{1n}]^T$, and choose $\beta_{1i} = \dot{y}_{di} - k_{1i} - \tau_i$, then (15) can be represented as

$$\dot{V}_1 = -R^T K_1 + R^T z_2 - R^T \tau - \sum_{i=1}^n [\frac{p_i \zeta_{b_i}}{h_{b_i}(1-\zeta_{b_i}^2)} \frac{\dot{h}_{b_i}}{h_{b_i}} + \frac{(1-p_i)\zeta_{a_i}}{h_{a_i}(1-\zeta_{a_i}^2)} \frac{\dot{h}_{a_i}}{h_{a_i}}] e_i \quad (18)$$

in which, $K_1 = [k_{11}, k_{12}, \dots, k_{1n}]^T$ with every constant $k_{1i} > 0$ be proposed by user. $z_2 = [z_{21}, z_{22}, \dots, z_{2n}]^T$, $\tau = [\tau_1, \tau_2, \dots, \tau_n]^T$ with $\tau_i = \sqrt{(\frac{\dot{h}_{a_i}}{h_{a_i}})^2 + (\frac{\dot{h}_{b_i}}{h_{b_i}})^2} + \varrho_i$, ϱ_i is a positive constant by designer.

Since $\tau_i + p_i \frac{\dot{h}_{b_i}}{h_{b_i}} + (1-p_i) \frac{\dot{h}_{a_i}}{h_{a_i}} \geq 0$, so it has

$$\dot{V}_1 \leq -R^T K_1 + R^T z_2 = -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1-\zeta_i^2} + R^T z_2 \quad (19)$$

The following RBFNNs adaptive switching controller is proposed according to the control task

$$u = \begin{cases} O_{n \times 1}, \|z_2\| > \alpha|\rho| \\ -M(x_1)[K_2 z_2 + \sum_{q=1}^r \hat{W}_q^T \phi_q(\frac{x}{\rho})], \|z_2\| \leq \alpha|\rho| \end{cases} \quad (20)$$

where O represents a matrix with all elements zero, $K_2 = \text{diag}\{k_{21}, k_{22}, \dots, k_{2n}\}$ with $k_{2i} > 0$, $\hat{W}_q = [\hat{W}_{q1}, \hat{W}_{q2}, \dots, \hat{W}_{qn}]^T$, $\phi_q(\frac{x}{\rho}) = [\phi_{q1}(\frac{x}{\rho}), \phi_{q2}(\frac{x}{\rho}), \dots, \phi_{qn}(\frac{x}{\rho})]^T$.

The updated laws be designed as

$$\dot{\rho} = \begin{cases} \frac{1}{\alpha^2 \bar{\rho}} [L + (\|R\| + \sum_{q=1}^r \bar{\varphi}_q) \|z_2\|], \|z_2\| > \alpha|\rho| \\ -\frac{\gamma}{2} \rho - \mu \alpha (\|R\| + \sum_{q=1}^r |\rho|^{k_q} \hat{\varepsilon}_q) \bar{\rho}, \|z_2\| \leq \alpha|\rho| \end{cases} \quad (21)$$

$$\dot{\hat{\varepsilon}}_q = \begin{cases} 0, \|z_2\| > \alpha|\rho| \\ -\delta_q \hat{\varepsilon}_q + \eta_q \alpha |\rho|^{k_q}, \|z_2\| \leq \alpha|\rho| \end{cases} \quad (22)$$

$$\dot{\hat{W}}_{qi} = \begin{cases} 0, \|z_2\| > \alpha|\rho| \\ \sigma_{qi} [z_{2i} \phi_{qi}(\frac{x}{\rho}) - \vartheta_{qi} \hat{W}_{qi}], \|z_2\| \leq \alpha|\rho| \end{cases} \quad (23)$$

where $\bar{\rho} = \begin{cases} -1, \bar{\rho} \leq 0 \\ 1, \bar{\rho} > 0 \end{cases}$, $\bar{\varphi}_q$ ($q = 1, 2, \dots, r$) are some known positive bounded constants, and the parameter $\alpha, L, \gamma, \mu, \delta_q, \eta_q, \sigma_{qi}, \vartheta_{qi}$ are proposed suitable positive constants.

Theorem 1: Consider the robotic manipulator system (1) with the Assumption 1-4, if the initial output $\underline{h}_s(0) < y(0) < \bar{h}_s(0)$ is satisfied, then the adaptive RBFNN control (20) with the adaptation laws (21)-(23) can guarantee that all the signals are global uniformly ultimately bounded in the closed-loop system, and the tracking errors can be ensured to enter into a small zero domain field.

The proof for Theorem 1 be designed as the following two folds:

Case (1): $\|z_2\| > \alpha|\rho|$

With this condition, open control $u = O_{n \times 1}$ is adopted. The following sliding model surface is defined:

$$s = s(x_1^T, x_2^T, \beta^T, \dot{\beta}^T, \tilde{\varepsilon}_q^T, \tilde{W}^T)^T$$

$$= V_1 + \frac{1}{2}z_2^T z_2 - \frac{1}{2}\alpha^2 \rho^2 + \frac{1}{2} \sum_{q=1}^r \tilde{\varepsilon}_q^2 + \frac{1}{2} \sum_{i=1}^n \sum_{q=1}^r \tilde{W}_{qi}^T \tilde{W}_{qi} \quad (24)$$

From (24), we know that $s > 0$, and select the positive function $\tilde{V} = \frac{1}{2}s^2$, then the differentiation $\dot{\tilde{V}}$ leads to

$$\begin{aligned} \dot{\tilde{V}} &= s\{\dot{V}_1 + z_2^T z_2 - \alpha^2 \rho \dot{\rho} + \sum_{q=1}^r \tilde{\varepsilon}_q \dot{\tilde{\varepsilon}}_q + \sum_{i=1}^n \sum_{q=1}^r \tilde{W}_{qi}^T \dot{\tilde{W}}_{qi}\} \\ &= s\{\dot{V}_1 + z_2^T [M^{-1}(x_1)u + \varphi(X)] - \alpha^2 \rho \dot{\rho} + \sum_{q=1}^r \tilde{\varepsilon}_q \dot{\tilde{\varepsilon}}_q + \sum_{i=1}^n \sum_{q=1}^r \tilde{W}_{qi}^T \dot{\tilde{W}}_{qi}\} \end{aligned} \quad (25)$$

where $\varphi(X) = -M^{-1}(x_1)[J^T(x_1)F + C(x_1, x_2)x_2 + G(x_2)] - \beta_1$, which is assumed to be an unstructured model. We denote $\varphi(X) = \sum_{q=1}^r \varphi_q(X)$ on the compact set U_0 , in which φ_q denotes a series unknown continuous homogeneous functions vector

with q degree such as $\varphi_q = [\varphi_{q1}, \varphi_{q2}, \dots, \varphi_{qn}]^T$. We know $\sup_{X \in U_0} \|(\varphi_{q1}, \varphi_{q2}, \dots, \varphi_{qn})\| \leq \sqrt{\sum_{i=1}^n \varphi_{qi}} \triangleq \bar{\varphi}_q$, $\bar{\varphi}_q$ is a given known bounded value, and hence that

$$\begin{aligned} \dot{\tilde{V}} &\leq s\{-\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1 - \zeta_i^2} + \|R\| \cdot \|z_2\| + \|z_2\| \sum_{q=1}^r \bar{\varphi}_q - \alpha^2 \rho \dot{\rho} + \sum_{q=1}^r \tilde{\varepsilon}_q \dot{\tilde{\varepsilon}}_q + \sum_{i=1}^n \sum_{q=1}^r \tilde{W}_{qi}^T \dot{\tilde{W}}_{qi}\} \\ &\leq -s\{\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1 - \zeta_i^2} + L\} \leq 0 \end{aligned} \quad (26)$$

On the basis of the results in [49], inequality (26) means that the state vector $Z = [x_1^T, x_2^T, \beta^T, \dot{\beta}^T, \tilde{\varepsilon}_q^T, \tilde{W}^T]^T$ can be reached the sliding surface $s = 0$ with limited times.

Case (2): $\|z_2\| \leq \alpha|\rho|$

In this case, $\|z_2\| \leq \alpha|\rho|$ indicates that $\|X\| \leq \alpha|\rho|$ is truth, and the following candidate Lyapunov function is considered

$$V_2 = V_1 + \frac{1}{2}z_2^T z_2 \quad (27)$$

the differential of V_2 becomes as

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2^T \dot{z}_2 \\ &\leq -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1 - \zeta_i^2} + R^T z_2 + z_2^T M^{-1}(x_1)u + z_2^T \varphi(X) \end{aligned} \quad (28)$$

according to the controller (20) with this case, it gets

$$\begin{aligned} &M^{-1}(x_1)u + \varphi(X) \\ &= -K_2 z_2 - \sum_{q=1}^r \hat{W}_q^T \phi_q\left(\frac{X}{\rho}\right) + \sum_{q=1}^r \varphi_q(X) \end{aligned} \quad (29)$$

the inequality (28) can be transformed as the following form

$$\begin{aligned} \dot{V}_2 &\leq -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1 - \zeta_i^2} + R^T z_2 - z_2^T K_2 z_2 + z_2^T \sum_{q=1}^r [\varphi_q(X) - \hat{W}_q^T \phi_q\left(\frac{X}{\rho}\right)] \\ &= -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1 - \zeta_i^2} + R^T z_2 - z_2^T K_2 z_2 + z_2^T \sum_{q=1}^r [W_q^T \phi_q\left(\frac{X}{\rho}\right) + \overline{|\rho|^{k_q}} \bar{\varepsilon}_q - \hat{W}_q^T \phi_q\left(\frac{X}{\rho}\right)] \end{aligned} \quad (30)$$

where $\overline{|\rho|^{k_q}} = [|\rho|^{k_{q1}}, |\rho|^{k_{q2}}, \dots, |\rho|^{k_{qn}}]^T$ and $\bar{\varepsilon}_q = [\varepsilon_{q1}, \varepsilon_{q2}, \dots, \varepsilon_{qn}]^T$. Because $\|\overline{|\rho|^{k_q}}\| \leq \sum_{i=1}^n |\rho|^{k_{qi}}$ and $\|\bar{\varepsilon}_q\| \leq \sum_{i=1}^n \varepsilon_{qi}$ can

be satisfied, if we denote $\sum_{i=1}^n |\rho|^{k_{qi}} \triangleq |\rho|^{k_q}$, $\sum_{i=1}^n \varepsilon_{qi} \triangleq \varepsilon_q$, so (30) yields the form as follows

$$\dot{V}_2 \leq -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1-\zeta_i^2} - \sum_{i=1}^n k_{2i} z_{2i}^2 + R^T z_2 + \|z_2\| \sum_{q=1}^r |\rho|^{k_q} \varepsilon_q - \sum_{i=1}^n \sum_{q=1}^r z_{2i} \tilde{W}_{qi}^T \phi_{qi} \left(\frac{X}{\rho} \right) \quad (31)$$

considering the following Lyapunov function

$$V_3 = V_2 + \frac{1}{2\mu} \rho^2 + \sum_{q=1}^r \frac{1}{2\eta_q} \tilde{\varepsilon}_q^2 + \frac{1}{2} \sum_{i=1}^n \sum_{q=1}^r \tilde{W}_{qi}^T \sigma_{qi}^{-1} \tilde{W}_{qi} \quad (32)$$

so the derivative of V_3 be read as

$$\begin{aligned} \dot{V}_3 &\leq -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1-\zeta_i^2} - \sum_{i=1}^n k_{2i} z_{2i}^2 + \alpha |\rho| \cdot \|R\| \\ &+ \alpha |\rho| \sum_{q=1}^r |\rho|^{k_q} \tilde{\varepsilon}_q + \mu^{-1} \rho \dot{\rho} + \sum_{q=1}^r \eta_q^{-1} \tilde{\varepsilon}_q \dot{\tilde{\varepsilon}}_q - \alpha |\rho| \sum_{q=1}^r |\rho|^{k_q} \tilde{\varepsilon}_q \\ &+ \sum_{i=1}^n \sum_{q=1}^r \tilde{W}_{qi}^T \sigma_{qi}^{-1} \dot{\tilde{W}}_{qi} - \sum_{i=1}^n \sum_{q=1}^r z_{2i} \tilde{W}_{qi}^T \phi_{qi} \left(\frac{X}{\rho} \right) \\ &\leq -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1-\zeta_i^2} - \sum_{i=1}^n k_{2i} z_{2i}^2 - \frac{\gamma}{2\mu} \rho^2 - \sum_{q=1}^r \frac{\delta_q}{\eta_q} \tilde{\varepsilon}_q \hat{\varepsilon}_q - \sum_{i=1}^n \sum_{q=1}^r \vartheta_{qi} \tilde{W}_{qi}^T \hat{W}_{qi} \end{aligned} \quad (33)$$

In virtue of the following inequality

$$-\frac{\delta_q}{\eta_q} \tilde{\varepsilon}_q \hat{\varepsilon}_q = -\frac{\delta_q}{\eta_q} \tilde{\varepsilon}_q^2 - \frac{\delta_q}{\eta_q} \tilde{\varepsilon}_q \varepsilon_q \leq -\frac{1}{2} \frac{\delta_q}{\eta_q} \tilde{\varepsilon}_q^2 + \frac{1}{2} \frac{\delta_q}{\eta_q} \varepsilon_q^2 \quad (34)$$

Similarly, we have

$$-\vartheta_{qi} \tilde{W}_{qi}^T \hat{W}_{qi} \leq -\frac{1}{2} \vartheta_{qi} \|\tilde{W}_{qi}\|^2 + \frac{1}{2} \vartheta_{qi} \|W_{qi}\|^2 \quad (35)$$

Substituting (34) and (35) into (33), it follows

$$\begin{aligned} \dot{V}_3 &\leq -\sum_{i=1}^n k_{1i} \frac{\zeta_i^2}{1-\zeta_i^2} - \sum_{i=1}^n k_{2i} z_{2i}^2 - \frac{\gamma}{2\mu} \rho^2 - \frac{1}{2} \sum_{q=1}^r \frac{\delta_q}{\eta_q} \tilde{\varepsilon}_q^2 - \sum_{i=1}^n \sum_{q=1}^r \frac{1}{2} \vartheta_{qi} \|\tilde{W}_{qi}\|^2 \\ &+ \frac{1}{2} \sum_{q=1}^r \frac{\delta_q}{\eta_j} \varepsilon_q^2 + \frac{1}{2} \sum_{i=1}^n \sum_{q=1}^r \vartheta_{qi} \|W_{qi}\|^2 \end{aligned} \quad (36)$$

Let $\varpi = \min\{k_{1i}, k_{2i}, \frac{\gamma}{\mu}, \frac{\delta_q}{\eta_q}, \vartheta_{qi} \sigma_{qi}\}$, and $v = \frac{1}{2} \sum_{q=1}^r \frac{\delta_q}{\eta_q} \varepsilon_q^2 + \frac{1}{2} \sum_{i=1}^n \sum_{q=1}^r \vartheta_{qi} \|W_{qi}\|^2$. With Lemma 1 and Lemma 2, then inequality (36) can be transformed as following

$$\dot{V}_3 \leq -\varpi V_3 + v \quad (37)$$

Multiplying $e^{\varpi t}$ in both sides of (37), and integrating over $[0, t]$, it becomes

$$V_3 \leq [V_3(0) - \frac{v}{\varpi}] e^{-\varpi t} + \frac{v}{\varpi} \leq V_3(0) + \frac{v}{\varpi} \quad (38)$$

Based on (7), then

$$-\underline{\Omega}_i(t) \leq z_1(t) \leq \bar{\Omega}_i(t) \quad (39)$$

where $\underline{\Omega}_i(t) = h_{a_i}(t) \{1 - e^{-2[V_3(0) + \frac{v}{\varpi}]} \}^{\frac{1}{2}}$, $\bar{\Omega}_i(t) = h_{b_i}(t) \{1 - e^{-2[V_3(0) + \frac{v}{\varpi}]} \}^{\frac{1}{2}}$. Thanks to Assumption 1-2, the bounded condition $\underline{h}_c(0) < y(0) < \bar{h}_c(0)$ can be satisfied, at the same time, $-h_{a_i}(0) < z_i(0) < h_{b_i}(0)$ be also hold based on the functions h_{a_i} and h_{b_i} , which means that $|\zeta_i(0)| < 1$. From the analysis of Lemma 2, $|\zeta_i(t)| < 1$ can be ensured, and $-h_{a_i}(t) < z_{1i}(t) < h_{b_i}(t)$ also can be guaranteed. Because of $x_{1i} = z_{1i} + y_{di}$, so it gets

$$y_{di}(t) - h_{a_i}(t) < x_{1i}(t) < h_{b_i}(t) + y_{di}(t) \quad (40)$$

With the inequality (40), we can get the results that the output of robot manipulator (1) can be guaranteed as $\underline{h}_s(t) \leq y(t) \leq$

$\bar{h}_s(t)$ ($\forall t \geq 0$). Under the condition of time-varying output constraints, we know that the output of the closed-loop system can not be violated.

From (17), it is obviously that the virtual control β_{1i} is bounded, and $\|z_2\| \leq 2\sqrt{V_3(0) + \frac{v}{\omega}}$ is also be bounded. Since the definition $z_2 = x_2 - \beta_1$, hence x_2 is bounded too. With the updated laws in (21)-(23), the parameters ρ , \hat{L} and $\hat{\varepsilon}$ can be ensured to be bounded. These completed the proof process of Theorem 1.

Remark 3: To summarize the RBFNNs adaptive control design method according to the analysis of Theorem 1, the Global Stability procedures of the control algorithm is described as the following two steps:

(i) The tracking errors will be drawn back to the sliding surface $s = 0$ by the updated laws (21)-(23) with the case of $\|z_2\| > \alpha|\rho|$, which implies $\|X\| > \alpha|\rho|$ because of $\|X\| > \|z_2\|$. With this sliding model surface, although the state of unknown robot manipulator system go out of the the defined domain that is designed in advance, which can be entered into the approximation domain field by applying adaptive laws.

(ii) Due to the sliding model surface $s = 0$, $\|z_2\| \leq \alpha|\rho|$ can be ensured. So on the approximation domain of $U_0 = \{X \mid \|X\| \leq |\rho|\alpha\}$, the RBFNNs as Fig.1 be utilized to approximate the unknown model of robot manipulators.

Accordingly, we have the conclusion that the tracking errors and the states of the closed-loop system can achieve GUUB whatever go outside or stay inside the approximation domain of RBFNNs by the above two steps.

IV. SIMULATION EXAMPLE

In this simulation studies section, the coordinate system of robot manipulator with 3-DOF as shown in Fig.3 to illustrate the RBFNNs adaptive controller. The dynamical robot manipulator system can be written as (1) with the parameters in Table 1.

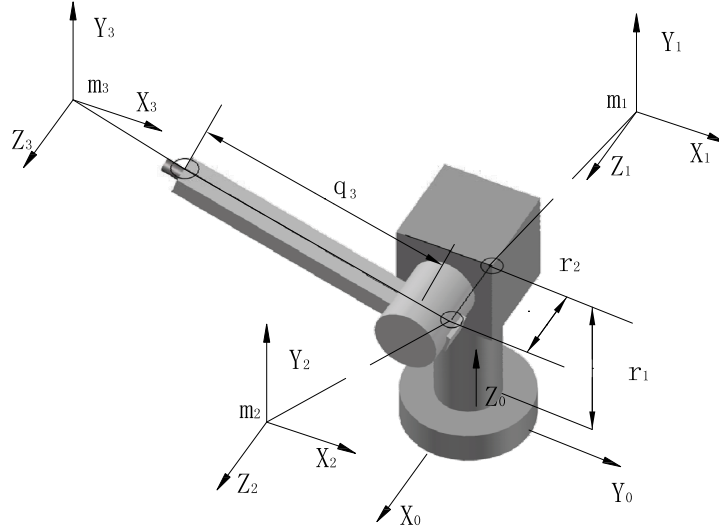


Fig. 3. The coordinates frame of robot manipulator with 3-DOF

$$M(q) = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} \end{bmatrix}, \tilde{F} = \begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \end{bmatrix}, C(q, \dot{q}) = \begin{bmatrix} \mathcal{C}_{11} & \mathcal{C}_{12} & \mathcal{C}_{13} \\ \mathcal{C}_{21} & \mathcal{C}_{22} & \mathcal{C}_{23} \\ \mathcal{C}_{31} & \mathcal{C}_{32} & \mathcal{C}_{33} \end{bmatrix}, G(q) = \begin{bmatrix} \mathcal{G}_{11} \\ \mathcal{G}_{21} \\ \mathcal{G}_{31} \end{bmatrix}, \text{ with } \mathcal{M}_{11} = m_3 q_3^2 \sin^2(q_2) + m_3 r_1^2 + m_2 r_1^2 + \frac{1}{4} m_1 r_1^2, \mathcal{M}_{12} = m_3 q_3 r_1 \cos(q_2), \mathcal{M}_{13} = m_3 r_1 \sin(q_2), \mathcal{M}_{21} = \mathcal{M}_{12}, \mathcal{M}_{22} = m_3 q_3^2 + \frac{1}{4} m_2 r_1^2, \mathcal{M}_{23} = \mathcal{M}_{32} = 0, \mathcal{M}_{31} = \mathcal{M}_{13}, \mathcal{M}_{33} = m_3, \mathcal{C}_{11} = m_3 q_3^2 \sin(q_2) \cos(q_2) \dot{q}_2 + m_3 q_3^2 \sin^2(q_2) \dot{q}_3, \mathcal{C}_{12} = m_3 q_3^2 \sin(q_2) \cos(q_2) \dot{q}_1 - m_3 r_1 q_3 \sin(q_2) \dot{q}_2 - m_3 r_1 q_3 \sin(q_2) \dot{q}_3, \mathcal{C}_{13} = m_3 q_3^2 \sin^2(q_2) \dot{q}_1 - m_3 r_1 q_3 \sin(q_2) \dot{q}_2, \mathcal{C}_{21} = -m_3 q_3 \sin(q_2) \cos(q_2) \dot{q}_1, \mathcal{C}_{22} = m_3 q_3 \dot{q}_3, \mathcal{C}_{23} = m_3 q_3 \dot{q}_3, \mathcal{C}_{31} = -m_3 q_3 \sin^2(q_2) \dot{q}_1 + m_3 r_1 \cos(q_2) \dot{q}_2, \mathcal{C}_{32} = m_3 r_1 \cos(q_2) \dot{q}_1 - m_3 q_3 \dot{q}_2, \mathcal{C}_{33} = 0, \mathcal{G}_{11} = 0, \mathcal{G}_{21} = -m_3 g q_3 \cos(q_2), \mathcal{G}_{31} = -m_3 g \sin(q_2). J(q) = I_{3 \times 3}, f_{11} = \sin(t) + 2.9, f_{21} = 2 \cos(t) + 1.2, f_{31} = 3 \sin(t) + 2. The output$$

TABLE I
THE PARAMETERS OF ROBOT MANIPULATORS.

r_1	r_2	m_1	m_2	m_3	g
0.3 m	0.4 m	2 kg	2 kg	1 kg	9.8 m/s ²

variable is denoted as $q = [q_1, q_2, q_3]^T = [x_{11}, x_{12}, x_{13}]^T$. The initial positions are chosen as $x_1(0) = [1, 0.5, 0.2]^T$, $x_2(0) = [0, 0, 0]^T$. The given desired trajectory beforehand is described as $y_{d1} = 1.2 \sin(2t)$, $y_{d2} = \sin(3t)$, $y_{d3} = 0.5 \sin(2t) + 0.6 \cos(t)$. the output constraints with time-varying are defined as $h_{s_1} = [h_{s_{11}}, h_{s_{12}}, h_{s_{13}}]^T = [-0.3 \sin(t) - 1.6, \sin(3t) - 0.5, -0.4 \sin(t) -$

$0.9]^T$, $h_{s_2} = [h_{s_{21}}, h_{s_{22}}, h_{s_{23}}]^T = [\cos(2t) + 0.5, 0.6 \sin(t) + 0.8, 0.3 \cos(2t) + 1.4]^T$. The original values of adaptive laws are given as $\rho(0) = 0.2$, $\hat{\varepsilon}_1(0) = 0.3$, $\hat{\varepsilon}_2(0) = 0.9$, $\hat{\varepsilon}_3(0) = 0.6$. The parameters in control selected as $k_{21} = 2000$, $k_{22} = 1500$, $k_{23} = 1200$, $L = 0.001$, $\alpha = 2$, $\gamma = 0.003$, $\mu = 0.0005$, $\delta_1 = 2$, $\delta_2 = 3$, $\delta_3 = 4$, $\eta_1 = 0.001$, $\eta_2 = 0.002$, $\eta_3 = 0.003$, $\varrho_1 = 0.002$, $\varrho_2 = 0.003$, $\varrho_3 = 0.004$. In this simulation, nine RBFNNs such as the structure of Fig.1 be used to approximate nine unknown homogeneous function. $\hat{W}_{11}^T \phi_{11}(\frac{X}{\rho})$, $\hat{W}_{12}^T \phi_{12}(\frac{X}{\rho})$ and $\hat{W}_{13}^T \phi_{13}(\frac{X}{\rho})$ are built to deal with the three unknown homogeneous function φ_{11} , φ_{12} and φ_{13} with degree 1. Another three RBFNNs $\hat{W}_{21}^T \phi_{21}(\frac{X}{\rho})$, $\hat{W}_{22}^T \phi_{22}(\frac{X}{\rho})$ and $\hat{W}_{23}^T \phi_{23}(\frac{X}{\rho})$ are also be used to compensate for other three unknown model homogeneous functions φ_{21} , φ_{22} and φ_{23} with degree 2. Equally, for the uncertain homogeneous functions φ_{31} , φ_{32} and φ_{33} with degree 3, $\hat{W}_{31}^T \phi_{31}(\frac{X}{\rho})$, $\hat{W}_{32}^T \phi_{32}(\frac{X}{\rho})$ and $\hat{W}_{33}^T \phi_{33}(\frac{X}{\rho})$ are employed. The known bounded positive constants are chosen as $\bar{\varphi}_1 = 5$, $\bar{\varphi}_2 = 10$, $\bar{\varphi}_3 = 15$, respectively. For each RBFNN, the number of nodes be chosen as 20 with center space in $[-0.4, 0.4] \times [-2.8, 2.8] \times [-2, 2] \times [-1.6, 1.6] \times [-0.6, 0.6] \times [-1.1, 1.1]$ and widths 5. $\sigma_{11} = 0.3I_{20 \times 20}$, $\sigma_{12} = 0.9I_{20 \times 20}$, $\sigma_{13} = 0.35I_{20 \times 20}$, $\sigma_{21} = 0.3I_{20 \times 20}$, $\sigma_{22} = 1.2I_{20 \times 20}$, $\sigma_{23} = 1.05I_{20 \times 20}$, $\sigma_{31} = 0.3I_{20 \times 20}$, $\sigma_{32} = 0.3I_{20 \times 20}$, $\sigma_{33} = 0.8I_{20 \times 20}$. The parameters ϑ_{ij} are selected as different values between 0.001 and 0.009 randomly. The corresponding simulation results of robot manipulators as shown in Figs. 4-10.

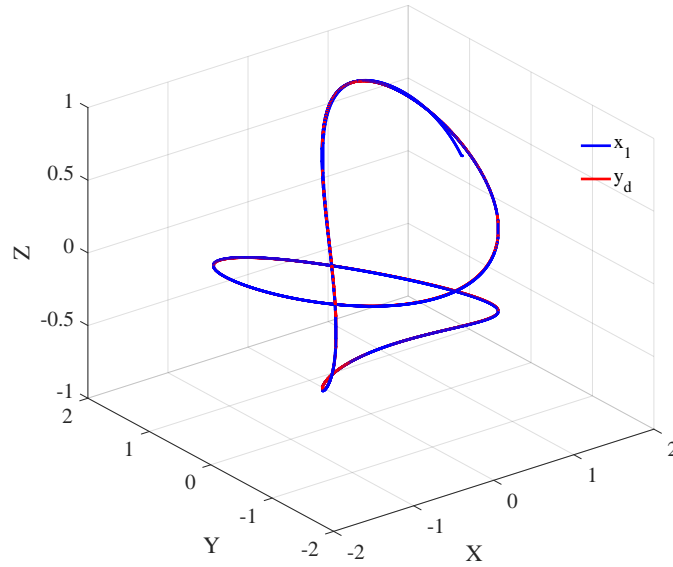


Fig. 4. The tracking curves of robot manipulators

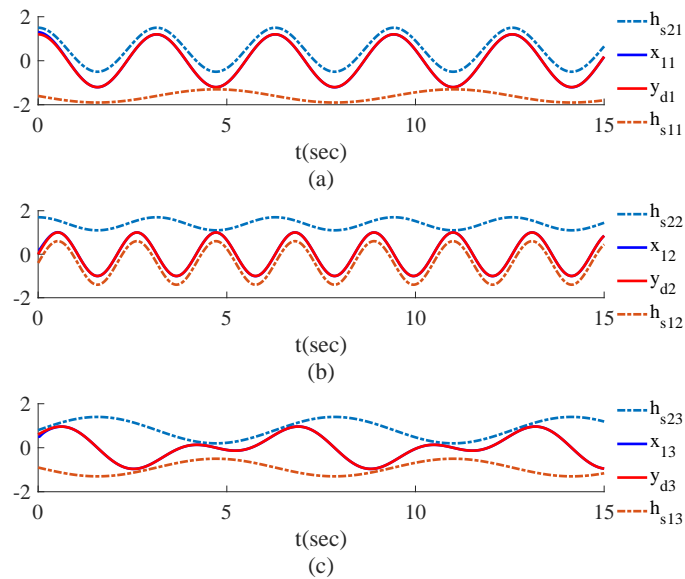


Fig. 5. The response curves of output x_1 with time-varying output constraint and reference trajectory y_d

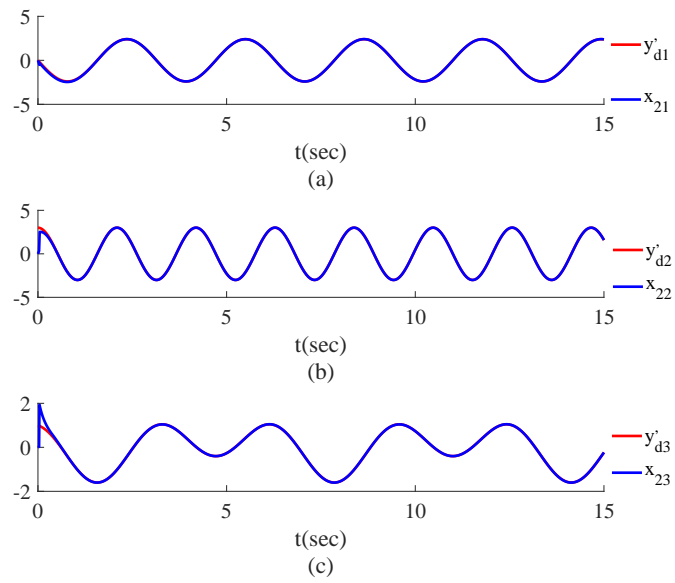


Fig. 6. The velocity response curves tracking trajectory

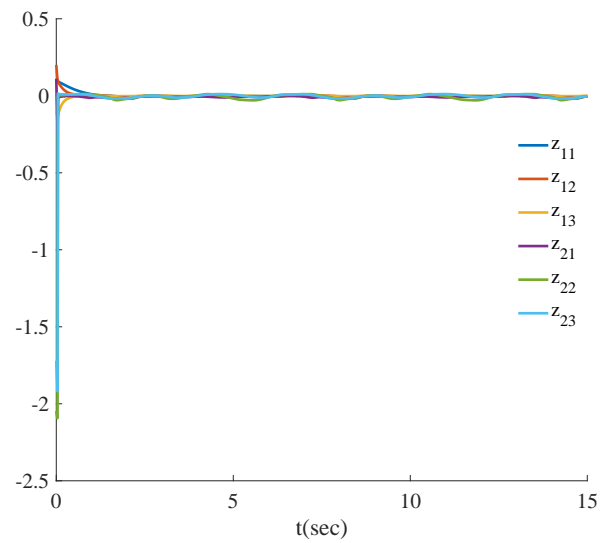


Fig. 7. The tracking error z_{11} , z_{12} , z_{13} , z_{21} , z_{22} and z_{23}

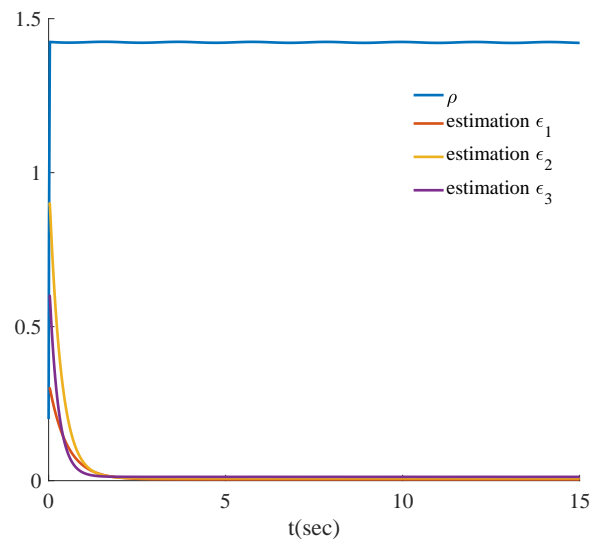


Fig. 8. The parameters response curves in controller (20) with (21)-(23)

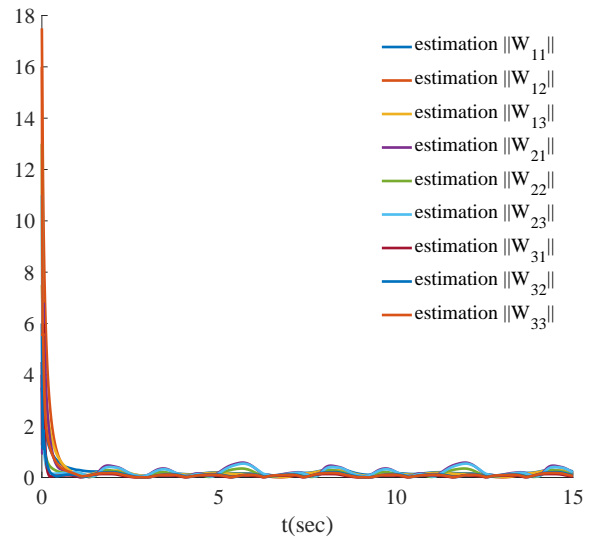


Fig. 9. The weight norm in RBFNNs controller (20)

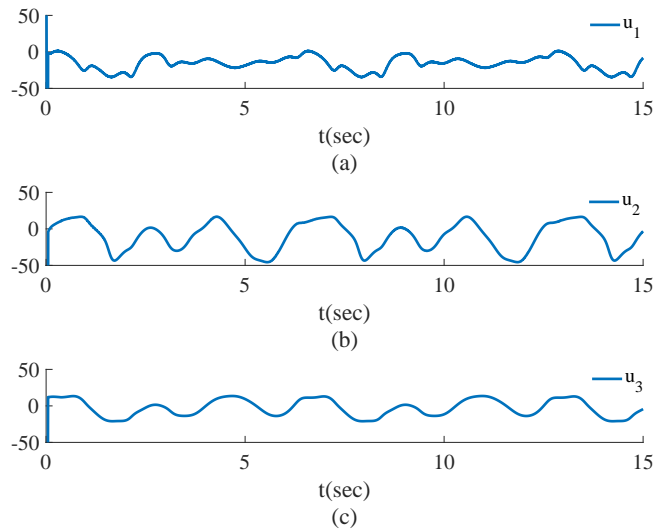


Fig. 10. Control input of RBFNNs controller (20)

Fig.4 shows that the output of the robot manipulators and the ideal references with good tracking performance. The output of the three robot manipulators with output time-varying constraints are shown in Fig.5, and the velocity of references are depicted shown in Fig.6, in which good tracking index be guaranteed. It is obviously that the output can not be destroyed. The output of the three robot manipulators and Fig.7 illustrate the performance of small tracking errors can be fulfilled to zero field. The adaptive parameters in RBFNN controller can be updated on-line automatically shown in Fig 8 with bounded, the norm of wight in RBFNNs and the input control are also be ensured uniformly ultimately bounded as shown in Fig.9 and Fig.10.

V. CONCLUSIONS

An adaptive RBFNNs controller with non-zero parameter in the device of RBFNN is proposed for a robot manipulators with the output time-varying constraints. RBFNN with time-varying parameter is used to deal with the unstructured uncertainties systems, in which the nature of homogeneous function can be used to deduce another originality universal approximation, the global stability can be achieved by the RBFNNs with non-zero updated parameter, with which the unknown model robot manipulators can be estimated and good tracking performance can be obtained by the design method in this paper. The approximation accuracies of RBFNNs can be on-line updated automatically compare with other RBFNN controls.

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