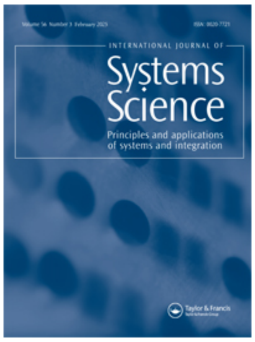

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U-control – a universal platform for control system design with inversion/cancellation of nonlinearity, dynamic and coupling through model-based to model-free procedures

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ABSTRACT

This paper presents the key components in the Universal(U)-control framework for designing dynamic control systems from model-based to model-free paradigms, in which the pillars include U-control system configuration, open-loop dynamic inversion, closed-loop dynamic inversion, general model-free cancellation of nonlinear-dynamic-coupling effect, model-free-sliding-mode control (MFSMC) and model-free composite nonlinear feedback (CNF) control. This paper is devoted to delivering intuitive and easy-to-understand explanations for the involved approaches with a series of schematic diagrams. Readers can refer to the corresponding publications for more details on the theoretical analyses. In addition, some showcase examples are provided in this paper to facilitate the understanding of certain important concepts.

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Universal(U)-control; U-model; dynamic inversion; decoupling; model-free-sliding-mode control (MFSMC); data-driven control; Lyapunov stability; Lyapunov differential inequality

1. Introduction

1.1. Configuration of Universal(U)-control

The U-control system for a general class of single-input and single-output (SISO) dynamic plants with reference input $r \in \mathbb{R}$ and system output $y \in \mathbb{R}$ is defined as

$$\Sigma_U : (\Psi, \Omega(C_{IV}, \hat{P}^{-1}), P) \Leftrightarrow (\Psi, C_{IV}, I_n) \Leftrightarrow (\Psi, C_{IV}) \tag{1}$$

where Ψ denotes the control system configuration, P is the nth-order dynamic plant which will be specified in Section 2, $\boldsymbol{\Omega}$ is the controller set with two functionalities for the dynamic inversion $\hat{P}^{-1}$ and the closed-loop performance specification C_{IV} in cascade, I_n is the identity matrix for the nth-order dynamic plants in state-space expressions and $I_n = 1$ for the plants in polynomial expressions. Consequently, the U-control represents a type of mapping from the reference input space to the system output space, $\Sigma_U : r \rightarrow y$.

There are two technique pillars, plant dynamic inversion (DI) and separation of control performance with dynamic plant, in the U-control configuration, which are structured with two design platforms to cover the whole model spectrum from model-based to model-free.

1.1.1. Model-based U-control

This is defined as

$$\Sigma_U : (\Psi, \Omega(C_{IV}, \hat{P}^{-1}), P) \Leftrightarrow (\Psi, C_{IV}, (P^{-1}, P) \in I_n) \Leftrightarrow (\Psi, C_{IV}, I_n) \Leftrightarrow (\Psi, C_{IV}) \tag{2}$$

Figure 1 illustrates the model-based U-control system (Zhu & Guo, 2002), in which $y \in \mathbb{R}$ and $r \in \mathbb{R}$ are the system output and reference input, respectively. P is the dynamic plant and $u \in \mathbb{R}$ is the plant input. As shown in the dotted block, the controller is composed of two cascaded function blocks, P^{-1} is the plant instant dynamic inverter (IDI) to achieve $P^{-1}P = 1$, C_{IV} is the invariant controller (normally a linear dynamic component) to specify the closed-loop performance $(Y(s) = C_{IV}(s)(1 + C_{IV}(s))^{-1}R(s)$ in terms of Laplace

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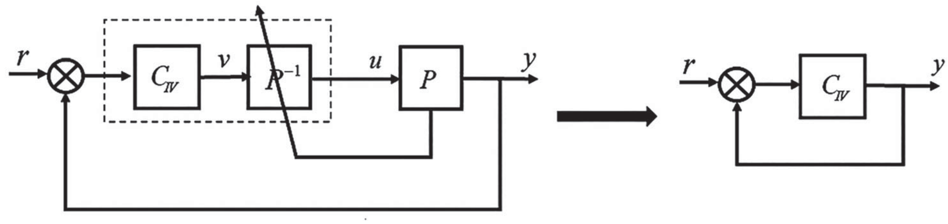

Figure 1. Model-based U-control system.

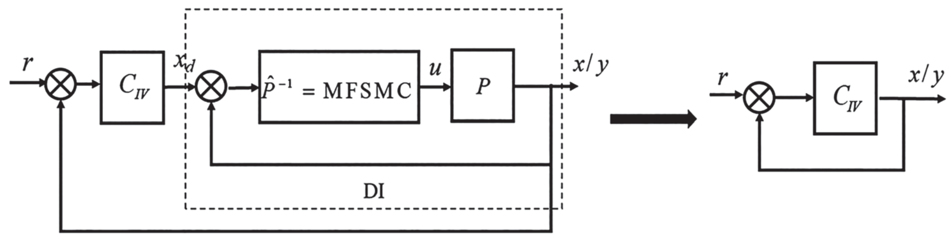

Figure 2. Model-free U-control system.

operator s) and $v \in \mathbb{R}$ is the controller output. The resultant system on the right side indicates that the plant has been cancelled into a unit, and therefore C_{IV} is invariant and independent of the plant models.

1.1.2. Model-free U-control

This is defined as

$$\begin{aligned}\Sigma_U : \ & (\Psi, \Omega(C_{IV}, \hat{P}^{-1}), P) \\ & \Leftrightarrow (\Psi, C_{IV}, C_{DI}(\hat{P}^{-1}, P) \in I_n) \\ & \Leftrightarrow (\Psi, C_{IV}, I_n) \Leftrightarrow (\Psi, C_{IV}) \end{aligned} \tag{3}$$

Figure 2 shows the configuration of the model-free U-control systems (Zhu, 2021). As shown in Figure 2, the plant P is regarded as a total uncertainty in a black box with enabling input u and obtainable state/output x/y. The asymptotic dynamic inversion (ADI) $x \rightarrow x_d, \forall C_{DI}(\hat{P}^{-1}, P)$ is obtained by an asymptotic stabilisation process in the inner closed loop, which $\hat{P}^{-1} = \text{MFSMC}$ is a model-free sliding mode controller/inverter (Zhu, 2021). The resultant system on the right side shows that the plant has been reversed to either a unit constant or a unit matrix. As such, C_{IV} is an invariant controller and is independent of the plant dynamics. While C_{IV} is a linear controller, the closed-loop performance is achieved with $Y(s) = C_{IV}(s)(1 + C_{IV}(s))^{-1}R(s)$ in terms of the Laplace operator s.

1.2. Characteristics of U-control

The phrase of Universal-control comes from the following considerations.

(1) The universality of the U-control design covers the whole model spectrum from model-based to model-free and separates the plant dynamic treatment from the control system performance specification. For example, refer to Figure 2, separate the robust stabilisation for the plant asymptotic dynamic inversion in the inner loop and control system performance specification in the outer loop independently. Another facilitation of the U-control is that it can be used for motion control (many mechanical systems) with a single inner loop (Li, Zhu, et al., 2023) and setpoint control (many industrial process operations) with the double loop (Geng et al., 2019) in Figure 2.
(2) Universal supplement to the conventional control system design approaches such as pole placement control (Zhu & Guo, 2002), Smith predictive control (Geng et al., 2019), Composite nonlinear feedback control (Zhu, Mobayen, et al., 2023), decentralise control (Zhu, Li, et al., 2023b), decoupling control (Zhu, Li, et al., 2023a), control with hard nonlinear inputs, underactuated control (Hussain et al., 2020), which have been expanded generically applicable to nonlinear non-affine dynamic systems besides augmenting the

control approach functionalities. For example, the composite nonlinear control (CNF), a fundamental approach, before U-control, the CFN has been uniquely designed to take the parallel control structure (Chen et al., 2003; Lin et al., 1998;). Now U-control has derived a type of cascade control for the CNF, which enhances the CNF in the form of model-free and applicable to nonlinear non-affine dynamic plants (Zhu, Mobayen, et al., 2023). In short, U-control is seamlessly integrated and complemented with many of the existing control methodologies.

(3) Generalised universal dynamic inversion with two new concepts and algorithms, instant dynamic inversion (IDI) for the model-based U-control and asymptotic dynamic inversion (ADI) for the model-free U-control. These could be the enhancements to the other existing dynamic inversion approaches. Furthermore, the dynamic inversion has expanded the conventional cancellation of nonlinearity and dynamics to include the decoupling, a type of cancellation of the coupling, in the fully actuated MIMO systems (Zhu, Li, et al., 2023a).

(4) Generalisation of model-free control. Regarding control system design, there have been two predominant methods: (1) offline physical-model-based design (such as pole placement control and linear quadratic regulation/LQR) (Fisher & Bhattacharya, 2009; Slotine & Li, 1991) and (2) data-driven adaptive (Astrom & Wittenmark, 1995) pointwise-model-based design (such as adaptive control, neural network enhanced control and the other traditional named model-free controls), which is conventionally called < model-free control > (Hou & Xiong, 2019). It should be noted that the conventional model-free control is < physical model-free control > but still uses < pointwise model – data-driven model > (Hou & Wang, 2013) to design the adaptive controllers. In brief, both methods use model-based control, the difference between them is if the physical model (offline model) or data-driven pointwise model (online model) is used. In addition, for those other data-driven control systems, even not using a pointwise model, they still use data iteration to make control decisions, which could be avoided in some way in the U-control. The model-free U-control proposes a new method, total-model-free robust control (TMFRC), which treats plants as a total uncertainty in a black box with enabled control inputs and attainable outputs (either measured or estimated). By removing the design requirement on a physical model, data-driven pointwise model and data iteration, the TMFRC provides merits with generality and simplicity in the control system design and robustness in the control functionality and the parameter tuning. In technique, the TMFRC uses a model-free sliding mode control (MFSMC) that has been developed by satisfying the Lyapunov differential inequality, rather than solving the Lyapunov differential equality in model-based SMC. Accordingly, the model-free U-control system is assessed with the Lyapunov robust stability conditions. Compared to the model-free PID controller tuning by trial and error, the TMFRC keeps the control system performance invariant by satisfying the Lyapunov differential inequality within the system boundary, rather like PID maintains the control system performance variation point by point even stability remained. A good example to illustrate is the Simulink PID auto-tuning function. Further if tuning by experience, such trial-error-based PID tuning is emendated with tedious tries of the three gain combinations.

(5) Expansion of robustness sensitivity function (RSF) analysis. As the total mode-free control is proposed, naturally how to evaluate its robustness is an issue to address. Conventional RSF is a model-based analysis. The U-control study has proposed a total robustness sensitivity function (TRSF) for the model-free U-control systems (Zhu, Li, et al., 2023a). The TRSF is still required for further justification and formulation in theoretical aspects.

(6) The feasibility and generality of U-control have been demonstrated by simulations and real systems in universities and companies across countries (Hussain et al., 2020; Wei et al., 2022, plus real bench testing papers under preparation in the leading author's team).

1.3. Literature review

For completeness, the following critical literature review is presented in comparison of the representative approaches in the related topics, which justifies the motivation of the proposed U-control.

1.3.1. Dynamic inversion for control

It has been observed that the most popular methodology in nonlinear control design is the dynamic inversion in one way or another (cancelling the dynamic model to generate a reference feedforward compensation action under the assumption that the referred plant dynamical model is an acceptable representation of real plant) (Isidori, 1995). Accordingly, dynamic inversion is one of the key components in nonlinear control system design (Steffensen et al., 2023; Vu, 1997; Zhang et al., 2020). During the implementation of the nonlinear dynamic inversion methods, the nonlinear dynamics are inverted or compensated for designing a control law that cancels out the nonlinear effects, which makes the system behave as if it were linear. The basic idea behind the dynamic inversion is to first design a controller with a specified linear dynamic system around a desired operating point, and then use such a controller as a starting point to compensate for the impacts of nonlinearities existing in the actual system. This kind of compensation is typically achieved by adding terms to the controller that counteracts the nonlinear effects and accordingly inverting their influence on the system behaviour. The dynamic inversion approach is particularly well-suited for systems with complex and highly nonlinear dynamics, to which the traditional linear control techniques may be inapplicable. In recent years, the dynamic inversion method has found successful applications in a diversity of areas such as aerospace, robotics, automotive control and other fields where the precise control of nonlinear systems is required (Acquatella et al., 2020; Li, Liu, et al., 2023; Steffensen et al., 2023; Taherinezhad & Ramirez-Serrano, 2023).

1.3.2. Model-based control

There are various model-based dynamic inversion approaches in the existing literature. Among them, the feedback linearisation control (FLC) has stood out as a typical method (Slotine & Li, 1991), where the nonlinear model is converted into a linear one through coordinate conversions. Another popular method is the so-called backstepping control (BSC) (Zhang et al., 2023), which is a Lyapunov-based recursive design approach and can accomplish the stabilisation and tracking tasks for a specific set of complex nonlinear systems. Nevertheless, the model-based dynamic inversion method has the following three shortcomings: (1) The performance heavily relies on the model's accuracy. Clearly, in most practical scenarios, it is by no means a trivial task to establish an accurate system model. In this context, it would be difficult to design proper controllers for nonlinear systems. (2) When it comes to the popular model-based dynamic inversion approaches, the FLC first utilises the complex coordinate transformation to design a linear control system and then transforms the designed controller to the original controller to generate the required controller output. Unfortunately, such a method requires strong Lie algebra conditions (Duan, 2021). For BSC, it is normally applicable to a class of triangle types of models. For example, the BSC is effective for second-order dynamics. However, the complexity would exponentially increase with the increase of the order of system dynamics (Echiheb et al., 2023). (3) So far, almost all the existing nonlinear dynamic inversion formulations have assumed linearity with the plant input (that is, nonlinear affine plants). Accordingly, a general and concise mode-based dynamic inversion method called model-based U-control has been put forward using the expression of U-model, which converts almost all the existing models into U-realisation without resorting to the coordinate transform and backstepping process (Zhang et al., 2020; Zhu & Guo, 2002). It is worth noting that the model-based U-control can significantly relieve the complexity in designing linear and nonlinear control systems, and the U-dynamic inversion and controller design are achieved separately. However, it should be noted that like all the other model-based dynamic inversion approaches, the U-dynamic inversion performance heavily relies on the model accuracy which robustness in dealing with model uncertainty is under study. Developing a model-based U-control method constitutes the first motivation for establishing the universal control system design platform.

1.3.3. Model-free control

In existing literature, many control strategies have been developed for the plants with unknown models, and some representatives include, but are not limited to, adaptive control (AC) (Chen & Astolfi, 2022; Tao, 2014), active disturbance rejection control (ADRC) (Fareh et al., 2021; Feng & Guo, 2017; Huang & Xue, 2014) and recently appeared model-free U-control (Zhu, 2021; Zhu, 2023; Zhu, Li, et al., 2023a; Zhu, Mobayen, et al., 2023). Considering whether the model is utilised for controller design, AC is a data-driven model-based approach since it needs to estimate the online model and ADRC needs to estimate an online non-parameter model output using an extended state observer (ESO). Recently, a new class of control methods named model-free control has attracted ever-increasing research attention. In the context of model-free control, the design of the controller is independent of the physical model of the plant to be controlled. Instead, the controller is designed by learning or adapting directly from the system's input-output data or through the trial-and-error approach. One popular method falling into this category is known as reinforcement learning (Vamvoudakis et al., 2021), where a learning agent can make decisions through its interaction with the environment and the feedback it receives in the form of rewards. As time passes by, the agent gains knowledge about which actions result in greater rewards and accordingly adapts its behaviour without requiring the explicit model of the environment. Notably, the model-free control methods are particularly suitable in the case where the system dynamics are complex or poorly understood. They can also be advantageous in certain scenarios where the system is subject to changes or uncertainties and the traditional model-based control methods are no longer effective. However, the model-free control approaches also have some inherent challenges, such as requiring large amounts of data for iterative learning and being more computationally intensive than the model-based methods. Additionally, such methods may not be effective when applied to new or unseen environments. The other most popular approach is the adaptive control (Astrom & Wittenmark, 1995; Hou & Wang, 2013), which does not require a plant physical model, but estimates a data-driven pointwise model. Strictly speaking adaptive control is still a type of model-based design approach, where the offline physical model is replaced by data-driven pointwise model. Like the model-based design, adaptive requires an extra step to estimate the online model. The unsolved challenging issue is model-free decoupling (a type of cancellation that could be treated as inversion formulation) with fully actuated MIMO dynamic plants.

Considering the above discussions, it has a space to propose a model-free dynamic inversion strategy with the U-control method, thereby improving the shortcomings of the current model-free control method. Referring to Figure 2, the model-free U-control has derived an asymptotic dynamic inversion approach (Zhu, 2021, 2023), by treating the model unknown plant as a total uncertainty with enabled inputs and attainable outputs, formulating model-free dynamic inversion in the form of a closed loop to achieve the asymptotic dynamic inversion, that is, the output asymptotically converges to the reference input in the closed loop. The U-control has expanded the dynamic inversion including the MIMO system decoupling (Zhu, Li, et al., 2023a). Developing a model-free U-control method constitutes the second motivation for establishing the universal control system design platform.

The primary contributions of this study are highlighted as follows:

(1) Provide a systematic summary of the U-control methodology and configuration with existing results, which can serve as a condensed collection of the theory, algorithms, simulation and applications.
(2) Present further enhanced model-based U-control schemes with a generic form.
(3) Summarise the latest model-free U-control results in a general framework.

The remainder of this paper is outlined as follows. Section 2 provides the preliminaries for presenting model-based and model-free U-control, which include a description of the general SISO plant, U-model realisation and dynamic inversions. Section 3 presents the model-based U-control schemes including pole placement control and Smith predictive control. Section 4 provides a concise introduction to the model-free U-control strategies including model-free sliding mode control (MFSMC), composite nonlinear feedback (CNF) control, decentralised control and decoupling control. Section 5 concludes this paper.

2. Preliminaries

This section includes the formulations of (1) dynamic plant for control, (2) U-model realisation and (3) dynamic inversion.

2.1. Dynamic plant for control

Consider a general set of the nth-order single-input single-output (SISO) nonlinear plants with the following dynamics:

$$P : y^{(n)} = f(y^{(0\sim n-1)}, \theta, u) \tag{4}$$

where $y \in \mathbb{R}$ and $u \in \mathbb{R}$ are defined as the system output and input, respectively, and $y^{(0\sim n-1)} = [y \cdots y^{(n-1)}]^T \in \mathbb{R}^n$ is the output derivative vector, $\theta \in \mathbb{R}^L$ is the parameter vector in proper dimension and $f : u \to y$ is a real function, representing the mapping from the input set to the output set and is sufficiently differentiable. The control input u is to be designed to change the system characteristics by specifications in a closed-loop framework. To design the control systems, assume that the plant satisfies the following constraints.

Remark 2.1: While f is known, it can be described by both polynomial and state-space equations. The corresponding canonical state-space equation can be derived by letting $x = y^{(0\sim n-1)} = [x_1 = y \quad x_2 = \dot{y} \quad \cdots \quad x_{n-1} = y^{(n-1)}]^T \in \mathbb{R}^n$ and $x_n = f(x, \theta, u)$ (Zhu, 2023).

Assumption 2.1: The plant is locally constrained by the Lipschitz condition $|f(x_1) - f(x_2)| \le K_L|x_1 - x_2|$, $K_L \in \mathbb{R}_{\ge 0}$. Consequently, the plant (4) can describe the discontinuous differential equation with piecewise continuous inputs as well.

Assumption 2.2: The plant is Bounded-Input-Bounded-Output (BIBO). Specifically, for the finite values $B_u > 0$ and $B_y > 0$, the plant's input and output signals satisfy $\exists B_u, \forall t(|y(t)| \le B_u)$ and $\exists B_y, \forall t(|y(t)| \le B_y)$.

Assumption 2.3: The generalisation of the controllability and observability of nonlinear control systems are formulated either based on linearisation around an equilibrium point or based on the concepts of differential geometry (Zabczyk, 2020). The plant is assumed to satisfy the observable/controllable conditions, in which the corresponding matrices derived from the Lie derivative have full ranks and proper dimensions in the linearisation formulation and proper algebra in the form of the differential geometry.

Assumption 2.4: The plant is a class of asymptotically stable zero dynamics, which have the minimum phase properties. Therefore, plant invertibility exists and is stable.

Assumption 2.5: For a disturbance $d \in \mathbb{R}$ added at plant (4) in the form of $y^{(n)} = f(y^{(0\sim n-1)}, u, d)$, the amplitude and derivative of the disturbance are bounded. That is, $\exists D, \forall t, |d(t)| \le D = \sup(|d|) \in \mathbb{R}^+$ and $\exists D_d, \forall t, |\dot{d}(t)| \le D_d = \sup(|\dot{d}|) \in \mathbb{R}^+$.

Assumption 2.6: While the plant (4) model is unknown, which is deemed as a total uncertainty in a black box, it is assumed that its input is enabled to drive the plant to generate the attainable outputs.

2.2. U-model

With reference to plant (4), the U-model (Zhu et al., 2016) is defined as an expanded polynomial from the nonlinear function $f(*)$ in the following expression of

$$\begin{aligned} U_m(u) : y^{(n)} &= U(\alpha(y^{(0\sim n-1)}, \theta), \mu(u)) \\ &= \sum_{i=0}^{M} \alpha_i \mu_i(u) \end{aligned} \tag{5}$$

where M is the number of the model input (controller output) function of u, $\mu_i(u)$ are the functions of u, such as u^3 and $\sin(u)$ and the time-varying parameter vector $\alpha(t) = [\ \alpha_0(t) \quad \cdots \quad \alpha_M(t)\ ] \in \mathbb{R}^{M+1}$ is a function of $y^{(0\sim n-1)}$ and the parameter vector $\theta = (\theta_0 \quad \cdots \quad \theta_L) \in \mathbb{R}^L$.

Property 2.1: Let $\psi : \mathbb{R}^{L+1} \to \mathbb{R}^{M+1}$ be the map from plant (4) to the U-model and its inverse be ψ^{-1}, that is, $f(y^{(0\sim n-1)}, \theta, u) \xrightarrow{\psi} U(\alpha(y^{(0\sim n-1)}, \theta), \mu(u))$. Then it has the following properties (Zhu et al., 2016)

(1) The map is injective (one-to-one)
(2) The map is surjective (onto)
(3) Therefore, the map is bijective as it is both injective and surjective.
(4) The map is invertible.
(5) The map does not change any of both model characteristics, such as output response, stability, dynamics and statics.

Remark 2.2: The U-model does not change the original plant (4) properties such as dynamics, stability and input and output external relationship. However, by such mapping (re-organisation) of the model expression of control input oriented with time-varying parameters, U-model provides a generic control-oriented prototype to facilitate dynamic inversion and separation of the plant dynamics from the control system design specifications to generate the invariant controller (Zhang et al., 2020).

Remark 2.3: It should be noted that the U-model is only used for model-based U-control. For model-free U-control plant (4) is treated as a total uncertainty in a black box with enabling input and measurable output (Zhu, 2021, 2023).

2.3. Dynamic inversions

There are two types of dynamic inversions associated with the U-control, instant dynamic inversion (IDI) for the model-free U-control and asymptotic dynamic inversion (ADI) for the model-free U-control.

The instant dynamic inversion (IDI) (Li et al., 2020) is a procedure to take the inversion of a given U-model (5) to determine the control input u with a specified desired output $y_d^{(n)}$, which is defined as follows:

$$U_m^{-1}(u) : u \in y_d^{(n)} - U(\alpha(y^{(0\sim n-1)}, \theta), \mu(u)) = y_d^{(n)} - \sum_{i=0}^{M} \alpha_i \mu_i(u) = 0 \tag{6}$$

Remark 2.4: The solution of the polynomial equation gives $y^{(n)} = y_d^{(n)} \leftrightarrow y^{(n)} - y_d^{(n)} = 0 \leftrightarrow y^{(n)}(y_d^{(n)})^{-1} = 1$, Therefore, it gives $U(u)U_m^{-1}(u) = U_m^{-1}(u)U(u) \to y^{(n)} = y_d^{(n)}$.

While the plant (4) model is unknown, deemed as a total uncertainty in a black box with enabled input and attainable output. The asymptotic dynamic inversion (ADI) has been proposed using a closed loop, the inner loop, as shown in Figure 2. The ADI is defined as follows:

$$x = C_{DI}(x_d, \hat{P}^{-1}, P) \overset{asymp}{\longrightarrow} x = x_d \leftrightarrow x - x_d = 0_n \leftrightarrow x(x_d)^{-1} = I_n \tag{7}$$

where x and x_d are the plant state vector and the desired state vector, respectively. As shown in Figure 2, C_{DI} denotes a closed-look dynamic inversion, P is the model unknown plant, $\hat{P}^{-1}$ is a dynamic inversion controller, currently, a model-free sliding mode controller is used (Zhu, 2021).

Remark 2.5: The dynamic inversions share two algebra properties, $U(u)U_m^{-1}(u) = 1$ in multiplication and $U(u) - (U_m^{-1}(u))^{-1} = 0$ in summation. They are equivalent in terms of inversion operation.

Remark 2.6: As the IDI is derived from the accurate model, its robustness against model uncertainty is still an open topic associated with the study of the model-based U-control.

To explain the U-model realisation (5) from (4) and its model-based dynamic inversion, let us pick up a few exemplary models.

(1) Linear continuous time model: $\ddot{y} = -2\dot{y} - 3y + 4u$. The U-realisation is expressed as $\ddot{y} = \alpha_0 + \alpha_1 u, \forall \alpha_0 = -2\dot{y} - 3y, \alpha_1 = 4$. Consequently, the inversion is formulated by $u = \frac{\ddot{y} - \alpha_0}{\alpha_1}$.
(2) Nonlinear discrete-time (here t is used for the discrete-time instant) rational model (Zhu et al., 2018): $y(t) = \frac{0.5y(t-1) + \sin(u(t-1)) + u(t-1)}{1 + exp(-y^2(t-1))}$. The U-realisation is expressed as $y(t) = \alpha_0 + \alpha_1 \sin(u(t-1)) + \alpha_2 u(t-1)$, $\forall \alpha_0 = \frac{0.5y(t-1)}{1+exp(-y^2(t-1))}, \alpha_{1,2} = \frac{1}{1+exp(-y^2(t-1))}$. Consequently, the inversion is formulated by $u : y(t) - \alpha_0 + \alpha_1 \sin(u(t-1)) + \alpha_2 u(t-1) = 0$.
(3) Linear state-space model: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 7 \end{bmatrix} u$. To determine the control u, express the U-realisation as $\dot{x}_2 = \alpha_0 + \alpha_1 u, \forall \alpha_0 = -5x_1 - 6x_2, \alpha_1 = 7$. Consequently, the inversion is formulated by $u = \frac{x_2 - \alpha_0}{\alpha_1}$.
(4) Non-affine nonlinear dynamic model (Zhu, 2023): $\dddot{\xi} + \xi\dot{\xi} + \upsilon\dot{\xi} + 0.6\xi - \sin(\upsilon) - 2\upsilon - \upsilon^3 = 0$, where (ξ, υ) denotes a pair of the model output and input. The U-realisation is expressed as $\dddot{\xi} = \sum_{i=0}^{4} \alpha_i \mu_i(\upsilon)$, where $(\alpha_0, \mu_0) \in (-\xi\dot{\xi} - 0.6\xi, 1)$, $(\alpha_1, \mu_1) \in (-\dot{\xi}, \upsilon)$, $(\alpha_2, \mu_2) \in (1, \sin(\upsilon))$, $(\alpha_3, \mu_3) \in (2, \upsilon)$ and $(\alpha_4, \mu_4) \in (1, \upsilon^3)$. Consequently, the inversion is formulated by $\upsilon : \dddot{\xi} - \sum_{i=0}^{4} \alpha_i \mu_i(\upsilon) = 0$.

Remark 2.7: The U-model can be deduced by resorting to the models from physical laws and/or the identified models from data. The time-varying parameters $\alpha(y^{(0\sim n-1)}, \theta) \in \mathbb{R}^{M+1}$ are used to absorb the non-current-control variables in discrete-time models and the non-control variables in continuous-time models. Consequently, the u variable polynomial is named the U-model, which is a type of control-oriented model since it can facilitate the design of control systems.

Figure 3 shows the paradigm structure of the model-based dynamic inversion, where $y_d^{(n)}$ is the desired output response. There are three types of dynamic inversion algorithms to determine the control u, which drives the plant output to achieve the desired $y_d^{(n)}$ at output.

(1) Numerical iterative Newton Raphson (N-R) root solver (Chong & Żak, 2013). Referring to the U-model equation (6), the corresponding N-R solver is formulated by

$$u_{k+1} = u_k - (y_d^{(n)} - U(u_k)) \times \left(\frac{\partial (y_d^{(n)} - U(u_k))}{\partial u_k} \right)^{-1} \tag{8}$$

where $k \geq 0$ is the iteration pointer when executing recursive calculations.

For commonly used power polynomials $y^{(n)} = \sum_{i=0}^{M} \alpha_i u^i$, (8) can be expressed as (Zhu & Guo, 2002)

$$u_{k+1} = u_k - \frac{y_d^{(n)} - \sum_{j=0}^{M} \alpha_j u_k^j}{\frac{\partial \left[y_d^{(n)} - \sum_{j=0}^{M} \alpha_j u_k^j \right]}{\partial u_k}} \tag{9}$$

(2) Functional equation root solver. Write the U-model equation (6) in the form of a transcendental polynomial equation $y_d^{(n)} - U(\alpha(y^{(0\sim n-1)}, \theta)$, $\mu(u)) = 0$. Solving such an equation requires certain solvers in Matlab or Maple software programs, such as the Matlab function used to determine the symbolic and numerical roots.

(3) U-neural network has been developed to facilitate the dynamic inversion (Zhu et al., 2019). Although such a method is initially developed for model-based computation, it has the potential for the model-free dynamic inversion by integrating the neuro-computing method with iterative training/learning to adapt the pointwise solutions.

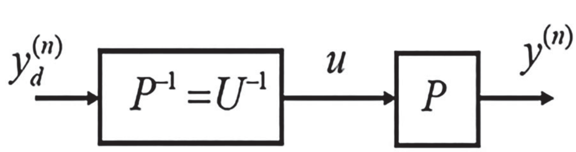

Figure 3. Model-based dynamic inversion.

3. Model-based U-control

3.1. Pole placement U-control

Pole placement control (Astrom & Wittenmark, 1995) is a technique used in control theory to design controllers for linear time-invariant (LTI) systems. The goal is to assign the closed-loop system's poles at desired positions in the Laplace coordinate system (a type of complex plane) to obtain specific properties, such as stability, transient response and robustness. In general, the implementation of the pole placement control method includes the following steps.

(1) Plant representation: This step aims to describe the dynamic behaviour of the plant as an LTI model. This model is often represented by transfer functions in the complex domain or state-space equations in the time domain.
(2) Desired pole locations: Based on the desired performance objectives, such as settling time, overshoot and stability margins, the engineer chooses the desired locations for the closed-loop poles in the complex plane.
(3) Controller design: Based on the desired pole locations, design a controller that will shift the open-loop poles to the desired positions when the feedback loop is closed. In the case of state-space representation, a full state feedback gain vector can be designed to assign the desired eigenvalues/poles in the closed-loop system. The transfer-function-based representation and the state-space-based representation, respectively, correspond to the dynamic controller and the constant gain controller.
(4) Designed system analysis: Once the controller is determined, the closed-loop system's stability, transient response and other performance metrics are analysed to ensure that the required system performance is satisfied.

It should be noted that the U-control can expand the linear plant into a general nonlinear plant in both polynomial and state-space models while keeping the closed-loop control performances achieved based on the conventional linear plant models. Moreover, the U-control method also separates its controller design and dynamic inversion, which avoids using the plant model in solving the Diophantine equation (Zhu & Guo, 2002).

3.1.1. Expanded Astrom approach (discrete time)

Figure 4 depicts the control system configuration of the Astrom approach (Astrom & Wittenmark, 1995). The control system design can be briefly explained as follows. For a given linear SISO discrete-time dynamic plant $P = \frac{B}{A}$ which B and A are the numerator polynomial and denominator polynomial, respectively in the discrete-time transfer function. To design a linear controller with $Ru = Tr - Sy$ which R, T and S are the polynomials to specify the control input u, the set point reference r and the system output y, respectively. For a given desired closed-loop system characteristic equation $A_c = 0$, solving the Diophantine equation $AR + BS = A_c$ to determine the controller polynomials R and S and $A_c(1) = T$ to achieve the zero steady-state error for a setpoint reference input. The factors in the characteristic equation are the poles for designing the closed-loop systems. This approach is only feasible for linear dynamic plants.

Figure 5 shows the expanded Astrom approach, named pole placement U-control (Zhu & Guo, 2002), which aims to make the Astrom approach generically applicable to nonlinear dynamic plants within the U-control framework. The solution to the Diophantine equation is reduced to $R + S = A_c$ due to the dynamic inversion making $P^{-1}P = 1$. Consequently, the design of pole placement U-control is independent of the plants, no matter whether they have linear or nonlinear dynamics.

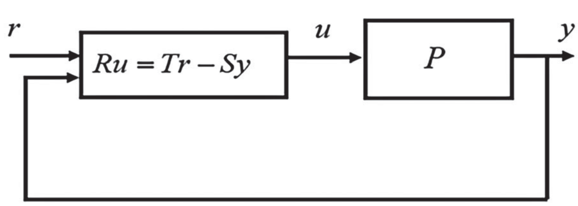

Figure 4. Astrom pole placement control system.

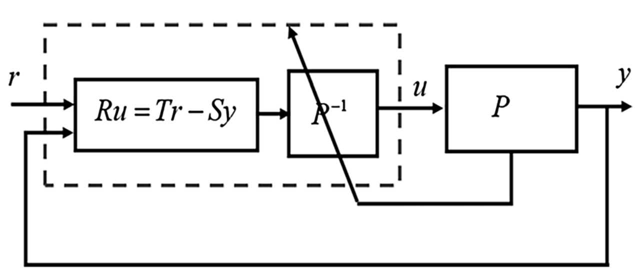

Figure 5. Expanded Astrom pole placement control system.

3.1.2. Direct pole placement (continuous time)

Figure 1 shows the structure of the direct pole placement U-control. The objectives in the design of the direct pole placement U-control include the following three aspects:

(1) Achieve model-based dynamic inversion for $P^{-1}P = 1$.
(2) Assign the control system performance in terms of closed-loop Laplace transfer function $\frac{Y}{R} = G = C_{IV}(1 + C_{IV})^{-1}$. The corresponding poles/eigenvalues are assigned to satisfy the closed-loop characteristic equation $1 + C_{IV} = 0$. Therefore, the invariant controller can be designed with $C_{IV} = [1 - G]^{-1}G$.
(3) For the steady-state performance to constant setpoints, one has $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (r(t) - y(t)) \to 0$.

3.2. Smith predictive U-control (discrete time)

Smith predictive control, also known as Smith predictor (Normey-Rico & Camacho, 2007), is an approach used to cope with process delays that appear in control systems with feedback loops. It is particularly useful for systems with significant dead time or transport delays, where the time consumed for a control action to affect the process output is non-negligible. It has been observed that in a standard feedback control system, delays would lead to instability, poor performance or even system failure. To this end, the Smith predictive control method has been developed to address such an issue by predicting the future behaviour of the process and incorporating this prediction into the control algorithm.

In general, the conventional Smith control method includes the following four aspects (Normey-Rico & Camacho, 2007).

(1) Process/plant model: This step aims to establish the mathematical models for the underlying plant/process, which are necessary for the design of control systems. The established models should capture the dynamics of the process, including any delays.
(2) Delay compensation: The predictor includes a delay model in the control configuration, and this delay model can predict the system's future output

with reference to the instant control input and the past outputs.

(3) Control algorithm: The control algorithm combines the predicted future process output with the desired setpoint to generate the control signal, which is achieved using some well-known control methods, such as PID control and model predictive control.
(4) Feedback: The actual process output is continuously monitored, and the feedback is used to update the control signal. The feedback loop ensures that the control action can be adjusted based on the difference between the predicted and actual process outputs.

The Smith predictive control also has some limitations, such as the strong dependence on model accuracy. In this regard, such a method is mainly used for linear plants. Fortunately, the U-control method can expand the time-delayed linear plant into a general time-delayed nonlinear plant while keeping the closed-loop control performances achieved based on the conventional linear plant models (Geng et al., 2019).

Figure 6 illustrates the paradigm structure of the expanded Smith predictive control. The dynamic inverter P^{-1} makes the Smith predictive control method (originally developed for linear models) applicable to non-linear models (Geng et al., 2019). Most of the symbols used have been defined in previous sections, except the plant P and its nominal model $\hat{P}$ are in the discrete-time domain and the q^{-1} operator makes $q^{-k}y(n) = y(n-k)$ (n is the sampling instance and $-k$ denotes the k step time delay). The cancellation of the plant by dynamic inversion makes the Smith predictive control generally applicable to non-linear systems in either polynomial or state-space models. It should be noted that the U-control requires the same model accuracy for better control effects.

3.3. Summary of model-based U-control

Based on the above discussions, we can give a summary of the model-based U-control methods as follows:

(1) It is straightforward to implement the model-based U-control methods in terms of control system configuration and controller design.
(2) Two controllers are used in the cascade structure to achieve dynamic inversion and control system performance specification.
(3) The design method of the linear control system is also applicable to the nonlinear dynamic plants (even for nonlinear non-affine systems) since the dynamic inversion has made the design process independent of the plant models.
(4) The downside of the model-base control is that it heavily relies on the model accuracy even though the feedback control can cope with tiny uncertainties in the neighbourhood of the nominal model. As such, it is quite necessary to develop model-free U-control methods and hence enhance the control robustness.

4. Model-free U-control

4.1. Design procedure

The general structure of the model-free U-control is shown in Figure 2. The model of plant (4) is

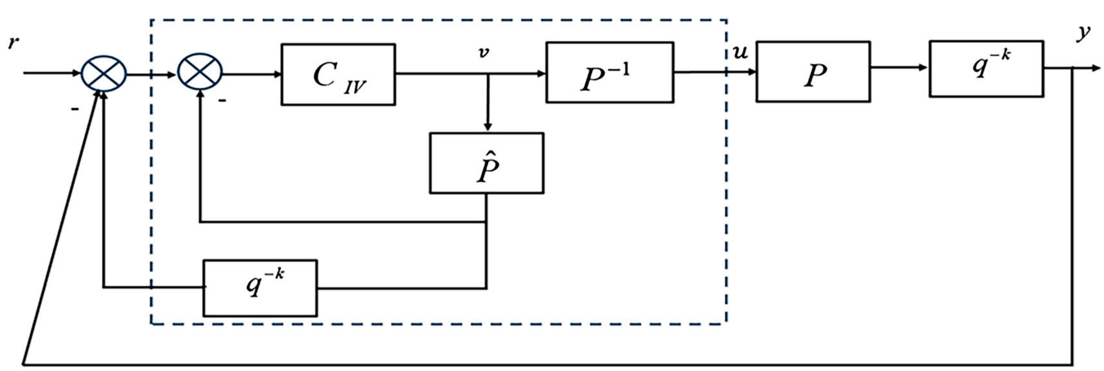

Figure 6. Smith predictive U-control.

assumingly unavailable in this case. That is, plant (4) is treated as a total uncertainty in a black box with enabling input and measurable output. The model-free U-control is defined in (3). The design procedure is explained below.

(1) *Robust dynamic inversion of plant*: The inner loop aims to use MFSMC to produce an nth-order identity matrix or a unit constant and then cancel the plant interaction in the control system design, which can guarantee robust stability in the dynamic inversion process. The inner loop is not intended to claim additional specifications of the system response. As expected, if the dynamic inversion is achieved, the performance controller in the outer loop can be independently assigned regardless of the plant/model. This outer loop controller is known as the invariant controller. If the full states are not available for sliding mode control (SMC), the state observers can be employed. For example, the ESO (Guo & Zhao, 2011) can be used to obtain the state vector.
(2) *Whole performance specification*: Setting up the outer loop has three objectives: (1) Achieve the whole system control performances. For instance, a required linear dynamic response in the system can be assigned with a Laplace transfer function in a complex domain, which can be achieved by resorting to the closed-loop implementation with a performance controller. This approach is more robust than the open loop implementation, especially when dealing with the frequently encountered constant disturbances. (2) Narrow down the closed-loop bandwidth to facilitate the process of plant inversion. (3) Use an invariant controller to provide the desired state vector for the inner loop SMC.
(3) *Detachable motion control and setpoint control*: The inner loop can be used for the motion control and both loops are suitable for the setpoint control. The invariant controller can be specified by the sliding mode PID (SMPID) (a type of U-PID Zhu 2023) and the CNF control (a new type of cascade U-CNF control that can generate variable damping ratio in comparison with the conventional parallel CNF configuration and hence guarantee the overshoot-free monotonic response with the fast-damping ratio).

4.2. MFSMC and model-free extended state observer (MFESO)

The SMC is a robust control methodology used to stabilise systems and track reference trajectories against uncertainties and disturbances. The first step of the SMC is to select a sliding surface in the state coordinate system such that the desirable performance can be achieved when the system trajectory is constrained to this surface. Then, proper feedback gains should be designed to guarantee that the system trajectory stays on this surface despite the existence of disturbances or uncertainties. It has been reviewed that most conventional SMC methods are based on the models and the controller outputs are determined by solving a sliding mode derivative equality (Yan et al., 2017). The U-control strategy has developed a procedure to design the MFSMC without using an adaptive data-driven pointwise model, which simplifies the design procedure and increases the global robustness (Zhu, 2021, 2023).

The MFSMC (Zhu, 2021, 2023) constitutes the core technique to establish a model-free control framework. The procedure of designing the MFSMC is explained as follows.

(1) Assign a linear sliding mode manifold $\sigma(e) = \sigma(x - x_d) = \sigma$, where (x, x_d) is the pair for the plant state vector and the desired state vector and σ is defined by (Slotine & Li, 1991):

$$\sigma = \left(\frac{d}{dt} + c\right)^{n-1} e, \ \mathbb{R}^n \to \mathbb{R} \qquad (10)$$

where the strictly positive constant $c \in \mathbb{R}^+$ denotes the slope (that is, the exponential convergence rate) of the sliding manifold, making the polynomial σ satisfy the Hurwitz stability conditions. The sliding mode $\sigma(*) = 0$ specifies a time-varying $\mathbb{R}^n$ hyperplane to guarantee that the tracking error trajectory asymptotically exponentially converges to zero.
(2) Design the model-free controllers. Specifically, the switching control u_{sw} and the equivalent

control u_{eq} are designed by

$$\begin{aligned} \dot{\sigma} &= u = u_{eq} + u_{sw} \\ u_{sw} &= -k_g \text{sgn}(\sigma)|\dot{\sigma}\sigma + \alpha\sigma^2 \leq 0 \quad \forall|\sigma| > \delta \\ u_{eq} &= \rho(\sigma|\dot{\sigma}\sigma + \alpha\sigma^2 \leq 0) \quad \forall|\sigma| \leq \delta \end{aligned} \tag{11}$$

where δ is the sliding mode boundary distance function (also known as the boundary thickness) to the sliding mode $\sigma(e) = 0$ with L^2 norm $\delta = ||\sigma(e)||_2$. The constant gain $k_g \in R_+$ in the switching control u_{sw} and $\rho(\sigma)$ in the equivalent control u_{eq} is designed to satisfy the differential inequality $\dot{V} + \alpha V \leq 0$ or $\dot{V} = \dot{\sigma}\sigma < 0, (\alpha = 0)$ in which $\alpha \in \mathbb{R}^+$ is the exponential convergence rate for the Lyapunov differential equation; $V = \frac{1}{2}\sigma^2$ and $\dot{V} = \dot{\sigma}\sigma$ are the quadratic control Lyapunov function and its derivative, respectively.

Remark 4.1: There are various options to assign the decreasing control function $\rho(\sigma)$. This study selects two typical controllers $u_{eq} = \rho(\sigma)$ for MFSMCs, called the augmented proportional and integral (PI) MFSMC and the continuous MFSMC (Zhu 2023). The details are given as follows:

4.2.1. PI function

$$\dot{\sigma} = u = \begin{cases} u_{sw} = -k_g \text{sgn}(\sigma)|\dot{\sigma}\sigma \\ \qquad + \alpha\sigma^2 \leq 0 \qquad \forall|\sigma| > \delta \\ u_{eq} = -k_p\sigma - k_i \displaystyle\int \sigma|\dot{\sigma}\sigma \\ \qquad + \alpha\sigma^2 \leq 0 \qquad \forall|\sigma| \leq \delta \end{cases} \tag{12}$$

where the coefficients $k_g, k_p, k_i \in \mathbb{R}^+$ are the gains chosen in line with the Lyapunov asymptotic stability. To accommodate the system bound $|\text{sup}(k)|$, these gains need to be assigned to satisfy that $|\text{sup}(k)| < k_g$, $|\text{sup}(k)| < (k_p + k_i)$, and $|(k_p)\sigma^2| > \left|(k_i)\sigma \int \sigma\right|, \forall k_p > k_i$. If the bound is unknown, take the trial-and-error approach to determine these gains.

4.2.2. Continuous MFSMC

Various monotonic nonlinear functions can be adopted for the continuous MFSMC. Among them, the most typical one is the sigmoid/logistic function, based on which the controller can be expressed as

$$\begin{aligned} \dot{\sigma} &= u = \rho(\sigma) \\ \rho(\sigma) &= -k_s \text{sigmoid}(k_0\sigma) + \frac{k_s}{2}|\dot{\sigma}\sigma + \alpha\sigma^2 \\ &\leq 0, \quad |\text{sup}(k)| < k_s, k_0 \in \mathbb{R}^+ \end{aligned} \tag{13}$$

where $\text{sigmoid}(k_0\sigma) = \frac{1}{1+e^{-k_0\sigma}}$, the gain k_s regulates the convergence direction and speed for the equivalent sliding mode, and the width gain k_0 is used to adjust the equivalent sliding mode boundary thickness.

Remark 4.2: The model-free SMC covers the model-based SMC as a special case. For the former, the controller output is obtained by solving the differential inequality $\dot{\sigma}\sigma < 0$, while for the latter, the differential equality $\dot{\sigma} = 0$ is utilised.

Figure 7 (Zhu, 2021, 2023) shows the model-based sliding mode and the model-free sliding modes in the coordinate system with respect to the $(\sigma, \dot{\sigma})$ plane.

The MFSMC is a type of control method requiring all state signals. If the state vector is not fully measurable, some state observers should be incorporated to estimate the state vector. To this end, the ESO (Guo & Zhao, 2011) has been revised within the U-control framework to develop a kind of MFESO.

Referring to plant (4) with unknown bounded disturbance d, assign the estimated state vector $z = y^{(0\sim n-1)} = [\, z_1{=}y \;\; z_2{=}\dot{y} \;\; \cdots \;\; z_{n-1}{=}y^{(n-1)} \,]^T \in \mathbb{R}^n$ and $z_n = f(z, u, d)$ to formulate the MFESO in the following form:

$$\begin{aligned} \dot{z}_j &= z_{j+1} + \beta_j e_1, \quad j = 1, 2, \cdots n-1 \\ \dot{z}_n &= z_{n+1} + \beta_n e_1 + u \\ \dot{z}_{n+1} &= \beta_{n+1} e_1 \end{aligned} \tag{14}$$

where $e_1 = (y - z_1)$ is the error, $\beta_j = \omega^j c_j$, $j \in [1, 2, \cdots, n+1]$ are the observer gains to be adjusted, ω is the observer bandwidth and c_j, $j \in 1, \cdots, n+1$ are the associated constants assigned by $c_j = \frac{(n+1)!}{j!(n+1-j)!}$, with which the Hurwitz stability constrains $(s+1)^{n+1} = \sum_{j=0}^{n+1} c_j s^{n+1-j}$ are satisfied.

Figure 8 shows the location of the MFESO in the U-control scheme.

When it comes to the theoretical analyses of the MFSMC, the theoretical derivation and the proof of robust stability have been provided by Zhu (2023) and

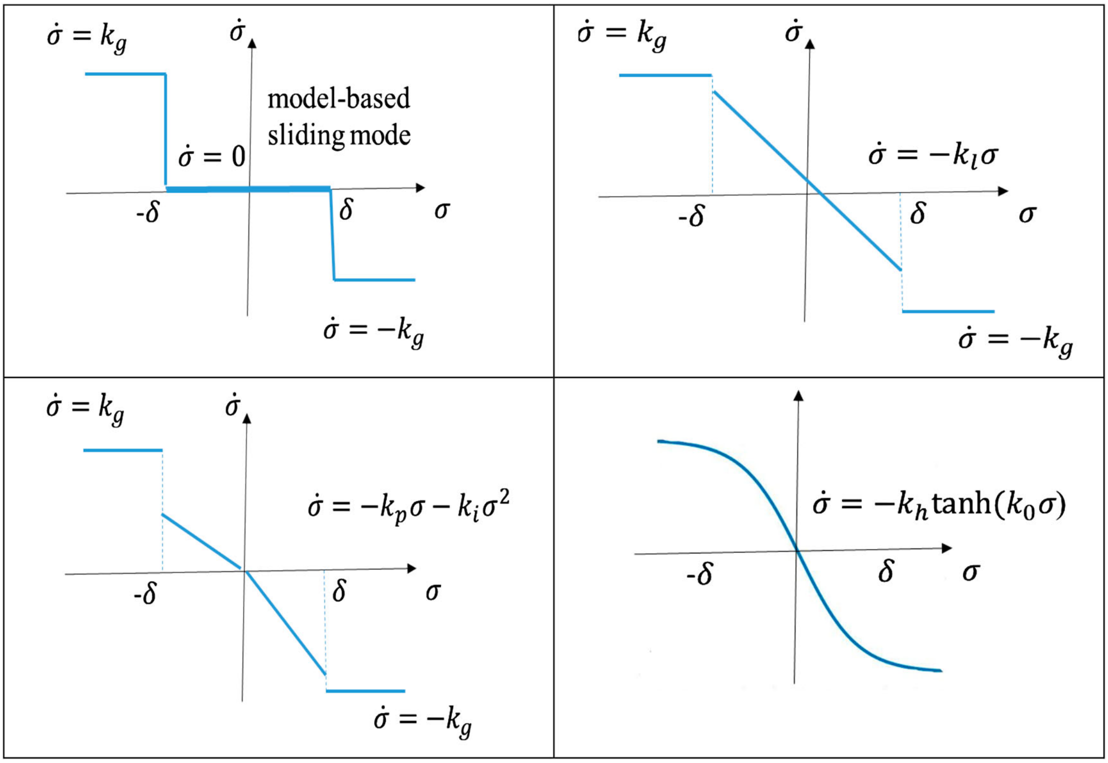

Figure 7. Sliding modes.

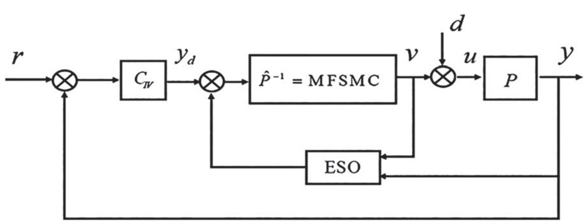

Figure 8. Model-free U-control with MFESO.

the total robustness based on the system sensitivity has been analysed by Zhu, Li, et al. (2023a).

It should be pointed out that the non-affine nonlinear plant $P: \dddot{\xi} + \xi\ddot{\xi} + \upsilon\dot{\xi} + 0.6\xi - \sin(\upsilon) - 2\upsilon - \upsilon^3 - d = 0$ (where (ξ, υ) is the pair for the plant output and input and d is an unknown but bounded external disturbance) has been used as a benchmark example to test the performance of MFSMC in various publications (Zhu, 2021, 2023; Zhu, Li, et al., 2023a; Zhu, Li, et al., 2023b; Zhu et al., 2023c; Zhu, Mobayen, et al., 2023).

4.3. Expanded CNF control

The CNF control is used to regulate the nonlinear dynamical systems, particularly the systems with uncertain or time-varying dynamics. Such a scheme usually combines multiple linear and/or nonlinear control techniques to achieve desired performance objectives such as stability, tracking and disturbance rejection. The basic idea behind CNF control is to decompose the whole control task into multiple components, each of which is responsible for addressing a specific aspect of the system's behaviour. These components are then combined in a systematic way to formulate a composite control strategy that can effectively regulate the control operation. It should be emphasised that the CNF control can provide robust control performance even in the case of internal uncertainties and external disturbances, which indicates that the integration of linear and nonlinear control techniques enables better performance than using either approach alone (Lu & Lan, 2019).

The implementation of the conventional CNF control scheme includes the following five steps.

(1) System modelling: Modelling of the plant dynamics serves as a cornerstone for the design of the control system. Two typical methods are linearising nonlinear models and using data-driven approaches to identify the system dynamics.

(2) Decomposition: In this step, the whole control task is decomposed into multiple control objectives, such as stability, tracking and disturbance rejection. Each control objective is achieved by resorting to a specific control component.
(3) Controller design: For each control objective, appropriate control techniques are selected and implemented, which include, but are not limited to, linear PID control, state feedback pole placement, sliding mode control and backstepping control in certain nonlinear control toolboxes.
(4) Composite control law: The individual control components are combined into a composite control law using appropriate combination rules, such as simple linear combinations, weighted sums and other more sophisticated techniques (e.g. feedback linearisation and adaptive control).
(5) Implementation and tuning: The composite control law is implemented in the control system, and its parameters are tuned to achieve desired performance objectives. To this end, simulation studies and experimental validation are usually conducted to ensure that the control performance is achieved under various operating conditions.

The conventional CNF control methods share two commonalities: (1) the design of almost all the conventional CNF control approaches is dependent on a plant/process model even though most adaptive strategies, neural networks and other model-free approaches are based on the pointwise online data-fitting models; (2) a parallel control structure, including the linear control and nonlinear control, is exploited, which usually involves complicated numerical calculations. This is particularly true for the nonlinear systems since the Lyapunov equation needs to be solved (Chen et al., 2003). Based on the above discussions, it makes practical sense to develop the model-free CNF control scheme within the U-control framework, which is abbreviated as U-CNF control.

Figure 9 shows the structure diagrams of conventional CNF control and U-CNF control. In the U-CNF control, two controls u_l and u_f (existing in the conventional CNF control) are merged into one control $C_{IV}(\zeta)$. By choosing the proper time-varying damping ratio ζ according to the error e, the desired dynamics and steady-state performance can be achieved without the need to solve the Lyapunov equation due to the advantage of model-free design (Zhu, Mobayen, et al., 2023).

4.4. Multi-input multi-output (MIMO) U-control

The MIMO U-control approach, usually abbreviated as the U-MIMO approach, is essentially a model-free design scheme and aims to cope with the interconnected coupling effects in designing the fully actuated MIMO control systems. Considering the interdependencies between inputs and outputs, two approaches have been proposed by treating the MIMO plants as a total uncertainty in a black box with two external ends for driving force and response measurements.

4.4.1. Decentralised U-control

In this method, the individual input-output pairs are regulated independently through the SISO controllers. In fact, the model-based design has been the prevalent approach in recent years, where the control systems are designed with specified control objectives by taking the nominal system models as references. The uncertainties (arising from the inaccuracies and/or variations in model parameters) are considered in this approach (Chen et al., 2022). It is worth mentioning that the model-free design should also be fully investigated due to its significant advantages in practice. The key idea is treating the plant as in a black box with an

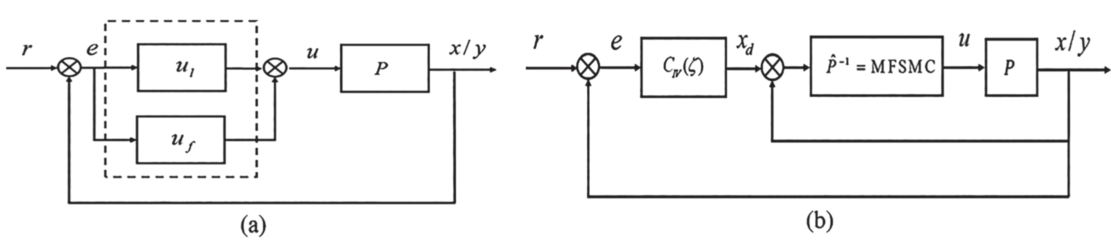

Figure 9. (a) Conventional CNF control and (b) U-CNF control.

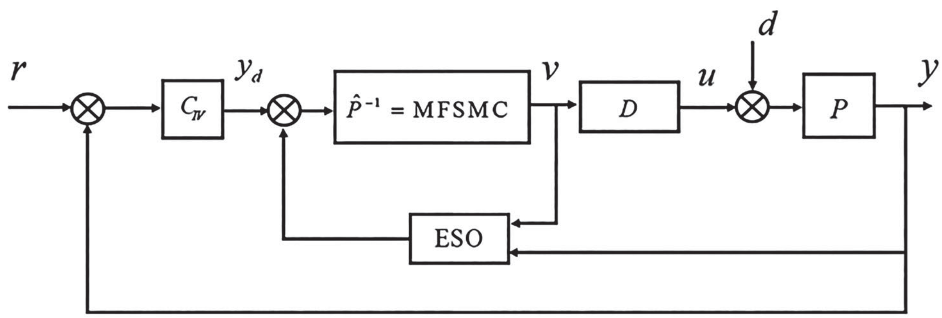

Figure 10. Decoupling U-control.

equal number of enabling inputs and measurable outputs. For example, when it comes to the fully actuated MIMO plant, the m input/output (I/O) subsystems can be designed as m SISO control systems such that the invariant control matrix is diagonal and the Lyapunov stability of the whole system is dependent on each subsystem's stability (Zhu, Li, et al., 2023b).

4.4.2. Decoupling U-control

Model-based decoupling control is a relatively mature technique, which is used to mitigate the interferences among the inputs and outputs in multivariable control systems. In the MIMO systems, it is often the case that the control variables influence multiple outputs simultaneously, thereby leading to unwanted coupling effects that can degrade performance or incur instability. Model-based decoupling control aims to design controllers that can decouple the system's inputs and outputs effectively, under which each input only affects its corresponding output (Bhattacharyya & Keel, 2022). The decoupling U-control (Zhu, Li, et al., 2023a) aims to provide a general model-free decoupling scheme for non-affine nonlinear MIMO plants, which is a nontrivial task since no reference model is available and there are difficulties in decoupling such nonlinear plants. Figure 10 illustrates the paradigm structure of the decoupling U-control system, where the D block serves the purpose of decoupling. The I/O coupling matrix function can facilitate the derivation of two types of decouplers, namely, U-decoupler/functional inversion and D-decoupler/static matrix inversion, which also provide evidence for the existing theorem of the MFSMC and form the basis for the nonlinear-dynamic-coupling inversion. Additionally, a simulated bench test is conducted on a non-affine two-input two-output nonlinear plant with the following system dynamics: $P_{2x2}: \begin{cases} \ddot{y}_1 + y_2\dot{y}_1 + u_2\dot{y}_2 + 0.6y_1 \\ \quad - \sin(u_1) - 2u_2 - u_1^3 = 0 \\ \ddot{y}_2 - 1.5(1 - y_1^2)\dot{y}_2 + y_1 \\ \quad - 2u_1 + u_2^3 = 0 \end{cases}$ and the comparative results are also provided for the U-control and a model-based dynamic decoupling control with respect to a two-input two-output linear plant.

5. Conclusions

The U-control framework has provided an effective supplementary approach to enhance the performance of various existing approaches applicable within the whole spectrum from model-based to model-free. More importantly, the U-control can be applied to a diversity of scenarios, from motion control to set-point control. The pursuit of the U-control is a type of applied method to achieve concise design of control systems, effective tuning of control parameters and promote practical applications. The integration of U-control with other conventional approaches can be described by the problem-method relationships, as shown in Figure 11 and Table 1 lists some of the considered future work. Although the authors and their

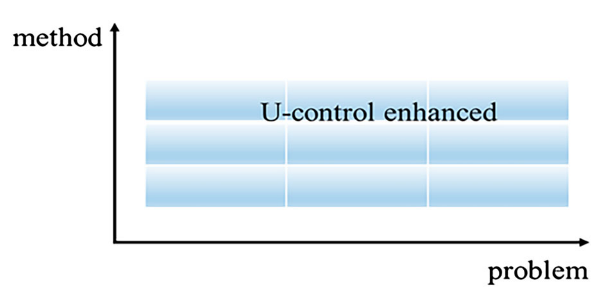

Figure 11. Integration of U-control and the other methods.

Table 1. Future U-control work integrated with the existing methods.

Problems	Conventional methods	U-control to supplement
SISO plants	Pole place, Smith predictive control, neural network control, composite nonlinear feedback control, event trigger control, control of nonlinear dynamics with discontinuous control inputs	U-control will be integrated in forms of model-based and model-free to enhance dynamic inversion, dealing with uncertainties and disturbance. Some algorithm enhancements could be the model-free terminal SMC to remove the singularity and chattering effects encountered in the most conventional algorithms. The other expansion could be the integration of control Lyapunov function and control barrier function for both stability and safety in model-free U-control.
MIMO plants	Underactuated control	Model-based and model-free sliding mode U-controls could be expanded by coupling the underactuated control inputs so that this generalisation could remove the coordinate transform request converting nth order, m plant outputs into $n*m$ lines of the state-space model. Consequently, the U-control directly uses the Lagrangian equation model for control system design, which is meaningful in design effectiveness and physical implementation.
Application bench tests	Under actuated control systems, particularly those motion control systems in robots, unmanned aerial vehicles' operation is described by Lagrangian equations.	Model-free U-control with simulations and real bench tests is aimed to increase the generality, conciseness in design and robustness in tuning and control. The development could start from inverted pendulums, unmanned aerial vehicles and many others that appeared in mechatronic systems.
	Nonlinear dynamic systems with non-continuous inputs (such as dead zone, friction and hysteresis) frequently appeared in mechatronic systems.	Model-free U-control in dealing with such complete uncertainties with piecewise smoothness.
	Co-design of emerging man-made products and systems, such as robots, unmanned aerial vehicles (UAVs), etc.	Regarding the codesign of UAVs, it could involve the integrated development of hardware and software systems to optimise performance, functionality and efficiency. Codesign ensures that all components of the UAV are designed to work seamlessly together, addressing various design challenges and requirements. Surely, U-control, in principle of separation of model and plant and total model-free, would provide new insight and prototype with the related applications

research teams have published a series of results on the U-control, such a method has not received sufficient attention from the control and related communities. Hopefully, this paper can pave the way for the theoretical research and practical applications of the U-control framework.

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No potential conflict of interest was reported by the author(s).

Data availability statement

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

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