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Probabilistic-based approach for evaluating the thermal response of concrete slabs under fire loading

G. Karaki[a*], R.A. Hawileh[b], V.K.R. Kodur[c]

[1]*Assistant Professor, Department of Civil & Environmental Engineering, Birzeit University, P.O. Box 14, Birzeit, Palestine. e-mail: gkaraki@birzeit.du*

[2] *Professor, Department of Civil Engineering, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates. e-mail: rhaweeleh@aus.edu*

[3]*University Distinguished Professor, Department of Civil and Environmental Engineering, Michigan State University, East Lansing, United States of America. Email: kodur@egr.msu.edu*

**corresponding author*

E-mail address: gkaraki@birzeit.edu

Abstract

Performance-based design for fire safety has been introduced in several international design frameworks. As the fire models and simulations include various assumptions and simplifications, and the current fire resistance evaluation is based on deterministic approaches, this leads to uncertainties in the performance of the structural members exposed to fire. An alternative to this is the application of probabilistic methodologies to assess fire resistance of the structural members. The authors present the application of an efficient probabilistic methodology to perform a sensitivity analysis to identify the critical variables of a thermal model of a structural element exposed to the characteristic fire loading. Furthermore, the methodology determines the reliability of the structural element. The methodology combines the elementary effects method with variance-based methods to rank the influence of the governing variables of the thermal and fire models on the thermal performance of a RC slab and to determine their uncertainty

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contribution to the time-dependent thermal response. Furthermore, Monte Carlo method is applied to calculate the probability of failure and reliability index of the structural member exposed to fire loading. It is found that the critical governing variables are, from the fire model; firefighting measures index which accounts for firefighting measures used in the compartment (FFMi), characteristic fuel load density ($q_{f,k}$), opening factor of the compartment (O), and ratio of floor area to total area of the compartment (A_f/A_t), and from the thermal model; coefficient of convection (h), concrete specific heat (c_c), concrete density (d_c), concrete conductivity (k_c). As one moves away from the exposed surface, h, $q_{f,k}$, and A_f/A_t are not as influential on the thermal response. It is also observed that the uncertainty of FFMi, O, c_c, and h are the primary sources of the thermal response's uncertainty. Considering the variability of the input variables, low-reliability index is determined for buildings with no basic firefighting measures, and adding intervention measures, sprinkler systems, and detection system will increase the reliability index by 53%, 85%, and 89%, respectively

Keywords: Concrete slabs; fire resistance; thermal analysis; sensitivity analysis; reliability analysis.

Introduction

Performance-based design for fire safety assessment has been introduced into several design frameworks (Hurley and Rosenbaum 2015; Hurley and Rosenbaum 2016). This framework for fire safety requires the designer to demonstrate that performance criteria are met for relevant fire scenarios, acceptance criteria, and simulations that adequately model the behavior of the structure under fire loading. As the fire models and simulations include assumptions and simplifications, and the current fire resistance evaluation is based on deterministic approaches, this leads to uncertainties in the thermal and mechanical performance of the structure. Therefore, in recent years, probabilistic approaches have been introduced to fire engineering, e.g., probabilistic risk analysis (PRA) and more extended performance-based structural fire engineering (PSFE). The framework (PRA) recommends designs based on the level of safety required, measured by the estimation of the probability of failure (Van Coile et al., 2011; Van Coile et al., 2019). The probability of failure is determined by first-order-reliability method (FORM), second-order reliability method (SORM), or the Monte Carlo approach (Guo and Jeffers 2014; Haideri et al. 2019). A performance-based probabilistic design approach (PSFE) considers multiple hazard levels and gives credence to all governing factors in performance evaluation and thus estimates probable damage, and consequently, losses (Rini and Lamont 2008; Lange et al. 2014; Hopkin et al. 2018; Van Coile et al. 2019).

There are different categorizations for the types of uncertainty considered in the engineering models; the definitions and the descriptions provided in the literature, e.g. (Der Kiureghian A.; Dirlevsen O. 2009) help the modelers in defining the categories of uncertainty in their models. The uncertainties in simulated engineering problems may be of two categories: model and parameter uncertainties. Model uncertainty is related to the mathematical model of the engineering problem, and limited data sources for the modeled scenario, while the parameter/variable uncertainty is linked to uncertainty in the variable estimates related to the amount and quality of collected information for the variable. Both uncertainties can be controlled, and the engineering model can be refined and improved by correlating with large set of experimental data. These types of uncertainties exist in the fire and in the structural element models, which may lead to significant variability in the thermal and structural performance and thus inconsistent levels of fire safety for the structural member in a building. Considering the uncertainties would allow the designers to quantify the proposed design or solution's reliability, which is useful for making proper informed judgement during decision-making processes. Sensitivity analysis is often used to characterize and quantify the significance of model’s input variables and processes and their uncertainties on the

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engineering performance measures. This can be extended to optimize the engineering design and assess its reliability, examples of such studies; Saltelli et al. (2019), Spagnol et al. (2019), Karaki (2013), Karaki (2011), Castillo et al. (2008), among others.

Existing research in fire engineering has focused mainly on uncertainty in fuel load density, thermo-mechanical properties of structural members, insulating materials and heat transfer process to structural members (Kodur et al. 2010; Iqbal and Harichandran 2010; Gernay et al. 2016; Olsson et al. 2017; Ribeiro et al. 2016; Heidari et al. 2016; Gao and Jeffers 2014). A review of different methods of treating the uncertainties in performance-based fire safety design can be found in (Hurley and Rosenbaum 2015). However, it was stated by Hurley and Rosenbaum (2015) that there is no single accepted methodology for dealing with uncertainty in the fire analysis and fire design processes. The current literature does not yet offer a comprehensive approach on performing global sensitivity analysis that can be integrated into PRA and PSFE frameworks.

This paper presents a methodology to characterize input variables in terms of their significance and uncertainty contribution that need to be considered in a chosen fire resistance design framework. Two aspects of the application of sensitivity analysis in fire engineering are considered; time-dependent model outputs and the computational efficiency of the used sensitivity analysis technique. This methodology could help support the decision for more examinations or simplifications for a number of input variables defining the heat transfer mechanisms and fire models.

Methodology

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The overall methodology for performing sensitivity and reliability analysis for the thermal performance of RC slabs comprises of the following steps:

- The elementary effects method is used to identify the important input variables affecting the thermal performance of a RC slab in case of a fully developed parametric fire.

- The method is extended to calculate the total sensitivity indices, which measure the contribution of the variables' and models' uncertainty to the total uncertainty of the thermal performance of a RC slab exposed to fire loading.

- Finally, Monte Carlo simulation is performed to investigate the fire resistance and reliability of the RC slab probabilistically, accounting for uncertainties in the fire and heat transfer models, in the case of a fully developed parametric fire.

This methodology incorporates all the salient factors governing probabilistic-based performance evaluation. It can be effectively applied to determine the input variables that dominate the uncertainty of performance measures and quantify the possible variations that could result in an acceptable/unacceptable outcome, which is needed to guide further analysis and design processes.

Sensitivity Analysis

With the recent advancement of computing power, decision-making procedures in building design frequently use numerical models and simulations that combine multiple processes. However, increasingly complex models require more information and definitions for the input variables, and typically this information is not well specified nor defined. Therefore, it is essential to examine the impact of the input

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variables and their uncertainties on the model's output in order to use the models effectively in the decision-making procedures. Global Sensitivity Analysis (GSA) refers to the methods that evaluate the effect of an input variable on the output, by varying not only the parameter in question but all other input parameters chosen for analysis. GSA uses a probabilistic framework that considers the values and types of the inputs' probability distribution functions and requires that the model output be evaluated multiple times for input samples randomly selected from the created input space. Therefore, a large number of Monte Carlo-based evaluations of the model are required. A group of GSA methods is the so-called screening-based methods or elementary effects method, which mainly ranks input variables by their importance and influence in descending order, using only a relatively small number of model evaluations is an attractive alternative for running a GSA. Another group of GSA methods is the so-called variance-based methods that are considered computationally expensive. These methods decompose the variance of the model’s outputs and quantify the input variables’ contribution to the total variance. A popular variance-based GSA method is the method of Sobol, which estimates sensitivity indices that describe the first-order effects and total effects index of the input variables variances on the output variance (Saltelli et al. 2008; Sobol 1993). The total effect index indicates the contribution of the input variable and its interactions on the output’s variance. The elementary effect method and its extension to variance-based methods are used in this study. The following is the mathematical description for the methods that were implemented and used.

Elementary Effects Method

Screening methods, also known as elementary effects belong to the class of One-at-Time (OAT) designs. However, they overcome the shortcomings of typical derivative-based approaches as they offer a wider variation for the input variables and averaging over many local measures (Saltelli et al. 2008). These methods are attractive as they are computationally inexpensive and ideal for models with a large number of input variables. This study adopts the elementary effects method to perform a sensitivity analysis for the thermal response of RC slab to identify the critical variables affecting the fire-resisting performance of the slab. Furthermore, it extends the application of the method to assess the uncertainty of the slab's thermal performance and quantify the contribution of the input variables to this uncertainty.

The radial-like configuration for the development of the samples required by the elementary effects method is used. Such configuration showed better performance as it requires a lower number of samples to get reliable sensitivity measures (Campolongo et al. 2011). Table 1 presents the radial-like configuration; two samples are created **A** and **B**, which are two different k-dimensional random vectors that can be used to realize the so-called X_i steps, which is a vector containing a complete set of the *k* input variables. $\mathbf{X_i}$ step is made of two points, which are apart only for one coordinate, i.e., only for variable x_i, all others being the same. In the radial design, one goes back to the first point (A_1, A_2, A_3, …., A_k) after each step. One can call **A** entries as the baseline point and B entries as the auxiliary point. $\mathbf{X_i}$ step is used for the computation of an elementary effect (EE_i) for that variable x_i. EE_i is calculated using Eq. (1)

$$EE_{ij} = \left|\frac{y(x_i^A x_{\sim i}^A) - y(x_i^B x_{\sim i}^A)}{x_i^A - x_i^B}\right|_j \qquad (1)$$

where $y\left(x_i^A x_{\sim i}^A\right)$ is the output considering only variables of base vector **A** (0th row in Table 1), $y\left(x_i^B x_{\sim i}^A\right)$ is the output considering the variables of base vector **A** except for x_i chosen from auxiliary vector **B** (ith row in Table 1).

Table 1. Map for creating the samples required by the elementary effect method

Radial sampling, k is the number of input variables	Auxiliary samples for the tests, J varies from 1 to r. r is number of tests (repetitions)

$A_1, A_2, A_3, \ldots.., A_k$	i=0	$B_1^1, B_2^1, B_3^1, \ldots, B_k^1$
$B_1^J, A_2, A_3, \ldots.., A_k$	i=1	$B_1^2, B_2^2, B_3^2, \ldots, B_k^2$
$A_1, B_2^J, A_3, \ldots.., A_k$	i=2	$B_1^3, B_2^3, B_3^3, \ldots, B_k^3$
…	…..	…
$A_1, A_2, A_3, \ldots.., B_k^J$	i=k	$B_1^J, B_2^J, B_3^J, \ldots, B_k^J$

A series of such steps allows an estimate of k-factors of the variables’ elementary effects. Repetitions (r) for the entire process allows the choice of different base and auxiliary points which covers the entire space of the input variables. For every J (varies from 1 to r), the elementary effects (EE_{ij}) for input variable (x_i) is calculated, and a general estimate for the elementary effect of input variable i is determined as μ_i using Eq. (2), which is used to rank the input variables following their importance.

$$\mu_i = \frac{\sum_{j=1}^{r} EE_{ij}}{r} \tag{2}$$

Furthermore, the standard deviation (σ_i) of EE_{ij} values is calculated using Eq. (3) and it indicates the interactions between the input variable i and the other variables considered in the analysis.

$$\sigma_i = \left[\frac{\sum_{j=1}^{r}\left(EE_{ij}-\mu_i\right)^2}{r-1}\right]^{0.5} \tag{3}$$

Sobol’s quasi-random sequences is used for the sampling of base and auxiliary points as it outperforms crude Monte Carlo sampling in the estimation of multi-dimensional integrals (Campolongo et al. 2011). The elementary effects method is used to rank input variables following their importance, identify interactions between the variables, and pinpoints the non-influential ones. However, uncertainty exists in the values of the input variables, and thus it is essential to quantify the contribution of the input variables uncertainty to the total uncertainty of the output, since this is important for the reliability analysis to be meaningful. Generally, global variance-based method by Sobol is used for such a purpose, and it is based on the decomposition of the total unconditional variance (as a measure of the uncertainty) of the model's output V(Y). The total unconditional model variance V(Y) (Saltelli et al. 2008) is represented by Eq. (4).

$$V(Y) = V_{X_{\sim i}}\left(E_{X_i}(Y|X_{\sim i})\right) + E_{X_{\sim i}}\left(V_{X_i}(Y|X_{\sim i})\right) \tag{4}$$

The first term is the variance explained conditioned on input parameter x_i (this indicates the first-order effect), and the second term is the remaining variance. The inner operator of the second term is the variance of Y taken over all possible values of the input matrix **X** except for one x_i. And the outer expectation E is taken over all possible values of x_i. The total effects sensitivity index (Saltelli et al. 2008) that determines the effect of the i^{th} input variable and its interactions is expressed by Eq. (5) as:

$$S_{T_i} = \frac{E_{X_{\sim i}}\left(V_{X_i}(Y|X_{\sim i})\right)}{V(Y)} \tag{5}$$

Total sensitivity indices can be calculated using the developed algorithm for the elementary effects as long as enough repetitions (r) are performed. For this purpose, the estimator of EEi is replaced by the estimation of $E_{X_{\sim i}}\left(V_{X_i}(Y|X_{\sim i})\right)$ following the estimator of Jansen as expressed by Eq. (6) (Campolongo et al. 2011)

$$E_{X_{\sim i}}\left(V_{X_i}(Y|X_{\sim i})\right) = \frac{1}{2r}\sum_{j=1}^{r}\left(y\left(a_1^j, a_2^j, \ldots, a_k^j\right) - y\left(a_1^j, a_2^j, \ldots, b_i^j, \ldots, a_k^j\right)\right)^2 \tag{6}$$

As mentioned above, the elementary effects method was applied for the thermal analysis of a concrete slab to identify the influential input variables on the thermal response. The values of input variables, i.e., density, thermal conductivity, and specific heat are temperature-dependent, and this dependency is included in the probabilistic model and analysis. However, for each variable the variation with temperature, e.g., concrete conductivity decreases as temperature increases, needs to be maintained for realistic modeling. Furthermore, the input variables have different ranges of values, and the elementary effects are calculated by dividing the output variation by Δ, which will differ based on the value of input variable, and thus this will affect the ranks of input variables. Therefore, to address these two points, a database for the input variables is developed following their developed probabilistic models. For a given analysis run, a set of input variables determined following a quantile value, and this quantile is kept constant across all temperatures. This solves the first problem of temperature-dependent variables. The Δ is calculated using the quantile values of the samples, this represents a sampling step in the range of [0, 1] for all variables, and this solves the second problem of the different scales of input variables. Saltelli et al. (2008) presented the advantages of using the quantiles to map the input variables. The use of the quantiles is adopted and adapted in this study as it fitted the nature of the considered input variables of the thermal model.

Reliability Analysis

This analysis seeks to examine the reliability of the thermal performance of the RC slab. In general, a failure criterion is defined in terms of a limit state function defined for the target performance measure, e.g. for the thermal analysis $g(X) = R - F$, where R is the actual thermal response (resistance) of the slab, and F is the thermal-failure criteria.

The failure probability is expressed in Eq. (7) and defined as the probability that the limit state function attains non-positive values

$$P_f = Prob[g(x_1, x_1, \dots, x_n) \leq 0] = \int_{g(X)\leq 0} \dots \int f_X(X)dX \tag{7}$$

The computational challenge is in determining the integral. This integral is determined using Monte Carlo simulation. The reliability analysis accounts for the uncertainties of the variables defining the

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characteristic fire model and the variables defining the heat transfer mechanisms. In Monte Carlo simulations, a random value is selected for each of the input variables based on the developed probabilistic models, and a failure criterion is assigned for a response function. The probability of failure (P_f) is calculated using Eq. (8).

$$P_f = {}^{n_f}\!/_{n} \tag{8}$$

where n_f is the number of samples exceeding the failure criterion, and n is the total number of run samples. The model is run repeatedly in a Monte Carlo simulation until the value of the outputs converges. The output of Monte Carlo simulation is used to determine the reliability index (β), which indicates the margin of safety for the structural element's performance. Assuming a Gaussian response (Nowak and Collins, 2000), then

$$\beta = {}^{\bar{R} - \bar{F}}\Big/_{\sqrt{\sigma_R^2 + \sigma_F^2}} \tag{9}$$

where $\bar{R}$ and $\bar{F}$ are the mean value for resistance and failure limit, consequently, and σ_R^2 and σ_F^2 are the variance of resistance and failure limit. If the limit state function is not Gaussian, Eq. (9) is only an approximation for the reliability index (β). In case the limit state function follows a lognormal distribution, Eq. (10) was proposed by Withiam et al. (1998) to calculate β

$$\beta = {}^{ln[\bar{R}/\bar{F}\sqrt{(1+COV_F^2)/(1+COV_R^2)}]}\Big/_{\sqrt{ln[(1+COV_F^2)(1+COV_R^2)]}} \tag{10}$$

where COV_R coefficient of variation for the resistance and COV_F coefficient of variation for the failure limit.

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Modeling and analysis results

Description of the Developed Numerical Model

Finite element analysis is used extensively to evaluate the thermal behavior of structural elements exposed to different fire scenarios (Hawileh et al. 2009; Hawileh et al. 2011; Hawileh et al. 2012; Naser et al. 2014; Naser et al. 2015; Hawileh and Rasheed 2017). More recently a numerical finite element (FE) model was developed by Hawileh and Kodur (2018) using the finite element software, ANSYS (ANSYS 2019) to predict the performance of RC slabs subjected to severe fire conditions. The developed model is based on a simply supported slab specimen tested by Cooke (2001) in a previous experimental investigation. The total length, span length, width, and thickness of the tested slab specimen are 4700, 4500, 930, and 150 mm, respectively. This slab was made of normal weight concrete using siliceous aggregates with a density and characteristic cube strength at room temperature of 2400 kg/m^{3}and 30 MPa, respectively. The slab is reinforced with 10 steel deformed longitudinal bars (BS 4449 Type 2) having a diameter and yield strength of 8 mm and 460 MPa, respectively. The concrete cover from the slab's soffit to the longitudinal steel is 25 mm.

The FE model for the quarter slab specimen is developed using ANSYS version 14.5 (ANSYS 2013); and it is shown in Fig. 1. Hawileh and Kodur (2018) performed a sequentially coupled thermo-mechanical analysis for the RC slab's fire response. The analysis was conducted in two parts. The first part was the nonlinear transient thermal analysis performed independently to obtain nodal temperature histories. The second part was then performed, which is the stress analysis incorporating nodal temperature histories from heat transfer analysis.

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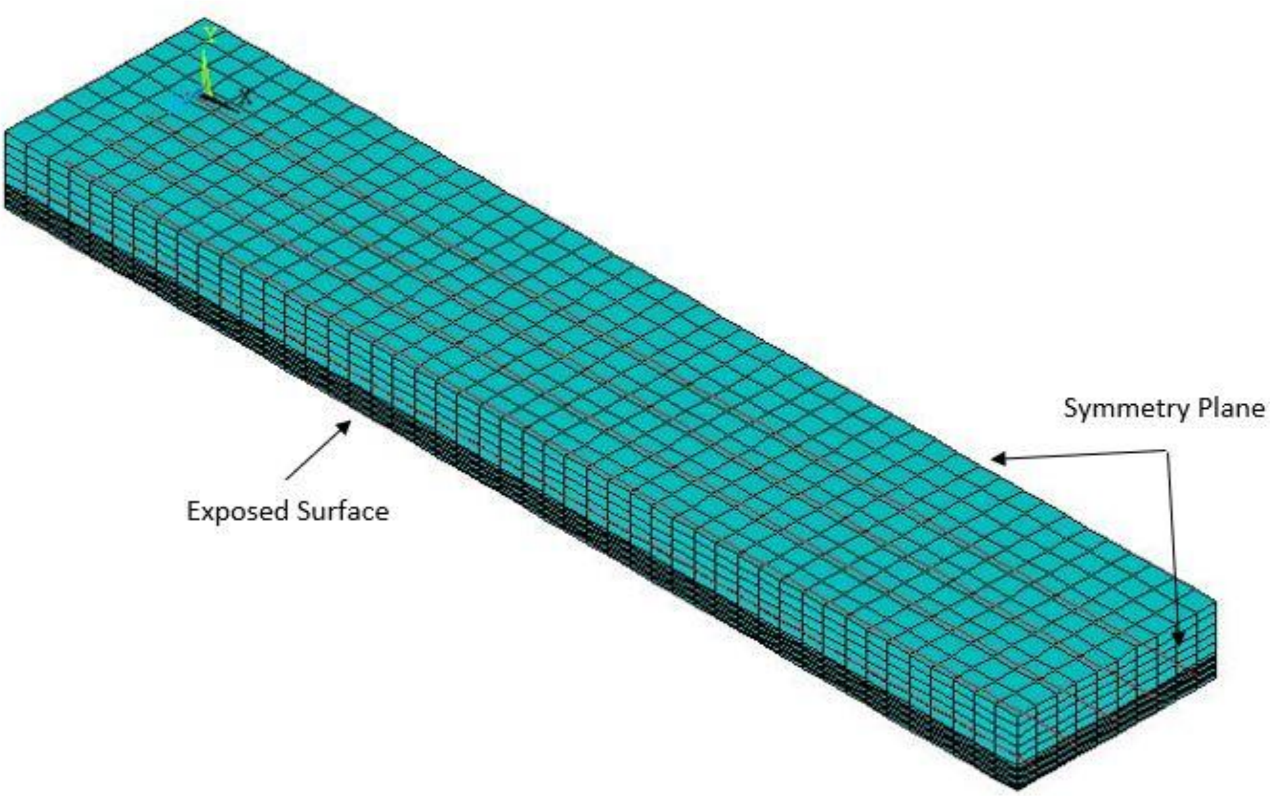
Quarter FE models are adequate to simulate the behavior of the slab, due to the symmetry in the geometry, materials, structural and fire loading, and boundary conditions of the tested slab. The use of a quarter model to simulate the slab behavior leads to a significant reduction in computational time and effort. Thermal symmetry needs to be achieved, which means no heat will flow across the symmetrical plane. Therefore, no boundary conditions nor constraints were defined on the symmetry plane. The element types used to discretize the concrete core and steel reinforcement bars in the thermal model are SOLID70 and LINK33, respectively. These elements can conduct heat throughout the slab's model due to transient heating resulting from the fire applied at the bottom surface of the slab. The 3D brick SOLID70 element, used for thermal discretization, has a total of eight nodes. Each node of the SOLID70 element has one degree of freedom (*dof*), namely temperature. The 3D spar uniaxial thermal LINK33 element is defined by two nodes, each with a temperature *dof* as well. The SOLID70 and LINK333 element types can be used in both steady-state or transient thermal analysis.

Two fire scenarios are applied; ISO834 standard (ISO834-1975 1975) fire and NPD-Hydrocarbon fire (Cooke 2001). Transient thermal analysis is performed for which conduction is the mechanism to describe the heat flow through the solid media, and convection and radiation are the main mechanisms for net heat flux applied on the boundary surface. Measured and predicted temperature profiles of RC slab in addition to measured and predicted temperatures in the steel reinforcement rebars during the test fire models are shown in Fig. 2(a) and 2(b). There is a reasonable agreement between the experimental and numerical model analysis results, which means to some extent that the model uncertainty is controlled. The full record of the developed numerical model of the slab along with the thermal and mechanical properties used for its validation are found in (Hawileh and Kodur 2018). It was observed by Hawileh and Kodur (2018) that the temperature's histories have significant influence on the structural response of the RC slab.

Therefore, to properly evaluate the fire resistance of the RC slab, the nodal temperature history should be perceived and understood. The propagation of the heat along the member depends on thermal properties of concrete and steel, e.g., specific heat, conductivity, and density, and fire scenario. Therefore, the RC slab's thermal model was examined thoroughly using sensitivity and reliability analysis in this paper. The effect of the input variables defining the RC slab's thermal behavior and fire models on temperatures' histories was performed and inferred.



(a)

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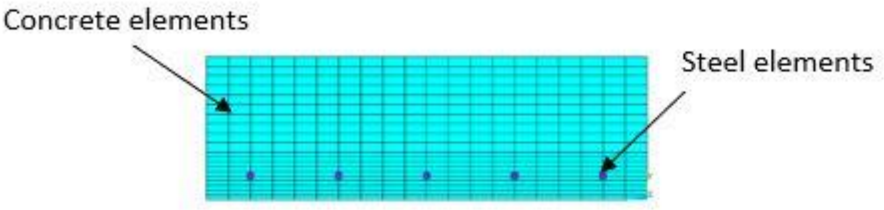
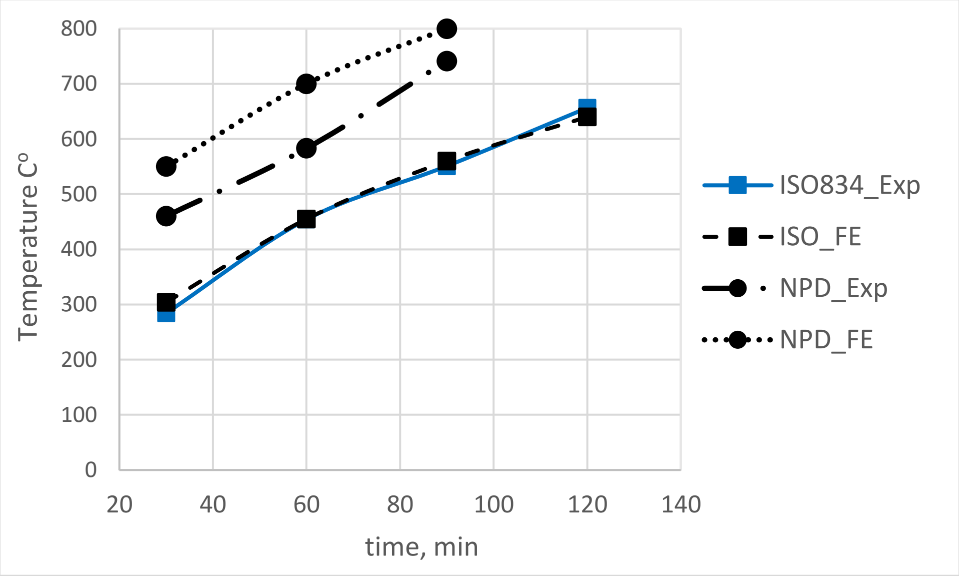
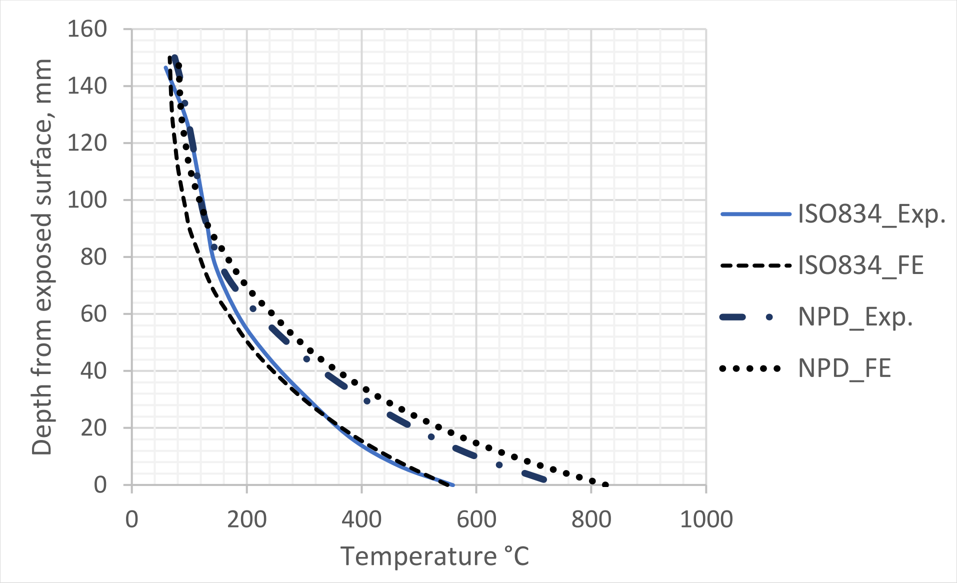


(b)

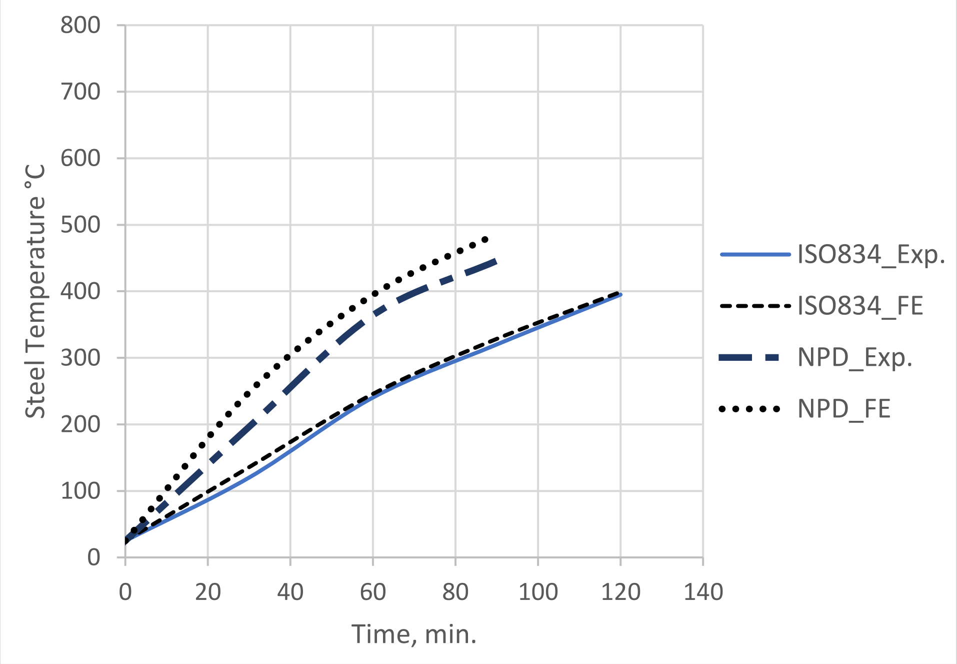
Fig. 1. (a) Isometric view of the discretized slab; (b) Front view of the discretized slab; (c) Cross-sectional view of the discretized slab



(a)



(b)



(c)

Fig. 2. (a) Measured and predicted temperature profiles at different time increments for the exposed surface; (b) Measured and predicted temperature profiles across the depth of RC slab at duration of

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exposure of 90 min. (c) Measured and predicted temperatures in the steel rebars at different time increments (Hawileh and Kodur 2018)

Fire Models

Fire scenarios for a fully developed fire were formed based on a range of values of input variables such as fuel load density, ventilation size, contribution of fire protection systems, boundary material properties, floor, and total compartment areas. A set of temperature-time curves was produced in accordance with the EC1 (2002) parametric fire method. The analytical equation given in EC1 to calculate the fire temperature is given by Eq. (11):

$$T_g = 1325[1 - 0.324\exp(-0.2t^*) - 0.204\exp(-1.7t^*) - 0.472\exp(-19t^*)]\ ({}^oC)$$

$$t^* = t \cdot \Gamma(h) \tag{11}$$

where t is the time (h), Γ is given as

$$\Gamma = [O/b]^2/(0.04/1160)^2 \tag{12}$$

where b is the thermal inertia of the enclosure boundary ($J/m^2s^{1/2}K$), O is the opening factor of the fire compartment ($m^{1/2}$), which represents the characteristics of vertical openings in the compartment.

The maximum temperature occurs at t^*_{max} which is calculated as in Eq. 13,

$$t_{max} = max\big[\big(0.2 \cdot 10^{-3} \cdot q_{t,d}/O\big); t_{lim}\big](h) \tag{13}$$

Assuming a medium fire rate, the limiting temperature t_{lim} is taken as 20 minutes. $q_{t,d}$ is the design value of the fire load density related to the total surface area A_t of the enclosure (MJ/m^2), and $q_{f,d}$ is the design value of the fire load density related to the surface area A_f of the floor (MJ/m^2).

$$q_{t,d} = q_{f,d} \cdot A_f / A_t \tag{14}$$

The cooling phase of fire starts after t_{max}, and the temperature-time curve during this phase depends on whether the fire is fuel controlled or ventilation controlled. These curves are described in EC1, and t_{max} is used to categorize the fire as fuel- or ventilation-controlled. If t_{max} is controlled by t_{lim} then the fire is fuel-controlled, and if $(0.2 \cdot 10^{-3} \cdot q_{t,d}/O)$ controls t_{max} then the fire is ventilation-controlled.

The design value of the fire $q_{f,d}$ is defined as

$$q_{f,d} = q_{f,k} \cdot m \cdot \delta_{q1} \cdot \delta_{q2} \cdot \delta_n \tag{15}$$

where m is the combustion factor taken as 0.8, δ_{q1} is a factor taking into account the fire activation risk due to the size of the compartment taken as 1, δ_{q2} is a factor taking into account the fire activation risk due to the type of occupancy taken as 1.5, and δ_n is a factor taking into account the different active firefighting measures, e.g. detection systems or sprinkler systems among others; it is also referred to as Firefighting measures index (FFMi) as in Heidari et al. (2019). The value $q_{f,k}$ is the characteristic fire load density per unit floor area (MJ/m^2), EC1 gives typical values classified according to the occupancy.

Table 2 presents the variables of the fire model considered in the sensitivity and reliability analyses. Their probabilistic values are also presented in Table 2. The input variables and fire models were created through a MATLAB code that ran ANSYS and applied the developed fire model.

Table 2. Parameters defining the probabilistic model of the characteristic fire

Parameter	**Probabilistic values**	**Notes**	**Reference**
Characteristic fuel load density (q_{fk})	Mean =780MJ/m^2, coefficient of variance =0.3, Gumbel distribution	The value corresponds to the fuel load density of dwellings following EC1 with a mean value of 780MJ/m^2 and 80th percentile of 980MJ/m^2.	EC1

FireFighting Measures Index accounts for the different active firefighting measures (FFMi)	Discrete values are calculated for FFMi, range [0.148-3.37]	The range values cover the possible firefighting measures representing sprinklers, auto detections, safe access routes, and firefighting devices.	EC1
Opening factor (O)	Uniform distribution [0.02-0.2]	Range taken following the limits assigned in EC1. This accounts for uncertainty in the glass breakage and falling out	EC1
Thermal inertia (b)	Uniform distribution [1150-2200]	Range taken to represent the extent of concrete thermal conductivities, specific heats and densities for normal weight concrete.	----
A_f/A_t	Uniform distribution [0.18-0.35]	Assumed range for the possibilities of the floor area in relation to the enclosure area	----

Thermal Material Properties

The material properties considered for the RC slab are temperature-dependent. The probabilistic models for these properties must also be temperature-dependent. Two classes of models can be found in the literature, models defined by a probability distribution function (PDF) with temperature-dependent parameters obtained through a polynomial fit, and models defined by continuous logistic functions (Qureshi et al. 2020). For the first class of models, closed-form equations are determined for the distribution parameters as a function of temperature. During the probabilistic analysis, the temperature-dependent distribution parameters are evaluated, and probability distribution functions are created. A user-input quantile is used to obtain a point on the created PDF. The second class of models is based on logistic approaches; the procedure for the probabilistic analysis is similar except that the value of the standard normal distribution parameter (ε) is used instead of the quantiles (Qureshi et al. 2020).

There are no probabilistic models available for the thermal properties of concrete in the literature. Therefore, such models are developed for the thermal conductivity and specific heat of concrete according to the first class of models explained above following the general framework offered by Qurashi et al. (2020). A data set is gathered from available literature documenting the thermal properties of concrete at different temperatures. Around 75-130 data points were collected for each of the considered properties from the work of Shin et al. (2002), Kodur (2014) and Kodur and Khaliq (2011). The property data is examined at intervals of 100^{o}C. Due to the limited number of data points, data positioned ±30^{o}C are considered for the examined interval. The data is fitted to basic distribution functions, which are functions that require a small number of parameters to define them. The following distribution functions are considered; normal, lognormal, Weibull, and Gamma. Bayesian Information Criterion (BIC) is used to determine the best distribution for the examined data at different temperatures. It was found that Weibull distribution is the best fit for the data of concrete conductivity, and Gamma distribution is the best fit for the data of concrete specific heat. Both distributions are defined by a shape and a scale factor. A second-order polynomial fit is used to express the parameters required for the chosen distributions, which are used to re-create the data used in the probabilistic analysis.

The probabilistic model of the concrete thermal conductivity k_c as a function of temperature follows Weibull distribution is presented in Fig. 3(a). The following parameters of Weibull distribution; A is the scale factor, and B is the shape are defined as

$$A = 1.493 \cdot 10^{-6} \cdot T^2 - 2.4 \cdot 10^{-3} \cdot T + 1.953 \tag{16}$$

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$$B = -8.485 \cdot 10^{-6} \cdot T^2 + 8.8 \cdot 10^{-3} \cdot T + 2.653 \qquad (17)$$

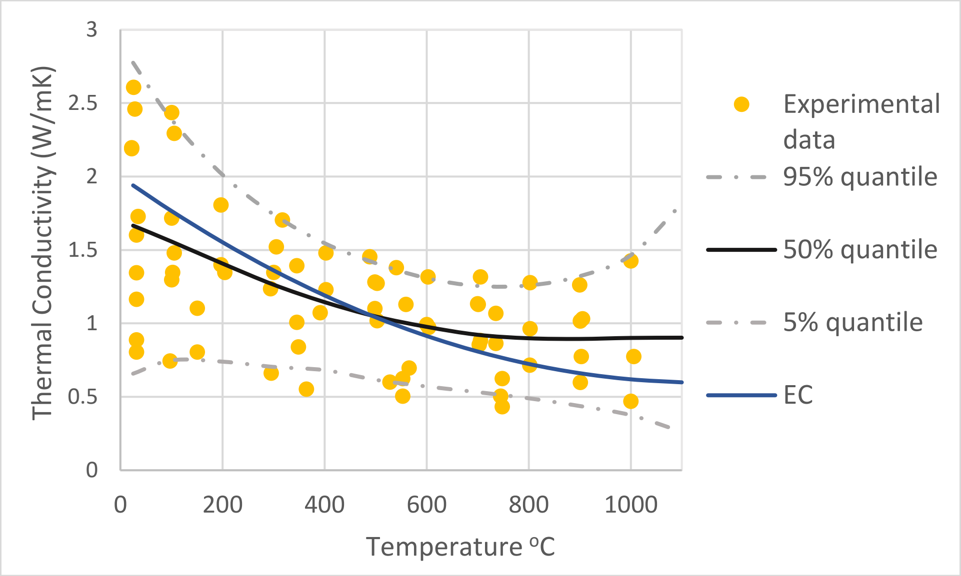
For the specific heat c_c there is a lack of data points to get consistent results for the fitted distribution; therefore, the model is developed based on the data points up to 700°C, Fig. 3(b). The data is fitted to Gamma distribution; its defining parameters; a-shape factor and b-scale factor, are expressed using Eq. (18) and Eq. (19). The developed model is then used to create the material property variation up to 1100°C.

$$a = 2.442 \cdot 10^{-6} \cdot T^2 - 5.246 \cdot 10^{-4} \cdot T + 5.389 \qquad (18)$$

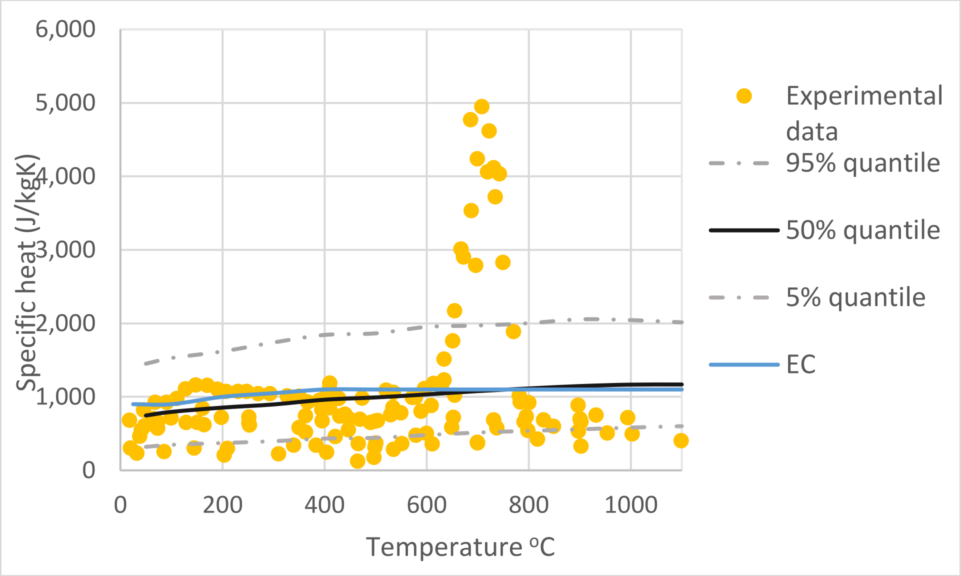
$$b = -1.082 \cdot 10^{-4} \cdot T^2 + 0.128 \cdot T + 146.78 \qquad (19)$$

The probabilistic models are developed based on collected data points, and their quality is affected by the number of points and their covered range of temperatures. However, the developed models cover the possible variation of the thermal properties' values, which is satisfactory for the purpose of this study.

As data is not available for the concrete density-temperature relation, the curve of density-temperature defined by EC2 (2004) is considered as the mean value for the probabilistic model and an assumed coefficient of variation of 0.25 is considered for the different temperatures, Fig. 3(c).


Thermal Conductivity (W/mK)
3
2.5
2
1.5
1
0.5
0
0
200
400
600
800
1000
Temperature °C
Experimental data
95% quantile
50% quantile
5% quantile
EC

(a)


Specific heat (J/kgK)
6,000
5,000
4,000
3,000
2,000
1,000
0
0
200
400
600
800
1000
Temperature °C
Experimental data
95% quantile
50% quantile
5% quantile
EC

(b)

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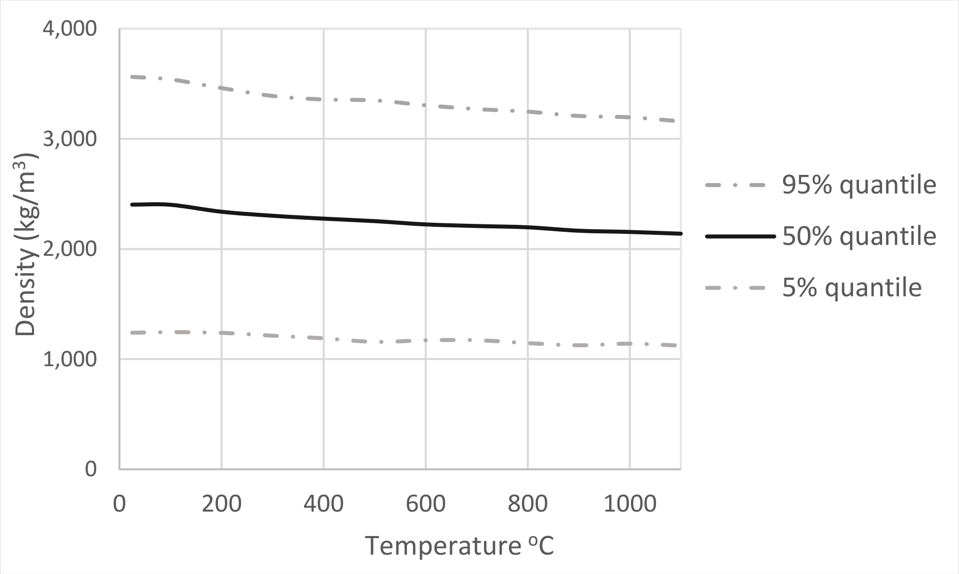

(c)

Fig. 3. (a) Thermal conductivity of the concrete; (b) Specific heat of the concrete; (c) Density of the concrete

The probabilistic model of the thermal conductivity of steel (k_s), Fig. 4(a), is of a logistic class (Khorasani et al. 2015) and is defined as

$$k_s = \begin{cases} 60 - \exp\left[ln\left(60 - \hat{k}_s\right) + 145.4 \cdot 10^{-3} - 0.5 \cdot 10^{-3} \cdot T + 0.206 \cdot \varepsilon\right] & T \leq 800^oC \\ 60 - \exp[3.23 + 0.206 \cdot \varepsilon] & T > 800^oC \end{cases}$$

(20)

where $\hat{k}_s$ is the temperature-dependent values of the thermal conductivity of steel defined in EC3 (2005).

Probabilistic models for the steel's specific heat are not available in the literature; however, the experimental data documented by Kodur et al. (2010) are used to obtain the upper and lower limits for the created samples of steel's specific heat using temperature intervals of 100°C considering the points with ±30°C within the examined temperature interval. A uniform distribution is assumed for the specific heat

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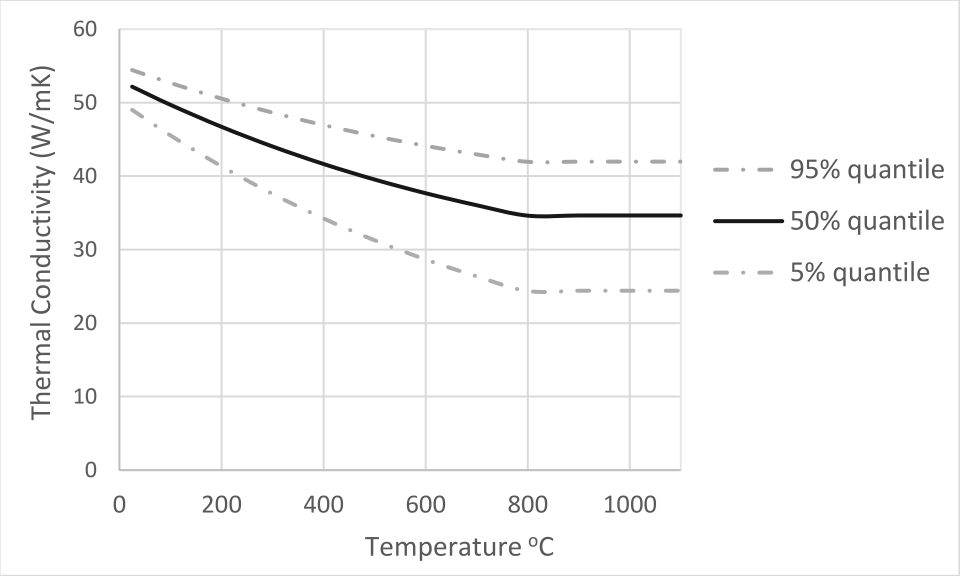
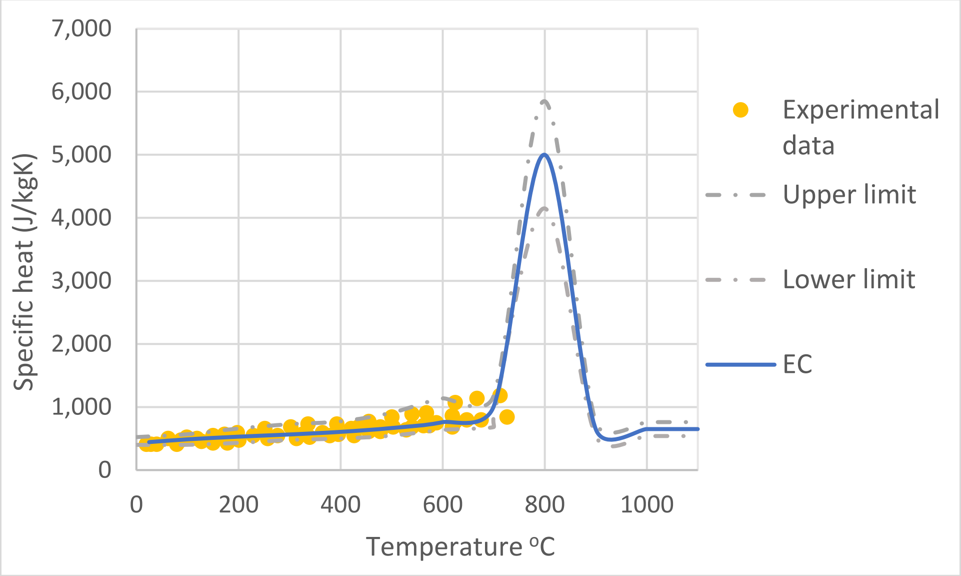
using the upper and lower limits. Furthermore, the coefficient of variation (COV) of data points at every considered temperature was calculated, and the values ranged between [0.1-0.17], where the COV increases as temperature increases. Therefore, for the temperatures with no data points, the specific heat-temperature relation offered by EC3 (2005) was used as a mean value assuming a COV of 0.17. Fig. 4(b) shows the model for the steel specific heat. Finally, the steel's density was assumed to follow a normal distribution with a mean value of 7800kg/m^3, coefficient of variation of 0.1, and the values were assumed not to be temperature-dependent.



(a)



(b)

Fig. 4 (a) Thermal conductivity of the steel; (b) Specific heat of the steel

Heat transfer model

The boundary heat transfer model composes of convection and radiation, the uncertainty in the convection coefficient was modeled using an assumed uniform distribution for the following range [10-100] W/m^{2}K, (Jowsey 2006), and the uncertainty in the emissivity was modeled using an assumed uniform distribution for the following range [0.2-0.95] (Stern-Gottfried and Rein 2012).

The input variables of the thermal model and fire models, in total 13 variables, were created in MATLAB, which ran ANSYS for the modeled input variables for the heat transfer model.

Failure Indicators

The fire resistance of RC slabs is evaluated based on the thermal-failure criteria specified in ASTM E119 guidelines. This failure indicator of RC slab thermal model is defined when one of the following is reached:

(1) The temperature of the steel reinforcing bars exceeds the critical temperature of 593°C (ASTM Test Method E119 2002).

(2) The temperature of the unexposed slab's top surface exceeds 140°C (ASTM Test Method E119 2002).

Results and Discussion

This section presents the three-part analysis of the sensitivity and reliability analyses of the fire-resisting performance of a concrete slab following the explained methodology.

Screening of the Input Variables

The method of elementary effects had been used, the analysis was run 14 times for the 13 considered input variables, and EEs of the variables were calculated. This method requires a small number of repetitions to get good results for the ranks of input variables, often 10-50 repetitions are used to calculate μ and σ. Therefore, for the first stage of the variables' screening, 50 repetitions were carried out, a total number of 700 transient-nonlinear thermal analysis was run. The mean value μ and standard deviation σ of EEs were calculated, μ indicates the variable's rank, and σ indicates the variable's interactions with other variables. These measures were calculated for the temperatures of the concrete slab at different positions; bottom, middle, and top, and the temperature of the bottom reinforcement. All measurements were calculated at different time points. Following Eq. (2) μ indicates the average change caused by the variation of input variable on the fire performance measure. Therefore, the input variable that had μ_i equal to or larger than 10% of the maximum μ for the examined performance measure was considered influential. Furthermore, a ratio ($\sigma_i/\mu_i \leq 0.1$) indicates that the variable has no interactions with other variables. The adopted limit is similar to the one found in (Sanchez et al., 2012).

Fig. 5 depicts μ–σ values of the input variables' considering the temperatures of the bottom surface of the concrete slab (fire exposed surface) and the temperature of the bottom reinforcement considering their maximum temperature. For the fire exposed surface of RC slab, Fig. 5(a), it can be seen that the important input variables are as the following: from the fire model; firefighting measures index (FFMi), characteristic fuel load density ($q_{f,k}$), area ratio of the compartment (A_f/A_t), and the opening factor (O), from boundary heat transfer mechanisms, convection coefficient (h), and from the thermal model of the slab; concrete specific heat (c_c), concrete conductivity (k_c), and concrete density (d_c). Interactions for the variables are the highest for characteristic fuel load density ($q_{f,k}$), firefighting measures index FFMi, Opening factor (O), and convection coefficient (h). This may be explained by the fact that the interaction between (FFMi, q_{fk}, and O) decides whether the fire is fuel- or ventilation-controlled, which consequently affects the thermal analysis, and the convection mechanism decides the transfer of the fire heat to the slab’s exposed surface. The screening of the input variables affecting the temperature of the steel reinforcement is similar to the one of the exposed surface of the slab, Fig. 5(b). The only difference is in the ranking of the following input variables; the opening factor (O), concrete specific heat (c_c), and concrete density (d_c), which have higher ranks of influence on the steel’s temperature. The higher ranks of concrete specific heat (c_c) and concrete density (d_c) characterize the effect of the concrete mass engulfing the steel rebars on their temperature gradient. The sensitivity measures (μ and σ) of steel density (d_s), steel conductivity (k_s), steel specific heat (c_s), and emissivity (e_{ms}) were lower than the previously assigned limits for μ and σ/μ. Therefore, these variables were considered non-influential variables on the

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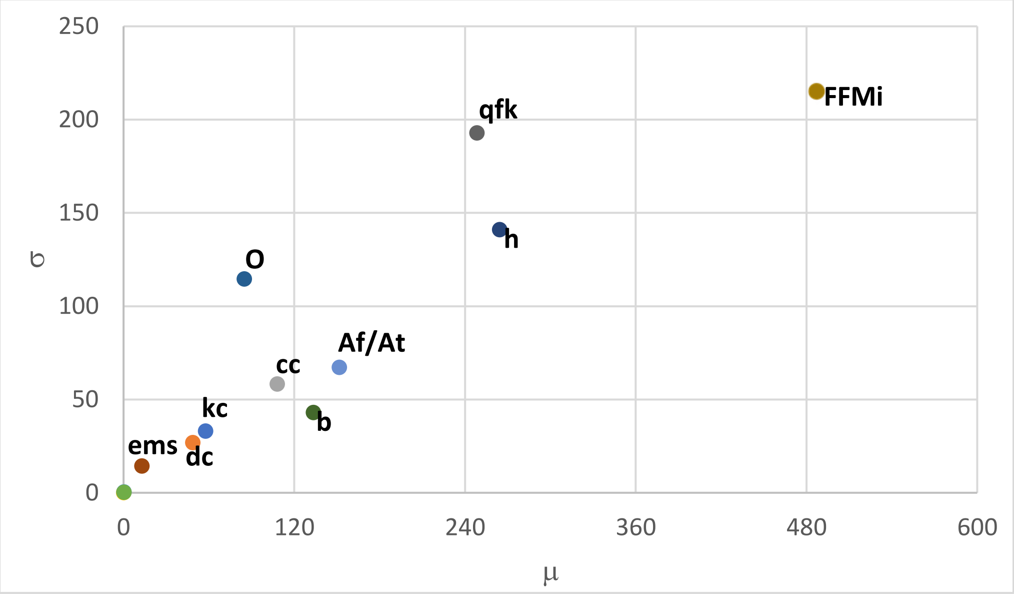
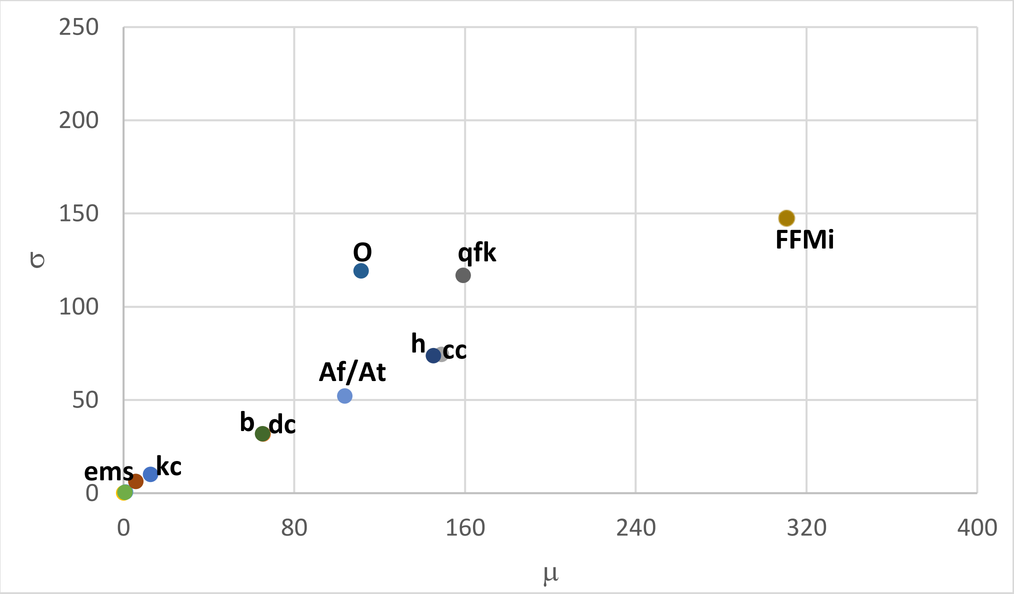
examined thermal responses. Furthermore, their sensitivity measures were too small to be added in Fig. 5.



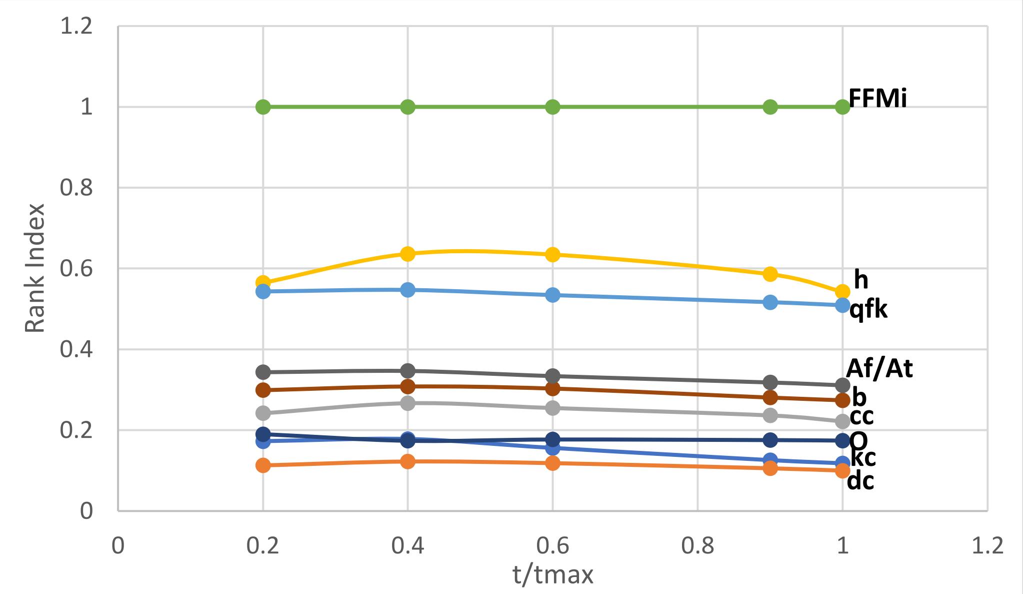
(a)



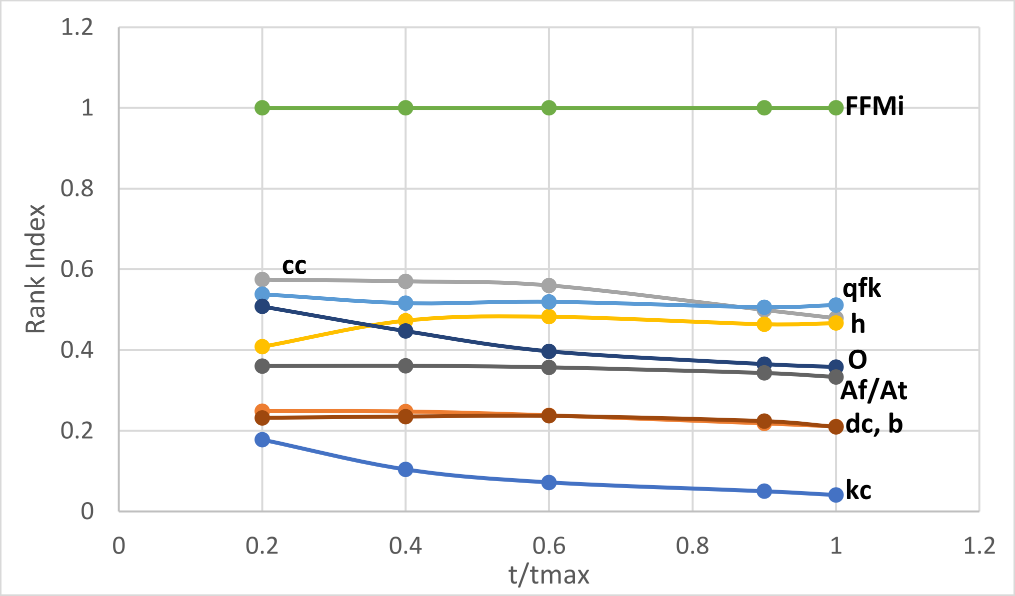
(b)

Fig. 5. (a) μ-σ of the input variables considering the exposed surface; (b) μ-σ of the input variables considering the steel rebars

Furthermore, the ranks of input variables during the time duration of exposure are examined and presented in Fig. 6. In order to compare the results from the thermal analysis for different fire exposure durations, the time is normalized using t_{max} of the fire model. For the slab's exposed surface, shown in Fig. 6(a), it can be seen that there is no significant change of the ranks with time. Fig. 6(b). for the steel rebars shows that the ranks c_c, h, and O change slightly with time. The rank of h increases with time and the rank of O and c_c decreases as the fire progresses for the steel rebars.



(a)



(b)

Fig. 6. (a) Rank index of the input variables with time considering the temperature of the exposed surface; (b) Rank index of the input variables with time considering the temperature of steel rebars

Fig. 7 depicts the screening of the variables affecting the progression of sectional temperature in the slab, i.e. the temperature of the middle and the top surfaces of the slab. The variables affecting the temperature of the middle surface, Fig. 7(a), are firefighting measures index (FFMi), concrete specific heat (c_c), opening factor (O), characteristic fuel load density ($q_{f,k}$). Compartment area ratio (A_f/A_t), concrete density (d_c), concrete conductivity (k_c), and convection coefficient (h) are identified with intermediate ranks of influence. It is observed that the variables defining part of the thermal model, specific heat (c_c), density (d_c), and conductivity (k_c) have higher ranks of influence on the thermal performance of the middle surface, whereas the rank of (h) is decreasing. The same observation for the steel rebars regarding the decreasing rank of opening factor (O) and concrete specific heat (c_c) with time is found for the middle surface of concrete, Fig. 8(a).

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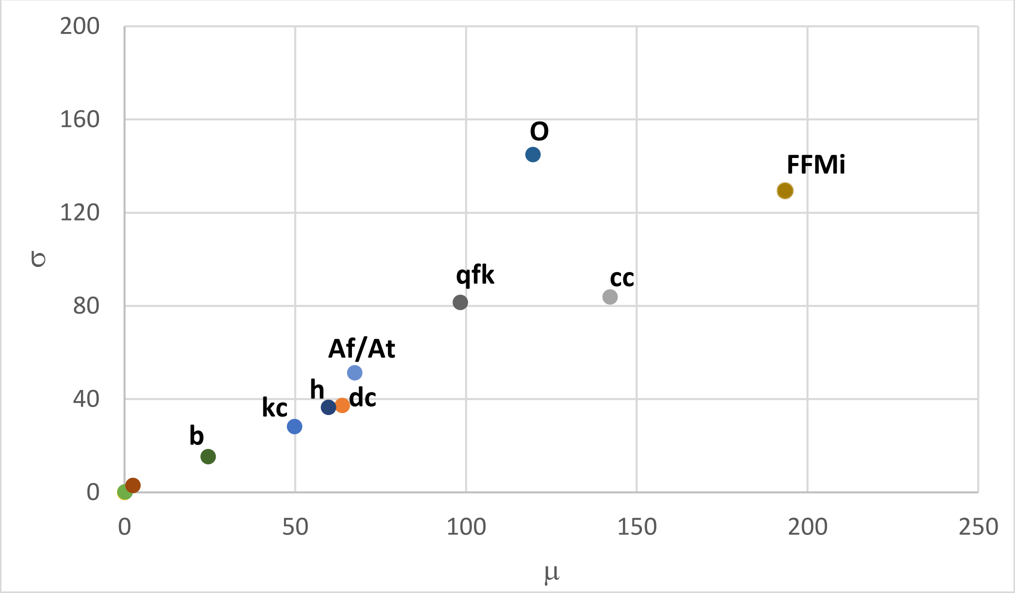
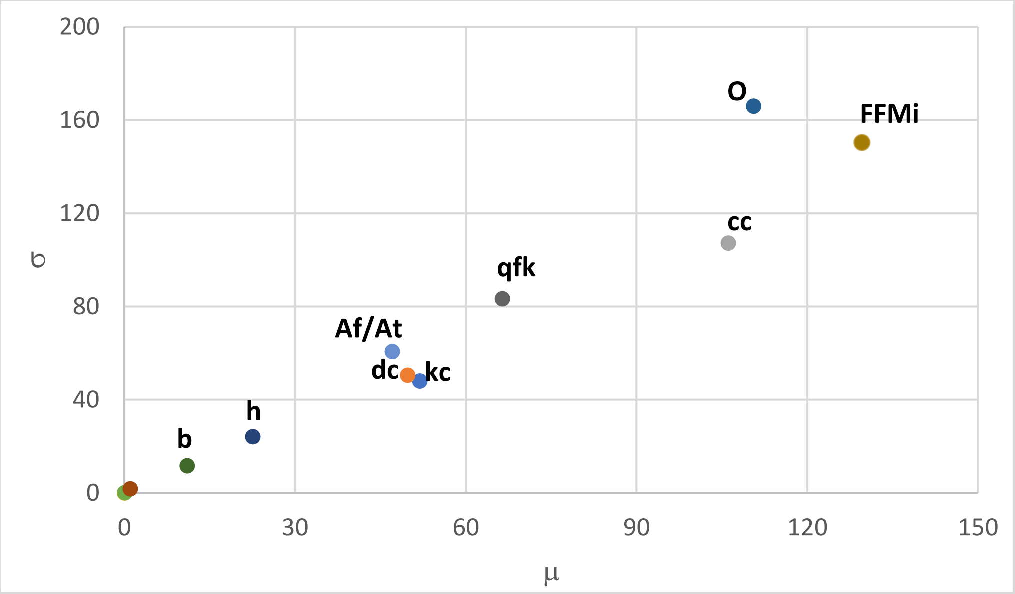


(a)



(b)

Fig. 7. (a) μ-σ of the input variables considering middle layer of the RC slab; (b) μ-σ of the input variables considering top layer of the RC slab

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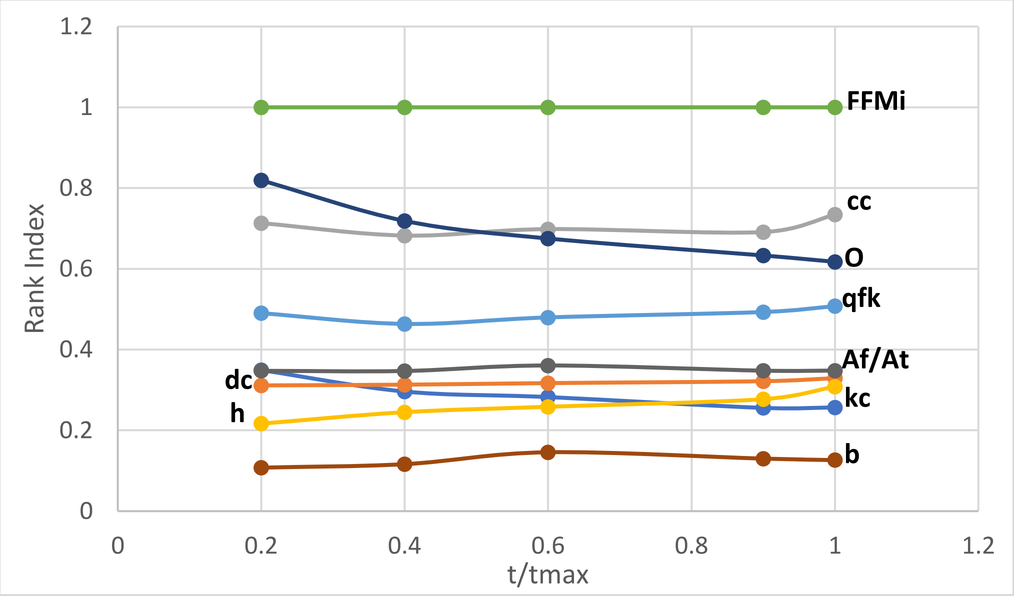
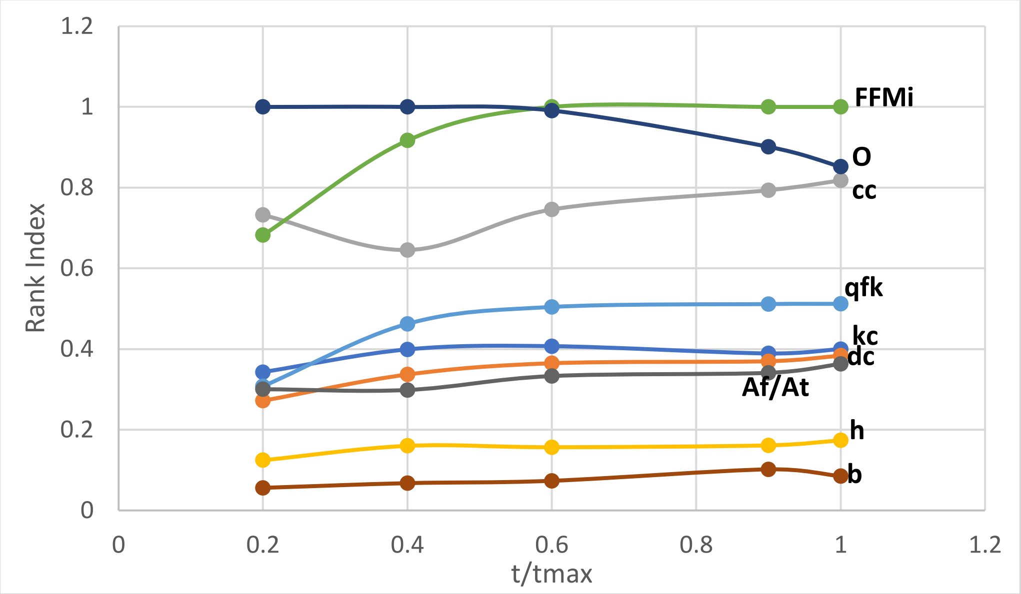
The ranks of the input variables affecting the top surface (unexposed surface) of RC slab are similar to the ones for the concrete middle surface, Fig. 7(b). However, the ranks of k_c and d_c are higher for this layer. Furthermore, examining the ranks as a function of the exposure time, the opening factor (O) has the highest rank of influence, and as the fire progresses its rank decreases and fire-fighting measures (FFMi) rank increases, Fig. 8(b). The heat convection (h), thermal inertia of the compartment (b), compartment area ratio (A_f/A_t) are not as influential on the temperatures of these layers, middle and top, when compared with their effect on the exposed surface and steel rebars. In general, as one moves away from the exposed surface, FFMi and O from the fire model, and c_c, d_c, and k_c from the thermal model are the influential input variables on the thermal performance. Steel conductivity (k_s), steel specific heat (c_s), and emissivity (e_{ms}) were identified as non-influential variables on all calculated thermal responses.



(a)



(b)

Fig. 8. (a) Rank index of the input variables with time considering the temperature of the middle surface; (b) Rank index of the input variables with time considering the temperature of the top surface;

Further examination of the thermal performance using μ was done by taking advantage of running a higher number of repetitions to calculate the total sensitivity indices as explained in the methodology. The extended analysis (second-stage of variables' screening) was run for the identified input variables excluding the non-influential variables on all measured thermal responses, which means that the steel density (d_s), steel conductivity (k_s), steel specific heat (c_s), and emissivity (e_{ms}) had been excluded from this stage of analysis. A total number of 500 repetitions was performed, a total number of 6000 samples were run. The required number of repetitions for the stability of the results of the elementary effects method is tested in Fig. 9. It can be seen that running 200 repetitions and above showed stability in the results. Therefore, 500 repetitions are enough to use μ as an indication of the relative importance between the input variables, not only identifying their ranks. This is used to examine the effect of input variables when considering fuel-controlled and ventilation-controlled fires. The temperature-responses used in the calculation of the reliability index are used for this stage of analysis, which are the temperature of steel rebars and the top surface of RC slab.

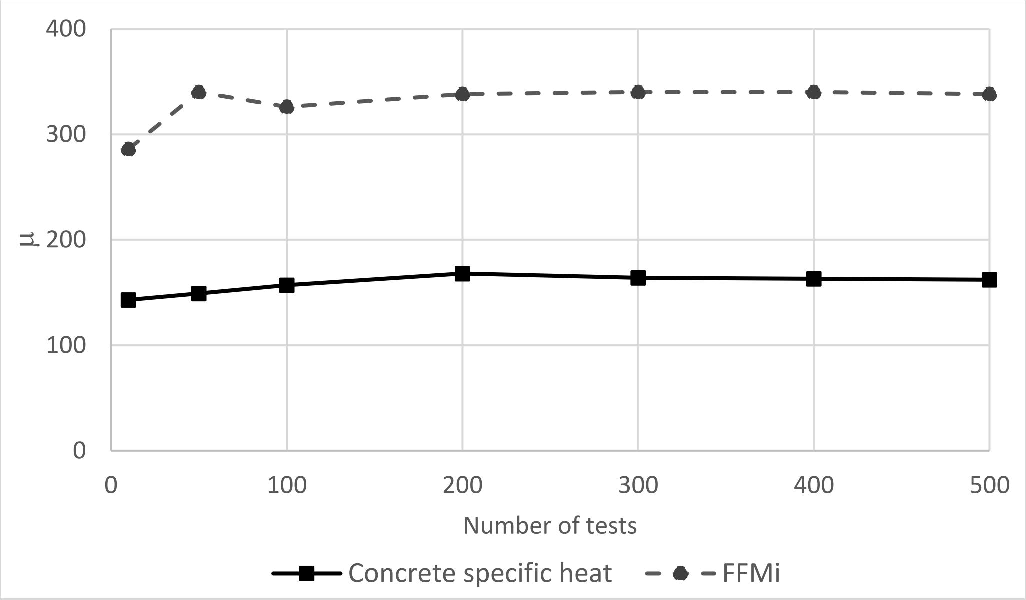


Fig. 9. Number of tests (repetitions) required for stable results for screening measure (μ) considered temperature of the steel rebars

Fig. 10. shows the screening measure μ for the input variables considered; fuel-controlled fire and ventilation-controlled fire. Fire-fighting measures (FFMi) is the dominating factor considering the temperature of the steel rebars. However, the effect of the thermal model variables, especially concrete specific heat (c_c), concrete conductivity (k_c), concrete density (d_c), and convective coefficient (h) is larger for the ventilation-controlled fire, Fig. 10(a). This is because these ventilation-controlled fires are longer in duration and have a plateau at high peak temperature levels. Therefore, if the slab is exposed longer to

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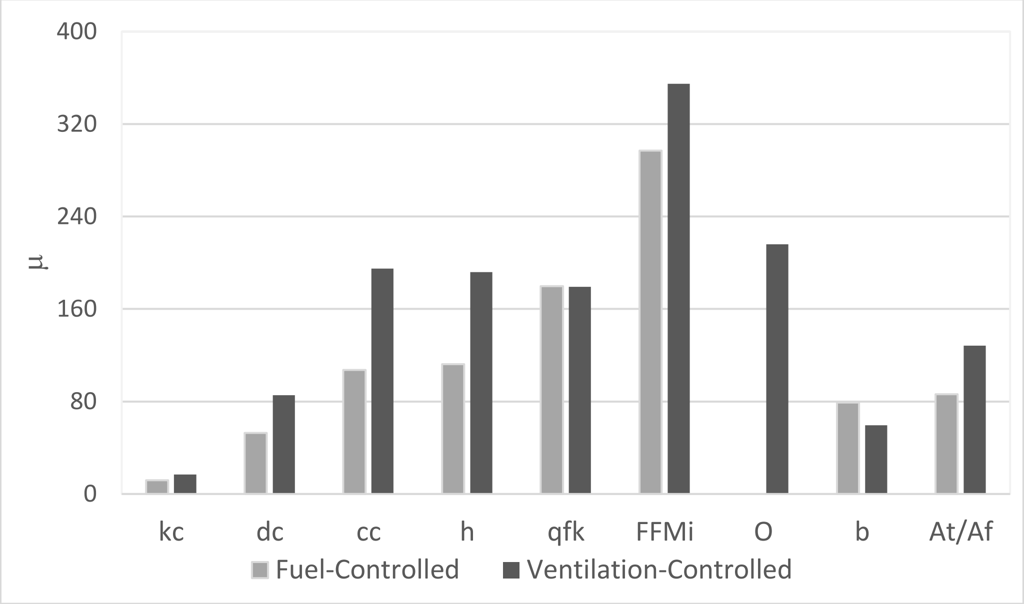
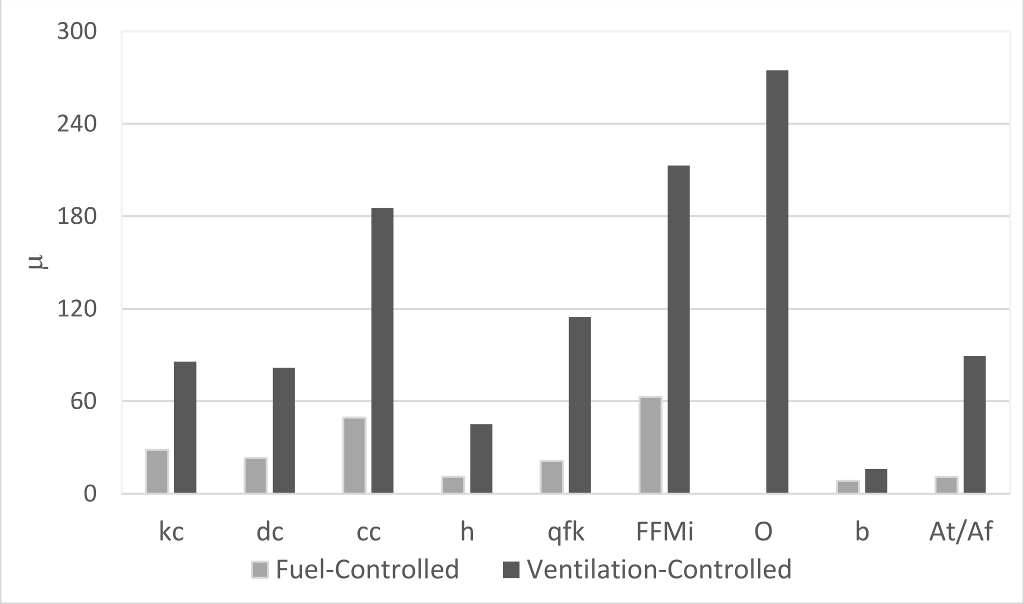
high temperatures, and heat is transferred via convection and radiation at the boundary and by conduction within the slab's body, the variables defining these mechanisms will have a significant influence. The temperature of steel rebars considering fuel- or ventilation-controlled fires is affected by the firefighting measures index (FFMi), characteristic fuel load density ($q_{f,k}$), and compartment area ratio (A_f/A_t) from the fire model.

Fig. 10(b) presents the screening measure μ for the top surface (unexposed surface) of the RC slab. The identified input variables presented in order are as the following; FFMi, c_c, k_c, d_c, q_{fk}, and h for fuel-controlled fire and as the following O, FFMi, c_c, q_{fk}, A_f/A_t, k_c, d_c, and h for ventilation-controlled fire. The pattern noticed is that thermal response to fuel-controlled fires is influenced by fuel load density and firefighting measures index, which affect the peak of temperature-time curves of the fire. Whereas for the thermal response due to ventilation-controlled fires, opening factor and A_f/At are as influential as firefighting measures index and fuel load density, as they affect not only the peak but also the duration of temperature-time curves of the fire. Furthermore, the concrete thermal properties are influential for fuel- and ventilation-controlled fires and the heat convection coefficient is more influential for ventilation-controlled fires for the same reason explained for steel rebars.



(a)



(b)

Fig. 10. (a) The sensitivity measures considering fuel-controlled and ventilation-controlled fires for the temperature of the steel rebars; (b) The sensitivity measure considering fuel-controlled and ventilation-controlled fires for the temperature of the top surface of the RC slab

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The findings of the variables' screening show position-dependency and time-dependency, noticed for the steel reinforcement and the unexposed surface of the slab. Such an examination of screening measures is needed and required to guide better the further investigation of the behavior of the slab under fire loading and assess the criteria of performance or failure depending on the used framework of design.

Total Sensitivity indices of Input Variables

The elementary effects are calculated using the variation of the response due to change in the input variables, whereas the total sensitivity indices (S_T) are calculated using the variances of the output. This means that S_T indicates mainly the contribution of the variable's uncertainty and its interaction with other variables to the output uncertainty. Such measurement of S_T indices shed light on the uncertainty of the output and its constituents. This is vital for running the reliability analysis. Following the explained methodology, the technique to create the samples for the elementary effects method was extended and using modified estimators, the total sensitivity indices were calculated. The total sensitivity indices were calculated for the influential input variables identified from the screening analysis, which meant that the steel density, steel conductivity, and steel specific heat, and emissivity were excluded from this sensitivity analysis. The number of samples is important to the stability of the values calculated for S_T; Fig. 11 examines the number of samples with calculated S_T. It is noticed that above 400 tests (repetitions), the measurement of the indices is stable; in this analysis 500 tests (repetitions) were used. The sensitivity indices (S_{Ti}) were calculated at different time steps. Their median value was calculated and shown in Fig. 12 as an overall representation of the variables influence on the thermal response at the different monitoring points. It is found that the uncertainty of firefighting measures index (FFMi), opening factor

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(O), concrete specific heat (c_c), heat convection (h), fuel load density (q_{fk}), compartment area ratio (A_f/A_t), and concrete density (d_c), are the main sources of the output uncertainty for the considered thermal responses. Moreover, looking more closely, the following is observed:

- The summation of the total sensitivity indices for the considered input variables is larger than one, which means that interactions between input variables exist and this agrees with the finding of the variables' screening
- Higher uncertainty contribution for $c_{c,}$ d_c, and k_c is observed as one moves away from the exposed surface
- Higher uncertainty contribution for O is observed as one moves away from the exposed surface
- Lower uncertainty contribution of h, and b is observed as one moves away from the exposed surface

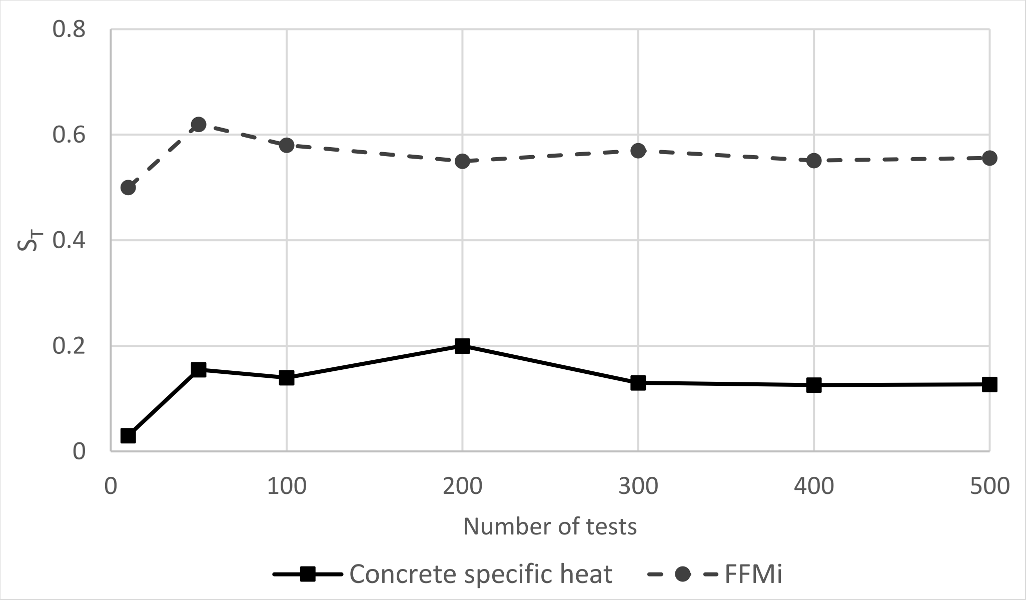


Fig. 11. Number of tests (repetitions) required for stable results for the total sensitivity indices (S_T) considering the temperature of steel rebars

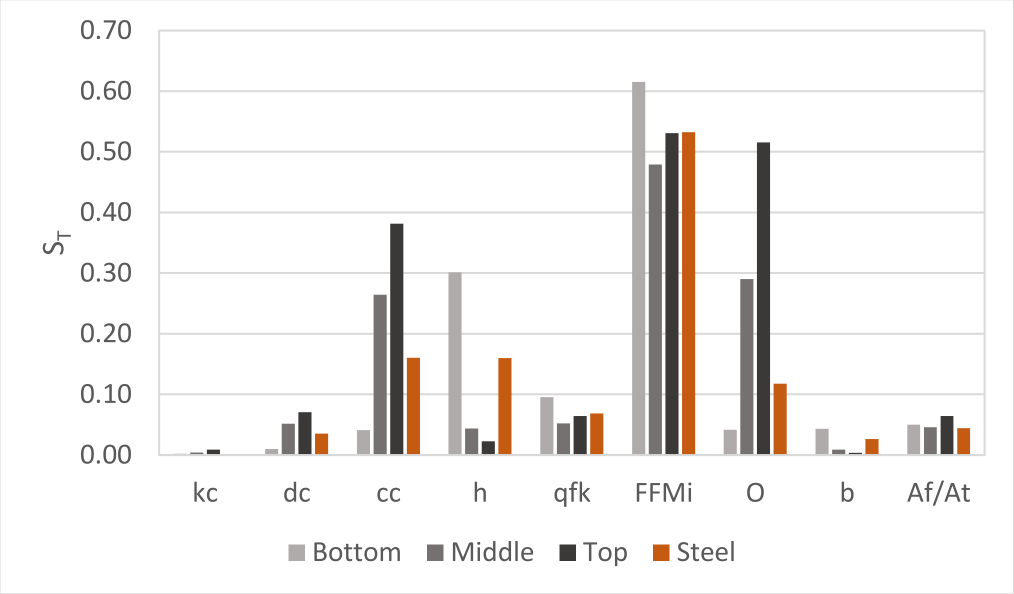
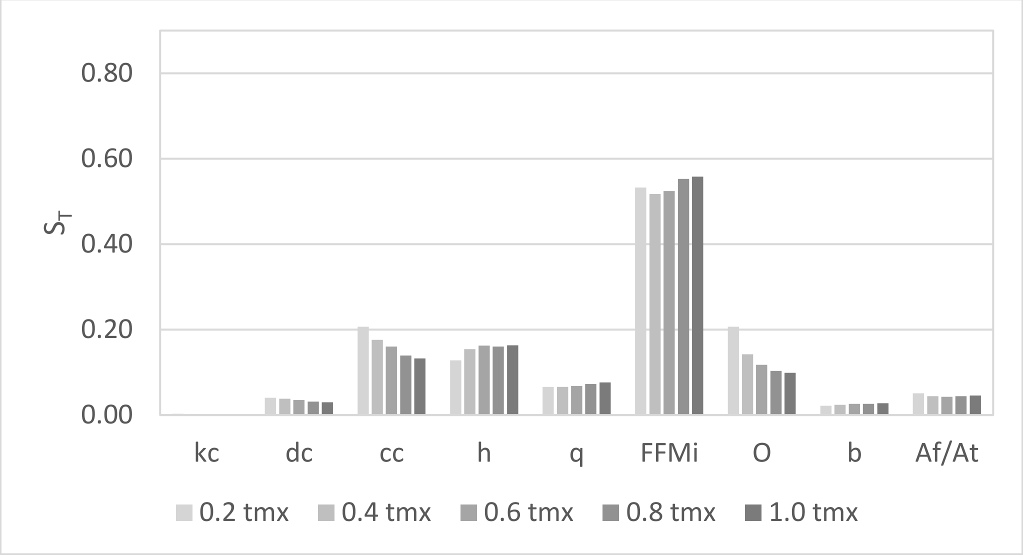
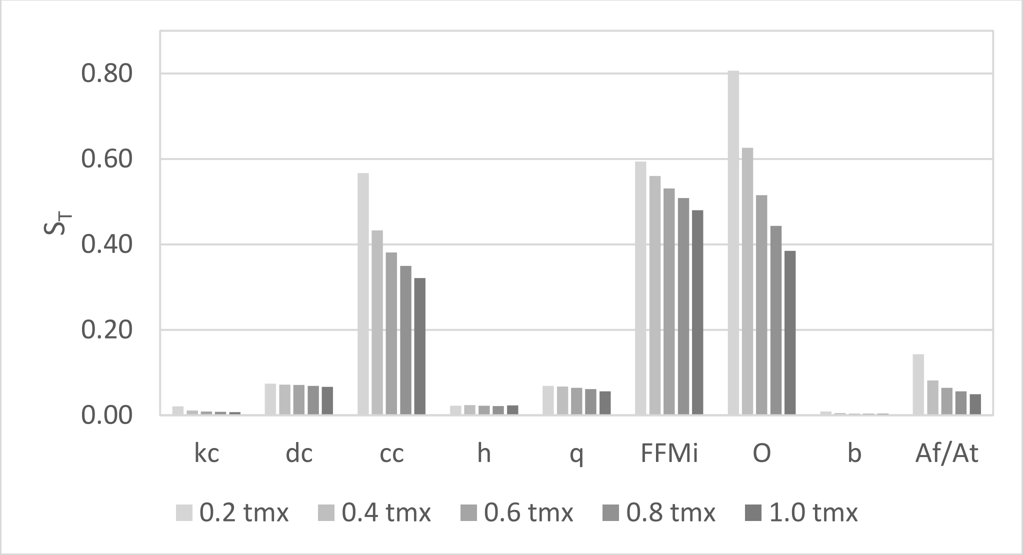


Fig. 12. The total sensitivity index (median value) of the input variables influencing the thermal performance of the slab

The above observations may pinpoint the dominant heat transfer mechanism at the different depths of the slab. As noticed in the variables' screening analysis, where conduction is dominant uncertainty of thermal properties of concrete are significant, and where it is a combination between convection and conduction, then the variables defining all mechanisms are significant. Furthermore, the sensitivity indices of the input variables during the fire duration are examined in Fig. 13 for the responses used in the reliability analysis. The time is normalized using t_{max} of the fire model in order to compare the analysis of the different fire-curves durations. Total sensitivity indices (S_T) are time-dependent, e.g., for the temperature of steel reinforcement, c_c and O have a decreasing contribution with a prolonged exposure to fire, and h has an increasing contribution with a prolonged exposure to fire. The same is noticed for the temperature of the unexposed top surface, furthermore, firefighting measures index has a decreasing contribution with a prolonged exposure to fire.



(a)



(b)

Fig. 13. (a)The total sensitivity index of the input variables for temperature of steel rebars; (b) The total sensitivity index of the input variables for temperature of top surface of RC slab

In order to have a comprehensive examination of the uncertainty constituents, one would like to identify the effect of the chosen type of fire scenario (Characteristic fire, standards fire, and Hydrocarbon fire),

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fire duration (e.g. 60, 90, 120 minutes), and variability in the slab's thermal model (nominal values/random values) on the thermal response. The setting of this examination is done by keeping the maximum temperature the same for all modeled fire curves. The same algorithm to develop the samples for the determination of the total sensitivity indices was used. However, the quantiles are mapped into discrete numbers presenting the fire model type, duration, and variability in the thermal model of the slab, a similar approach as the one in Keitel et al. (2011). The mapping is explained in Table 3, for example, if the randomly generated samples for the three variables are as the following (0.2, 0.5, 0.7) then for this sample a characteristic fire model with a duration of 90 minutes is modeled and used in the analysis considering the nominal values of the variables defining the thermal model of the slab. The variability of the thermal model is considered through the variables related to the heat transfer by convection and radiation within the boundary and the conduction within the concrete slab; and they are steel's and concrete's specific heat, conductivity and density, heat convection coefficient and emissivity.

Table 3. Mapping of the discrete input variables

Model Indicator	**1**	**2**	**3**
Fire Model Type Quantile Range	Characteristic fire $0 < Q \leq 0.33$	Hydrocarbon fire $0.33 < Q \leq 0.67$	Standard fire $0.67 < Q \leq 1.0$
Fire Duration Quantile Range	60 minutes $0 < Q \leq 0.33$	90 minutes $0.33 < Q \leq 0.67$	120 minutes $0.67 < Q \leq 1.0$
Variability in Thermal Model Quantile Range	Uncertainty considered $0 < Q \leq 0.50$	Nominal values considered $0.50 < Q \leq 1.0$	------

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In this analysis, 500 tests (repetitions) were performed, a total number of 4000 simulations were run. The sensitivity indices were calculated and presented in Table 4. The chosen fire model has the highest effect on steel's temperature; while, the duration affects more the middle and top surfaces. This is consistent with the sensitivity analysis of input variables defining the temperature-time curves, as the variables affecting the fire duration were more significant for the unexposed top surface of the slab. Furthermore, the sensitivity analysis for input variables identified that h is more influential on the thermal response of the exposed surface, and the thermal properties of the slab are more influential on the thermal response of surfaces away from the exposed surface, and the uncertainty contribution of c_c is decreasing with longer fire exposures. The combination between these observations may explain the increase in the sensitivity index for variability in the thermal model and then the decrease as one moves away from the exposed surface.

The analysis of the influential input variables and their sensitivity indices answers essential questions regarding the reliability of the thermal performance of the slab. Moreover, this methodology can be easily implemented within the chosen design framework. It requires a reasonable number of simulations which depends on the purpose of analysis; essential screening requires the lowest number of simulations; a more detailed analysis requires higher number of simulations to determine quantified ranking measurements and sensitivity indices. The results of the sensitivity analysis are used to identify the variables used in the reliability analysis and better guide the experimental work to develop models for the input variables and processes. These are valuable information when shifting fire-resisting engineering from conventional to performance-based design.

Table 4. Sensitivity indices S_T for the choice of fire model, duration and variability in the thermal model

Total Sensitivity Indices S_T	Temperature of Slab's Exposed Surface	Temperature of Bottom Steel Reinforcement	Temperature of Slab's Middle Surface	Temperature of Slab's Top Surface
Fire Model Type	0.83	0.77	0.53	0.24
Fire Duration	0.34	0.47	0.78	0.92
Variability in Thermal Model	0.24	0.31	0.39	0.26

Reliability Analysis

Using Monte Carlo simulation, a value is selected at random for each of the input variables based on the developed distributions. The maximum rebar temperature in the concrete slab and the temperature on the concrete slab's unexposed top surface are obtained after performing the transient thermal analysis. The process is repeated; and the probability of exceeding the thermal-failure criteria is calculated. This calculated probability of failure is a conditional probability upon the occurrence of the occupancy-specific fire scenario used in the analysis. The thermal properties of steel and emissivity were considered at their nominal values as they were identified as non-influential. Furthermore, the analysis is run at certain levels of FFMi; four cases are chosen to represent typical firefighting measures installed in typical existing housing. Table 5 presents the considered cases. The gaussianity of the limit state function was tested. It was found that the steel rebars temperatures and unexposed surface temperatures follow a lognormal distribution. The test was performed using Bayesian Information Criterion (BIC). Fig. 14 compares the cumulative distribution function of the actual data points and the fitted data points to a lognormal distribution for the temperature of steel rebars and exposed surface of the concrete, and a linear correlation is noticed which supports the finding of BIC test. Therefore, the reliability indices are calculated using

Eq. 10. The calculated reliability indices are conditional upon the occurrence of the occupancy-specific fire scenario used in the analysis.

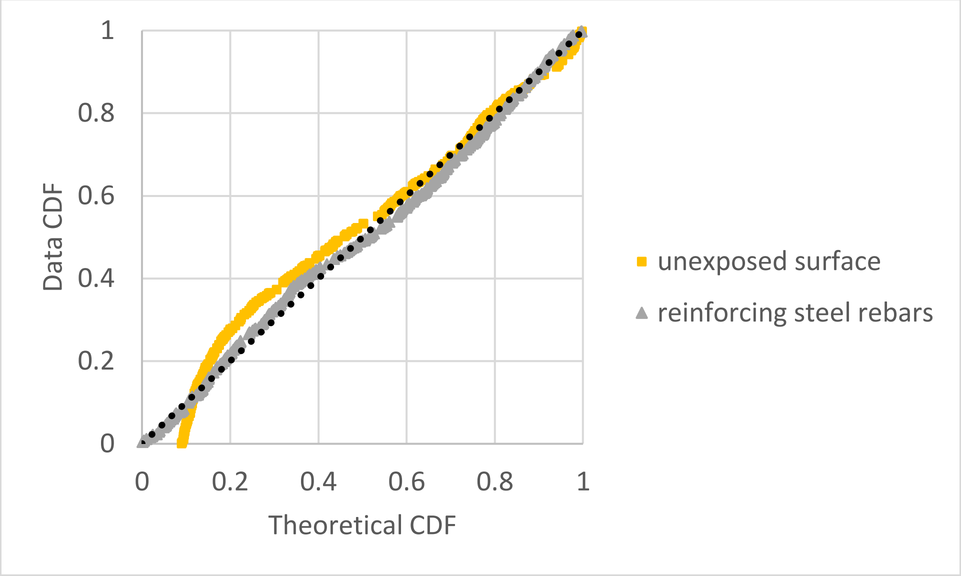

Fig. 14. The probability-probability plot for the temperature of the steel rebars and unexposed surface fitted to lognormal distribution for Case IV

Table 5. Reliability indices following different firefighting measures

Case #	Detection system available	Sprinkler system available	Fire-fighting intervention available	Calculated FFMi	Probability of thermal-failure	Reliability Index (β)
I	No	No	No	3.37	0.565	0.151
II	No	No	Yes	2.25	0.342	0.321
III	No	Yes	Yes	1.37	0.143	1.033
IV	Yes	Yes	Yes	0.99	0.064	1.448

As the number of considered samples is essential to trust the values of the reliability analysis, a test of the required number of samples was carried out and its results are shown in Fig. 15. The stability of the results

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of the probability of failure is observed for a sample count above 500. The number of samples used for the results in Table 5 was 1000.

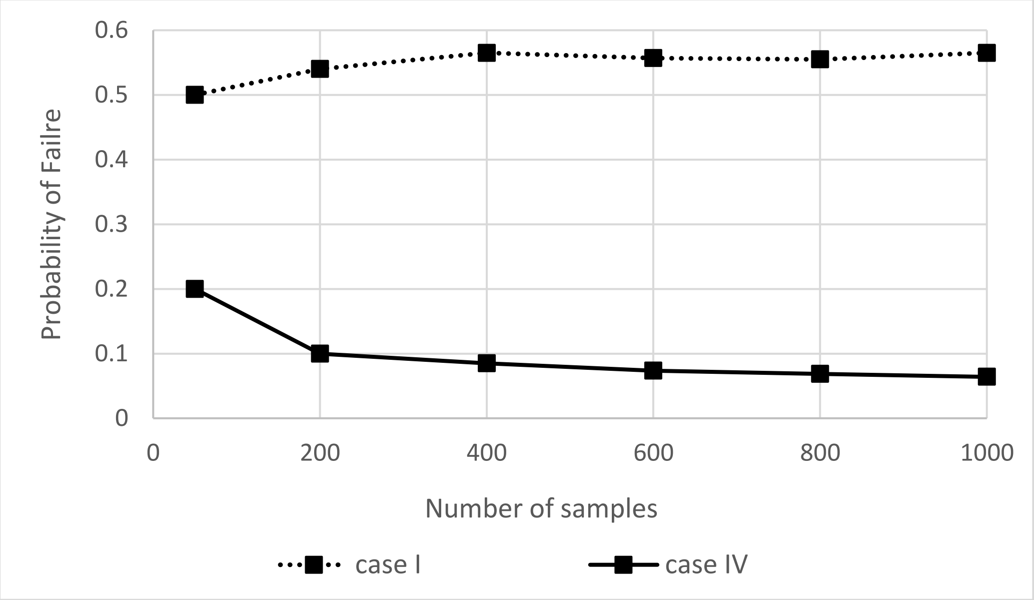

Fig. 15. Number of samples required for stable results for the reliability indices

From Table 5., it can be observed that a low-reliability index of 0.151 is expected for the flooring systems in buildings of category I, however, adding intervention measures, sprinkler systems, and detection systems will increase the reliability index by 53%, 85%, and 89% respectively.

Conclusions

The following observations and conclusions can be drawn from the results of this study:

- The temperature rise during fire exposure at the exposed bottom surface of the slab and the bottom layer of steel rebars are influenced by these critical input variables; firefighting measures index (FFMi), characteristic fuel load density ($q_{f,k}$), opening factor (O), and area ratio

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of the compartment (A_f/A_t), from the thermal model; convection coefficient (h), concrete specific heat (c_c), concrete density (d_c), concrete conductivity (k_c).

- As one moves away from the exposed surface, FFMi and O from the fire model and c_c, k_c and d_c from the thermal model are the influential input variables on the thermal performance of RC slab.
- In general, for the middle and unexposed top surfaces, the heat transfer is controlled by conduction, and that is the reason behind the increasing effect of the variables related to this thermal mechanism; specific heat, conductivity, and density, and more evident for ventilation-controlled fires. The findings of the variables' screening show position-dependency and time-dependency.
- It is found that the uncertainty of firefighting measures (FFMi), opening factor (O), concrete specific heat (c_c), and convection coefficient (h) are the primary sources of the output uncertainty for the considered thermal responses.
- Flooring systems of residential buildings with no basic firefighting measures have a low-reliability index of 0.151, and adding intervention measures, sprinkler systems, and detection system will lower the probability of failure and increase the reliability index by 53%, 85%, and 89%.

The performed sensitivity analysis justifies the decision for more examinations or simplifications for a number of input variables or processes defining the heat transfer mechanisms and fire models. This is essential to inform the reliability analysis for the fire resistance performance of RC slabs, which is required for advanced fire-resistance design frameworks such as PRA and PSFE.

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Limitations and Ongoing research

The sensitivity and reliability analyses were performed for the thermal model of RC slab. Therefore, the effect of variables defining the mechanical model and consideration of strength failure criteria for RC slab were not within the scope of this study. The challenge of running a probabilistic analysis for the thermo-mechanical model is its demanding computational power, storage, and time. Thus, there is a research need to introduce meta-models, mathematical models for the prediction of the output, in fire engineering to represent the response of the thermo-mechanical model; these models are to be developed using the available and often limited experimental and numerical data points. The research must aim to investigate the types of suitable meta-models, requirements on data points, and criteria to assess the quality of these model's predictions. The developed meta-models will allow further investigations for the thermo-mechanical responses of a structural element in a timely fashion. These models can be used in running sensitivity and reliability analyses, and their output can guide decision-making processes in design stage.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

- Collected data points for the thermal properties of the concrete
- MATLAB code files for the algorithms used for the probabilistic analysis
- ANSYS files for the thermal analysis of RC slab

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