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A conditional higher-moment CAPM

Vasco Vendrame *, Cherif Guermat, Jon Tucker

University of the West of England, United Kingdom

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ABSTRACT

This paper investigates whether dynamic and moment extensions to the traditional CAPM can improve its empirical performance and offer some alternative explanation to the cross-section of average returns on portfolios of stocks double sorted on book-to-market ratios and size. We consider three extensions. First, we introduce time-varying factor loadings obtained from a multivariate GARCH and dynamic conditional correlations. Second, we extend the model to a four-moment CAPM, which incorporates coskewness and cokurtosis. Finally, we allow for time-varying risk premia, based on a Markov-switching process. Our results confirm that the higher-moment CAPM does not perform well in its unconditional version, but its performance is significantly improved when we introduce a conditional version that accounts for both time-varying factor loadings and time-varying risk premia. The four-moment CAPM tests lead to a positive total risk premium estimate of 0.67% per month over the period 1926–2021, with all risk premia (beta, coskewness, and cokurtosis) exhibiting the expected theoretical signs.

1. Introduction

The traditional Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966) has generally failed to satisfactorily explain the cross section of average returns after several decades of empirical testing (Black, Jensen, & Scholes, 1972; Douglas, 1969; Fama & French, 1992; Fama & MacBeth, 1973; Lintner, 1965). Yet, its simplicity and theoretical appeal have cemented its position as the most popular model used by US companies to estimate the cost of equity capital (Graham & Harvey, 2001).

Attempts to improve on the simple CAPM while retaining the spirit of the systematic risk principle have led to two different streams, the conditional and the unconditional. The conditional stream recognises that the CAPM is actually a conditional model and may not hold unconditionally even if it holds conditionally (Jagannathan & Wang, 1996). Conditional versions of the CAPM have so far presented mixed results (Lettau & Ludvigson, 2001; Vendrame, Guermat, & Tucker, 2018).

The second stream focuses on the specification of the unconditional model by adding more proxies for systematic risk. Within this stream, there are two approaches. One approach implies that the market portfolio is not the only relevant risk factor. The most prominent models include the three-factor model of Fama and French (1993) and the four-factor model of Carhart (1997). Unfortunately, despite their popularity, and despite several attempts to create additional factors, these types of models have not been able to successfully explain expected returns (Fama and French, 2004).

Critics of this approach argue that the problem lies in the (in)ability of the second moment to fully account for systematic risk. The problem is the choice of moments rather than the choice of (market versus non-market) portfolios. Kraus and Litzenberger (1976) pioneered extensions based on increasing the moments of the investor optimisation problem and introduced skewness as an additional term to the CAPM, while Dittmar (2002) introduced the fourth moment, kurtosis. Empirical evidence on the relevance of skewness and kurtosis within the unconditional stream is provided by Vendrame, Tucker & Guermat (2016).

This paper combines both streams by considering higher moments in a conditional context. Unlike the multifactor models, higher moment models are not ad-hoc and share with the CAPM the fact that they are grounded on sound economic theory. They are also more intuitive and simpler to apply in practice. Indeed, higher moment models only require the market portfolio, making it straightforward to apply in emerging markets. Higher moment models are also supported by ample evidence that stock returns exhibit skewness and a heavy-tailed distribution (Taylor, 2005). Vendrame et al. (2016) discuss various advantages of the higher moment model over the standard CAPM.

During the past few decades, financial markets have seen extreme gains and losses in episodes of crises and recoveries, starting with the 1987 crash and ending with the 2019 Covid-19 pandemic, and at a rate of at least one major crisis every decade. This increased frequency and

* Corresponding author at: Bristol Business School, University of the West of England, Frenchay Campus, Coldharbour Lane, Bristol BS16 1QY, United Kingdom.
E-mail addresses: Vasco.Vendrame@uwe.ac.uk (V. Vendrame), Cherif.Guermat@uwe.ac.uk (C. Guermat), Jon.Tucker@uwe.ac.uk (J. Tucker).

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amplitude of financial market turbulence calls for a greater understanding of market risk. Considering return distributions with higher moments is one way of capturing the complexity of market risk. Further, the fact that markets sometimes shift from states of relative calm to states of extreme turmoil gives conditional asset pricing models more power to account for time varying systematic risks and risk premia.

Indeed, unconditional asset pricing models unrealistically assume that systematic risks and risk premia do not vary across time. For example, the marginal utility of consumption does vary over the business cycle and so then should the risk premium (Cochrane, 2001; Yogo, 2006). Several studies have also confirmed that systematic risks are time-varying (Adrian & Franzoni, 2009; Avramov & Chordia, 2005).

Our paper makes several contributions to the literature, including extending both factors and moments in asset pricing models, and exploring these moments and factors in a conditional setting. This enables us to deal with a number of complexities. First, the true market portfolio is unobservable and replacing it with a proxy can lead to missing factors that may be correlated with portfolios such as the Fama and French (1993) small-minus-big (SMB) and high-minus-low (HML) portfolios. Second, using a static (unconditional) CAPM, when the true model is conditional, can also give rise to a second factor (Jagannathan & Wang, 1996). Thus, even if investors are optimising in a mean-variance universe, beta alone may not be sufficient to explain average returns. Third, investors may, in reality, be optimising in a mean-variance-skewness-kurtosis world, thereby giving rise to systematic risks which are missing from the empirical CAPM. Employing a four-moment CAPM should mitigate this limitation.

In this paper we test a conditional four-moment CAPM with time-varying betas, obtained via a dynamic conditional correlation (DCC) model, and time-varying risk premia. The betas and risk premia are assumed to follow a dynamic process and to evolve over the business cycle. More specifically, we assume the presence of two distinct sets of risk premia: one associated with bullish markets and another with bearish markets. It is argued that a large risk premium attached to an extremely bearish market might help explain the empirical anomalies of the CAPM.

The remainder of the paper is structured as follows. Section 2 discusses the literature underpinning the higher moment CAPM, followed by the literature concerning conditional pricing models in Section 3. In Section 4 the general four-moment CAPM is briefly outlined together with the methodology. The data and the empirical results are presented and discussed in Section 5. Section 6 summarises and concludes.

2. Literature review

A significant body of the finance literature highlights the importance of extending the standard CAPM for both skewness and kurtosis. Despite the predominance of the mean-variance approach in asset pricing, the literature is abundant with arguments in favour of incorporating higher moments. Not long after the CAPM was developed, researchers argued that rational risk-averse investors have a preference for positive skewness (Arditti and Levy, 1972) and an aversion to kurtosis (Scott and Horvath, 1980).

The first known work to incorporate skewness in the standard CAPM is Kraus and Litzenberger (1976). Consistent with expectations, they find a significant positive beta premium, and a significant and negative coskewness premium. Friend and Westerfield (1980), Lim (1989) and Harvey and Siddique (2000a, 2000b) test conditional versions of the three-moment CAPM. The latter authors show that adding skewness to the standard CAPM increases the adjusted R-squared.

Fang and Lai (1997) and Athayde and Flores (1997) appear to be the first to test the four-moment CAPM. Fang and Lai (1997) triple-sort portfolios on beta, coskewness, and cokurtosis, and show an improved R-squared for the four-moment CAPM and a positive and significant risk premium for cokurtosis. In contrast, Athayde and Flores (1997) conclude that skewness is more important than kurtosis.

Hwang and Satchell (1999) estimate an unconditional four-moment CAPM for emerging market stocks and find that the multi-moment model shows better explanation of the variability of average stock returns. Dittmar (2002) is the first to test a conditional version of the four-moment CAPM and shows a significant reduction in the pricing errors. UK stocks are tested by Kostakis, Muhammad, and Siganos (2012), who find a negative risk premium for coskewness and a positive risk premium for cokurtosis risk. Again, the addition of coskewness and cokurtosis improves the explanatory power of the FF and momentum factors. In the US, Lambert and Hubner (2013) extend the four-moment CAPM to include the Fama and French factors. They find that cokurtosis risk is more relevant for low book-to-market portfolios, whereas small portfolios are more sensitive to coskewness risk. More importantly, they find that coskewness and cokurtosis complement rather than replace the Fama and French factors. In any case, adding higher moments always results in increased R-squared. Young, Christoffersen, and Jacobs (2013) examine US daily data and show that high exposure to market skewness (kurtosis) yields lower (higher) returns. The same conclusion is reached by Moreno and Rodriguez (2009) for coskewness.

3. The conditional model literature

Essentially, asset pricing models express expected excess returns in terms of the product of systematic risks (or sensitivities) and the prices of those risks (risk premia). For example, for the simple CAPM we have

$$E\left(R_{it} - R_f\right) = \lambda\beta_i \tag{1}$$

where β is systematic risk, λ is the price of risk (risk premium), R_{it} is the return for stock i at time t, and R_f is the risk free rate. However, although these models may hold period by period, they do not necessarily hold unconditionally. In other words, expected returns, risk premium and systematic risk vary over time.

$$E\left(R_{it} - R_f \mid t-1\right) = \lambda_t\beta_{it} \tag{2}$$

Conditional models differ in the way they deal with time-varying systematic risks (e.g. β_{it}) and risk premia (e.g. λ_t). Cochrane (2001) argues that asset price variations are mainly due to the expectation of future returns. Campbell and Shiller (1988) argue that risk premia are related to the business cycle. Finally, Ferson and Harvey (1991) argue that most of the variation in expected returns is due to changes in risk premia rather than betas.

In practice, conditional models need to address two main issues. The first is the choice of the time-varying parameters. For example, should λ or β or both be conditioned? As we see below, many studies have adopted approaches that condition both systematic risks and risk premia. The second, and more important, issue is how to model the time-varying parameters. That is, how λ_t evolves from t to $t+1$.

Some authors use instrumental variables to model time-varying parameters, including macroeconomic variables and firm specific characteristics. Jagannathan and Wang (1996) models conditional risk premium as a linear function of the default premium. Lettau and Ludvigson (2001) use the consumption to aggregate wealth ratio to capture time variation. Both studies favour the conditional CAPM over the static CAPM. Bauer, Cosemans, and Schotman (2010) use the default spread, size, and the book-to-market ratio. Their conditional version of the three-factor model offers better explanation of expected returns than the static version. Avramov and Chordia (2005) relate sensitivities to firm characteristics and macro variables. They conclude that the conditional three-factor Fama and French model provides the best explanation of average returns.

Other researchers have adopted statistical techniques to model the time variation. Bodurtha and Nelson (1991) employ an autoregressive process to model the risk premium and the systematic risk and firmly reject the static CAPM. Bali and Engle (2010) use a dynamic conditional correlation (DCC) to model conditional covariance. Their model explains most asset pricing anomalies with the exception of momentum.

The Ang and Chen (2007) version of the conditional CAPM models conditional betas as an endogenous AR(1) process, and the risk premium as a mean-reverting process. They find that betas are highly time-varying and positively correlated with the risk premium. Lewellen and Nagel (2006) calculate conditional betas via a short window time-series rolling regression approach. While their conditional betas are time-varying, they do not explain the large pricing errors of the unconditional CAPM. Finally, Jostova and Philipov (2005) use a stochastic mean-reverting process to model conditional betas. Their approach appears to outperform GARCH, rolling regressions, constant beta, and other conditional specifications of beta.

A different statistical approach focuses on the idea that risk premia change with the states of the market or market regimes. Pettengill, Sundaram, and Mathur (1995) were the first to indicate that upward markets are associated with positive risk premia, while downward markets are associated with negative risk premia. However, these authors use an ad-hoc rule to determine the state of the market. An improved version is proposed by Vendrame, Guermat, & Tucker (2018) who use a Markov switching model to determine the probability of up and down markets. These authors find that upmarkets (bull markets) are associated with a positive realised risk premium, while downmarkets (bear markets) are associated with a negative realised risk premium.

4. Method

Let R_{it} and R_{mt} be the return of stock i and market return, respectively, at time t. The (unconditional) four-moment CAPM can be obtained via a mean-variance-skewness-kurtosis optimisation (Jurczenko & Maillet, 2001), which implies that expected returns are given by

$$\begin{aligned} E(R_{it}-R_f) &= \gamma^{\beta}\frac{E(r_{it}\bullet r_{mt})}{E(r_{mt}^2)}+\gamma^{s}E(r_{it}\bullet r_{mt}^2)+\gamma^{k}\frac{E(r_{it}\bullet r_{mt}^3)}{E(r_{mt}^4)} \\ &= \gamma^{\beta}\beta_i+\gamma^{s}s_i+\gamma^{k}k_i \end{aligned} \tag{3}$$

where $r_{it} = R_{it} - E(R_{it})$, $E(.)$ is the time expectation, and R_f is the risk-free rate. The expected excess return is a function of covariance (β), coskewness (s) and cokurtosis (k) systematic risks.

The risk premia γ^{β} and γ^{k} are expected to be positive because investors prefer lower second and fourth moments. The skewness premium, γ^{s}, is expected to be negative as investors have preference for higher skewness (Vendrame et al., 2016). Note that the coskewness is not standardized (by $E(r_m^3)$) due to the possibility of a symmetric distribution for the market return (Vendrame et al., 2016).

The expected excess return for the market portfolio is

$$E(R_m-R_f)=\gamma^{\beta}+\gamma^{s}s_m+\gamma^{k} \tag{4}$$

Following the argument of Pettengill et al. (1995), we assume that there are two states: bull and bear regimes. At any point in time, the market is in a bull state with probability p_t, and a bear state with probability $q_t = 1 - p_t$. These state probabilities are obtained from a Markov switching model as explained below. Similarly, given a risk premium for bull markets (γ_u), and another for bear markets (γ_d), the conditional expectation of the risk premium at any point in time is $\Gamma_t = p_t\gamma_u + q_t\gamma_d$. A time series test can then be carried out on Γ_t to test specific hypotheses implied by the conditional four-moment CAPM.

Although the expected risk premium is time-varying, the bull and bear market premia are constant. This specification does not impose ad-hoc rules in the determination of states, but instead considers that the state is a stochastic process whose probability can be estimated via a Markov switching process.

The bull and bear premiums cannot be estimated via the Black et al. (1972) cross section approach, or the Fama and MacBeth (1973) time series approach. The reason is that we have two parameters and one explanatory variable (beta risk). Instead, γ_u and γ_d can be estimated directly from a fixed effects panel regression. For example, for the conditional CAPM we have

$$R_{it}=\gamma_u^{\beta}p_t\beta_{it}+\gamma_d^{\beta}q_t\beta_{it}+\varepsilon_{it} \tag{5}$$

where γ_u^{β} and γ_d^{β} are estimated and tested directly from the regression. The hypothesis that the up and down premia are equal is tested via the fixed effects panel regression

$$R_{it}=\gamma_{ud}^{\beta}p_t\beta_{it}+\gamma_d^{\beta}\beta_{it}+\varepsilon_{it} \tag{6}$$

where $\gamma_{ud}^{\beta}=\gamma_u^{\beta}-\gamma_d^{\beta}$ is tested directly.

Similarly, the up and down premia for the conditional four-moment CAPM can be estimated and tested via a panel regression with conditional beta, coskewness, and cokurtosis as regressors

$$R_{it}-R_{ft}=\Gamma_{\beta t}\beta_{it}+\Gamma_{st}s_{it}+\Gamma_{kt}k_{it}+\varepsilon_{it} \tag{7}$$

where $\Gamma_{\beta t} = p_t\gamma_u^{\beta} + q_t\gamma_d^{\beta}$, $\Gamma_{st} = p_t\gamma_u^{s} + q_t\gamma_d^{s}$, and $\Gamma_{kt} = p_t\gamma_u^{k} + q_t\gamma_d^{k}$ are, respectively, the probability weighted covariance, coskewness, and cokurtosis premia.

4.1. Hypotheses

The first hypothesis for the four-moment CAPM regards the unconditional test of the model, that is, whether the up and down risk premia for the co-moments are equal:

$$H_0:\begin{cases}\gamma_u^{\beta}-\gamma_d^{\beta}=0\\ \gamma_u^{s}-\gamma_d^{s}=0\\ \gamma_u^{k}-\gamma_d^{k}=0\end{cases}$$

versus the alternative that at least one equality is rejected:

$$H_A:\begin{cases}\gamma_u^{\beta}-\gamma_d^{\beta}\neq 0\\ \gamma_u^{s}-\gamma_d^{s}\neq 0\\ \gamma_u^{k}-\gamma_d^{k}\neq 0\end{cases}$$

The hypotheses for the equality of up and down premia are tested by estimating a panel regression model as in Eq. (6), namely:

$$R_{it}=\gamma_0+\gamma_{ud}^{\beta}p_t\beta_i+\gamma_d^{\beta}\beta_i+\gamma_{ud}^{s}p_ts_i+\gamma_d^{s}s_i+\gamma_{ud}^{k}p_tk_i+\gamma_d^{k}k_i+\varepsilon_{it} \tag{8}$$

where the coefficients $\gamma_{ud}^{j}=\gamma_u^{j}-\gamma_d^{j}$ for $j = \beta, s, k$ are tested directly.

The second hypothesis relates to the sign of the three co-moments. The premia for the standardized covariance and cokurtosis are expected to be positive, whereas coskewness should have a negative premium, as investors have a preference for positive skewness and an aversion to variance and kurtosis

$$H_0:\begin{cases}E(\Gamma_{\beta t})=0\\ E(\Gamma_{st})=0\\ E(\Gamma_{kt})=0\end{cases}$$

against

$$H_A:\begin{cases}E(\Gamma_{\beta t})>0\\ E(\Gamma_{st})<0\\ E(\Gamma_{kt})>0\end{cases}$$

The rejection of the null confirms partially or fully the four-moment

Table 1
Descriptive statistics for the Fama and French 25 ME/BM portfolios.

	B1	B2	B3	B4	B5	B1	B2	B3	B4	B5
	Panel A. 1926–2021									
	Means					Standard Deviation				
S1	0.64	0.74	1.01	1.17	1.35	12.11	9.80	8.94	8.33	9.30
S2	0.68	0.97	1.00	1.05	1.24	7.98	7.50	7.25	7.42	8.70
S3	0.76	0.92	0.94	1.02	1.13	7.40	6.48	6.49	6.92	8.50
S4	0.77	0.79	0.86	0.96	1.04	6.23	6.09	6.38	6.86	8.68
S5	0.67	0.64	0.72	0.65	0.94	5.34	5.27	5.61	6.60	8.56
	Skewness					Excess Kurtosis				
S1	3.10	2.91	2.16	2.71	3.05	32.56	33.56	18.85	31.25	30.66
S2	0.73	1.48	1.77	2.05	1.75	9.27	17.55	20.02	22.40	18.17
S3	0.82	0.11	1.01	1.69	1.82	9.09	5.71	13.27	19.47	18.39
S4	−0.19	0.49	1.31	1.51	1.96	3.85	9.92	17.19	18.49	22.37
S5	−0.13	0.35	0.91	1.12	1.74	4.97	8.16	16.14	16.77	22.21
	Panel B. 1980–2021									
	Means					Standard Deviation				
S1	0.28	0.92	0.85	1.04	1.07	7.93	6.89	5.78	5.59	5.99
S2	0.64	0.95	0.95	0.93	0.95	7.13	5.90	5.32	5.24	6.16
S3	0.73	0.95	0.82	0.93	1.05	6.56	5.45	4.97	5.11	5.75
S4	0.89	0.85	0.79	0.84	0.90	5.91	5.18	5.04	4.99	5.72
S5	0.78	0.73	0.74	0.54	0.79	4.65	4.52	4.47	4.91	5.91
	Skewness					Excess Kurtosis				
S1	0.02	0.21	−0.52	−0.53	−0.19	2.50	4.66	2.62	4.03	6.30
S2	−0.39	−0.72	−0.85	−0.84	−0.90	1.66	2.91	3.23	2.94	4.14
S3	−0.49	−0.62	−0.70	−0.72	−0.90	1.64	2.96	2.48	3.49	4.42
S4	−0.29	−0.79	−0.85	−0.91	−0.92	2.04	3.56	3.56	4.89	4.45
S5	−0.39	−0.57	−0.52	−0.94	−0.49	1.34	2.27	2.59	4.10	2.51

The table shows the descriptive statistics for portfolios double sorted on market capitalization and the book-to-market ratio from July 1926 to February 2021 and from July 1926 to February 2021. S1 through S5 show the five quintiles (from the smallest to the largest) in terms of market capitalization. B1 through B5 show the five quintiles (from the lowest to the highest) in terms of the book-to-market ratio. The means are the average excess returns over the risk-free rate of a Treasury bill.

conditional CAPM. The above test is performed by first constructing three time series $\Gamma_{jt} = p_t \gamma_u^j + q_t \gamma_d^j$, for $j = \beta, s, k$ and then testing the mean of each series via robust regression.

4.2. Estimating state probabilities

The up and down states are determined by a Markov switching model applied to the market excess return. We assume following the stochastic process:

$$R_M - R_f = \mu_{Mi} + \sigma_{Mi}\varepsilon_i \tag{9}$$

The coefficients, μ_{Mi}, and σ_{Mi}, $i = u, d$ take one of two values, depending on the state, and ε_i is a random disturbance assumed to be normally distributed. Details can be found in Hamilton (1989).

Briefly, the up and down states evolve across time via a first-order Markov chain. A given state at any time can take one of two values, $S_t = u, d$. Let $p_{uu} = Prob(S_t = u|S_{t-1} = u)$ be the probability of staying in state u, and $p_{ud} = Prob(S_t = u|S_{t-1} = d)$ be the probability of moving from state d to state u. The probabilities and the likelihood functions are then calculated recursively:

$$\pi_{t|t-1} = p_{uu}\pi_{t-1|t-1} + p_{ud}(1 - \pi_{t-1|t-1}) \tag{10}$$

$$LogLik_t = log\{\pi_{t|t-1} f_u(R_{mt}|\Omega_{t-1}, \theta) + (1 - \pi_{t|t-1}) f_d(R_{mt}|\Omega_{t-1}, \theta)\} \tag{11}$$

The updated probabilities are obtained from the likelihood function

$$\pi_{t|t} = \frac{\pi_{t|t-1} f_u(R_{mt}|\Omega_{t-1}, \theta)}{\pi_{t|t-1} f_u(R_{mt}|\Omega_{t-1}, \theta) + (1 - \pi_{t|t-1}) f_d(R_{mt}|\Omega_{t-1}, \theta)} \tag{12}$$

where θ is the vector of parameters in the likelihood function. These are estimated via non-linear maximum likelihood method. We assume a conditional normal density for $f_u(.)$ and $f_d(.)$ representing the two sets of mean and variance in eq. (9).

The (filtered) probability of a bull regime, p_t, is obtained using the Expected Maximization (EM) algorithm of Hamilton (1989). These probabilities are used in eq. (8) to obtain the up and down risk premia for each of the three co-moments.

5. Empirical results

The conditional models discussed in Section 3 are estimated using the 25 ME/BM portfolios for both the full sample (1926–2021) and the more recent subsample (1980–2021). Subsequently, we augment the 25 portfolios with 40 industry portfolios for robustness checks. The recent subsample presents the greatest challenge for the CAPM. Fama and French (1993) show that the CAPM is vulnerable to the size and value anomalies, especially in the period following the 1970s. The subsample choice is justified for two main reasons. First, the shorter period of 1980–2021 is more likely to have seen a limited number of regimes. Thus, our simple two regimes Markov switching model is likely to be more appropriate for the subsample than the full sample. Second, the recent 40 years represent the greatest challenge to the CAPM.

5.1. Data and descriptive statistics

In this paper we employ the 25 Fama and French portfolios, which are sorted on market capitalization and the book-to-market ratio. These portfolios are formed by intersecting five size (ME, market value equity) portfolios and five book-to-market (BM, book equity-to-market value equity) portfolios. The size breakpoints are based on market equity quintiles at the end of June in each year. Similarly, the BM breakpoints are quintiles obtained from the ratio of end of fiscal year book equity to the end of December market equity of the same year. Portfolios are

Table 2
Test of the unconditional CAPM using Fama and MacBeth (1973).

–	1926–2021	1980–2021
	Standard CAPM	
α	0.0064	0.0101
	(2.60)* [0.00]	(3.17)* [0.00]
γ^{β}	0.0026 (0.89) [0.19]	−0.0021 (−0.56) [0.69]
	Four-moment CAPM	
α	0.0089	0.0104
	(2.99)* [0.00]	(3.23)* [0.00]
γ^{β}	0.0092	0.0015
	(1.30) [0.09]	(0.19) [0.45]
γ^{s}	52.32	1.11
	(1.00) [0.83]	(0.01) [0.50]
γ^{k}	−0.0132	−0.0080
	(−1.73) [0.96]	(−0.77) [0.77]
$\gamma^{\beta}+\gamma^{s}s_m+\gamma^{k}$	−0.0007	−0.0028
	(−0.25) [0.60]	(−0.77) [0.77]

This table reports the results of the monthly cross-sectional regressions for the 25 ME/BM portfolios on the CAPM and the four-moment CAPM. The coefficients are reported for the beta premium, and for skewness and kurtosis premia for the four-moment CAPM. The last coefficient, $\gamma^{\beta}+\gamma^{s}s_m+\gamma^{k}$, is the estimated market risk premium for the four-moment CAPM. The time series of the risk premia for the CAPM, γ_t^{β}, is obtained from the cross section regression $R_{i,\,t}-R_{f,\,t}=\gamma_t^{\beta}\beta_{i,\,t-1}$. Similarly, the covariance, skewness and kurtosis premia are obtained from the cross section regression: $R_{i,\,t}-R_{f,\,t}=\gamma_t^{\beta}\beta_{i,\,t-1}+\gamma_t^{s}s_{i,\,t-1}+\gamma_t^{k}k_{i,\,t-1}$ for the four moment CAPM. The t-statistics are reported in parentheses. The tests are based on sample averages and t-statistics of the time series of risk premia. The bootstrap *p*-values are shown in square brackets.

rebalanced in July of each year.

Descriptive statistics for the 25 double sorted portfolios are shown in Table 1. Panel A shows statistics for the full sample (1926–2021). The average excess return increases with the BM ratios for four of the five size (S1-S4) portfolios. For example, for the bottom size quintile (S1), excess returns range from 0.64% for the lowest book-to-market ratio (B1) to 1.35% for the highest book-to-market ratios (B5). For the top size quintile excess returns tend to increase but not monotonically. For all sizes, excess returns are clearly an increasing function of the BM ratio. On the other hand, excess returns are strictly decreasing with size for the top three BM ratios (B3-B5). For the bottom two BM ratios, the pattern is not clear but appears to show a hump-shaped pattern. In any case, for all BM portfolios, excess returns are generally decreasing with size.

There is a different pattern for total risk (standard deviation). While the standard deviation is (mostly) strictly decreasing with size, it shows a smile pattern across the BM quintiles. For example, for S1, the standard deviation decreases from 12.11 (B1) to 8.33 (B4) and then increases to 9.30 (B5). Note, however, that the pattern is not the same for all sizes. For S2 the decrease is between B1 and B3, while for S3-S5 the decrease is only between B1 and B2.

Finally, both skewness and kurtosis generally decrease with size and increase with BM. This suggests that higher moments (especially cokurtosis) might help explain the observed returns.

Panel B shows the descriptive statistics for the 25 ME/BM portfolios for the subsample (1980–2021). The evidence of a value premium is less obvious than for the full sample. Although returns show some increase with BM ratios, it is not monotonic and it is only for the bottom three size quintiles. The excess returns are increasing with size for the top four BM quintiles, but are decreasing for the lowest BM quintile.

The standard deviations, skewness and kurtosis are lower for all portfolios compared with the full sample, suggesting that this period is much less volatile than the 1926–1979 period. Otherwise, the general pattern is unchanged. The standard deviations decrease with market capitalization for all BM quintiles, suggesting a greater volatility of small stocks. Similar to the full period, we observe a smile pattern for the BM

Table 3
Dual test of Pettengill et al. (1995) for the four-moment CAPM.

	1926–2021		1980–2021	
	Bull	Bear	Bull	Bear
α	0.0140	−0.0013	0.0161	0.0038
	(5.43)*	(−0.43)	(4.84)*	(0.74)
γ^{β}	0.0270	−0.0397	0.0210 (5.606)	−0.0417
	(8.378)*	(−12.76)*	*	(−8.191)*
γ^{s}	22.7918	16.2572	−21.0973	12.2451
	(1.325)	(0.991)	(−1.243)	(0.6202)
γ^{k}	−0.0006	0.0000	−0.0151	0.0008
	(−0.828)	(0.038)	(−2.551)*	(0.936)
$\gamma^{\beta}+\gamma^{s}s_m$	0.0262	−0.0396	0.01925	−0.0409
$+\gamma^{k}$	(8.871)*	(−13.12)*	(5.299)*	(−8.187)*

This table reports the results of the monthly cross-sectional regressions for the 25 ME/BM portfolios on the four-moment CAPM. The coefficients are averages of the premiums for conditional beta, conditional coskewness, and conditional cokurtosis. The last coefficient is the estimated overall premium for the four-moment CAPM. The t-statistics are reported in parentheses, and significant coefficients at the 5% level are indicated with an asterisk. The time series of risk premia are obtained from the cross-sectional regression $R_{i,\,t}-R_{f,\,t}=\gamma_t^{\beta}\beta_{i,\,t-1}+\gamma_t^{s}s_{i,\,t-1}+\gamma_t^{k}k_{i,\,t-1}$ at each point in time. The bull (bear) premium is obtained by splitting the time series sample of premia (γ_t^{β}, γ_t^{s}, and γ_t^{k}) into positive (negative) market excess return. The same procedure is applied on the market risk premium, $\gamma^{\beta}+\gamma^{s}s_m+\gamma^{k}$.

quintile. Again, the smile effect is different for different size quintiles. Skewness figures are mostly negative and small in terms of absolute value. There is no obvious correlation between skewness or kurtosis and size. Skewness is decreasing in BM for the bottom four size quintiles (S1-S4), but no obvious pattern is noted for the top size quintile (S5). In contrast, kurtosis is generally increasing in BM for all size quintiles.

5.2. Fama-MacBeth test

The results of the standard Fama and MacBeth methodology (Fama and MacBeth, 1973) to test the CAPM and four-moment CAPM are reported in Table 2 for the periods 1926–2021 and 1980–2021, respectively. The conditional co-moments are estimated as sample co-moments based on rolling windows as in FM. At each point in time, premia are obtained via cross-sectional regressions. A *t*-test is then conducted on the time series of each premium.

The static CAPM is rejected for both the full sample and the subsample. The market premium is positive but not significant for the full sample and even negative, though insignificant, for the subsample 1980–2021. For both the full sample and the subsample, the four-moment CAPM is rejected as the risk premium, $\gamma^{\beta}+\gamma^{s}s_m+\gamma^{k}$, is negative though insignificant. The unconditional four-moment CAPM, thus does not perform well using the Fama and MacBeth methodology. The introduction of higher co-moments does not rescue the model in its static form.

5.3. Pettengill et al. (1995) test

Table 3 shows the introduction of two regimes, bull and bear, based on the ex-post excess market return, using the Pettengill et al. (1995) test of the CAPM.

Following Pettengill et al. (1995), we split the sample into upmarket and downmarket periods, defined as months with positive or negative ex-post market excess returns, respectively. Having estimated beta, coskewness and cokurtosis from a first pass, we define a conditional four-moment CAPM as:

$$R_{it}-R_{ft}=\widehat{\alpha}_{0t}+\widehat{\gamma}^{\beta}_{1t}\bullet\delta_t\bullet\beta_t+\widehat{\gamma}^{s}_{1t}\bullet\delta_t\bullet s_{it}+\widehat{\gamma}^{k}_{1t}\bullet\delta_t\bullet k_t+\widehat{\gamma}^{\beta}_{2t}\bullet(1-\delta_t)\bullet\beta_t+\widehat{\gamma}^{s}_{2t}\bullet(1-\delta_t)\bullet s_{it}+\widehat{\gamma}^{k}_{2t}\bullet(1-\delta_t)\bullet k_t+\varepsilon_{it} \quad (13)$$

Table 4
Markov switching parameters for the market model.

Parameters	Coeff.	T-Stat.
μ_1	0.0100	7.71
μ_2	0.0	–
p_{12}	0.0179	3.13
p_{21}	0.1114	3.40
σ_1	0.0378	37.07
σ_2	0.1056	22.88

This table shows the parameters of the Markov switching process for the market model. The parameters reported are the two means, the transition probabilities, and the standard deviations.

where $\delta_t = 1$ if the realised market excess return is positive, and 0 otherwise.

The model is estimated for each t, that is, there are T cross sectional regressions, yielding T risk premia. These are then split into two samples depending on whether the excess market return implies an upmarket regime (positive) or a downmarket regime (negative). A systematic conditional relationship between the co-moments and realised returns is supported if.

$\gamma^\beta + \gamma^s s_m + \gamma^k > 0$ in upmarket and $\gamma^\beta + \gamma^s s_m + \gamma^k < 0$ in downmarket.

The results confirm the asymmetry in market premia, with a positive and significant return of 2.62% in bull markets and a significant negative return of −3.96% in bear markets for the full period 1926–2021. For the subsample period 1980–2021, the bull and bear premia are similar at 1.93% and −4.09%, respectively.

The market premia are virtually identical to the beta premia, suggesting that skewness and kurtosis are irrelevant. Indeed, all but one of the skewness and kurtosis premia are small and insignificant. There is a clear asymmetric beta premium, with a significant positive beta premium in bull markets and a larger significant negative beta premium in bear markets. This asymmetry is slightly more pronounced in the subsample 1980–2021 with a greater spread between the bull and bear premia.

5.4. Markov switching model

A well-known limitation of the Pettengill et al. (1995) approach is that the sign of the market return is not necessarily associated with a bull or bear market. Indeed, an overall positive (negative) price trend may well contain negative (positive) returns. However, the Markov Switching approach allows for this possibility. More importantly, given that the bull and bear regimes are latent processes, we can only estimate the probability of their realisation.

In this section, we apply the conditional methodology based on the Markov Switching model described in Section 4. The two regimes are estimated with a different mean and standard deviation for each regime. Table 4 shows the results for the regime switching model applied to the market return. The results show that the bull regime is more likely. The probability of moving from bull to bear market is only 0.0179, whereas the probability of moving from bear to bull market is 0.1114. However, both transition probabilities are small, which suggests that both regimes are persistent.

The bull market shows a positive and statistically significant average return of 1% and a standard deviation of 3.78%. For the bear market we initially obtain a negative but insignificant (at the 5% level) average return, μ_2. We therefore re-estimate the model, imposing a zero average bear market return. The results reveal a low (zero) return on average but a high standard deviation of 10.56%. Thus, the bull market is characterized by a positive return and low volatility, suggesting a steady regime. In contrast, the bear market is characterized by high volatility and low (zero) average returns. This is typical of a highly unstable regime that contains boom-and-bust cycles.

Fig. 1 shows the filtered probabilities of the up and down markets for the full period (1926–2021). The market is in a bullish regime most of the time. The bullish regime is typical of the 1940s, 1950s, and the 1980s, and is especially prolonged in both the 1990s and the recovery following the dotcom crisis in the early 2000s. The bear regime is typical of the year 1929, the mid 1970s, the early 1980s, the end of the 1980s, the high volatility period of the late 1990s, the early 2000s, the financial crisis of 2007, and the recent Covid-related crisis.

5.5. Individual-fixed effects panel for the CAPM and the four-moment CAPM

In individual fixed-effects panel data, the intercepts are allowed to vary across individual assets (the 25 ME/BM-sorted portfolios), but not over time. The intercepts will therefore capture an individual effect that drives the portfolios but does not change over time. The effect is that the intercept is removed, with only the two risk premia remaining. This allows us to focus on: (i) the coefficients of the risk premia; (ii) the tests of the unconditional four-moment CAPM; and (iii) the weighted average risk premium.

The conditional models are obtained by first using a DCC GARCH approach covering the full sample, and then using a panel for the period 1926–2021. More specifically, the conditional betas are first estimated using a multivariate DCC GARCH approach for the full period 1926–2021 for the estimation process. The results are reported in Table 5.

Similar to Vendrame, Guermat, & Tucker, 2018, the conditional CAPM is able to rescue the model. The risk premium is 1.01% and highly significant for the whole sample. The static CAPM is rejected as the risk premia for bullish and bearish markets are significantly different. The risk premium is positive in the bullish market (1.27%) and negative in the bearish markets, as expected (−0.57%). This gives a highly significant difference of 1.84%, which gives further support for the regime based conditional CAPM.

Table 5 also shows the results for the four-moment CAPM. Focusing on bull markets, all risk premia are significant and have the expected sign. The risk premiums for beta is 1.37%, −280.81 for coskewness and 0.21% for cokurtosis. In bear markets the coefficients are significant and have the expected signs, with the beta risk premium of −3% and coskewness positively rewarded at 65.42. The only exception is for cokurtosis that is positive even in bear markets, although very small at only 0.02%. The symmetry hypotheses are clearly rejected for all co-moments. For example, the difference between the up and down beta premia is 4.36% and is highly significant.

For the weighted co-moment risk premia, a time series regression with Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors using a Newey-West window with four lags is used to test for the means. Our choice is driven by the fact that the estimated risk premia have low dispersion and are highly serially correlated.

All weighted risk premia are significant and have the expected sign. The beta risk premium is significant and positive, with a coefficient of 0.75% per month and with a t-statistic of 10.98. The coskewness premium is negative and significant (as expected), and the cokurtosis premium is positive and significant, though very small (0.18%). Therefore, the conditional coefficients of the risk premia are consistent with theory. Investors require a risk reward for covariance and cokurtosis risks and are willing to forego some returns in exchange for positive coskewness.

The overall market risk premium is positive (0.67%) and highly significant. More importantly, the addition of higher moments to the simple CAPM reduces the market risk premium from 1.01% to 0.67% per month. Overall, the conditional four moment CAPM seems to be a better description of average returns than the conditional CAPM.

To check that our results are robust to the number of portfolios used, we repeat the same procedure using 65 portfolios (25 book-to-market and size sorted portfolios plus 40 industry portfolios). The results are also reported in Table 5. For the simple CAPM, the results are qualitatively and quantitatively similar. Indeed, the sign of up and down

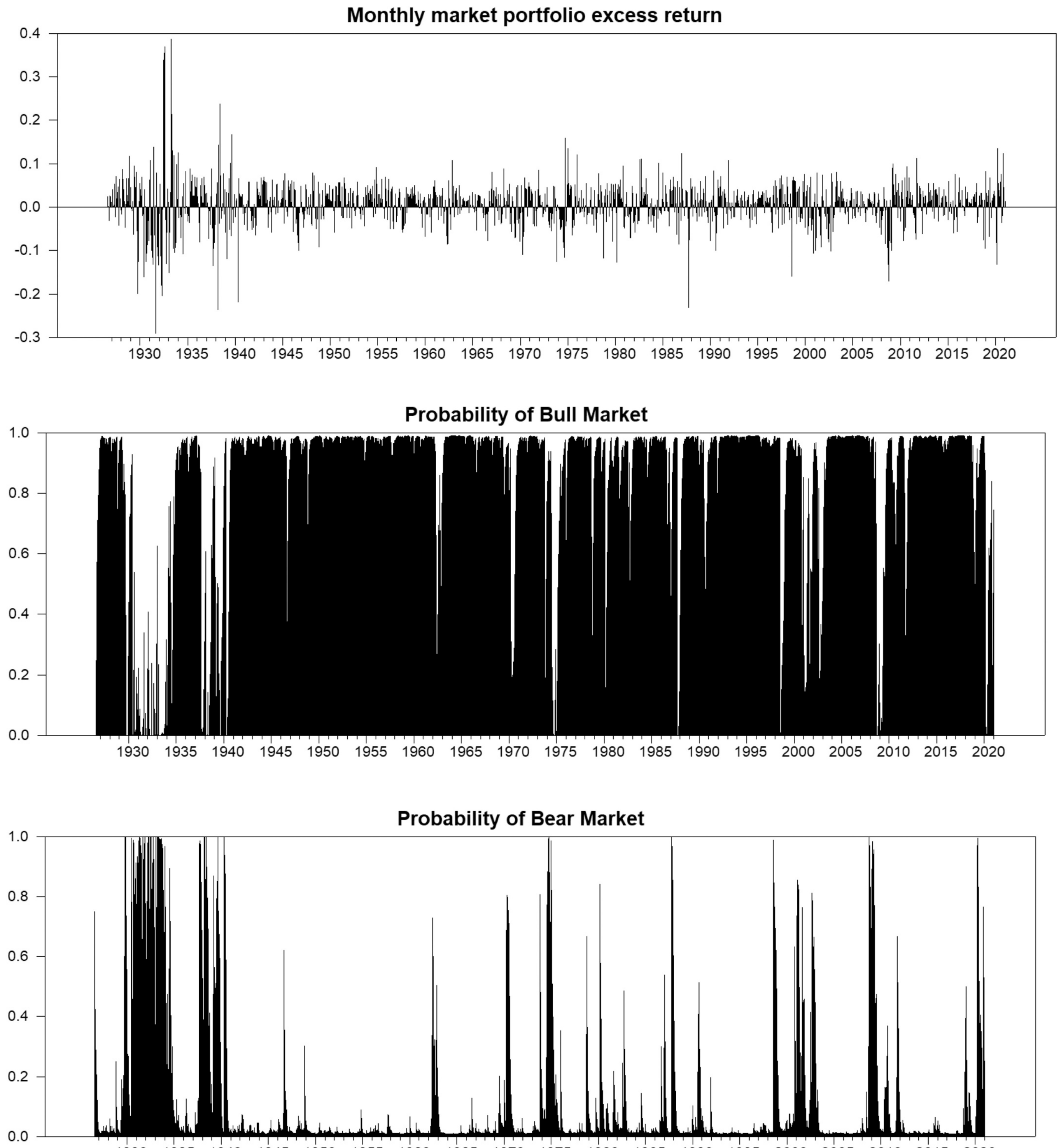

Fig. 1. Filtered probabilities of the bull and bear regimes for the period 1926–2021.

markets (beta) risk premia is unchanged and so is the statistical significance of asymmetry and the market risk premium. The static CAPM is firmly rejected as the risk premia are statistically different in bull and bear markets. Moreover, the coefficient estimates are very similar. For example, the overall risk premium is positive and significant at 0.62%.

The same applies to the four-moment CAPM. The beta risk premium is 0.86% in the bullish market and − 2.52% in the bearish market (both significant), coskewness is negatively rewarded in the bullish market and positively rewarded in the bearish market, whereas cokurtosis is always positively and significantly rewarded. The weighted average risk premium is 0.37% and significant, coskewness is negatively rewarded as theoretically expected and it is −140.13, whereas cokurtosis is positively rewarded as theoretically expected and it is 0.16% per month. Beta is also positively rewarded with 0.38% per month. The introduction of the higher moments produces a slightly lower risk premium from 0.38% to 0.37% but, overall, the beta-CAPM seems to be enough to

Table 5
Test of the MS conditional CAPM (25 ME/BM Portfolios/All Portfolios).

	25 ME/BM		All Portfolios	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0127 (6.60)*	−0.0057 (−2.80)*	0.0091 (7.94)*	−0.0113 (−8.51)*
$\gamma_u^\beta - \gamma_d^\beta$	0.0184 (13.33)*		0.0204 (22.71)*	
$\Gamma_{\beta t} = p_t\gamma_u^\beta + q_t\gamma_d^\beta$	0.0101 (34.96)* [0.00]		0.0062 (19.54)* [0.00]	

	Four-moment CAPM 1926–2021		Four-moment CAPM 1926–2021	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0137 (7.10)*	−0.0300 (−13.50)*	0.0086 (7.48)*	−0.0252 (−18.06)*
$\gamma_u^s \mid \gamma_d^s$	−280.81 (−24.61)*	65.42 (40.01)*	−169.38 (−30.04)*	37.48 (47.25)*
$\gamma_u^k \mid \gamma_d^k$	0.0021 (8.35)*	0.0002 (2.44)*	0.0017 (7.99)*	0.0006 (6.51)*
$\gamma_u^\beta - \gamma_d^\beta$	0.0436 (27.32)*		0.0338 (34.10)*	
$\gamma_u^s - \gamma_d^s$	−346.22 (−27.85)*		−206.86 (−37.48)*	
$\gamma_u^k - \gamma_d^k$	0.0018 (6.22)*		0.0012 (4.55)*	
Γ_β	0.0075 (10.98)* [0.00]		0.0038 (7.17)* [0.00]	
Γ_s	−231.85 (−42.70)* [0.00]		−140.13 (−43.20)* [0.00]	
Γ_k	0.0018 (63.20)* [0.00]		0.0016 (85.20)* [0.00]	
Γ_m	0.0067 (10.83)* [0.00]		0.0037 (7.60)* [0.00]	

This table reports the results for the regime switching conditional CAPM for the 25 ME/BM portfolios and All Portfolios (25 ME/BM Portfolios plus 40 Industries Portfolios) over the period 1926–2021. The up and down probabilities, p_t and $q_t = 1 - p_t$, are obtained from a Markov switching model. The conditional co-moments (covariance, coskewness and cokurtosis) are obtained from DCC GARCH models. The up and down risk premia, and their differences, are estimated and tested based on panel data regressions with individual-fixed effects. The weighted average risk premia, Γ_β, Γ_s and Γ_k are first computed as the average of the time series $\Gamma_{jt} = p_t\gamma_u^j + q_t\gamma_d^j$, for $j = \beta, s, k$. These averages are then tested for significance using a time series regression with HAC standard errors using a Newey-West window with four lags. The same procedure is applied for the market risk premium, Γ_m, which is computed as the average of the time series $\Gamma_t = \Gamma_{\beta t} + \Gamma_{st}s_{mt} + \Gamma_{kt}$. The t-statistics are reported in parentheses and significant coefficients at the 5% level are indicated with an asterisk. The bootstrap p-values are shown in square brackets.

explain the cross section of average returns.

5.6. Time series and cross-sectional explanation of returns

Although the statistical significance of the estimated conditional risk premia is useful evidence that our conditional model is a reasonable improvement on the static CAPM and static four-moment CAPM (henceforth 4-CAPM), especially for the whole sample, this evidence is nevertheless incomplete and should be complemented by contrasting the cross sectional and time series performance of the conditional model with that of the static model. In this section, we first provide some time series results on the premiums associated with size, value, and momentum. We then discuss the pricing errors of the static and conditional models (for both the CAPM and the 4-CAPM).

If our conditional model explains size, value, and momentum, then the loadings from the three factors should not be priced. Thus, we run T cross-sectional regressions with the three factors on net returns:

$$R_{it}^{net} = \lambda_t^0 + \lambda_t^s\beta_{it}^{smb} + \lambda_t^h\beta_{it}^{hml} + \lambda_t^m\beta_{it}^{mom} + \varepsilon_{it} \quad (14)$$

The time series test is then performed on sample means of λ_t^s, λ_t^h, and λ_t^m. The net returns are given by $R_{it}^{net} = R_{it} - R_{ft} - \Gamma_{\beta t}\beta_{it} - \Gamma_{st}s_{it} - \Gamma_{kt}k_{it}$. For the conditional 4-CAPM (the conditional betas are obtained from a DCC model). For the static 4-CAPM, the net returns are given by $R_{it}^{net} = R_{it} - R_{ft} - \lambda_\beta\beta_{it} - \lambda_s s_{it} - \lambda_k k_{it}$, where the betas are obtained from a five year rolling window and lambdas are the standard 4-CAPM estimated risk premia. The loadings, β_{it}^{smb}, β_{it}^{hml} and β_{it}^{mom} are estimated from time series models using five year rolling windows

$$R_{it}^e = \beta_0 + \beta_{it}^{mkt}R_{mt}^e + \beta_{it}^{smb}SMB_t + \beta_{it}^{hml}HML_t + \beta_{it}^{mom}MOM_t + \varepsilon_{it} \quad (15)$$

where R^e indicates excess return. Table 6 provides tests for the (time) average risk premia for the three factor sensitivities. For the conditional 4-CAPM, only the value premium is significant. It is important to note that the intercept becomes insignificant compared to the static 4-CAPM. Size and momentum appear to be explained by the model. On the other hand, for the static 4-CAPM size remains unexplained with a significant large intercept. For the cross-sectional comparison, we calculate the pricing errors from the two models as follows:

$$\begin{aligned}\varepsilon_i^{Cond} = & E(R_{it} - R_{ft}) - E(\Gamma_{\beta t})E(\beta_{it}) - E(\Gamma_{st})E(s_{it}) - E(\Gamma_{kt})E(k_{it}) \\ & - Cov(\Gamma_{\beta t}, \beta_{it}) - Cov(\Gamma_{st}, s_{it}) - Cov(\Gamma_{kt}, k_{it})\end{aligned} \quad (16)$$

$$\varepsilon_i^{Stand} = E(R_{it} - R_{ft}) - \lambda_\beta E(\beta_{it}) - \lambda_s E(s_{it}) - \lambda_k E(k_{it}) \quad (17)$$

where, as before, the conditional betas are from a DCC model, the standard 4-CAPM betas are from five year rolling univariate regressions, and λ_β, λ_s, λ_k are the standard 4-CAPM estimated risk premia. The pricing error from the static CAPM and 4-CAPM are much larger than that of the conditional 4-CAPM. The average absolute error is 0.20% for the conditional 4-CAPM model, against 1.45% for the static model. Another way of looking at the pricing error is by visually comparing the performance of the two models.

In Fig. 2, the fitted expected returns from the conditional 4-CAPM and CAPM are plotted against realised average returns. We do the same for the static CAPM. Ideally, the fitted returns should be close to the 45-degree line. However, none of the fitted models achieve a perfect fit. Nevertheless, the conditional models show substantial improvements over the static model. The static CAPM suggests returns that are way below their average realised returns. It is clear from the figure that the returns predicted by the static CAPM bear no relation to realised returns. The slope is indistinguishable from a flat line, reflecting an insignificant risk premium.

Table 6
Time series explanation of size, value, and momentum.

Panel A.	Static Four-Moment CAPM 1926–2021		Conditional Four-Moment CAPM 1926–2021	
	Mean	T-stat (p-val)	Mean	T-stat (p-val)
Intercept	0.0083	3.547 (0.000)*	−0.0022	−1.422 (0.155)
size	0.0022	2.001 (0.046)*	0.0002	0.293 (0.770)
value	0.0032	2.036 (0.042)*	0.0025	2.510 (0.012)*
momentum	−0.0037	−1.179 (0.239)	−0.0009	−0.463 (0.643)

Panel B.	Static CAPM 1926–2021		Conditional CAPM 1926–2021	
	Mean	T-stat (p-val)	Mean	T-stat (p-val)
Intercept	0.0100	4.222 (0.000)*	−0.0024	−1.489 (0.137)
size	0.0020	2.205 (0.028)*	0.0000	0.049 (0.961)
value	0.0028	2.499 (0.013)*	0.0031	3.144 (0.002)*
momentum	−0.0009	−0.410 (0.682)	0.0013	0.630 (0.529)

This table reports the test on sample means of the loadings λ_t^s, λ_t^h, and λ_t^mobtained from T cross-sectional regressions based on the model: $R_{it}^{net} = \lambda_t^0 + \lambda_t^s\beta_{it}^{smb} + \lambda_t^h\beta_{it}^{hml} + \lambda_t^m\beta_{it}^{mom} + \varepsilon_{it}$ as described in Section 5.6. The tests are for the 25 ME/BM portfolios over the period 1926–2021.

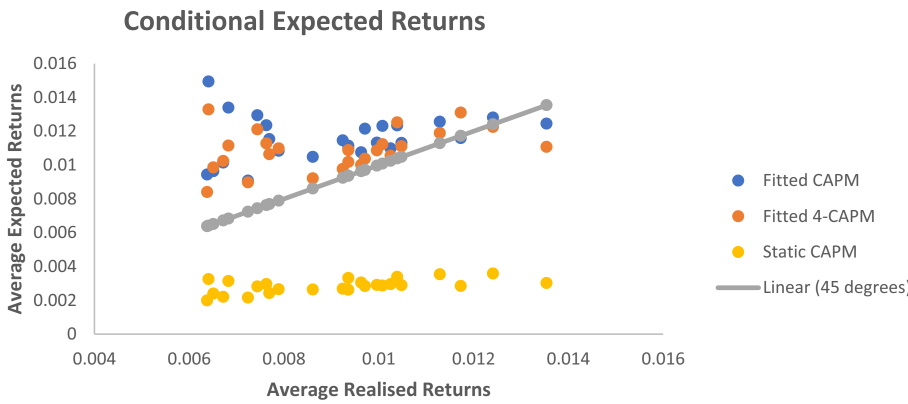

Fig. 2. Average fitted returns and average realised returns.

Table 7
Cross-sectional regression of factor sensitivities on pricing errors.

Panel A	Standard one-pass betas		Average rolling betas	
	Coefficient	t-stat (p-val)	Coefficient	t-stat (p-val)
Static CAPM				
Intercept	0.0051	12.352 (0.000)*	0.0050	13.770 (0.000)*
size	0.0010	2.275 (0.034)*	0.0015	3.450 (0.002)*
value	0.0035	5.404 (0.000)*	0.0036	6.006 (0.000)*
momentum	0.0165	3.010 (0.007)*	0.0272	3.647 (0.002)*
R^2 / Adj R^2	0.63/0.57		0.69/0.65	
Conditional CAPM				
Intercept	−0.0027	−5.524 (0.000)*	−0.0027	−6.038 (0.000)*
size	−0.0006	−1.152 (0.262)	−0.0001	−0.171 (0.866)
value	0.0043	5.622 (0.000)*	0.0044	5.983 (0.000)*
momentum	0.0246	3.818(0.001)*	0.0370	4.049 (0.001)*
R^2 / Adj R^2	0.64/0.59		0.67/0.63	

Panel B	Standard one-pass betas		Average rolling betas	
	Coefficient	t-stat (p-val)	Coefficient	t-stat (p-val)
Static 4-CAPM				
Intercept	0.0106	15.409 (0.000)*	0.0107	18.459 (0.000)*
size	0.0023	3.136 (0.005)*	0.0034	4.810 (0.000)*
value	0.0092	8.458 (0.000)*	0.0095	10.087 (0.000)*
momentum	0.0141	1.533 (0.140)	0.0281	2.374 (0.027)*
R^2 / Adj R^2	0.80/0.77		0.85/0.83	
Conditional 4-CAPM				
Intercept	−0.0022	−4.024 (0.001)*	−0.0022	−4.762 (0.000)*
size	−0.0002	−0.364 (0.720)	0.0004	0.650 (0.523)
value	0.0034	4.006 (0.001)*	0.0037	4.936 (0.000)*
momentum	0.0180	2.515 (0.020)*	0.0341	3.675 (0.001)*
R^2 / Adj R^2	0.46/0.38		0.59/0.53	

This table reports the results of a cross-sectional regression $\varepsilon_i = f(\beta^{smb}, \beta^{hml}, \beta^{mom})$, where ε_i is the pricing error obtained from the conditional CAPM (Eq. (16)) and the static CAPM (Eq. (17)). The standard one-pass multivariate betas are obtained from N time series four factor model (Eq. (18)). The average rolling betas are obtained from T regressions for each portfolio (Eq. (19)). The tests are for the 25 ME/BM portfolios over the period 1926–2021.

Table 8
Markov switching parameters for the market model 1980–2021.

Parameters	Coeff.	T-Stat.
μ_1	0.0117	5.46
μ_2	0.0	–
p_{12}	0.0459	2.05
p_{21}	0.0414	1.39
σ_1	0.0264	9.53
σ_2	0.0562	14.77

This table shows the parameters of the Markov switching process for the market model. The parameters reported are the two means, the transition probabilities, and the standard deviations.

For the conditional models, the fitted returns are closer to the realised average returns, though the models over-predict expected returns. The models do not perform particularly well for low average realised returns, and the slope is substantially less than the ideal 45 degrees, perhaps due to low variability in the co-moment estimates, which reduces the correlation between fitted and realised average returns. Nevertheless, for returns above 0.8% both the proximity of returns and the slope are good. The conditional 4-CAPM provides better fit than the conditional CAPM, producing less outliers and closer predictions to the 45 degrees line. Both types of outliers coincide with the bottom BM (growth) stocks, confirming the failure of the conditional models to explain the value anomaly. Overall, the conditional returns from the 4-CAPM fit the realised returns better than the conditional CAPM, confirming the improvement obtained by adding the higher moments to the conditional version of the CAPM.

Finally, we regress both sets of pricing errors on the size, value, and momentum sensitivities using two types of estimates. The first is standard multivariate betas, obtained from a single time series for each portfolio $i = 1, \ldots, N$

$$R_{it}^e = \beta_0 + \beta_i^{mkt} R_{mt}^e + \beta_i^{smb} SMB_t + \beta_i^{hml} HML_t + \beta_i^{mom} MOM_t + \varepsilon_{it} \tag{18}$$

The second set of sensitivities is obtained from five year rolling regressions for each portfolio, and averaging sensitivities. That is, for each portfolio $i = 1, \ldots, N$, we perform T regressions

$$R_{it}^e = \beta_0 + \beta_{it}^{mkt} R_{mt}^e + \beta_{it}^{smb} SMB_t + \beta_{it}^{hml} HML_t + \beta_{it}^{mom} MOM_t + \varepsilon_{it} \tag{19}$$

The cross section of sensitivities, $(\overline{\beta}^{smb}, \overline{\beta}^{hml}, \overline{\beta}^{mom})$ are obtained by averaging across time. Table 7 presents the cross-sectional regression results of pricing errors on the factor sensitivities. There is little difference between the multivariate and rolling betas, except that the rolling betas are slightly more correlated with the pricing errors as can be seen by the coefficient of determination. Regardless of the sensitivity estimates, the conditional 4-CAPM clearly explains away the size effect as

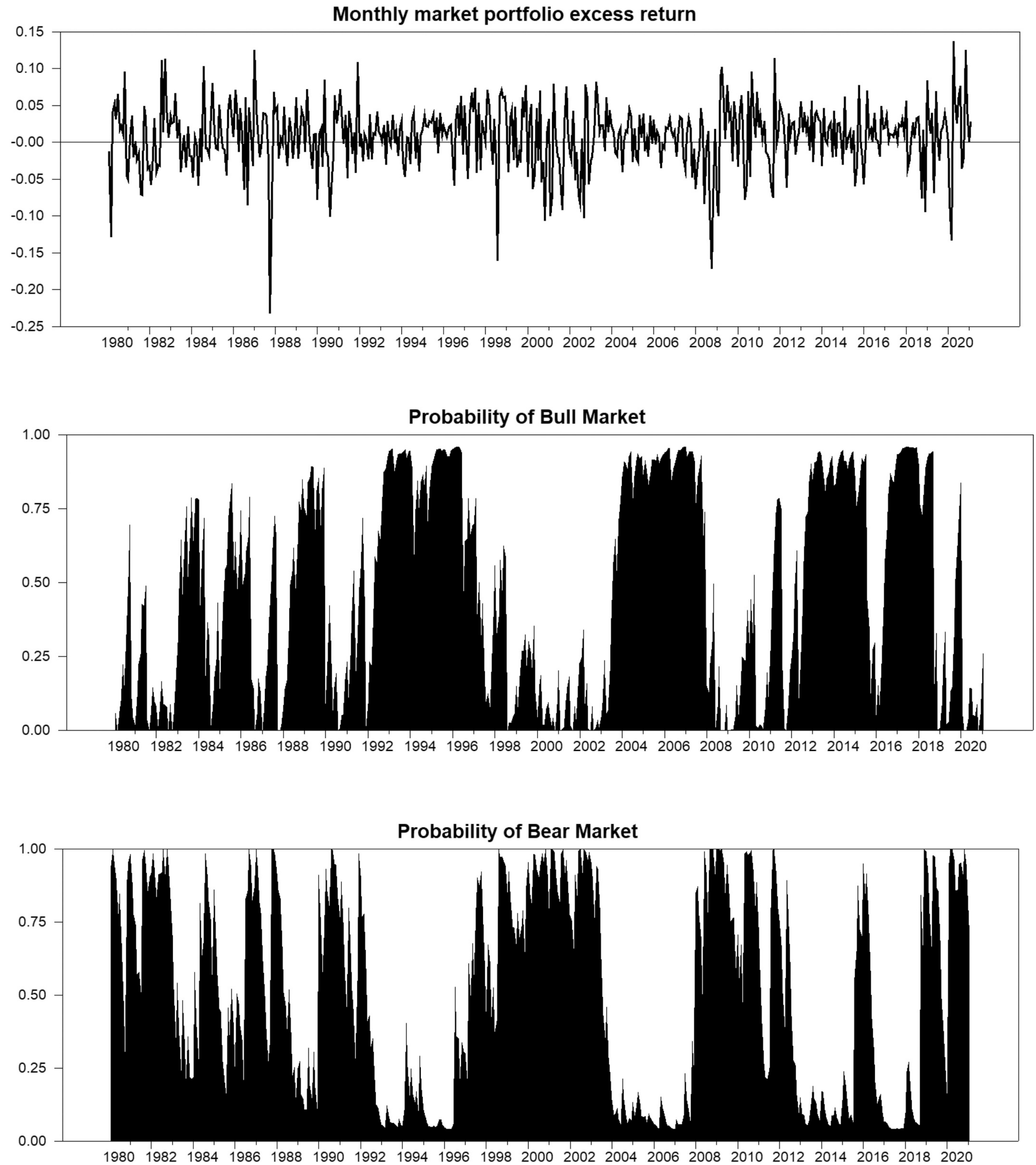

Fig. 3. Bull and Bear Market probabilities for the period 1980–2021.

the slope associated with the size beta is highly insignificant. The static CAPM and 4-CAPM, on the other hand, do not explain size. Both conditional and static models, however, fail to explain value and momentum effects in the cross section.

5.7. Conditional CAPM and conditional 4-CAPM for period 1980–2021

One of the possible ways to improve the conditional models over the last 40 years is to conduct a Markov switching for this period rather than for the full all sample 1926–2021. The market dynamics may have changed over such a long period of time, and the last 40 years' dynamics

Table 9
Test of the MS conditional CAPM (25 ME/BM Portfolios and All Portfolios).

	25 ME/BM		All Portfolios	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0104 (3.44)*	−0.0002 (−0.06)	0.0102 (5.99)*	−0.0004 (−0.24)
$\gamma_u^\beta - \gamma_d^\beta$	0.0106 (8.02)*		0.0106 (11.81)*	
$\Gamma_{\beta t}$	0.0049 (14.07)*		0.0047 (13.45)*	

	Four-moment CAPM		Four-moment CAPM	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0033 (0.756)	0.0029 (0.866)	−0.0018 (−0.69)	0.0034 (1.83)
$\gamma_u^s \mid \gamma_d^s$	−34.05 (−0.646)	37.27 (4.75)*	−95.68 (−2.79)*	54.89 (10.05)*
$\gamma_u^k \mid \gamma_d^k$	0.0031 (3.07)*	0.0002 (1.49)	0.0034 (4.95)*	0.0003 (2.92)*
$\gamma_u^\beta - \gamma_d^\beta$	0.0004 (0.10)		−0.0052 (−1.92)	
$\gamma_u^s - \gamma_d^s$	−71.32 (−1.32)		−150.56 (−4.26)*	
$\gamma_u^k - \gamma_d^k$	0.0029 (2.86)*		0.0031 (4.50)*	
Γ_β	0.0031 (227.29)* [0.00]		0.0009 (5.19)* [0.00]	
Γ_s	2.59 (1.10) [0.99]		−18.32 (−3.67)* [0.00]	
Γ_k	0.0016 (16.56)* [0.00]		0.0018 (17.22)* [0.00]	
Γ_m	0.0040 (14.20)* [0.00]		0.0029 (10.96)* [0.00]	

This table reports the results for the regime switching conditional CAPM for the 25 ME/BM portfolios and All Portfolios (25 ME/BM plus 40 Industries Portfolios) over the period 1980–2021. The up and down probabilities, p_t and $q_t = 1 - p_t$, are obtained from a Markov switching model. The conditional co-moments (covariance, coskewness and cokurtosis) are obtained from DCC GARCH models. The up and down risk premia, and their differences, are estimated and tested based on panel data regressions with individual-fixed effects. The weighted average risk premia, Γ_β, Γ_s and Γ_k are first computed as the average of the time series $\Gamma_{jt} = p_t\gamma_u^j + q_t\gamma_d^j$, for $j = \beta, s, k$. These averages are then tested for significance using a time series regression with HAC standard errors using a Newey-West window with four lags. The same procedure is applied for the market risk premium, Γ_m, which is computed as the average of the time series $\Gamma_t = \Gamma_{\beta t} + \Gamma_{st}s_{mt} + \Gamma_{kt}$. The t-statistics are reported in parentheses and significant coefficients at the 5% level are indicated with an asterisk. The bootstrap p-values are shown in square brackets.

may be better captured by two regimes specific to such a subsample. Therefore, we repeat the Markov switching regime methodology for the period 1980–2021, giving the results shown in Table 8.

The bullish regime has a positive return of 1.17% whereas the bearish regime is characterized by null return and a higher volatility of 5.62%. The two regimes are very sticky as can be observed from Fig. 3. It is now apparent that the market has been characterized by alternate regimes. Of particular interest is the bearish regime brought about by the Covid-19 pandemic. We therefore repeat our analysis for the conditional CAPM and conditional 4-CAPM for the last four decades.

Interestingly, for the 25 double sorted portfolios, the risk premium is positive and significant with a reasonable magnitude. We observe a risk premium of 0.49%, with a bull risk premium of 1% and an insignificant risk premium in bear markets (the effect of the boom-and-bust cycles). This result is also confirmed by the 65 portfolios in Table 9. For the conditional 4-CAPM we obtain a risk premium of 0.40% with positive beta and cokurtosis premia of 0.31% and 0.16%, respectively, both of which are significant. However, the coskewness premium is insignificant. The risk premium demanded for cokurtosis and therefore for extreme returns cannot be ignored if one is to explain the cross section of returns.

When repeating the analysis for the 65 portfolios the results show a positive and significant risk premium 0.29%, with a positive beta and cokurtosis premium and a negative and significant coskewness premium, as theory predicts.

Based on the 25 ME/BM portfolios and the extended 65 portfolios samples, the full sample and the more recent sample, it is clear that beta alone cannot explain the cross section of returns. Both skewness and kurtosis are priced. However, although the pricing sign is as expected when we use the full 1926–2021 sample, the recent 40 years sample shows one important discrepancy, namely that the coskewness premium is insignificant for the 25 ME/BM portfolios. However, this could be explained by the high cokurtosis premium. We could tentatively speculate that in the last 40 years investors have been particularly averse to extreme losses and demanded high premium for kurtosis.

Overall, the implementation of the Markov switching regime for the last 40 years improves both models, and, in particular, the conditional CAPM (for the 25 double-sorted portfolios). Most interestingly, perhaps, the conditional 4-CAPM now exhibits an insignificant and much lower intercept than in the case of the conditional CAPM. In Fig. 4, we report the fitted conditional returns of both the conditional CAPM and the conditional 4-CAPM against the realised returns, together with the static CAPM average returns. The fitted returns for the conditional 4-CAPM seem to fit much closer the 45-degree line. The conditional 4-CAPM has an average conditional pricing error of 0.29% against 0.33% for the conditional CAPM and 1.12% for the static CAPM.

5.8. Robustness tests

As pointed out by a referee, our results may be sensitive to the choice of breakpoints. As a robustness test, we re-run the previous set of tests by

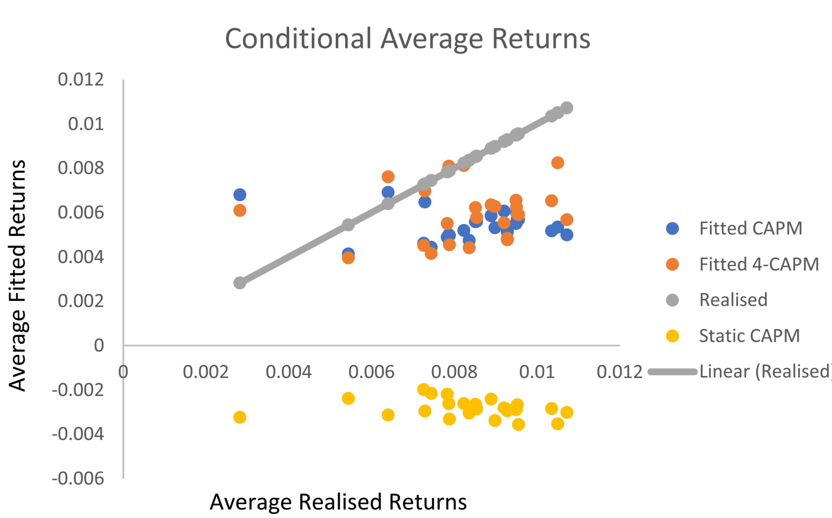

Fig. 4. Average fitted returns and average realised returns.

Table 10
Test of the MS conditional CAPM (Decile Breakpoints Portfolios. 1926–2021).

	ME/BM		All Portfolios	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0129 (11.92)*	−0.0047 (−3.94)*	0.0110 (12.71)*	−0.0080 (−8.09)*
$\gamma_u^\beta - \gamma_d^\beta$	0.0176 (20.63)*		0.0190 (27.40)*	
$\Gamma_{\beta t} = p_t\gamma_u^\beta + q_t\gamma_d^\beta$	0.0104 (37.60)* [0.00]		0.0083 (27.83)* [0.00]	

	Four-moment CAPM 1926–2021		Four-moment CAPM 1926–2021	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0125 (11.59) *	−0.0233 (−17.97)*	0.0104 (12.02) *	−0.0237 (−22.52)*
$\gamma_u^s \mid \gamma_d^s$	−166.83 (−35.86)*	34.63 (52.96)*	−154.01 (−40.93)*	32.63 (61.79)*
$\gamma_u^k \mid \gamma_d^k$	0.0002 (1.16)	0.0007 (10.72) *	0.0004 (2.89)*	0.0007 (11.97) *
$\gamma_u^\beta - \gamma_d^\beta$	0.0359 (37.32)*		0.0340 (44.42)*	
$\gamma_u^s - \gamma_d^s$	−201.46 (−39.61)*		−186.64 (−45.36)*	
$\gamma_u^k - \gamma_d^k$	−0.0005 (−2.57)*		−0.0003 (−1.49)	
Γ_β	0.0075 (13.28)* [0.00]		0.0055 (10.39)* [0.00]	
Γ_s	−138.34 (−43.79)* [0.00]		−127.62 (−43.61)* [0.00]	
Γ_k	0.0002 (33.39)* [0.00]		0.0005 (114.11)* [0.00]	
Γ_m	0.0061 (12.14)* [0.00]		0.0045 (9.41)* [0.00]	

This table reports the results for the regime switching conditional CAPM for the 70 ME/BM portfolios using decile breakpoints and All Portfolios (70 ME/BM Portfolios plus 40 Industries Portfolios) over the period 1926–2021. The up and down probabilities, p_t and $q_t = 1 - p_t$, are obtained from a Markov switching model. The conditional co-moments (covariance, coskewness and cokurtosis) are obtained from DCC GARCH models. The up and down risk premia, and their differences, are estimated and tested based on panel data regressions with individual-fixed effects. The weighted average risk premia, Γ_β, Γ_s and Γ_k are first computed as the average of the time series $\Gamma_{jt} = p_t\gamma_u^j + q_t\gamma_d^j$, for $j = \beta, s, k$. These averages are then tested for significance using a time series regression with HAC standard errors using a Newey-West window with four lags. The same procedure is applied for the market risk premium, Γ_m, which is computed as the average of the time series $\Gamma_t = \Gamma_{\beta t} + \Gamma_{st}s_{mt} + \Gamma_{kt}$. The t-statistics are reported in parentheses and significant coefficients at the 5% level are indicated with an asterisk. The bootstrap p-values are shown in square brackets.

Table 11
Test of the MS conditional CAPM (Decile Breakpoints Portfolios. 1980–2021).

	ME/BM		All Portfolios	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	0.0088 (5.84)*	−0.0011 (−0.66)	0.0092 (7.59)*	−0.0008 (−0.65)
$\gamma_u^\beta - \gamma_d^\beta$	0.0098 (13.66)*		0.0100 (16.23)*	
$\Gamma_{\beta t} = p_t\gamma_u^\beta + q_t\gamma_d^\beta$	0.0037 (11.40)* [0.00]		0.0041 (12.17)* [0.00]	

	Four-moment CAPM 1980–2021		Four-moment CAPM 1980–2021	
	Bull	Bear	Bull	Bear
$\gamma_u^\beta \mid \gamma_d^\beta$	−0.0012 (−0.52)	0.0026 (1.55)	−0.0021 (−1.15)	0.0031 (2.26) *
$\gamma_u^s \mid \gamma_d^s$	−81.35 (−2.80) *	34.08 (8.39)*	−96.72 (−3.96) *	42.19 (11.79) *
$\gamma_u^k \mid \gamma_d^k$	0.0038 (6.92)*	0.0025 (3.13) *	0.0038 (7.94)*	0.0003 (3.86) *
$\gamma_u^\beta - \gamma_d^\beta$	−0.0038 (−1.70)		−0.0052 (−2.75)*	
$\gamma_u^s - \gamma_d^s$	−115.43 (−3.86)*		−138.91 (−5.53)*	
$\gamma_u^k - \gamma_d^k$	0.0035 (6.34)*		0.0035 (7.24)*	
Γ_β	0.0008 (6.21)* [0.00]		0.0005 (6.66)* [0.00]	
Γ_s	−22.05 (−5.75)* [0.00]		−25.35 (−5.50)* [0.00]	
Γ_k	0.0020 (16.80)* [0.00]		0.0020 (16.94)* [0.00]	
Γ_m	0.0034 (15.24)* [0.00]		0.0032 (14.09)* [0.00]	

This table reports the results for the regime switching conditional CAPM for the 96 ME/BM portfolios using decile breakpoints and All Portfolios (96 ME/BM Portfolios plus 40 Industries Portfolios) over the period 1980–2021. The up and down probabilities, p_t and $q_t = 1 - p_t$, are obtained from a Markov switching model. The conditional co-moments (covariance, coskewness and cokurtosis) are obtained from DCC GARCH models. The up and down risk premia, and their differences, are estimated and tested based on panel data regressions with individual-fixed effects. The weighted average risk premia, Γ_β, Γ_s and Γ_k are first computed as the average of the time series $\Gamma_{jt} = p_t\gamma_u^j + q_t\gamma_d^j$, for $j = \beta, s, k$. These averages are then tested for significance using a time series regression with HAC standard errors using a Newey-West window with four lags. The same procedure is applied for the market risk premium, Γ_m, which is computed as the average of the time series $\Gamma_t = \Gamma_{\beta t} + \Gamma_{st}s_{mt} + \Gamma_{kt}$. The t-statistics are reported in parentheses and significant coefficients at the 5% level are indicated with an asterisk. The bootstrap p-values are shown in square brackets.

using different breakpoints to ascertain that the results are not affected by the breakpoint choice. We test the conditional versions of the CAPM and Higher-moment CAPM using ME/BM portfolios formed using deciles as breakpoints as opposed to quintiles. Furthermore, we the 100 ME/BM portfolios formed using decile breakpoints.

However, unlike the 25 ME/BM portfolios, for which full returns data are available, both the full sample (1926–2021) and the subsample (1980–2021) suffer from data attrition due to missing values. Indeed, 30 of the 100 ME/BM portfolios formed using decile breakpoints have missing values throughout the full sample period, and 4 portfolios in the subsample have continuous missing values. It is perhaps for this reason that the 25 ME/BM portfolios have kickstarted the current debate and research in asset pricing. Nevertheless, it is interesting to test the robustness of our findings in earlier sections. The robustness results for the ME/BM and Full portfolios are reported in Tables 10 and 11 for the full sample and the sub-sample respectively. The Full portfolios include the ME/BM and the industry portfolios.

By contrasting Table 5 and Table 10 it is apparent that the results are qualitatively identical. The coefficient signs, scale, and test conclusions are unchanged. The estimates are very similar. For example, the bull premium for ME/BM portfolios changed marginally from 0.0127 to 0.0129, whereas the bear premium changed from −0.0057 to −0.0047. The risk premium for the conditional CAPM changed from 0.0101 to 0.0104.

The Four-Moment conditional tests are also unchanged. For ME/BM portfolios, the market risk premium drops marginally from 0.0067 to 0.0061. The beta premium is unchanged at 0.0075, while the skewness premium increases from −231.85 to −138.34. However, the only large difference is the kurtosis premium, which drops substantially from 0.0018 to 0.0002. Nevertheless, the kurtosis remains positive and significant in the decile portfolios. The same goes for the extended portfolio set.

The sub-sample results are also largely similar. By comparing Table 9 and Table 11, it is seen that the sign and scale of estimates are unchanged, although we note some marginal differences. Focusing on the ME/BM portfolios, the four-moment beta premium drops from 0.0031 to 0.0008 when we use the deciles breakpoints. In contrast, the skewness premium becomes negative and significant at −22.05 (similar to the ‘All portfolios’ sample for both quintile and decile breakpoints). Thus, it seems that the choice of breakpoint might have some impact on the results for the more recent sample (1980–2021). In contrast to the full (1926–2021) sample, the kurtosis premia are virtually unchanged across the two tables, changing marginally from 0.0016 to 0.0020. Finally, the market risk premium drops marginally but remains positive and statistically highly significant.

Overall, the results confirm our previous analysis. The conditional CAPM is a good explanation of the cross-section of returns. More particularly, the coskewness and beta remain the two most important factors in the explanation of the cross-section of returns in the full 1926–2021 sample, with kurtosis playing a greater role in the more recent sub-sample (1980–2021).

We also repeated the time series tests for size, value and momentum shown in Table 6, as well as the cross-sectional tests on the pricing errors shown in Table 7, using decile breakpoints. For the sake of space, we do not report the results in this paper (results are available upon request). For the time series tests the conclusions reported in Table 6 are unchanged apart from marginal change in the significance of the average value loading for the statistic four-moment CAPM (the *p*-value changes from 4.2% in the quintile breakpoint portfolios to 6.8% in the decile breakpoint portfolios).

For the cross-sectional tests on the pricing errors, the 32 tests shown on Table 7 are mostly unchanged, except from two important cases. For the conditional 4-CAPM, the size is not significant for the quintile portfolio case (Table 7) for both one-pass and rolling beta. This is reversed for the decile portfolios where both size coefficients become highly significant.

6. Conclusion

This paper aims to investigate whether further extensions to the traditional CAPM can improve its empirical performance, and to offer some alternative explanation to the average cross-section of returns on portfolios of stocks double sorted on book-to-market ratios and size. In particular, we make the following contributions to current asset pricing theory: (i) we use time-varying factor loadings obtained from a multivariate GARCH and dynamic conditional correlations; (ii) we introduce coskewness and cokurtosis; and (iii) we use time-varying risk premia, which are assumed to change according to the regime of the market and with regimes defined by a Markov-switching process.

The assumption of a constant required rate of return may be too strong for the real world. Risk premia should be related to uncertainty, commonly measured as volatility, and to risk aversion. Therefore, it is reasonable to assume that risk aversion is time-varying. More specifically, investors are expected to be less (more) risk averse and more optimistic (pessimistic) when financial market performance and economic news are both positive (negative), and hence more risk averse at those times when volatility is also expected to be greater given underlying uncertainty.

The assumption is made in this paper that there are two regimes, each with a probability that is returned by a Markov switching process, and it is assumed that there are two distinct sets of risk premia for each regime. Whereas the factor loadings are still conditional and determined through a multivariate GARCH, the risk premia are estimated in a panel data regression, and the average risk premia are calculated as the mean of the time series of the weighted average of the two risk premia where the weights are given by the probability of being in each regime. A further objective of this paper was to investigate whether the addition of time-varying factor loadings and time-varying risk premia can explain the cross-section of US average stock returns.

Our results confirm that the higher-moment CAPM does not perform well in its unconditional version, but its performance is significantly improved by introducing a conditional version that accounts for both time-varying factor loadings and time-varying risk premia. Both conditionings are required to rescue unconditional models. The four-moment CAPM tests lead to a positive total risk premium estimate of 0.67% per month over the period 1926–2021, with all factors (beta, coskewness, and cokurtosis) exhibiting the expected theoretical sign, more specifically, a beta premium of 0.75%, a non-standardized coskewness premium of -231.85, and a cokurtosis premium of 0.18%. More interestingly, the model shows a positive return of 0.40% over the later subsample period 1980–2021, with a positive premium for all of the factors apart from coskewness (insignificant, while coskewness should in theory have a negative return reward). Importantly, accounting for cokurtosis allows us to find a positive and significant risk premium whereas a beta-CAPM produces an insignificant risk premium. One clear result that emerges is that unconditional versions of asset pricing models are rejected in our study in favour of conditional versions, and that cokurtosis has become a critical risk factor in the last few decades, probably due to the high market volatility experienced during that time.

The implication of our study for portfolio managers and investors is fairly straightforward: stocks with higher betas (or a small market capitalization given the positive correlation between SMB and beta), with higher exposure to market kurtosis, and with lower coskewness do indeed perform better on average and are rewarded commensurately by the market.

Our relatively simple model offers significant improvement over the static CAPM, thanks to a combination of higher moments and conditional models. However, some limitations remain. One main limitation is the error-in-variable problem. Indeed, our models are based on estimated regime probabilities and co-moments. This is a well-known limitation of all two-step regressions, which are usually mitigated via instrumental variables method. For example, lagged co-moments could be used as instruments which are correlated with contemporaneous co-moments but not with the error term. However, this is not straightforward in the context of panel data. We leave this question for further studies. A second limitation relates to small-growth portfolios. Although our model reduces the pricing errors, some portfolios such as small-growth remain difficult to explain.

Our study could be extended in at least two directions. First, we use DCC to determine the time-varying co-moments. However, as Caporin and McAleer (2013) stated, the DCC has many shortcomings, including that it does not yield dynamic conditional correlations, and that it gives inconsistent two step estimators. Thus, future work should consider other estimation alternatives such as short-window rolling regressions, or high frequency approaches. Second, although a large number of new factors have been proposed as complements to or replacement of the original three Fama-French factors, liquidity based extensions seem to be promising. For example, Virk and Butt (2022) propose a three-factor model based on sensitivity to changes in market liquidity. Future work could consider extending the Fama-French three factor model with momentum and liquidity portfolios.

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