An Alternative Test to Check the Validity of Convergence Results

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Abstract

This paper presents a simple method for identifying distributional dynamic properties of economies using the ideas of concordance and discordance. It can be employed to examine the strength and validity of the results of other methods. The method has the advantage of comparing distributions at two points without relying on intermediary data between the two time points. We present results that suggest there is more 'strong-divergence' than 'strong-convergence' in GDP between countries over the time period 1960-2000 although the distribution exhibits both convergent and divergent characteristics.

JEL Classification: C14; F19

Keywords: Convergence; Concordance; Income per capita.

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Abstract

This paper presents a new method for identifying distributional dynamic properties of economies using the ideas of concordance and discordance. It can be employed to examine the strength and validity of the results of other methods. The method has the advantage of comparing distributions at two points without relying on intermediary data between the two time points. We present results that suggest there is more 'strong-divergence' than 'strong-convergence' in GDP between countries over the time period 1960-2000 although the distribution exhibits both convergent and divergent characteristics.

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1. Introduction

Neoclassical growth theory suggests an inverse relationship between the capital-labour ratio and the productivity of capital. The Solow-Swan model of economic growth suggests that income per capita will converge due to differences in the rates of return on capital and capital will move from economies with lower rates of return to those with higher rates of return, with convergence driven by movements in the wage rates (see Solow, 1956; Swan, 1956; or Barro and Sala-i-Martin, 1995, for a review of the literature). This leads to the proposition that poor economies should grow faster than rich economies and that output per capita should converge across economies.

Five distinct empirical tests on long-term convergence in economic conditions can be identified. First, *σ-convergence* is where there is a reduction of the dispersion of the variable concerned over time. Second, *β-convergence* is the most common form of estimation which relates long-term growth with initial levels of income and follows on from Mankiw *et al*. (1992). Here, the variables are assumed to be in transition towards their limiting distributions and will ultimately converge to a position in the distribution conditional on some other variable. Third, the Durlauf and Quah (1999) approach tests for the presence of deterministic or stochastic trend components. This focuses on the equalisation of variables to a steady state level, with sample means taken to be good proxies of asymptotic means, and variables assumed to be near their limiting distributions. A fourth approach, suggested by Quah (1996), proposes a non-parametric method of estimation of the shape of the cross-economy distribution of transition probabilities of this distribution among different long-term stochastic basins of attraction. The fifth, and most recent proposed method, examines convergence empirics using the ideas of concordance and discordance (Webber and White, 2003; Webber *et al.*, 2005).

However, most analyses of the dynamics of distributions of economic variables do not find that cross-sectional distributions collapse through time even though growth miracles do exist. Some of the empirical literature, such as Quah (1997), identify that convergence is not necessarily the dominant force for a distribution of economies and highlight that other distributional dynamic properties are also evident.

Although there have been important developments in this field, the results generated through such methods should stand up to external scrutiny. To increase confidence in the results of these methods, such results should be validated either by each other or by an alternative and easy to use method. Each method has its own strengths and weaknesses and a robust method is needed that can be easily employed to help assist in validating such results.

This paper presents a new method for identifying distributional dynamic properties of economies using the ideas of concordance and discordance which is both theoretically and empirically simply and it can be employed as a quick check of empirical results generated by other methods. The method has the advantage of comparing distributions at two points in time without relying on intermediary data between the two time points. We illustrate the use of this method by performing an empirical investigation on real GDP per capita figures for 97 countries between 1960 and 2000, which has been carried out by numerous empirical economists. The analysis suggests that there is more ‘strong-divergence’ than ‘strong-convergence’ between countries although the distribution exhibits both convergent and divergent characteristics.

2. Data and Model

Convergence can be understood to mean several different things. Let $s_{i,t}$ be a metric for output of country *i* at time *t*. For instance, convergence could be inferred to be the gradual reduction in the magnitude of the difference between a richer economy (*i*) and a poorer economy (*j*) between two periods of time (t and $t+k$) i.e. without loss of generality assume $s_{i,t} > s_{j,t}$. Under this condition, if $(s_{i,t} - s_{j,t}) > (s_{i,t+k} - s_{j,t+k})$ then this could be taken as "convergence without switching". Convergence using this reduction in 'the difference' necessarily requires a slower rate of growth for *i* than for *j*. Other definitions of convergence without switching could be based on the ratio of rewards. Again assume without loss of generality that $s_{i,t} > s_{j,t}$. Under this condition, if $s_{i,t} / s_{j,t} > s_{i,t+k} / s_{j,t+k} > 1$ then this could be taken as "convergence without switching". If considering convergence irrespective of whether switching has taken place then similar definitions may be proposed based on the 'absolute difference' $|s_{i,t} - s_{j,t}|$ or based on the maximum ratio of rewards i.e. $\max\{s_{i,t}, s_{j,t}\} / \min\{s_{i,t+k}, s_{j,t+k}\}$. The data in empirical investigations of convergence could be either in 'ratios' (based on the original data) or in 'differences' possibly with the original data being transformed using a new baseline value to reduce the bias due to natural growth.

For development purposes consider two economies indexed by *i* and *j* and without loss of generality assume $s_{i,t} > s_{j,t} > 0$ and define $X_{i,t}$ to be the solution of:

$$\left(\frac{s_{it}}{s_{jt}}\right)^{X_{ij}} = \left(\frac{s_{i,t+k}}{s_{j,t+k}}\right)$$

i.e. for $s_{i,t} > s_{j,t} > 0$

$$X_{i,j} = \frac{\log[s_{i,t+k}] - \log[s_{j,t+k}]}{\log[s_{i,t}] - \log[s_{j,t}]}$$

If:

$X_{i,j} > 1$ then countries *i* and *j* exhibit divergence in ratio without switching

$0 < X_{i,j} < 1$ then countries *i* and *j* exhibit convergence in ratio without switching

$X_{i,j} = 0$ then there cannot be any further convergence: the economies have converged

$-1 < X_{i,j} < 0$ then countries *i* and *j* exhibit convergence in ratio with switching

$X_{i,j} < -1$ then countries *i* and *j* exhibit divergence in ratio with switching

Similarly let $r_{i,t}$ denote a normalising transformation of $s_{i,t}$ (e.g. $r_{i,t} = (s_{i,t} - \bar{s}_t)/\bar{s}_t$ where $\bar{s}_t$ is the sample mean at time *t*) and define $Y_{i,j} = \frac{r_{i,t+k} - r_{j,t+k}}{r_{i,t} - r_{j,t}}$. If:

$Y_{i,j} > 1$ then countries *i* and *j* exhibit divergence in difference without switching

$0 < Y_{i,j} < 1$ then countries *i* and *j* exhibit convergence in difference without switching

$-1 < Y_{i,j} < 0$ then countries *i* and *j* exhibit convergence in difference with switching

$Y_{i,j} < -1$ then countries *i* and *j* exhibit divergence in difference with switching

The calculation of all possible pairwise combinations of ($X_{i,j}, Y_{i,j}$) may then be summarised by counting instances of convergence and divergence with and without switching in a matrix as shown in Table 1. This method can be employed irrespective of the size of the sample. Note that the matrix in Table 1 contains 8 shaded cells that

denote infeasible combinations in that they involve the ratio but not the difference switching, or vice versa.

{Insert Table 1 about here}

3. Empirical Results

The data used in the following exploratory analysis is country level real GDP per capita (in 1995 constant $US) obtained from the World Bank. Countries were included in the sample if and only if data for their GDP per capita was available for ten-year intervals between 1960 and 2000. The countries included in the data set maximise both the time period (1960-2000) and the number of countries (97) subject to the constraint that there are observations for each country at each point in time.

The variables $X_{i,j}$ were estimated using GDP per capita ($s_{i,t}$) and the variables $Y_{i,t}$ were estimated using $r_{i,t}=(s_{i,t}-\bar{s}_t)/\bar{s}_t$. Enumeration of $X_{i,j}$ and $Y_{i,t}$ for all pairs of countries is summarised in matrix form given in the appendix. Tables A1 – A4 present the results relative to 1960, while Tables A5 – A7, A8 – A9, and A10 present the results relative to 1970, 1980 and 1990 respectively. The numbers representing switching are in the bottom left quadrant of the tables.

Several patterns can be identified from the tables. First, the longer the period of time under examination then the greater the number of pairs of economies that have switched their position. This suggests that the growth path of a national economy is neither uniform nor smooth across the sample: some countries grow faster than others and an economy can overtake another.

Second, there is more evidence of divergence than there is of convergence. The number of pairs of economies that have strongly diverged (irrespective of whether or not they switched) are summarised in Table 2, while the number of pairs of economies

that have strongly converged (irrespective of whether or not they have switched) are summarised in Table 3.

{Insert Table 2 about here}

{Insert Table 3 about here}

As the total possible number of converging pairs for a sample of 97 economies is equal to 4656, the evidence does not support the proposition that there was strong convergence throughout the period. When the frequencies are converted into percentages, it is easily identifiable that the maximum number of 'strongly converging' pairs is only 34.9% of the entire sample whereas the minimum number of 'strongly diverging' pairs is 40.1%.

The above analysis has been repeated replacing country *i* and country *j* with the *i*-th and *j*-th percentile of the distributions i.e. working directly with the properties of the empirical distribution rather than the countries which make up the empirical distribution. The analysis based on percentiles leads to the same broad conclusions (e.g. when comparing 1960 with 2000 there are 2783 pairs of percentiles for which $X_{(i),(j)}>1$ and $Y_{(i),(j)}>1$, 1300 pairs of percentile for which $X_{(i),(j)}>1$ and $0<Y_{(i),(j)}<1$, 543 pairs of percentiles for which $0<X_{(i),(j)}<1$ and $0<Y_{(i),(j)}<+1$, and 30 pairs of percentiles for which $0<X_{(i),(j)}<1$ and $Y_{(i),(j)}>1$. The evidence provided in these analyses does not support the proposition that the sample is strongly-converging over time and questions the realism of the traditional Solow-Swan neo-classical growth model.

4. Conclusion

This paper has presented a new method for identifying the convergence and divergence empirics of a sample that permits the researcher to identify switching simultaneously. When these methods are employed to identify some convergent properties of 97 countries' real GDP per capita between 1960 and 2000, the results suggest that *div*ergence is the dominant property, but there is evidence of convergence and switching in the sample also. The results question empirically the traditional theoretical approach to growth, which suggest that economies should converge to the long-run steady-state. Moreover, it questions empirically the role of capital mobility in equalising income levels and reinforces the need to find other explanations to understand cross-economies patterns of growth.

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Appendix:

The countries in the sample are: Algeria, Argentina, Australia, Austria, The Bahamas, Bangladesh, Barbados, Belgium, Belize, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Chile, China, Colombia, Rep. of Congo, Costa Rica, Cote d'Ivoire, Denmark, Dominican Republic, Ecuador, Arab Rep. of Egypt, El Salvador, Fiji, Finland, France, Gabon, Ghana, Greece, Guatemala, Guyana, Haiti, Honduras, Hong Kong, China, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Kenya, Rep. of Korea, Lesotho, Luxembourg, Madagascar, Malawi, Malaysia, Malta, Mauritania, Mauritius, Mexico, Morocco, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Portugal, Rwanda, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, South Africa, Spain, Sri Lanka, Sudan, Sweden, Switzerland, Syrian Arab Republic, Thailand, Togo, Trinidad and Tobago, United Kingdom, United States, Uruguay, Venezuela, Zambia and Zimbabwe.

Table 1: All combinations of outcomes

Differences Axis	+1			Converged, Ratio, No Switching Diverged, Differences, No Switching	Diverged, Ratio, No Switching Diverged, Differences, No Switching
				Converged, Ratio, No Switching Converged, Differences, No Switching	Diverged, Ratio, No Switching Converged, Differences, No Switching
	-1	Diverged, Ratio, Switching Converged, Differences, Switching	Converged, Ratio, Switching Converged, Differences, Switching		
		Diverged, Ratio, Switching Diverged, Differences, Switching	Converged, Ratio, Switching Diverged, Differences, Switching		
		-1		+1	
		Ratio Axis			

Table 2: Number of Strongly Diverging Pairs of Economies

	1970	**1980**	**1990**	**2000**
1960	2104 (45.2%)	2220 (47.7%)	2125 (45.6%)	2152 (46.2%)
1970	-	2155 (46.3%)	2063 (44.3%)	2006 (43.1%)
1980	-	-	2002 (43.0%)	1869 (40.1%)
1990	-	-	-	1932 (41.5%)

Table 3: Number of Strong Converging Pairs of Economies

	1970	**1980**	**1990**	**2000**
1960	1446 (31.1%)	1462 (31.4%)	1411 (30.3%)	1355 (29.1%)
1970	-	1623 (34.9%)	1570 (33.7%)	1465 (31.5%)
1980	-	-	1625 (34.9%)	1518 (32.6%)
1990	-	-	-	1619 (34.8%)

Table A1: 1960 to 1970

*********	*********	91	2025
*********	*********	1363	1006
4	85	******	******
79	3	******	******

Table A2: 1960 to 1980

*********	*********	167	2069
*********	*********	1315	788
14	147	******	******
151	5	******	******

Table A3: 1960 to 1990

*********	*********	169	1918
*********	*********	1212	912
30	199	******	******
207	9	******	******

Table A4: 1960 to 2000

*********	*********	132	1914
*********	*********	1143	958
43	212	******	******
238	16	******	******

Table A5: 1970 to 1980

*********	*********	299	2048
*********	*********	1514	569
7	109	******	******
107	3	******	******

Table A6: 1970 to 1990

*********	*********	190	1892
*********	*********	1404	804
17	166	******	******
171	12	******	******

Table A7: 1970 to 2000

*********	*********	158	1789
*********	*********	1268	993
20	197	******	******
217	14	******	******

Table A8: 1980 to 1990

*********	*********	199	1901
*********	*********	1537	821
5	88	******	******
101	4	******	******

Table A9: 1980 to 2000

*********	*********	139	1720
*********	*********	1389	1102
20	129	******	******
149	8	******	******

Table A10: 1990 to 2000

*********	*********	199	1840
*********	*********	1549	900
6	70	******	******
92	0	******	******